Moule2
Unit 2.1

Probability and Probability Distribution

Random experiment: Any action which gives one or more results is called a random experiment. Each result of the experiment is called an outcome of the experiment.

Sample space: The set of all possible outcomes of an experiment is called the sample space of that experiment and is denoted by S

Examples of sample

1) Toss a fair coin   S={H,T}
2) Toss 2 coins   S={HH,HT,TH,TT}
3) Tossing of a cubic die gives S= {1, 2,3 ,4,5,6}

Note :

If A= {a ,b ,c}

If order is important then we use permutation.

(I) 2-permutations of A are ab, ba, ac,ca,bc,cb

i.e. 3_{p_2}=6

(II) 2-combination of A are ab, ac, bc

i.e. 3_{c_2}=3

Examples of sample space with its cardinality

1) If 2 cards are drawn from 52 cards then

   $|S|= 52_{c_2}$

   $= (52*51)/2$

2) If 3 cards are drawn from 52 cards then

   $|S|=52_{c_3}$

3) If 3 balls are drawn from 10 balls then

   $|S|=10_{c_3}$

4)If 2 cards are from well-shuffled pack $|S|=52_{c_2}$

In well –shuffled pack :

13 cards of diamond ,13 card of heart, 13 card of club, 13 cards of spade

Examples of sample with its cardinality

5) If 2 unbiased dice are thrown then write sample space

S={ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

|S | = 36

Event:

In general any happening is a subset of S and is called an “Event”

Example 1. Consider the experiment of throwing a cubic die

Then S={ 1,2,3,4,5,6} is sample space

we want E- event as an even number to come up

E={2,4,6}

Example 2. Consider the tossing of a die, then n(S)=6

1) If A is event that an odd number comes up

A= {1,3,5}

2) If B is event that a number is divisible by 3 comes up

B= {3,6}

3) If C is event that a number is less than 4 comes up

C= {1,2,3}

Example 3.A card is drawn from pack ok 52 cards

If A is event that a spade is drawn |A|=13

B is event that a king is drawn |B|=4

C is event that a king of spade is drawn |C|=1

D is event that a red card is drawn |D|=26

Algebra of Events

Since events represent subsets, therefore operations of set theory are very useful to define algebra of events and gives sound approach to probability.

Union of two events: Let A and B be any two events defined on the sample space S. Then union of two events is denoted by $A \cup B$ and is defined as occurrence of at least one event i.e. either event A occurs or event B occurs.

i.e. $A \cup B = \{x / x \in A \text{ or } x \in B \text{ or both}\}$

Intersection of two events: Let A and B be any two events defined on the sample space S. Then intersection of two events is denoted by $A \cap B$ or AB and is defined as simultaneous occurrence of both the events.

i.e. $A \cap B = \{x \,/\, x \in A \text{ and } x \in B\}$

Note: If two events A and B defined on the sample space S are mutually exclusive and exhaustive, then they are said to be **complimentary events**.

i.e. if $A \cup B = S$ and $A \cap B = \varnothing$ then A and B are complimentary events

Mutually exclusive events

Two events A and B defined on the sample space are said to be mutually exclusive if they cannot occur simultaneously in a single trial i.e. occurrence of any
one event prevents the occurrence of the other event i.e.
$A \cap B = \varnothing$ i.e. A and B are distinct or disjoint sets.

Exhaustive events

Two events A and B defined on the sample space S are said to be exhaustive if $A \cup B = S$.

Note: If two events A and B defined on the sample space S are mutually exclusive and exhaustive, then they are said to be complimentary events.
i.e. if $A \cup B = S$ and $A \cap B = \varnothing$ then A and B are complimentary events

Concept of Probability

In any random experiment there is always uncertainty as to whether a particular event will or will not occur. It is convenient to assign a number between 0 and 1, as a measure of the chance, with which we can except the event to occur. If, for example, the chance (or probability) of particular event is 1/4, we would say that there is 25% chance it will occur and a 75% chance that it will not occur. Assignment of real numbers (between 0 and 1) to the events defined in a sample space S is known as the probability measure

Definition:

If S is finite set of sample space of an experiment then probability that E takes place is $P(E) = n(E)/n(S) = |E|/|S|$

Q.1) A cubic die is thrown once

1) What is probability of obtaining odd number?
2) What is probability of obtaining even number?
3) What is probability of obtaining number divisible by 3?

Ans: 1/2,1/2,1/3

Q.2 If two coins are tossed,

1) What is probability of obtaining 1 head?
2) What is probability of obtaining two heads?
3) What is probability of obtaining two tails?

Ans: 1/2,1/4,1/4

Addition theorem

Statement: Let S be a sample space associated with the given random experiment. Let A and B be any two events defined on the sample space S. Then the probability of occurrence of at least one event is denoted by $p(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary 1: If two events A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B)$$

Corollary 2: If A and A¢ are complimentary events, then P(A) + P(A¢) = 1

Q.3 If two dies are rolled,

1) What is probability of that the sum of numbers of uppermost faces is even number or perfect square?

2) What is probability of that the sum of numbers of uppermost faces is divisible by 2 or 3?

3) What is probability of that the sum of numbers of uppermost faces is divisible by 2 or 4?

Ans: 11/18

Q.4: In a family there are 4 children what is the probability of having exactly 2 boys

$|S|=16$

E= { BBGG,BGBG,BGGB,GBBG,GGBB,GBGB}

P(E)=6/16=3/8

Q.5: In a family there are 2 children what is the probability of having exactly 1) 1boy and 2) 2 girls

$S = \{ BB, BG, GB, GG \}$

1) $A = \{1boy\} = \{ BG, GB \}$

$P(A) = |A| / |S| = 2/4 = 1/2$

2) $B = \{2girls\} = \{ G, G \}$

$P(B) = |B| / |S| = 1/4 = 1/4$

Q.6: If two cards are drawn from a well shuffled pack find probability that

(1) one is diamond card and one is spade card

(2) both are red cards

(3) one is red and one is black

(4) one is face card and one is an ace,

Ans: P(1)=13*13/52C2

P(2)= 26C2/52C2

P(3)= 26*26/52C2

P(4)= 12*4/52C2

Q.7 A bag contains 7W,5B and 4R balls If two ball are drawn at random from the bag I) find the probability that one is black and other is red.

E1 be the event that one black and other is red

P(E1)= 5C1*4C1/16C2=1/6

II) find the probability that one is black and other white

E2 be the event that one is black and other white

P(E2) = 5*7/16C2

III) find the probability both are black balls

E3 be the event that both are black balls

P(E3)= 5C2/16C2

IV) find the probability that both are black or both are white

=P(B)+P(W)=(5C2+7C2)/16C2

Q.8) A bag contains 3R,7W and 2B balls If one ball are drawn at random from the bag

I) find the probability that it is R.

II) find the probability that it is W

III) find the probability that it is either R or W .

Q.8) A bag contains 3R,7W and 2B balls If one ball are drawn at random from the bag

I) find the probability that it is R- E1

II) find the probability that it is W- E2

III) find the probability that it is either R or W -E3

P(E1)=3/12, P(E2)=7/12

P(E3)=P(ball is red)+P(ball is white)=3/12+7/12=10/12

Conditional Probability

Let S be a sample space associated with the given random experiment. Let A and B be any two events defined on the sample space S. Then the probability of occurrence of event A under the condition that event B has already occurred and $P(B) \neq 0$ is called the conditional probability of event A given B and is denoted by $P(A/B)$.

Theorem: Conditional probability of event A given that event B has already occurred is given by

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad , P(B) \neq 0$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad , P(A) \neq 0$$

Multiplication theorem:

Statement: Let S be a sample space associated with the given random experiment. Let A and B be any two events defined on the sample space S. Then the probability of occurrence of both the events is denoted by $P(A \cap B)$ or $P(AB)$ and is given by

$P(A \cap B)$ or $P(AB) = P(A) \times P(B/A) = P(B) \times P(A/B)$

Independence of Events.

Let S be a sample space associated with the given random experiment. Let A and B be any two events defined on the sample space S. If the occurrence of any one event does not depend on occurrence or non-occurrence of other event, then two events A and B are said to be independent.

i.e. if $P(A/B) = P(A/B^{¢}) = P(A)$ or

$P(B/A) = P(B/A^{¢}) = P(B)$

then A and B are independent events.

Remark: If A and B are independent events then
$P(A \cap B) = P(A) \times P(B)$
In general, if A1, A2, . . .,An are n independent events, then
$P(A1 \cap A2 \cap . . . \cap An) = P(A1) \times P(A2) \times . . . \times P(An)$

Theorem: If A and B are independent events then
a) A and B¢ are also independent
b) A¢ and B¢ are also independent.

Theorem If A1,A2 ---- An from partition of sample space and B is any other event of S
Then $P (B)=P(B \cap A1)+P(B \cap A2)+\text{-------}+P(B \cap An)$
$=P (A1)P(B/A1)+P(A2)P(B/A2)+\text{--------}$

Example A card is drawn from a well shuffled pack of 52 cards. If it is red card find the probability that it is a king.

Let A be the event that card is red

Let B be the event that card is king

To find P (B/A)

P (B/A) = $\frac{P(\mathrm{A} \cap \mathrm{B})}{P(A)}$ =$\frac{2/52}{26/52}$ =--

Example Consider the following table:

Sex	Employed (E)	Unemployed (U)	Total
Male (M)	250	50	300
Female (F)	150	100	250
Total	400	150	550

(a) If a person is male, what is the probability that he is unemployed? (b) If a person is female, what is the probability that she is employed? (c) If a person is employed, what is the probability that he is male?

Solution:

(a) $P(U/M) = \dfrac{\text{Unemployed and male}}{\text{Male}} = \dfrac{P(U \cap M)}{P(M)} = \dfrac{50}{300} = \dfrac{1}{6}$

(b) $P(E/F) = \dfrac{P(E \cap F)}{P(F)} = \dfrac{150}{250} = \dfrac{3}{5}$

(c) $P(M/E) = \dfrac{P(M \cap E)}{P(E)} = \dfrac{250}{400} = \dfrac{5}{8}$.

Example If we randomly pick two television sets in succession from a shipment of 240 television sets of which 15 are defective, what is the probability that they will be both defective?

Answer: Let A denote the event that the first television picked was defective.

Let B denote the event that the second television picked was defective.

Then $A \cap B$ will denote the event that both televisions picked were defective.

Using the conditional probability, we can calculate

$$P(A \cap B) = P(A)\, P(B/A)$$
$$= (15/240)(14/239) = 7/1912$$

Example

A purse contains 3 silver coins and 4 copper coins

Second purse contains 4 silver and 3 copper coins

I) if a coin is selected at random from one of the two purses, find the probability that it is a silver coin

Ans: S be the event that coin as silver

A be the event that 1st purse is selected

B be the event that 2nd purse is selected

$S = S \cap (A \cup B) = (S \cap A) \cup (S \cap B)$

$P(S) = P(S \cap A) + P(S \cap B)$

$P(S) = P(A)\,P(S/A) + P(B)\,P(S/B)$

$P(S) = (1/2)(2/6) + (1/2)(4/7) = 19/42$

Example

A bag I contains 3R and,2W balls , bag II contains 2R and 4W balls. One ball is selected at random from bag I and transferred to the bag II .Find the probability that ball drawn from bag II is R ball.

Let E be the event that R ball is drawn from bag II

R be the event that R ball is transferred from 1^{st} to 2^{nd}

W be the event that ball is transferred from 1^{st} to 2^{nd}

$E=E \cap (R \cup W)$

$P(E)= (E\cap R)+ (E \cap W)$

$P(E)= P(R)\, P(E/R)+P(W)P\,(E/W)$

$P(E)=3/5.3/7+2/5.2/7= 13/35$

Bays Theorem

If A,B,C are events of sample space S

{ such that A∩**B**,B∩ C,A∩ C are empty sets and A**U** B **U** C=S}

A,B,C from partition of S and x is any other event of S then

$$P(A/X) = \frac{P(X/A)P(A)}{P(X/A)P(A)+P(X/B)P(B)+P(X/C)P(C)}$$

Example: A box I contains 2 White and 3 Red balls ,box II contains 4 White and 1 Red balls ,box III contains 3 White and 4 Red balls .

I) A box is selected at random a ball is drawn. Find the probability of that ball is White.

II)A box is selected at random and a ball drawn at random to be white find the probability that box I was selected

III)A box is selected at random and a ball drawn random to be white find the probability that box II was selected

IV)A box is selected at random and a ball drawn random to be white find the probability that box III was selected.

Let I be the event that the box I is chosen

II be the event that the box II is chosen

III be the event that the box III is chosen

W be the event that white ball is drawn

B be the event that black ball is drawn

I) $P(W) = P(W \cap I)+P(W \cap II)+P(W \cap III)$

$= P(W/I)P(I)+P(W/II)P(II)+P(W/III)P(III)$

$=(2/5)(1/3)+(4/5)(1/3)+(3/7)(1/3)$

$= 1/3\ [6/5+3/7]=(1/3)(42+15)/35=57/35*3$

= total prob of drawing white ball

II) $P(I/W) = \dfrac{P(W/I)P(I)}{P(W/I)P(I)+P(W/II)P\,(II)+P(W/III)P(III)}$

$= \dfrac{(2/5)(1/3)}{(2/5)(1/3)+(4/5/\,(1/3)+(3/7)(1/3)} = 14/57$

III) $P(II/W)= \dfrac{P(W/II)P(II)}{P(W/I)P(I)+P(W/II)P\,(II)+P(W/III)P(III)}$

$= (4/5)(35/57)= 28/57$

IV) $P(III/W)= \dfrac{P(W/III)P(III)}{P(W/I)P(I)+P(W/II)P\,(II)+P(W/III)P(III)}$

$=(3/7)(1/3)/\ \{35/57\}$

$= 15/57$

V) $P(I/R)=21/57$

VI) $P(II/R)= 7/57$

Example (C) *In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total. If their output 5, 4 and 2 per cent are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B?*

Solution. A : bolt is manufactured by machine A.

B : bolt is manufactured by machine B.

C : bolt is manufactured by machine C.

$$P(A) = 0.25, \quad P(B) = 0.35, \quad P(C) = 0.40$$

The probability of drawing a defective bolt manufactures by machine A is $P(D/A) = 0.05$

Similarly, $P(D/B) = 0.04$ and $P(D/C) = 0.02$

By Baye's theorem

$$P(B/D) = \frac{P(B)\,P(D/B)}{P(A)\,P(D/A) + P(B)\,P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02} = 0.41$$

Example : In a certain college 4% of the boys and 1% of the girls are taller than 1.8 m. Furthermore 60% of the students are girls Now if a student is selected at random and taller than 1.8 m what is probability that the student is girl ?

Example :Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have probability 0.08 of having an accident, but new drivers with driver education have only a 0.05 probability of an accident. What is the probability a new driver has had driver education, given that the driver has had no accident the first year?

Answer: Let A represent the new driver who has had driver education and

B represents the new driver who has had an accident in his first year.

Let Ac and Bc be the complement of A and B, respectively.

We want to find the probability that a new driver has had driver education, given that the driver has had no accidents in the first year, that is P(A/Bc).

$$P(A/Bc) = \frac{P(Bc/A)P(A)}{P(BC/A)P(A)+P(Bc/Ac)P(Ac)}$$