

Classical Ciphers (CS-452)

Week 2

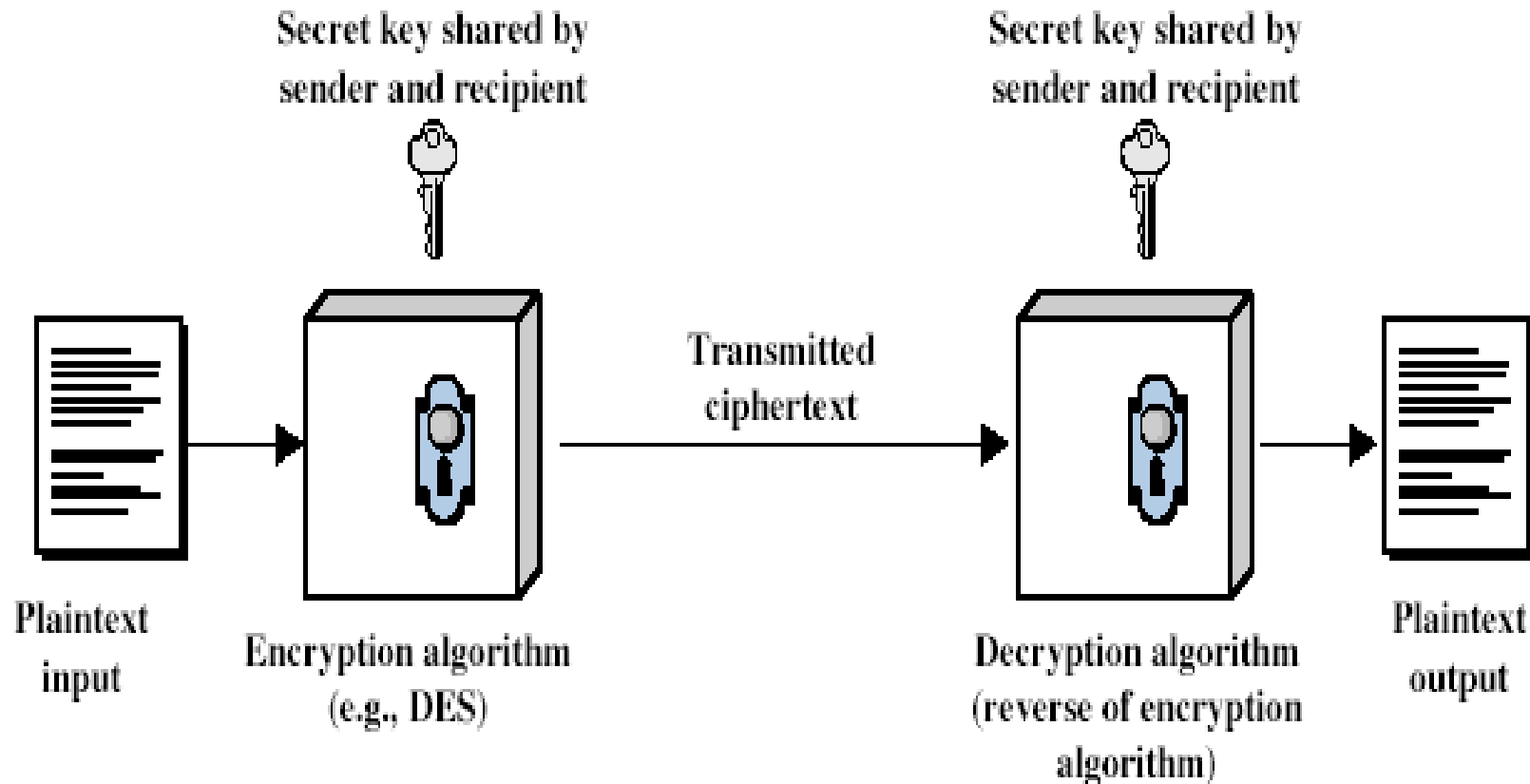
Cryptology Fundamentals and Classical Ciphers

Symmetric Encryption

- A form of cryptosystem in which encryption and decryption are performed using the same key – **single-key encryption**
- Was only type prior to invention of public-key in 1970's, and by far most widely used

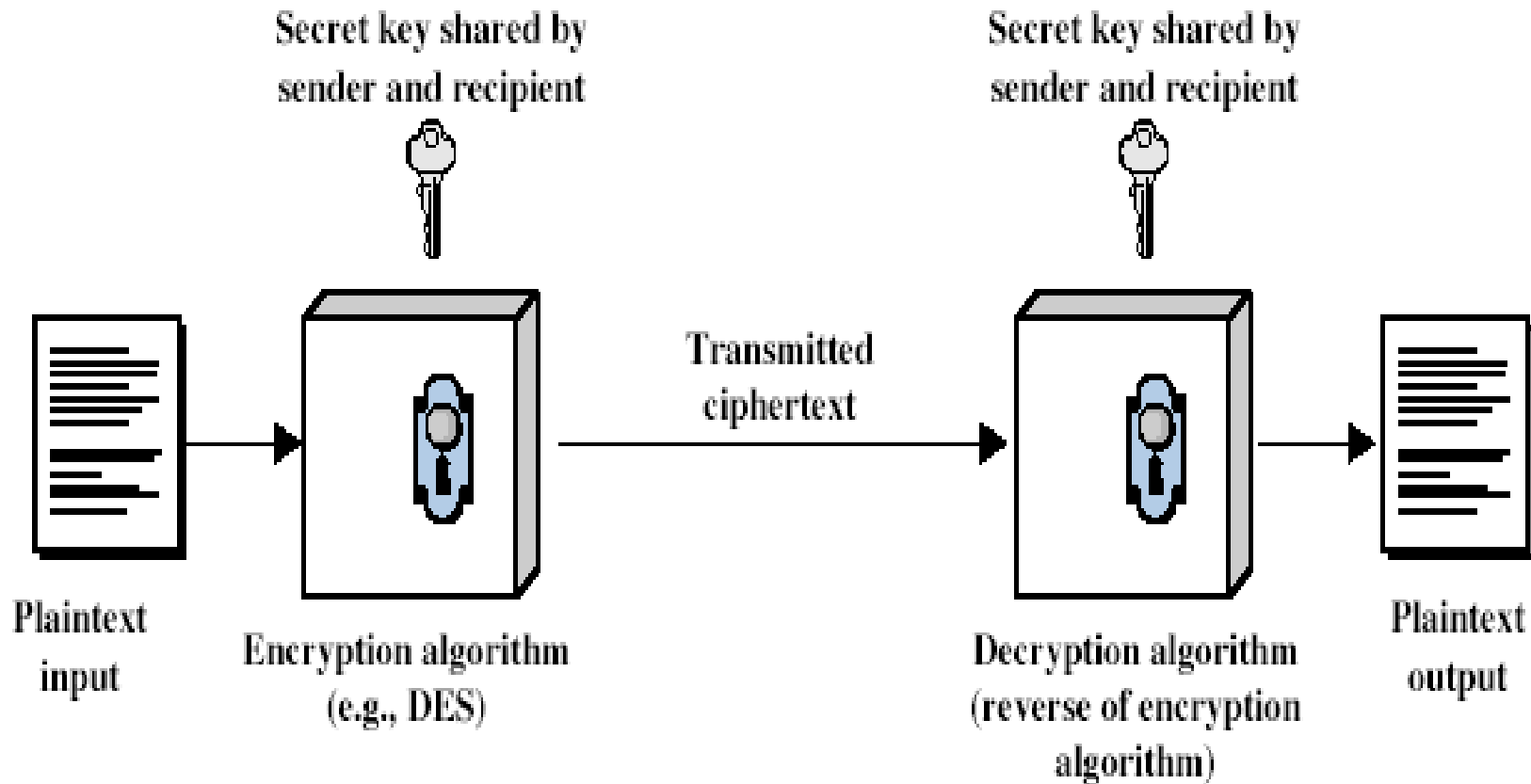
Symmetric Cipher Model

- **Encryption algorithm:** performs various substitutions and transformation on the plaintext.
- **Secret key:** the input of encryption algorithm. The key is independent of the plaintext and the alg..



Symmetric Cipher Model

- **Decryption algorithm:** the ciphertext and the secret key and produces the original plaintext



Requirements

• Two requirements for secure use of symmetric encryption:

- ◆ **A strong encryption algorithm**: the opponent should be unable to decrypt ciphertext or discover the key even if he/she has a number of ciphertexts and the plaintext that produced each ciphertext
- ◆ Assume encryption algorithm is known
- ◆ Mathematically have:

X: message, **K**: encryption key, **Y**: ciphertext

$$Y = E(K, X)$$

$$X = D(K, Y)$$

An opponent can observe Y , but do not have access to K or X .

Brute Force Search

- Simply try every key until an intelligible translation of the ciphertext into plaintext is obtained
- On average, **half** of all possible keys must be tried to achieve success - proportional to key size
- DES: 56-bit, triple DES: 168-bit, AES: > 128 bits
- Time required for various key spaces:

Key Size (bits)	Number of Alternative Keys	Time required at 1 decryption/ μ s	Time required at 10^6 decryptions/ μ s
32	$2^{32} = 4.3 \times 10^9$	$2^{31} \mu\text{s} = 35.8$ minutes	2.15 milliseconds
56	$2^{56} = 7.2 \times 10^{16}$	$2^{55} \mu\text{s} = 1142$ years	10.01 hours
128	$2^{128} = 3.4 \times 10^{38}$	$2^{127} \mu\text{s} = 5.4 \times 10^{24}$ years	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	$2^{167} \mu\text{s} = 5.9 \times 10^{36}$ years	5.9×10^{30} years

Brute Force Search

- Simply try every key until an intelligible translation of the ciphertext into plaintext is obtained.
- On average, **half** of all possible keys must be tried to achieve success - proportional to key size
- DES: 56-bit, triple DES: 168-bit, AES: > 128 bits
- Time required for various key spaces:

Key Size (bits)	Number of Alternative Keys	Time required at 1 decryption/ μ s	Time required at 10^6 decryptions/ μ s
32	$2^{32} = 4.3 \times 10^9$	$2^{31} \mu\text{s} = 35.8$ minutes	2.15 milliseconds
56	$2^{56} = 7.2 \times 10^{16}$	$2^{55} \mu\text{s} = 1142$ years	10.01 hours
128	$2^{128} = 3.4 \times 10^{38}$	$2^{127} \mu\text{s} = 5.4 \times 10^{24}$ years	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	$2^{167} \mu\text{s} = 5.9 \times 10^{36}$ years	5.9×10^{30} years

Types of Symmetric Ciphers

- Symmetric ciphers are characterized **by the techniques they use to transform plaintext to ciphertext**:
 - ◆ **Substitution Ciphers**: replace the elements of the plaintext with new elements
 - ◆ **Transposition Ciphers**: Rearrange the elements of the plaintext
 - ◆ **Product Ciphers**: do both (a bridge from classical to modern symmetric ciphers)
- Next, we will study the fundamental techniques of symmetric encryption by looking at some classical ciphers

Substitution Ciphers

Classical Substitution Ciphers

- Letters of plaintext are replaced by other letters or by numbers or symbols

Caesar Cipher

- The earliest known substitution cipher (by Julius Caesar)
- First attested use in military affairs
- Replaces each letter with the letter standing **K** places further down the alphabet
- When **K = 3**, can define transformation as:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Example:

Plaintext: meet me after the toga party

Caesar Cipher

- The earliest known substitution cipher (by Julius Caesar)
- First attested use in military affairs
- Replaces each letter with the letter standing **K** places further down the alphabet
- When **K = 3**, can define transformation as:

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

- Example:

Plaintext: meet me after the toga party

Ciphertext: PHHW PH DIWHU WKH WRJD SDUWB

Caesar Cipher

- Mathematically give each letter a number

a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	1	2	3	4	5	6	7	8	9	10	11	12	13
o	p	q	r	s	t	u	v	w	x	y	z		
14	15	16	17	18	19	20	21	22	23	24	25		

- Then have Caesar cipher as:

◆ Plaintext: pt, ciphertext: ct

Encryption: $ct = E(pt) = (pt + k) \bmod 26$

Decryption: $pt = D(ct) = (26 + (ct - k)) \bmod 26$

Cryptanalysis of Caesar Cipher

Cryptanalysis of Caesar Cipher

- A **brute force** search can be easily performed: simply try all the 25 possible keys - far from security
 - ◆ The **language** of the plaintext is known and easily recognizable
- The input may be compressed, make recognition difficult, e.g., E.g. .zip file

		PHHW	PH	DIWHU	WKH	WRJD	SDUWB
KEY							
	1	oggv	og	chvgt	vjg	vqic	rctva
	2	nffu	nf	bgufs	uif	uphb	qbsuz
	3	meet	me	after	the	toga	party
	4	ldds	ld	zesdq	sgd	snfz	ozqsx
	5	kccr	kc	ydrpc	rfc	rmey	nyprw
	6	jbbq	jb	xcqbo	qeb	qldx	mxoqv
	7	iaap	ia	wbpan	pda	pkcw	lwnpu
	8	hzzo	hz	vaozm	ocz	ojbv	kvmot
	9	gyyn	gy	uznyl	nby	niau	julns
	10	fxxm	fx	tymxk	max	mhzt	itkmr
	11	ewwl	ew	sxlwj	lzw	lgys	hsjlg
	12	dvvk	dv	rwkvi	kyv	kfxr	grikp
	13	cuuj	cu	qvjuh	jxu	jewq	fqhjo
	14	btti	bt	puitg	iwt	idvp	epgin
	15	assh	as	othsf	hvs	hcuo	dofhm
	16	zrrg	zr	nsgr	gur	gbtn	cnegl
	17	yqqf	yq	mrfqd	ftq	fasm	bmdfk
	18	xppe	xp	lqepc	esp	ezrl	alcej
	19	wood	wo	kpdob	dro	dyqk	zkbdi
	20	vnnc	vn	jocna	cqn	cxpj	yjach
	21	ummb	um	inbmz	bpm	bwoi	xizbg
	22	tlla	tl	hmaly	aol	avnh	whyaf
	23	skkz	sk	glzcx	znk	zumg	vgxze
	24	rjjy	rj	fkyjw	ymj	ytlf	ufwyd
	25	qiix	qi	ejxiv	xli	xske	tevxc

Figure 2.3 Brute-Force Cryptanalysis of Caesar Cipher

Monoalphabetic Cipher

- Allow an **arbitrary** substitution rather than just shifting the alphabet
- Each plaintext letter maps to a **random** ciphertext letter

Plain: abcdefghIjklmnopqrstuvwxyz

Cipher:DKVQFIBJWPESCXHTMYAUOLRGZN

E.g.,

If we wish to replace letters

Monoalphabetic Cipher

- Allow an **arbitrary** substitution rather than just shifting the alphabet
- Each plaintext letter maps to a **random** ciphertext letter

Plain: abcdefghIjklmnopqrstuvwxyz

Cipher:DKVQFIBJWPESCXHTMYAUOLRGZN

E.g.

If we wish to replace letters

WI RF RWAJ UH YFTSDVF SFUUFYA

Monoalphabetic Cipher Security

- The cipher line can be any permutation of the 26 alphabetic characters
 - ◆ $26! = 4 \cdot 10^{26}$ mappings
- With so many mappings, might think is secure

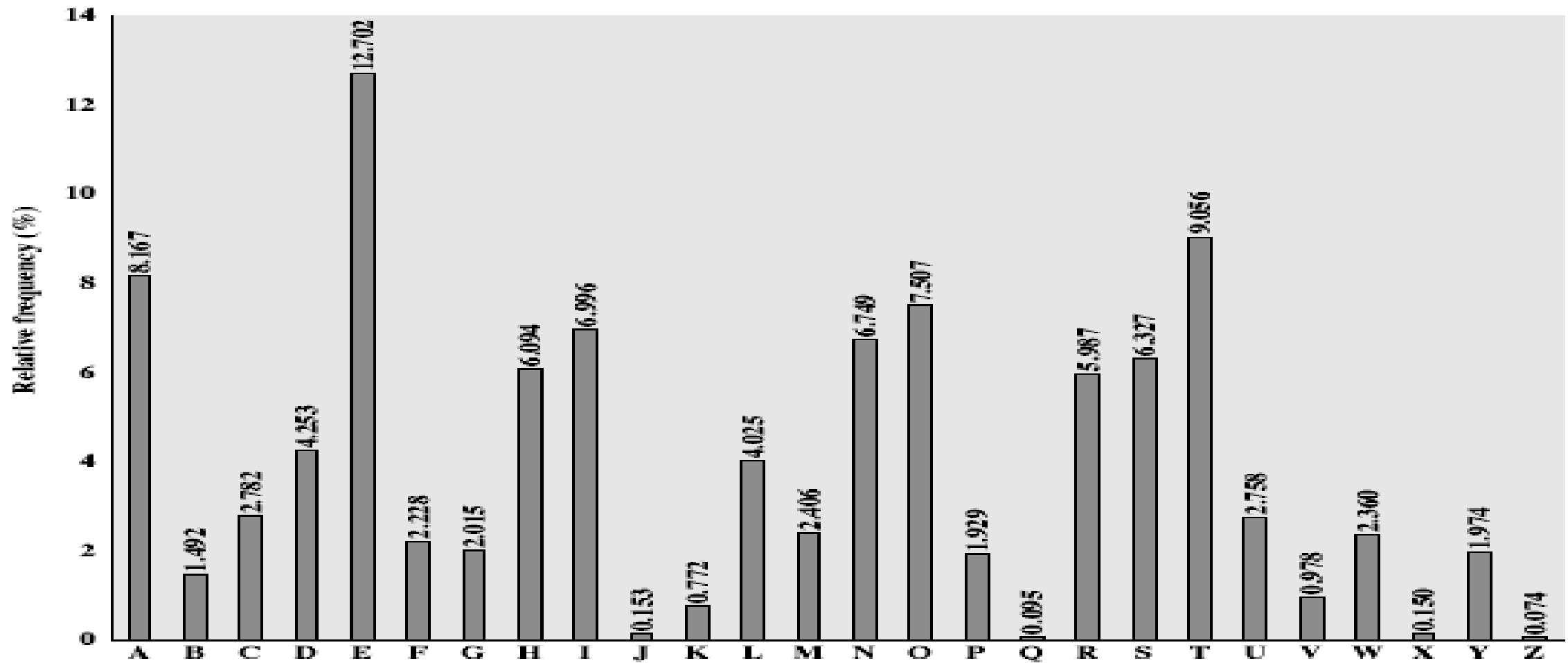
Monoalphabetic Cipher Security

- The cipher line can be any permutation of the 26 alphabetic characters
 - ◆ $26! = 4 \times 10^{26}$ keys
- With so many keys, might think is secure
- But would be **WRONG** – if the cryptanalyst knows the nature of the plaintext (e.g., noncompressed English text), then the analyst can exploit the regularities (**frequency of letters**) of the language

Language Redundancy and Cryptanalysis

- Human languages are redundant. Letters are not equally commonly used
- In English **E** is by far the most common letter, followed by **T,A,O,I,N,S,R**, other letters like **Z,J,K,Q,X** are fairly rare
 - ◆ If the message is **long** enough, this technique alone may be sufficient

English Letter Frequencies



Use in Cryptanalysis

- Main concept - monoalphabetic substitution ciphers do not change relative letter frequencies
- Calculate letter frequencies for ciphertext
- Compare counts/plots against known values

Example

● UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPPDTSVPQ
UZWYMXUZUHSXEPYEPDPDZSUFPOMBZWPFUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

Example

UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPPDTSVPQ
UZWYMXUZUHSEXEPYEPDPDZSUFPOMBZWPFUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

P → e

Example

● UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPPDTSVPQ
UZWYMXUZUHSXEPYEPDPZSUFPOMBZWPFUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

● P → e, Z → t

Example

UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMET SXAI ZVUEPHZHMDZSHZOWSFPAPPDTSVPQ
UZWYMXUZUHSEXEPYEP OPDZSUF POMBZWPFUPZHMDJUDTMOG MQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

P → e, Z → t, {S,U,O,M,H} → {a,h,i,n,o,r,s}

Example

UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPPDTSVPQ
UZWYMXUZUHSXEPYEPOPDZSUFPOMBZWPFPUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

- $P \rightarrow e, Z \rightarrow t, \{S, U, O, M, H\} \rightarrow \{a, h, i, n, o, r, s\}$
- Most common pair: $ZW \rightarrow$ and hence $ZWP \rightarrow$, $ZWSZ \rightarrow$

Example

- UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPDTSVPQU
ZWYMXUZUHSEXEPYEPDPDZSUFPOMBZWPFPUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

- P → e, Z → t, {S,U,O,M,H} → {a,h,i,n,o,r,s}
- Most common pair: ZW → th and hence ZWP → , ZWSZ →

Example

UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPPDTSVP
QUZWMYXUZUHSXEPYEPDPZSUFPOMBZWPFPUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

- $P \rightarrow e, Z \rightarrow t, \{S, U, O, M, H\} \rightarrow \{a, h, i, n, o, r, s\}$
- Most common pair: $ZW \rightarrow th$ and hence $ZWP \rightarrow the, ZWSZ \rightarrow$

Example

- UZQSOVUOHXMOPVGPOZPEVAGZWSZOPFPESXUDBMETSXAI ZVUEPHZHMDZSHZOWSFPAPPDTS
VPQUZWYMXUZUHSXEPYEPOPDZSUFPOMBZWPFPUPZHMDJUDTMOGMQ

P 13.33	H 5.83	F 3.33	B 1.67	C 0.00
Z 11.67	D 5.00	W 3.33	G 1.67	K 0.00
S 8.33	E 5.00	Q 2.50	Y 1.67	L 0.00
U 8.33	V 4.17	Tt 2.50	J 0.83	N 0.00
O 7.50	X 4.17	A 1.67	I 0.83	R 0.00
M 6.67				

- P → e, Z → t, {S,U,O,M,H} → {a,h,i,n,o,r,s}
- Most common pair: ZW → th and hence ZWP → the, ZWSZ → that
- Finally: it was disclosed yesterday that several informal but direct contacts have been made with political representatives of the Viet Cong in Moscow

Playfair Cipher

- Not even the large number of mappings (4×10^{26} mappings) in a monoalphabetic cipher provides security
- One approach to improving security is to **encrypt multiple letters** – e.g., **Playfair Cipher**

Playfair Key Matrix

- A **5X5** matrix of letters based on a keyword
- Fill in **letters of keyword** (minus duplicates) from left to right and from top to bottom
- Fill rest of matrix with **other letters**
- Eg. using the keyword **MONARCHY**
- **I** and **J** count as one letter

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Playfair Cipher: Encryption

- Plaintext is encrypted **two letters** at a time
 - ◆ If a pair is a repeated letter, insert filler **x**, e.g., **bolloon** → **bo lx lo on**
 - ◆ If both letters fall in the same row, replace each with **letter to right** with the first element of the row circularly following the last, e.g. **ar** → **rm**
 - ◆ If both letters fall in the same column, replace each with the letter below it (again wrapping to top from bottom), e.g. **mu** → **cm**

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Playfair Cipher: Encryption

- Plaintext is encrypted two letters at a time
 - ◆ Otherwise, each letter is replaced by the letter in the same row and in the column of the other letter of the pair, e.g., **hs** → **bp**, **ea** → **im** (or **jm**)

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Playfair Cipher: Encryption

- Plaintext is encrypted two letters at a time
 - ◆ Otherwise, each letter is replaced by the letter in the same row and in the column of the other letter of the pair, e.g. **hs** → **bp**, **ea** → **im** (or **jm**)

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Playfair Cipher: Decryption

- To decrypt, use the inverse of the encryption rules and drop any extra **x** that does not make sense in the final message.
- ◆ Decrypts two letters at a time
- ◆ If both letters fall in the same row, replace each with **letter to left**, e.g., **rm** → **ar**
- ◆ If both letters fall in the same column, replace each with the letter **above** it, e.g., **cm** → **mu**

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Playfair Cipher: Decryption

- Plaintext is encrypted two letters at a time
 - ◆ Otherwise, each letter is replaced by the letter in the same row and in the column of the other letter of the pair
 - ◆ E.g. bp → hs, im → ea

M	O	N	A	R
C	H	Y	B	D
E	F	G	I/J	K
L	P	Q	S	T
U	V	W	X	Z

Security of Playfair Cipher

- Security much improved over monoalphabetic
- Would need a **676 (26*26)** entry frequency table to analyse (verses 26 for a monoalphabetic)
- Was widely used for many years
 - ◆ eg. by US & British military in WW1
- It is **relatively easy** to break because it still leaves much of the structure of the plaintext language intact

Polyalphabetic Ciphers

- Improve security using **multiple monoalphabetic substitutions**
- Features
 - ◆ A set of related **monoalphabetic substitution rules** is used
 - ◆ A **key** determines which particular rule is chosen for a given transformation
- **Vigenère Cipher**: Simplest polyalphabetic substitution cipher

Modern Vigenère Tableau

		Plaintext																									
		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Key	a	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	b	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
	c	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
	d	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
	e	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
	f	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
	g	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
	h	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
	i	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
	j	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
	k	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
	l	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
	m	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
	n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
	o	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
	p	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	r	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
	s	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
	t	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
	u	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
	v	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
	w	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
	x	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
	y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
	z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

The set of related monoalphabetic substitution rules consists of **26 Caesar ciphers**

		Plaintext																									
		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Key	a	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	b	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
	c	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
	d	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
	e	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
	f	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
	g	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
	h	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
	i	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
	j	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
	k	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
	l	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
	m	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
	n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
	o	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
	p	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	r	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
	s	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
	t	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
	u	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
	v	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
	w	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
	x	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
	y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
	z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

- Each Caesar cipher is denoted by a **key** letter
- A normal alphabet for the plaintext runs across the top

		Plaintext																									
		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Key	a	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	b	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
	c	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
	d	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
	e	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
	f	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
	g	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
	h	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
	i	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
	j	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
	k	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
	l	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
	m	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
	n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
	o	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
	p	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	r	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
	s	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
	t	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
	u	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
	v	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
	w	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
	x	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
	y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
	z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Given a key letter **x** and a plaintext letter **y**, the ciphertext letter is at the intersection of row labeled **x** and the column labeled **y** → the ciphertext is **v**

		Plaintext																									
		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Key	a	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
	b	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
	c	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
	d	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
	e	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
	f	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
	g	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
	h	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
	i	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
	j	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
	k	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
	l	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
	m	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
	n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
	o	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
	p	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	r	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
	s	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
	t	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
	u	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
	v	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
	w	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
	x	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
	y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
	z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

• Write the **plaintext** out, write the **keyword** repeated above it

• E.g., using keyword *deceptive*

key: deceptivedeceptivedeceptive

plaintext: wearediscoveredsaveyourself

		Plaintext																											
		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z		
Key	a	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z		
	b	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A		
	c	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B		
	d	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C		
	e	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D		
	f	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E		
	g	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F		
	h	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G		
	i	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H		
	j	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I		
	k	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J		
	l	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K		
	m	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L		
	n	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M		
	o	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N		
	p	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O		
	q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P		
	r	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q		
	s	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R		
	t	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S		
	u	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T		
	v	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U		
	w	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V		
	x	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W		
	y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X		
	z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y		

• Write the **plaintext** out, write the **keyword** repeated above it

• Eg using keyword *deceptive*

key: deceptivedeceptivedeceptive

plaintext: wearediscoveredsaveyourself

ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ

Vigenère Cipher: Decryption

Vigenère Cipher: Decryption

• Decryption:

- ◆ The **key letter** identifies the **row**
- ◆ The position of the **ciphertext letter** in that row determines the **column**
- ◆ The plaintext letter is at the top of that column

Cryptoanalysis of Vigenère Ciphers

- Have multiple ciphertext letters for each plaintext letter. Hence letter frequencies are obscured
- But not totally lost
- Suppose that the opponent believes that the ciphertext was encrypted using Vigenère Ciphers
 - ◆ If two identical sequences of plaintext occur at a distance that is $\text{an integer} * \text{length of keyword}$, they will generate identical ciphertext sequences.

```
key:      deceptivedeceptivedeceptive
plaintext: wearediscoveredsaveyourself
ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ
```


Security of Vigenère Ciphers

- Have multiple ciphertext letters for each plaintext letter. Hence letter frequencies are obscured
- But not totally lost
- Suppose that the opponent believes that the ciphertext was encrypted using Vigenère Ciphers
 - ◆ If two identical sequences of plaintext occur at a distance that is $\text{an integer} * \text{length of keyword}$, they will generate identical ciphertext sequences.

```
key:      deceptivedeceptivedeceptive
plaintext: wearediscoveredsaveyourself
ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ
```

Cryptoanalysis of Vigenère Ciphers

- How can we make the Vigenère cipher a bit more resilient to cryptoanalysis?
- Vigenère with Autokey! (next slide)

Autokey Cipher: Encryption

- Ideally want a key as long as the message
- Vigenère proposed the **autokey** cipher
- A keyword is concatenated with the plaintext itself providing a running key
- E.g., given key **deceptive**

key: deceptivewearediscoveredsav

plaintext: wearediscoveredsaveyourself

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

How to decrypt a ciphertext?

Autokey Cipher: Decryption

• E.g., given

◆ key **deceptive**

◆ Ciphertext: **ZICVTWQNGKZEIIGASXSTSLVWLA**

key: deceptive

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

plaintext:

Autokey Cipher: Decryption

● E.g., given

◆ key **deceptive**

◆ Ciphertext: **ZICVTWQNGKZEIIGASXSTSLVWLA**

key: deceptive

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

plaintext: w

Autokey Cipher: Decryption

• E.g., given

◆ key **deceptive**

◆ Ciphertext: **ZICVTWQNGKZEIIGASXSTSLVWLA**

key: deceptive

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

plaintext: w

Autokey Cipher: Decryption

● E.g., given

◆ key **deceptive**

◆ Ciphertext: **ZICVTWQNGKZEIIGASXSTSLVWLA**

key: deceptive

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

plaintext: we

Autokey Cipher: Decryption

• E.g., given

◆ key **deceptive**

◆ Ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

key: deceptivewe

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

plaintext: we

Autokey Cipher: Decryption

• E.g., given

◆ key **deceptive**

◆ Ciphertext: **ZICVTWQNGKZEIIGASXSTSLVWLA**

key: deceptivewearediscoveredsav

ciphertext: ZICVTWQNGKZEIIGASXSTSLVWLA

plaintext: wearediscoveredsaveyourself

Transposition Ciphers

- Consider classical **transposition** or **permutation** ciphers
- Hide the message by **rearranging** the letter order without altering the actual letters used

Transposition Ciphers

- Consider classical **transposition** or **permutation** ciphers
- Hide the message by **rearranging** the letter order without altering the actual letters used
- Can recognize these since have the **same frequency distribution** as the original text

Rail Fence cipher

- Write message letters out diagonally over a number of rows
- Then read the letters row by row
- E.g., Encrypt the message “meet me after the toga party” with a rail fence of depth 2

m e m a t r h t g p r y
e t e f e t e o a a t

- Ciphertext: **MEMATRHTGPRYETEFETEOAAT**
- **Think:** how to encrypt the above message using rail fence cipher of depth 3?

Rail Fence cipher

- Write message letters out diagonally over a number of rows
- Then read the letters row by row
- E.g., Encrypt the message “meet me after the toga party” with a rail fence of depth 3

```
m  t  a  e  h  o  p  t
  e  m  f  r  e  g  a  y
    e  e  t  t  t  a  r
```

• Ciphertext: **MTAEHOPTFMFREGAYEETTAR**

• **Think:** how to decrypt a ciphertext

ciphertext: CPEERYOURCIMTSUT

Rail Fence cipher: Decryption

- How to decrypt a ciphertext
 - ❖ Let $|\text{row}|$ be the number of rows
 - ❖ Compute the length of the ciphertext $|\text{cipher}|$
 - ❖ Compute the number of letters of each row
 - ❖ Write down the ciphertext row by row
 - ❖ Read the ciphertext diagonally

Rail Fence cipher: Decryption

● Example:

ciphertext: CPEERYOURCIMTSUT

|row| = 3

❖ **|cipher| = 16**

❖ **$16/3 = 5$, $16 \bmod 3 = 1 \rightarrow$**

1st row: $5+1 = 6$ letters

2nd row: 5 letters, 3rd row: 5 letters

C P E E R Y

O U R C I

M T S U T

\rightarrow Plaintext: computersecurity

Rail Fence cipher: Decryption

● Example:

ciphertext: CPEERYOURCIMTSUT

$|row| = 3$

❖ $|cipher| = 16$

❖ $16/3 = 5, 16 \bmod 3 = 1 \rightarrow$

1st row: $5+1 = 6$ letters

2nd row: 5 letters, 3rd row: 5 letters

C P E E R Y

O U R C I

M T S U T

→ **Plaintext:** computersecurity

Steganography

- **Cryptography:** make the message unintelligible.
- **Steganography:** Conceal the existence of the message.
- **Example:**

“Dear George;

Greetings to all at Oxford. Many thanks for your [redacted]
letter and for the summer examination pack[redacted]age.
All Entry Forms and Fees Forms should be ready [redacted]
for final dispatch to the Syndicate by Friday [redacted]
20th or at the very latest, I’m told by the 21st. [redacted]
Admin has improved here, thought there’s room [redacted]
for improvement still, just give us all two or three [redacted]
more years and we’ll really show you! Please [redacted]
don’t let these wretched 16+ proposals destroy [redacted]
your basic O and A pattern. Certainly this [redacted]
sort of change, if implemented immediately, [redacted]
would bring chaos.”

Steganography

Example:

“PRESIDENT’S EMBARGO RULING SHOULD HAVE IMMEDIATE NOTICE. GRAVE SITUATION AFFECTING INTERNATIONAL LAW. STATEMENT FORESHADOWS RUIN OF MANY NEUTRALS. YELLOW JOURNALS UNIFYING NATIONAL EXCITEMENT IMMENSELY. “

Steganography

Example:

“PRESIDENT’S EMBARGO RULING SHOULD HAVE IMMEDIATE NOTICE. GRAVE SITUATION AFFECTING INTERNATIONAL LAW. STATEMENT FORESHADOWS RUIN OF MANY NEUTRALS. YELLOW JOURNALS UNIFYING NATIONAL EXCITEMENT IMMENSELY. “

PERSHING SAILS FROM NY JUNE 1

Steganography

Modern Examples:

- ◆ Encoding hidden message in the pixels of the image
- ◆ Encoding hidden messages in the bits of sound files

● **Advantage:** useful to parties who are more concerned with hiding the **fact** of their secret communication, rather than the contents of messages

Drawbacks:

- ◆ **Significant overhead** to hide a small amount of information
- ◆ Once the scheme is **known**, the system is useless

Row Transposition Ciphers

- A more complex transposition
- Write letters of message out in rows over a specified number of columns
- Then reorder the columns according to some key before reading off the columns

Key: 3 4 2 1 5 6 7

Plaintext: a t t a c k p
 o s t p o n e
 d u n t i l t
 w o a m x y z

Read the 3rd column

Ciphertext: TTNAAPTMTSUOAODWCOIXKNLYPETZ

- Think: how to decrypt a ciphertext?
ciphertext: ATHNIERIPTSIORPNSOCX

Row Transposition Ciphers: Decryption

- How to decrypt a ciphertext?

ciphertext: ATHNIERIPTSISORPNSOCX

◆ $|cipher| = 21$, $|key| = 7 \rightarrow |row| = 3$

Key: 3 4 2 1 5 6 7

Ciphertext: T R A N S P O
 S I T I O N C
 I P H E R S X

Plaintext: transpositionciphers

Product Ciphers

- Ciphers using **substitutions** or **transpositions** are not secure because of language characteristics
- Hence consider using several ciphers in succession to make harder
 - ◆ Two substitutions make a more complex substitution
 - ◆ Two transpositions make a more complex transposition
 - ◆ But a substitution followed by a transposition makes a new much harder cipher
 - This is bridge from classical to modern ciphers

Additional References

- <http://www.eventid.net/docs/desexample.htm>
- <http://www.iusmentis.com/technology/encryption/des/>
- <http://www.research.ibm.com/journal/rd/383/coppersmith.pdf>

Acknowledgement

- Some slides are borrowed from Dr. Ping Yang