Complex Systems & Electoral Swings in the Province of Ontario

A Report Submitted in Partial Fulfillment
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I Introduction

Change of government is a natural feature of democracy [1]. In the Province of Ontario, the percentage of seats held by a political party is determined by general elections every 4 years. The frequency and magnitude of these electoral swings can be studied through a complex systems perspective. In this paper, we explore the heterogeneous, interdependent and non-linear dynamics of electoral swings through the concepts of bifurcations, limit cycles and criticality.

Historical Context

A half-century after the British prevented American expansion into Upper Canada during the War of 1812, the British North America Act constituted Ontario as the English province of Canada [2]. It quickly rose as the eminent economic power for its agriculture, forestry, manufacturing, mining industries, and remains today for its energy generation, automotive, financial services and IT sectors. The Ontario election results since 1867 are plotted:

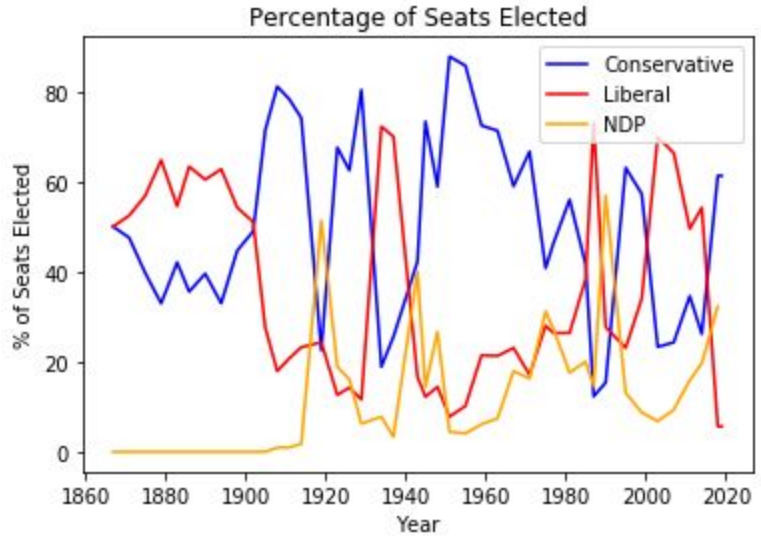

Figure 1: Percentage of seats elected for the three major parties in Ontario across time.

The population of Ontario has grown steadily from 1.5 to 13.5 million over 150 years; the ethnic demographics are prominently European with growing Asian and African communities since the

Federal Immigration Act of 1976 [3]. Southern Ontario makes up 95% of the total population with Toronto and Ottawa as the significant urban centres.

System Classification

The main state variables are the percentage of seats held by the government and whether or not an electoral swing took place that general election. Other variables such as the number of votes or term-length are also considered. While no formal model equations are defined in this analysis, parameters that might form the basis of a model like GDP, employment, population and Federal Government party are analyzed for their relationship with the system state.

Heterogenous and Stochastic Dynamics

Public support is subject to heterogeneous and ensemble dynamics. General election results depend on the party leaders, political climate, rhetoric, policy positions, as well as factors like coalitions of labour unions, population demographics, and incumbency [4]. These factors are often quoted as anecdotal evidence for an election result: for example, claims that the 1999 sale of Highway 407 resulted in the 2003 Liberal victory, campaign promises to fund faith-based schools resulted in the 2007 Conservative defeat, or that the accumulative scandals of the recent Liberal government resulted in their historic 2018 defeat [5]. To incorporate these difficult-to-capture dynamics in a model, subjective poll data or stochastic terms would need to be used.

Time and Spatial Non-Stationarity

The number of electoral districts and their boundaries has changed since Ontario's founding, and many different political parties have held seats. Like its Federal counterpart, Ontario is mostly a “two-party system” with a third NDP party, but these variables make the system non-stationary.

II Bifurcations

The time plot below shows (1) percentage of seats won by each party in a general election and (2) grey rectangles if a change of government took place over time. It is observed that a change of government does not occur at a regular frequency and relatively high magnitude.

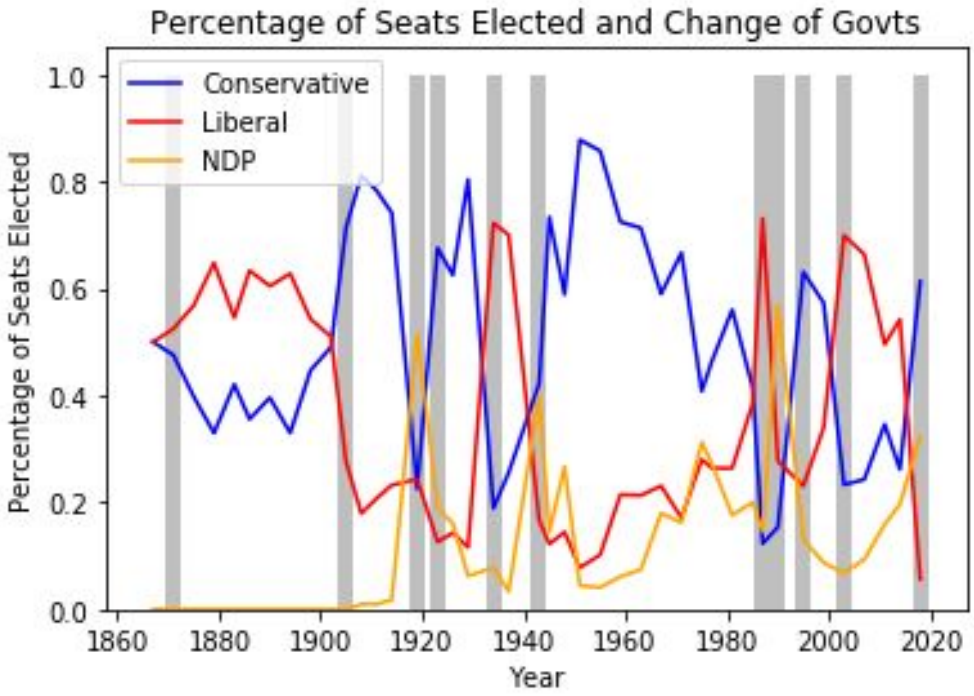

Figure 2: Percentage of seats elected (as a decimal) in Ontario for the three major parties across time, with changes in government marked with grey lines.

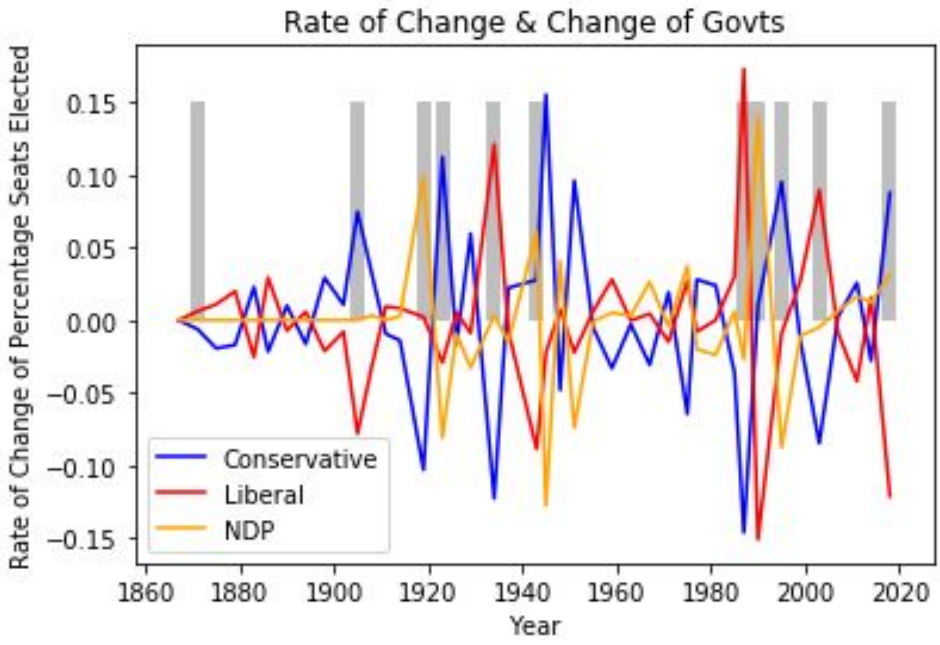

Figure 3: The rate of change of the percentage of seats elected for the three major parties in Ontario, with changes of government marked in grey

The time derivative plot of the state is more illustrative. It shows that when changes of government occur, the change in the percentage of seats held is large in magnitude. Instead of seeing an electoral swing for a few seats, representing linear behaviour, changes of government involve nonlinear electoral swings across many seats.

Parameters

Bifurcation theory states that abrupt changes in system behaviour, electoral swings, occur when certain system parameters are changed. There was a single liberal government throughout the 30 year period of 1875 to 1905 and a single conservative government throughout a 40 year period from 1945-1985. In contrast, the post-WW1 period from 1919 to 1923 saw 2 governments and the 8 year period from 1987 to 1995 saw 3 different governments. Below, some parameters that might have a coupling effect on these changing behaviours are explored.

Vote Count

The model parameter of *percentage of seats held* for each party is a lumped form of observing the state of the system, and while it does represent the truth, it may not contain details of the system which reveal more insights about its behaviour.

Examining the rules of the rules of the parliamentary system are important to consider: the *first-past-the-post* system which exists in many democratic systems. In this election system, each riding elects one candidate(the one with more votes than any other candidate), and the party with the most candidates forms a government [6]. However, this system may not closely represent the votes of its people. For example, a parliament has 100 ridings which each have even voting populations, and the voting across ridings is split between three parties as follows:

1. *Party 1* captures 34% of the votes from ridings 1 through 51, and 0% of the votes in the other 49 ridings. This means they capture 34% of 51% of the votes, or 17.34%.
2. *Party 2* captures 33% of the votes in ridings 1 through 51, and 100% of the votes in the other 49 ridings. This means they capture 65.83% of the votes.
3. *Party 3* captures 33% of the votes in ridings 1 through 51, and 0% of the votes in the other 49 ridings. This means they capture 16.83% of the votes.

In this theoretical example, Party 2 captures almost four times as much of the total votes as Party 1, yet Party 1 forms a majority government, capturing 51 seats. Party 3 captures 97% as many of the total votes as Party 1, yet captures no seats in parliament. These tiny margins across ridings 1 through 51 hold disproportionally large differences in the overall system, and these ridings are known as *swing ridings* [7].

This means that the lumped model of *percentage of total votes captured* generally has weak correlation with how many seats are captured by the winning party. Figure 4 shows this history across Ontario elections. An interesting example is the 1951 election: Leslie Frost's Conservative government captured a marginal amount more of the total vote (~7%) and captured almost 30% more seats in the provincial legislature than it held previously.

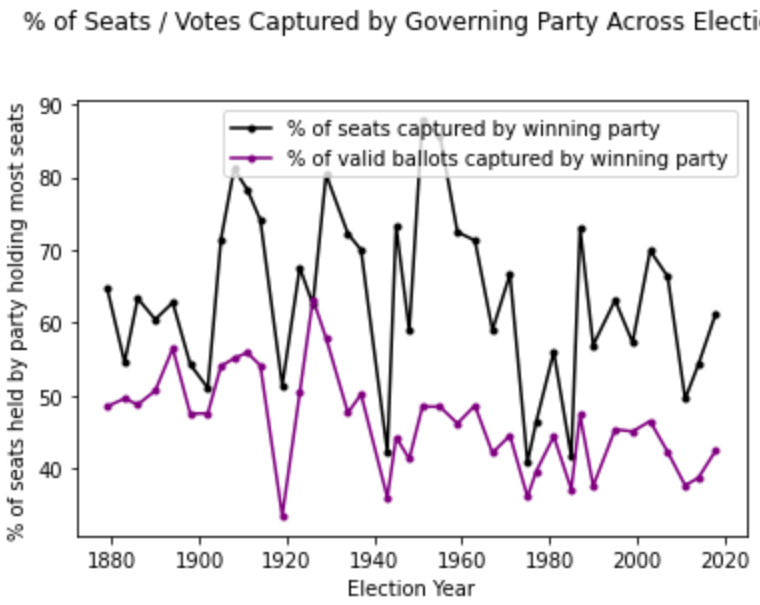

Figure 4: Percentage of total provincial parliamentary seats captured by winning party and percentage of total valid votes captured by winning party, across election years.

Similarly, the most recent federal election in 2019 saw the Liberal Party capture 33.07% of the total votes, but 39.47% of the total parliamentary seats, while the Conservative Party captured more total votes (34.41% of the total) but only 31.89% of the total parliamentary seats [8]. This illustrates that while there is obviously a relationship between capturing ballots and capturing seats to govern, there are other important factors at play.

A complex factor in elections is the way that the electorate is divided into ridings: the previous section showed the disproportional power that a few votes in key ridings may hold on the outcome of elections. While the detailed analysis across the geographical drawing of ridings and time is out of the scope of this paper, the potential effects brought by the number of seats in the legislature will be examined.

Figure 5 plots the total seats in the Ontario Legislature across time, with color codes for the party which held the most seats in each election. The number of seats in the legislature increased under both Liberal and Conservative governments. It has gradual increases, and sudden large

reductions in seat counts which get implemented across single elections. It should be noted: This chart shows "*X* party in power", which means the party was in power *after* the election for that year.

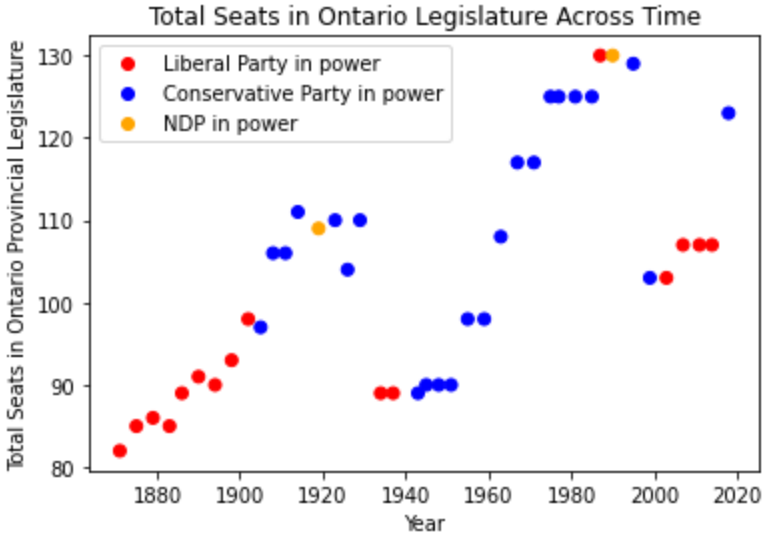

Figure 5: Total Seats in Ontario Legislature Across Time of General Elections, Colour-Coded by Major Party

A large amount of analytical complexity is brought about by ridings:

1. The spatial non-stationarity: ridings can occupy large or small areas (compare *Mushkegowuk—James Bay* with *Toronto Centre* in total area), and the vast geographical differences in ridings.
2. The population differences across ridings: *Kiiwetinoong*, a Northern Ontario riding with ~ 14,000 registered voters, compared with *Burlington*, a suburban GTA riding with ~100,000 registered voters [9]. This can cause increased nonlinearity in the relationship between number of votes and number of seats captured.
3. Time non-stationarity: the plot in Figure 5 above shows that ridings change very frequently to compensate for a changing population distribution across space.

While the boundaries of ridings certainly could offer insight into the election system for Ontario and other democratic systems, they are **excluded** from this lumped model due to a lack of historical data, as well as time for appropriate modelling of this data.

Federal Party

Federal general elections could have a direct or lagging impact on the provincial results. Ontario has been the most significant presence federally in terms of its economic output and population. Links between the respective federal and provincial parties might also increase the correlation.

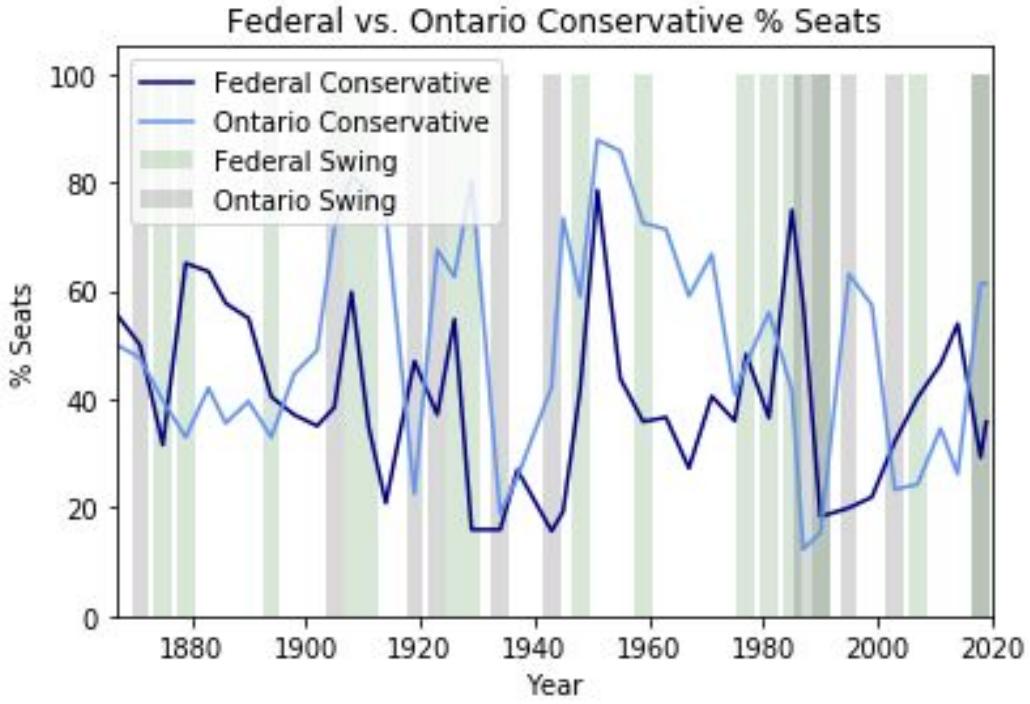

Figure 6: Federal and Ontario Provincial Conservative Party percentages of seats held across time, with federal swings in power marked in light green and provincial swings in power marked in grey.

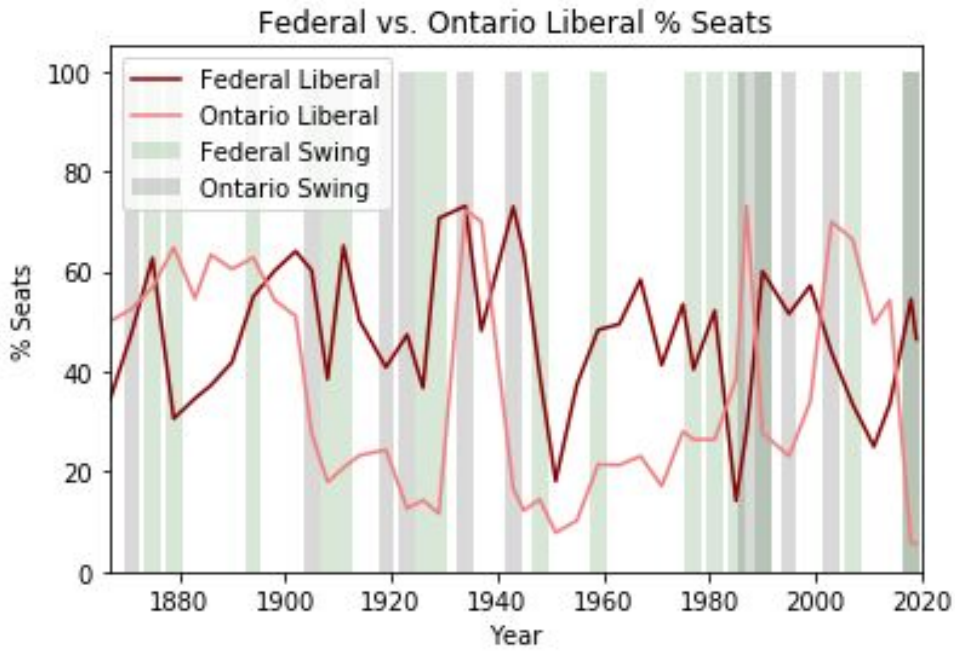

Figure 7: Federal and Ontario Provincial Liberal Party percentages of seats held across time, with federal swings in power marked in light green and provincial swings in power marked in grey.

From graphing federal election results with the Ontario election results, all three of leading, direct, and lagging relationships could be observed. There is a correlation between Federal and Ontario party seat percentage throughout history, especially in the early to mid-1900s, where the Ontario election results appear to amplify the Federal results. Before and after this period, there appears to be different behavior where the Ontario and Federal electorates elect opposing parties. In terms of changes of government, change of Federal government events do not seem to consistently predict or lag behind Ontario electoral swings, as can be observed from the random sequence of Federal (green rectangles) and Ontario (grey rectangles) swings in the background.

Population

The next parameter explored is Ontario's population and its percentage change. Steep changes to Ontario's population might influence changes in system state. Ontario's population, percentage change, and system state are plotted below from 1971 to 2018.

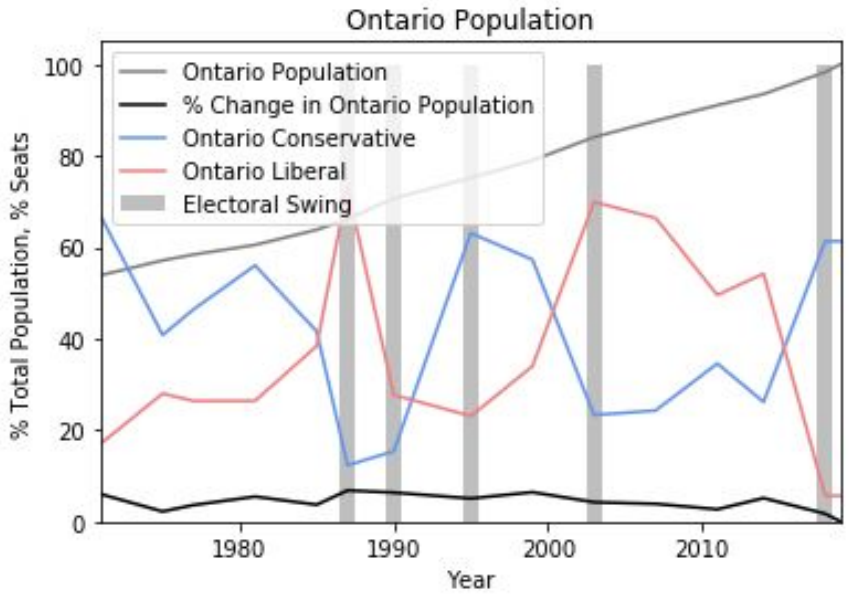

Figure 8: Percentages of seats held by the Ontario Conservative and Liberal parties, alongside changes in population across time. Changes in power between parties marked in grey.

It is observed that the rate of population increase is relatively steady over the past 30 years. As a result, there is little evidence that the population would be an effective parameter to describe electoral swing behaviour. Breaking down the population by demographic data, such as age, ethnicity or education level would be a next step here to explore if the population data does in fact influence electoral swings.

GDP

Similarly, the state of the economy is often quoted as a significant indicator of election outcomes. GDP measures aggregate economic output of the province. Below, Ontario's GDP, GDP percentage change, and system state are plotted below from 1981 to 2007.

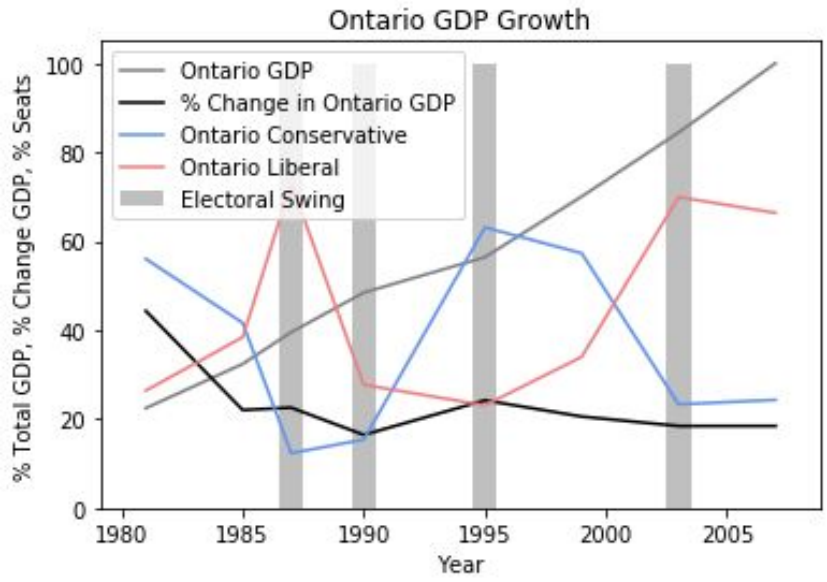

Figure 9: Percentages of seats held by the Ontario Conservative and Liberal parties, alongside changes in GDP growth across time. Changes in power between parties marked in grey. Note: changes in GDP growth are across election years.

In this case, while the number of data points is quite small to observe a meaningful correlation, electoral swings seem to correlate with slight decreases in GDP percent change. Similar to the population data, more granular analysis of economic sectors and labour groups might provide more insight into which parameter changes cause non-linear changes in seats held. While no strong GDP data is available, simple empirical data shows no strong relationship: following the 2008 recession, Ontario re-elected a Liberal government three times in ten years following the recession until 2018.

III Limit Cycles

The *Introduction to Complex Systems* textbook states, "A significant challenge in studying limit cycles is detecting them." In this case, where the state of the province's democracy has two (or three) possible states of a governing party, cycles between these states is an obvious limit cycle. Simple inspection of the first plot in this paper shows that there is obvious cyclic behaviour between the three main parties seeking power in the Ontario legislature. However, this limit

cycle is not simple to define with mathematical variables: the state does not follow any constant rate of change.

This highly variable rate of change is not simply due to the rate of change (provincial elections) being necessarily discrete. It is due to a *highly variable* rate of how long governing parties maintain power.

Table 1 shows the length of the consecutive terms of governing parties in power; both in years, and in number of general provincial elections. It shows high variance, with
It should be noted that the final data point in this table is not certain, as this government is ongoing and remains in power.

Table 1: Length of Historical Governments in Number of Elections with Most Seats and Years

Start of government	Length of government (elections)	Length of government (years)	Party with most seats
1871	9	34	liberal
1905	4	14	conservative
1919	1	4	ndp
1923	3	11	conservative
1934	2	9	liberal
1943	13	44	conservative
1987	1	3	liberal
1990	1	5	ndp
1995	2	8	conservative
2003	4	15	liberal
2018	1	4	conservative

Although there are only nine data points, plotting a histogram of the length of governments by number elections shows interesting behavior (shown in Figure 10). Beside this plot in Figure 10 is the Log-Log histogram of the same data. Although there are only 11 data points, this data does appear somewhat linear, indicating that government lengths could follow power law behavior in

their limit cycle behavior. In this case, there is the difficulty of having enough data to understand whether the data truly is fat-tailed. Since this is a measurement of governments across time, collecting this data would take a *very* long time.

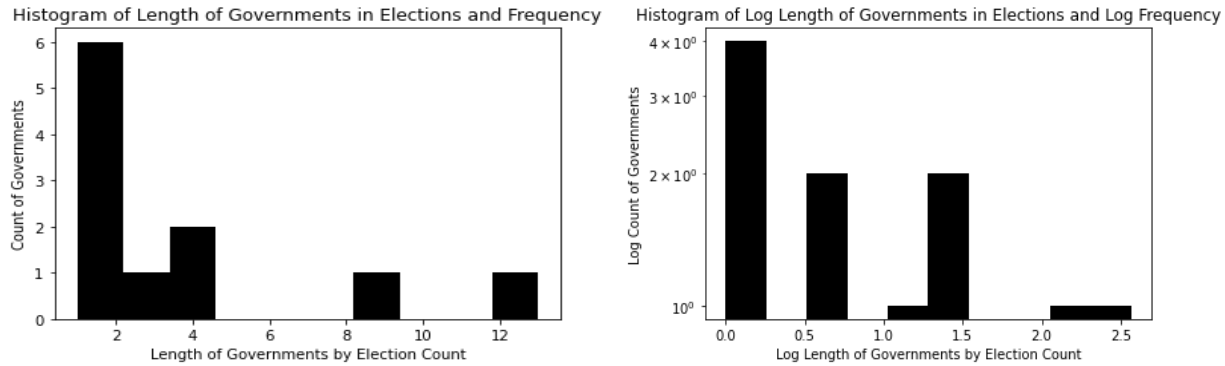

Figure 10: Left: Count of governments by length of government. Right: log-count of governments by log-length of government.

One other point of consideration, is how has the length of governments in Ontario changed with time? Figure 11 shows the lengths of 10 governments in Ontario plotted across their starting years.

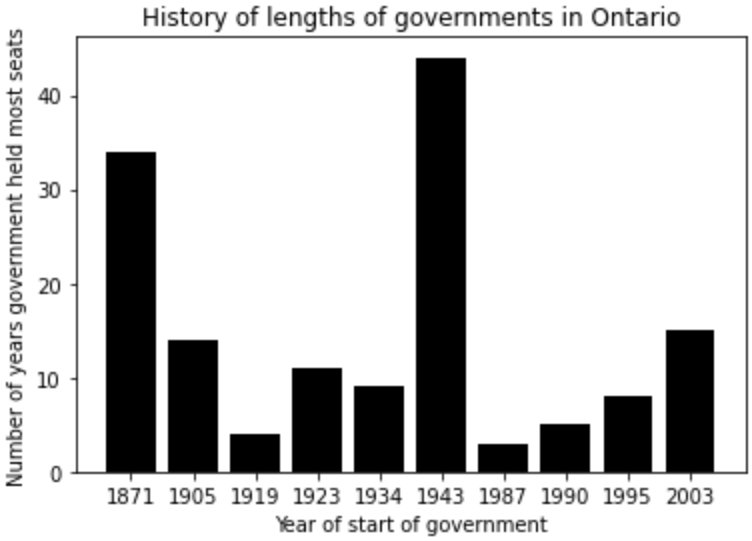

Figure 11: Length of parties consecutively holding the most seats. The x-axis is the year the government first took power.

The party holding the most seats from 1943 to 1987 is the most obvious extreme value; at 44 years, this government (Conservative) was a *dynasty*. This government took control near the end of World War II, and was in power until the other parties formed a coalition government. It is important to note that the Conservative party only governed until 1985, for in the 1985 election the NDP party and Liberal party formed a coalition to take control.

After this conservative dynasty ended, governments appear to have become shorter: the mean length of a government in power after that 7.75 years, while the mean length of a government in power before that being 19.3 years. It would be mere speculation to analyze whether the system has changed such that governments will have shorter periods of consecutive power: a two-sample T-test between these two time periods is inappropriate both due to the non-normal distribution of the data and the small sample size.

Why is the behaviour of the democratic system so cyclic? Why can a single governing party not maintain control of parliament for perpetuity? Aside from using power to build systemic barriers to having power removed (a sabotage of democracy), this is largely due to *self-organized criticality*, discussed next.

IV Power Laws & Self-Organized Criticality

Is the parliamentary democratic system an example of self-organized criticality? Does it follow power laws? Indeed, the shifting power in democracy has been compared to the SOC sand pile model [1], and different media sources have referred to the past three changes in the elected government of Ontario using the word “landslide” [10] [11] [12]. The reasons for self-organized criticality occurring in the system will be discussed afterwards.

First, the sizes of shifts in seats held by governments can be inspected. Previous plots show that these shifts vary greatly in size. Figure 12 is a plot of the change in percentage of total parliamentary seats held by the conservative party, with colour to indicate when the party lost, gained, maintained, or failed to gain power (having more seats than any other party).

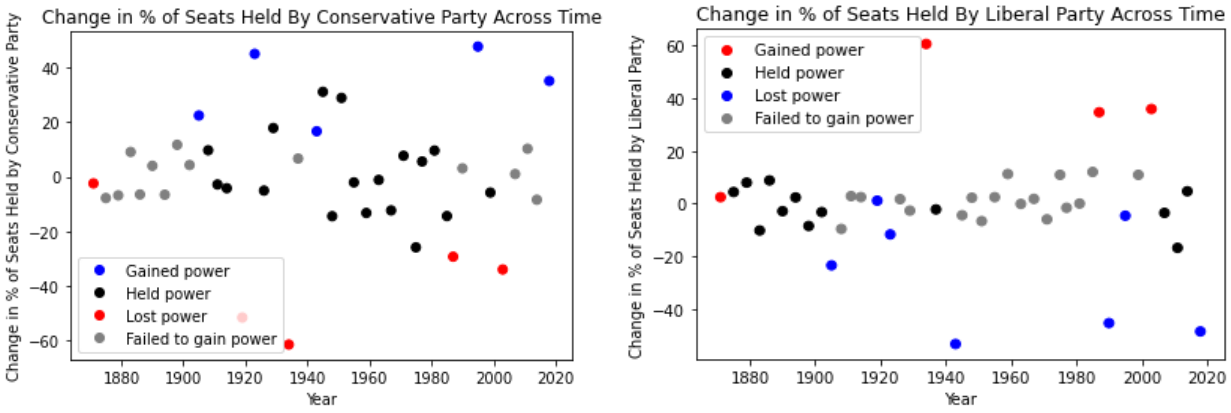

Figure 12: Color-coded changes in the percentage of seats won by the Conservative Party (left) and Liberal Party (right) in Ontario across elections in time.

tbehaviour about how changes of power occur. It shows an approximate minimum of one party gaining or losing 20% of the seats in order to gain power. The distribution of the percentage of seats captured in elections by the Conservative party is plotted in Figure 13, using a histogram. This plot has many elections with absolute changes of <20%, with a few very large percentage changes spread across the tail. Below it, the log frequency is plotted to examine the distribution for thick-tail behaviour.

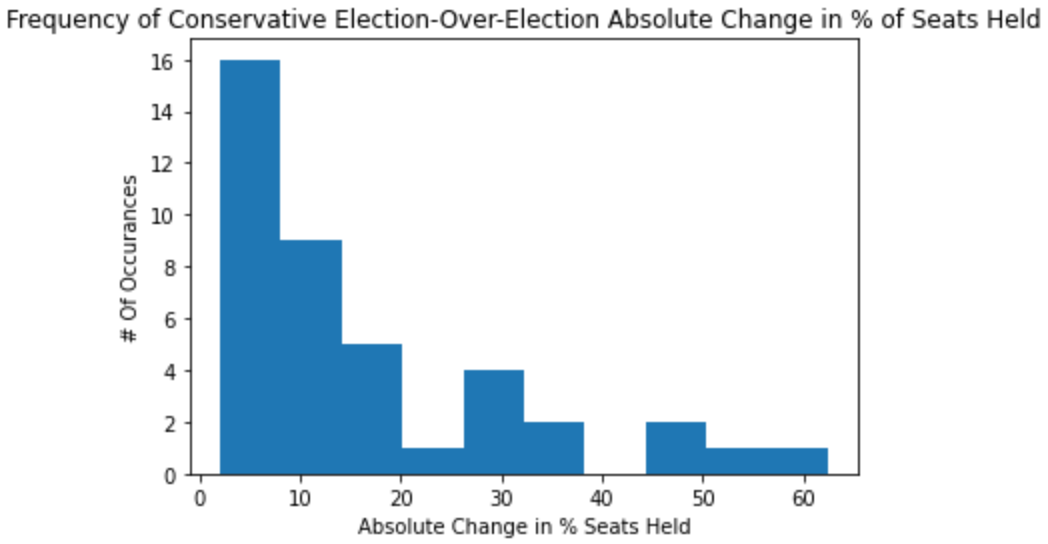

Figure 13: Frequency of Absolute Change in Percentage of Seats Held by Conservative Party

Figure 14: Log Frequency of Absolute Change in Percentage of Seats Held by Conservative Party

The reason Figure 14 is not plotted as a log-log plot is because the percentage of seat change is bounded; theoretically at 100, but more realistically at a number lower than 100. As well, a large percentage shift represents a *much* larger scale change in the system despite not being larger by some significant scaling factor.

It is this combination of power law behaviour and self-organized criticality which is of interest. In *Self-Organized Criticality: A New Theory of Political Behaviour and Some of Its Implications*, Brunk proposes that the reason self-organized criticality occurs is because of the increasing challenge in satisfying all of the partners in a political coalition [1]. Across some set of policies in time, a government enacts policies, each which do not perfectly satisfy any one party. Eventually, the total of all of these policies leads the coalition to fall apart as parties break away to find preferable coalitions. This is the forcing function. Brunk also notes a second component of the forcing function: the cascading effects of certain actions. In this case, a piece of potentially important information regarding the government ripples across an electorate population. Interesting psychological factors may appear in this forcing function of information cascading through a democratic society, including groupthink [13].

The limit in the system which leads to self-organized criticality in the system's behaviour, not mentioned anywhere by Brunk, is the ability of other parties to provide the electorate with an electable alternative. If voters have a strong alternative in any election, then a single scandal or unkept promise could cause a change of power; meaning that a very tight limit has been imposed on the government. An example of a loose limit on a government is the 2014 Ontario provincial election: Dalton McGuinty's Liberal government had held power for ten years; and was mired by scandal. However, the Conservative Party's leader Tim Hudak proposed extreme budget cutting measures; including 100,000 public sector job cuts [14]. The Ontario population chose to re-elect its existing provincial government, showing that the Conservative Party's extreme policies which were proposed increased the limits the population placed on the Liberal government.

There is a more nuanced dimension of the limits to a government in self-organized criticality: the individual leader of the governing party. Electorates are often willing to extend the limits of criticality if a government makes an appropriate change to its leader. In the longest Conservative government, from 1943 to 1981 the Conservative government saw four different leaders (Premiers), as did the Liberal government in the late 1800s.

In conclusion, the consistent observation of large changes in percentages of seats with changes of power is a strong indicator behaviour of self-organized criticality. The distribution of these changes seems to follow a power-law distribution, but the limited size of the data makes drawing a conclusion about power law behaviour unwise.

V Data Issues and Possible Further Analyses

The largest issue in modelling the change in state of a democratic system is the lack of granularity captured by a lumped model. Shown earlier, the *first-past-the-post* system means that the objective for each party should be to obtain the largest share of votes in each riding, rather than seeking to provide value to the largest pool of the electorate. The issue with developing a more granular model in this was largely due to a lack of data.

With regards to granularity in elections, there is an aspect of the system which was not included to its data complexity: by-elections. A by-election is an election for a specific riding in between general elections, where small shifts in power occur. Changes in government at the individual riding granularity could signal the strength of the governing party, and if a bifurcation was on the horizon.

There is a major question in the overall behavior of the system across time: is the change in the system time stationary, or would a state equation for the system involve time? (For example, $\dot{x} = F(x, t)$). Some possible factors which could affect the time stationarity of the model which were assessed include GDP and immigration, however, the data for these was only available for a more recent time period. A drastic number of other changes have occurred: the development of technology since 1867 is one obvious change, from radio to television to the internet.

Another possible factor in the behaviour of the system is the system rules itself; the laws which govern elections. The Election Act is the legislation which imposes restrictions on the way elections are conducted, and has had many changes since it was brought to order in 1990 [15].

The law that provides anonymity to voters in their ballots provided by the Election Act is a particular limit to possible modelling available to the system. Knowing the identities and data of individual voters would enable a variety of possibilities in modelling: rich data about demographics (for instance, age), spatial demographics (the wealth of voters could be estimated from voters' addresses) as well as the change of individual voters' choices in elections across time could be modelled. Having individual voter data could even open up the possibility of developing agent-based models. These models could look at each voter having some probability of influencing the decisions of each voter around them, as well as a probability of not voting whatsoever. Agent-based models could also lead to interesting insights into the self-organized criticality in elections.

Voter turnout is a significant factor in elections: many analysts cite poor voter turnout as the reason for Donald Trump's victory in the 2016 US Presidential Election [16]. The decision (or inability) of members of the electorate to not vote in swing-ridings increases the proportional influence of those who do vote. The province of Ontario has had many thin margins in ridings in past elections [9]. The data for voter turnout for the most recent approximate 100 years is available, however, its collection and analysis was determined to have too much complexity for this project.

In game theory, the electoral competition is a game where candidates must present their political positions on a set of dimensions. The winner is the candidate whose policy positions have the highest number of voters whose positions are closest to the candidate's policy. The modelling of candidate parties' electoral platforms in a quantifiable manner could yield interesting results. The example of the Conservative party's extreme policy in 2014 showed that the party had a profitable deviation of offering a more moderate platform, but instead opted for a platform which appealed to a smaller population. The quantifiable mathematical modelling of the extremeness policies of electoral platforms likely could assist in accurate modelling of the electoral system, but would require extensive analysis, which lies outside the scope of this project.

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