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Assignment-2 → Time and Space complexity

$\rightarrow T.C = O(n)$

1) for(int i=0; i<n; i++) {
for(int j=0; j*j < n; j++)
}

$j^2 < n \Rightarrow j < \sqrt{n}$

For inner loop
$T.C = O(\sqrt{n})$

~~j = 0, 1, 2,~~ Let ~~n = 10~~

~~j=0, j*j = 0~~

~~j=1, j*j = 1~~

~~j=2, j*j = 4~~

~~j=3, j*j = 9~~

~~j=4 j*j = 16 > n~~

Outloop Complexity = $O(n)$

Innerloop complexity = $O(\sqrt{n})$

Total Complexity = $O(n\sqrt{n})$

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$$\boxed{x^{\log_x y} = y}$$

$$2n\left(1-\frac{1}{n}\right) \Rightarrow 2n - \frac{2\cancel{n}}{\cancel{n}} \Rightarrow 2n-2$$

$$T.C. \Rightarrow O(2n-2) \Rightarrow O(2n) \Rightarrow O(n)$$

$$\boxed{T.C. \Rightarrow O(n)}$$

Que. 5 →

```
int c=0;
for(int i=1; i<n; i*=2){          O(logn)
    for(int j=n; j>i; j--){
        c++;
}
```

Let $i = 5$

$j = n, n-1, n-2, \ldots 6$

$i=1$, $j = n, n-1, n-2, \ldots 2 = n-1$ time

$i=2$, $j = n, n-1, n-2, \ldots 3 = n-2$ time

$i=4$, $j = n, n-1, n-2, \ldots 5 = n-4$ times

and ~~so~~, on

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$i = n, \frac{n}{2}, \frac{n}{4}, \text{---} 1$

Now for every 'i'

$i = n, \quad j = 0, 1, 2, \text{---} n-1 = n$ times

$i = \frac{n}{2}, \quad j = 0, 1, 2 \text{---} \frac{n}{2} - 1 = \frac{n}{2}$ times

$i = \frac{n}{4}, \quad j = \frac{n}{4}$

$\rightarrow \quad n + \frac{n}{2} + \frac{n}{4} + \text{---} 1$

$\hookrightarrow$ reducing G.P. formula $= a \frac{(1 - r^N)}{1 - r}$

a = first term $= n$

r = common ratio $= \frac{1}{2}$

$N = \log_2 n$

$$\Rightarrow \frac{n\left(1 - \left(\frac{1}{2}\right)^{\log_2 n}\right)}{1 - \frac{1}{2}} = 2n \left(1 - \left(\frac{1}{2}\right)^{\log_2 n}\right)$$

$$\Rightarrow 2n \left(1 - \frac{1}{2^{\log_2 n}}\right)$$

T. no. of operations

$$\Rightarrow \quad \underbrace{(n-1) + (n-2) + (n-4) + (n-8) \ldots}_{x = \log_2 n}$$

$$= \underbrace{(n + n + n + n \ldots)}_{\log n} - \underbrace{(1 + 2 + 3 + 4 \ldots)}_{\log_2 n \to \text{ no. of terms}}$$

$$= \quad n \cdot \log n - \left[\frac{1 \cdot (2^{\log_2 n} - 1)}{2 - 1} \right]$$

$$n \cdot \log n - \left[\frac{1 \cdot (n-1)}{\cancel{2-1}} \right]$$

$$n \cdot \log n - [n - 1]$$

$$\Rightarrow \quad n \cdot \log n - n + 1$$

$$T.C. = O(n \log n)$$

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$j = \cancel{1, 4,}\ 1, 2, 4, 8 \ldots \sqrt{n}$

Let $\sqrt{n} = k$ $\quad j = \underbrace{1, 2, 4, 8, \ldots k}_{x \text{ terms}}$

$= k = 1 \cdot 2^{x-1}$

$= x - 1 = \log_2 k$

$= x = \log_2 \sqrt{n}$

$= \log_2 n^{\frac{1}{2}}$

$= \frac{1}{2} \log_2 n \cong \log_2 n$

$\underline{T.C. = O(n \cdot \log_2 n)}$

Que. 4)

```
for (int i=n; i>0; i/=2){
    for( int j=0; j<i; j++ ){
        c++;
    }
}
```

$j = \underbrace{0, 1, 2, 3, 4 - - - - \sqrt{n}}$

Let no of terms $= x$

$x = \sqrt{n}$

Total complexity $= O(n\sqrt{n})$

Que 2 →

```
for (int i=0; i<n; i++){          → O(n)
    for(int j=1; j<n; j*=2){      → O(logn)
        c++;
    }
}
```

Time Complexity $= O(n \cdot \log n)$

Que 3

```
for (int i=0; i<n; i++) {
    for(int j=1; j*j<n; j*=2)
```

$j*j<n \rightarrow j < \sqrt{n}$

$j = 1, 2, 4, 16, 256 - - - \sqrt{n}$

$1, 2^1, 2^2, 2^4, 2^8, - - - \sqrt{n}$