DEPARTMENT OF MATHEMATICS
BIRLA INSTITUTE OF TECHNOLOGY MESRA, RANCHI

MA24103 Mathematics-II

Tutorial - I (Module I), SP 2025

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1. (a) Show that the Wronskian of $e^{ax}\cos(bx)$ and $e^{ax}\sin(bx)$ where $b \neq 0$, is b^2e^{2ax}.

   (b) Prove that the Wronskian of functions $e^{m_1x},\ e^{m_2x},\ e^{m_3x}$ is equal to $(m_1-m_2)(m_2-m_3)(m_3-m_1)e^{(m_1+m_2+m_3)x}$.

   (c) Show that $y_1(x) = \sin x$ and $y_2(x) = \sin x - \cos x$ are linearly independent solutions of $y'' + y = 0$.

   (d) Show that the Wronskian of the function $x^2,\ x^2\log(x)$ is non-zero. Can these functions be independent solution of an ordinary differential equation; if so determine this equation.

   (e) Check whether $e^x,\ xe^x,\ \sinh(x)$ are linearly independent or dependent.

   (f) Check whether $e^{ikx},\ e^{-ikx},\ \sin(kx)$ are linearly independent or not.

   (g) Show that any pair of these four functions are linearly independent: $e^{ikx},\ e^{-ikx},\ \sin(kx),\ \cos(kx)$.

2. Solve the following differential equations:

   (a) $y'' + 3y' - 10y = 0$.

   (b) $y''' - 9y'' + 23y' - 15y = 0$.

   (c) $y^{iv} + 8y'' + 16y = 0$.

   (d) $y^{iv} - 8y'' + 16y = 0$.

   (e) $y'' + 3y' + 4y = x^3 - 2x$.

   (f) $y'' - 3y' + 2y = e^{3x}$.

   (g) $y'' - 4y' + 4y = e^{2x} + x^3 + \cos(2x)$.

   (h) $y''' - 12y' + 16y = (e^x + e^{-2x})^2$.

   (i) $y''' - 5y'' + 7y' - 3y = e^{2x}\cosh(2x)$.

   (j) $y'' - 4y' + 4y = 8x^2e^{2x}\sin(2x)$.

   (k) $y'' - (a+b)y' + aby = e^{ax} + e^{bx}$.

   (l) $y^{iv} + 3y'' - 4y = \cos^2 x - \cosh x$.

   (m) $y''' - 7y' - 6y = e^{2x}(1+x)$.

   (n) $y''' - 3y'' + 4y' - 2y = e^x + \cos x$.

   (o) $y''' - 3y'' + 7y' - 5y = e^x + \cos x$.

   (p) $y'' + 4y = 2\cos^2 x + 10e^x$.

   (q) $y'' + y = \sin x + (1+x^2)e^x$.

   (r) $y'' - y = e^{-x}(\sin x + \cos x)$.

   (s) $y''' - 3y'' - y' + 3y = x^2e^x$.

   (t) $y'' + y' - 2y = e^x$.

   (u) $y'' - 6y' + 9 = 1 + x + x^2$.

   (v) $y'' + 9y = x\cos x$.

3. Solve the following Cauchy-Euler equations:

   (a) $y'' + \frac{1}{x}y' = 12 \log x / x^2$.
   (b) $x^2y'' - 4xy' + 6y = 42/x^4$.
   (c) $x^2y'' + 3xy' + y = 1/(1-x)^2$.
   (d) $x^2y'' + 2xy' - 12y = x^3 \log x$.
   (e) $x^2y'' - 2xy' - 4y = x^2 + 2 \log x$.
   (f) $x^2y'' - 2y = 2x + 6$.
   (g) $x^2y'' - 3xy' + 3y = 2 + 3 \ln x$.
   (h) $x^4y^{iv} + 6x^3y''' + 2x^2y'' - 4xy' + 4y = 10/x^3$.
   (i) $(3x+1)^2y'' + (3x+1)y' + y = 6x$.
   (j) $x^2y'' + 2xy' = \cos(\ln x)$.
   (k) $x^2y'' + 2xy' - 2y = x \sin(\ln x)$.
   (l) $4x^2y'' + 16xy' + 9y = 19 \cos(\ln x) + 22 \sin(\ln x)$.

4. Solve by the method of variation of parameters.

   (a) $y'' + y = x$.
   (b) $y'' + n^2y = \sec(nx)$.
   (c) $y'' + 9y = \sec(3x)$.
   (d) $y'' - y = \frac{2}{1+e^x}$.
   (e) $y'' + y = \tan x$.
   (f) $y'' + y = \sec x$.
   (g) $y'' + 2y' + y = e^{-x}$.
   (h) $y'' - 2y' + y = e^x$.
   (i) $y'' + 4y = 4\tan(2x)$.
   (j) $t^2y'' - t(t+2)y' + (t+2)y = 2t^3$, where $y_1(t) = t$ and $y_2(t) = te^t$ are two linearly independent solutions corresponding to the homogeneous part.