Name: Kushwanth Sai Bollepalli

UTA ID:- 1002156365

HOMEWORK-4

1) Given points $(x,y) = (-1,6), (0,2)$ and $(1,4)$

Find the Lagrange polynomial to interpolate through the given points

We know that $P_{n-1} = \sum_{i=1}^{n} y_i L_i(x)$, $L_i(x) = \prod_{j=1, i\neq j}^{n} \frac{(x-x_j)}{(x_i-x_j)}$

i.e $P_{n-1} = y_1 l_1(x) + y_2 l_2(x) + y_3 l_3(x)$

$P_{3-1} = P_2 = y_1 l_1(x) + y_2 l_2(x) + y_3 l_3(x)$

$$l_1(x) = \frac{x-x_2}{x_1-x_2} \times \frac{x-x_3}{x_1-x_3} = \frac{x-0}{-1-0} \times \frac{x-1}{-1-1} = \frac{x(x-1)}{2}$$

$$l_2(x) = \frac{x-x_1}{x_2-x_1} \times \frac{x-x_3}{x_2-x_3} = \frac{x+1}{0+1} \times \frac{x-1}{0-1} = \frac{(x+1)(x-1)}{-1} = -(x+1)(x-1)$$

$$l_3(x) = \frac{x-x_1}{x_3-x_1} \times \frac{x-x_2}{x_3-x_2} = \frac{x+1}{1+1} \times \frac{x-0}{1-0} = \frac{x(x+1)}{2}$$

$$P_2 = y_1\left(\frac{x(x-1)}{2}\right) + y_2\left(-(x+1)(x-1)\right) + y_3\left(\frac{x(x+1)}{2}\right)$$

$$= \overset{3}{\cancel{6}}\left(\frac{x^2-x}{\cancel{2}}\right) + 2\left(-(x+1)(x-1)\right) + \overset{2}{\cancel{4}}\left(\frac{x^2+x}{\cancel{2}}\right)$$

$$= 3(x^2-x) - 2(x^2-1) + 2(x^2+x)$$

$$= 3x^2 - 3x - \cancel{2x^2} + 2 + \cancel{2x^2} + 2x$$

$$= 3x^2 - x + 2$$

$\therefore P_2 = 3x^2 - x + 2$

$\therefore$ Lagrange polynomial to interpolate through the points $(x,y) = (-1,6), (0,2)$ and $(1,4)$ is $3x^2 - x + 2$

2)

a) Given points $(x,y) = (-1,6), (0,2)$ and $(1,4)$

Find Newton polynomial to interpolate through the given points

We know that, For n data points, the interpolant is of degree $n-1$ polynomial

$$y(x) = P_{n-1}(x) = a_1 + (x-x_1)a_2 + (x-x_1)(x-x_2)a_3 + \dots + (x-x_1)(x-x_2)\cdots(x-x_{n-1})a_n$$

Here, given 3 points

$$P_2(x) = a_1 + (x-x_1)a_2 + (x-x_1)(x-x_2)a_3$$

$$y_1 = a_1$$

$$y_2 = a_1 + (x_2 - x_1)a_2$$

$$y_3 = a_1 + (x_3 - x_1)a_2 + (x_3 - x_1)(x_3 - x_2)a_3$$

$$a_1 = y_1 = 6 \quad \Rightarrow a_1 = 6$$

$$y_2 = 2 = 6 + (0+1)a_2$$

$$2 = 6 + a_2$$

$$\Rightarrow a_2 = -4$$

$$y_3 = 4 = 6 + (1+1)(-4) + (1+1)(1-0)a_3$$

$$4 = 6 - 8 + 2a_3$$

$$4 = -2 + 2a_3$$

$$2 = -1 + a_3$$

$$\Rightarrow a_3 = 3$$

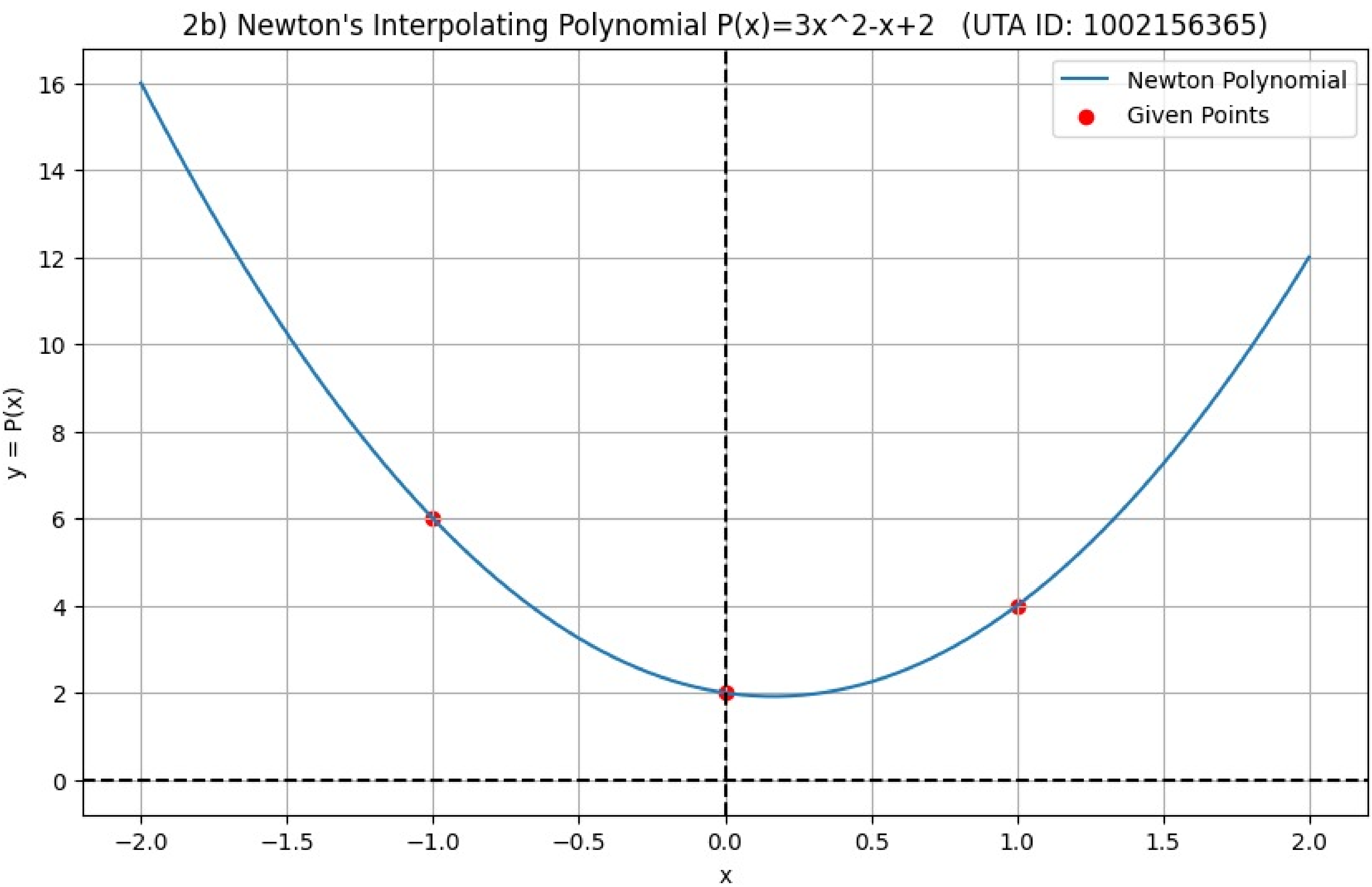
$$\therefore P_2(x) = 6 + (x+1)(-4) + (x+1)(x-0)(3)$$

$$= 6 - 4x - 4 + 3x^2 + 3x$$

$$P_2(x) = 3x^2 - x + 2$$

$\therefore$ Newton polynomial to interpolate through points $(x,y) = (-1,6), (0,2)$ and $(1,4)$ is

$$3x^2 - x + 2$$


2b) Newton's Interpolating Polynomial P(x)=3x^2-x+2   (UTA ID: 1002156365)
Newton Polynomial
Given Points
y = P(x)
x
16
14
12
10
8
6
4
2
0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

3)
a) Given points $(x,y) = (-1,6), (0,2)$ and $(1,4)$
Find natural piecewise cubic splines needed to interpolate through the given points

we know that
Cubic spline equation is:

$$S_i(x) = a_i + (x-x_i)b_i + (x-x_i)^2 c_i + (x-x_i)^3 d_i$$

we have to calculate 2 such splines for 3 points

$$S_0(x) = a_0 + (x-x_0)b_0 + (x-x_0)^2 c_0 + (x-x_0)^3 d_0$$

$$S_1(x) = a_1 + b_1(x-x_1) + (x-x_1)^2 c_1 + (x-x_1)^3 d_1$$

S_0 passes through $(-1,6)$ and $(0,2)$

S_1 passes through $(0,2)$ and $(1,4)$

Let's apply 4 conditions

1) Interpolation condition:-

$$S_i(x_i) = y_i \quad \text{and} \quad S_i(x_{i+1}) = y_{i+1}$$

$$S_0(x_0) = y_0$$

$$S_0(-1) = 6$$

$$a_0 + (\cancel{-1}\cancel{+1})b_0 + (\cancel{-1}+\cancel{1})^2 c_0 + (\cancel{-1}+\cancel{1})^3 d_0 = 6$$

$$\boxed{a_0 = 6}$$

$$S_0(x_1) = y_1$$

$$S_0(0) = y_1$$

$$S_0(0) = 2$$

$$a_0 + (0+1)b_0 + (0+1)^2 c_0 + (0+1)^3 d_0 = 2$$

$$6 + b_0 + c_0 + d_0 = 2$$

$$\boxed{b_0 + c_0 + d_0 = -4}$$

$S_1(x_1) = y_1$

$S_1(0) = 2$

$a_1 + (0-0)b_1 + (0-0)^2 c_1 + (0-0)^3 d_1 = 2$

$\boxed{a_1 = 2}$

$S_1(x_2) = y_2$

$S_1(1) = 4$

$a_1 + (1-0)b_1 + (1-0)^2 c_1 + (1-0)^3 d_1 = 4$

$2 + b_1 + c_1 + d_1 = 4$

$\boxed{b_1 + c_1 + d_1 = 2}$

2) Continuity of first derivative

$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})$

$S_0'(x_1) = S_1'(x_1)$

$S_i'(x) = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2$

$S_0'(x_1) = b_0 + 2c_0(x - x_0) + 3d_0(x - x_0)^2$

$= b_0 + 2c_0(0+1) + 3d_0(0+1)^2$

$= b_0 + 2c_0 + 3d_0$

$S_1'(x_1) = b_1 + 2c_1(x - x_1) + 3d_1(x - x_1)^2$

$= b_1 + 2c_1(0-0) + 3d_1(0-0)^2$

$= b_1$

$\Rightarrow \boxed{b_0 + 2c_0 + 3d_0 = b_1}$

3) Continuity of second derivative

$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})$

$S_i''(x) = 2c_i + 6d_i(x - x_i)$

$S_0''(0) = 2c_0 + 6d_0(0+1) = 2c_0 + 6d_0$

$S_1''(0) = 2c_1 + 6d_1(0-0) = 2c_1$

$\Rightarrow \boxed{2c_0 + 6d_0 = 2c_1}$

4) Boundary Conditions

Given Natural Spline

So, $S''(x_0) = S''(x_n) = 0$

$S_0''(x_0) = S_1''(x_2) = 0$

$S_0''(-1) = 2c_0 + 6d_0(-1+1) = 2c_0$

$2c_0 = 0$

$\boxed{c_0 = 0}$

$S_1''(1) = 2c_1 + 6d_1(1-0) = 2c_1 + 6d_1$

$\boxed{2c_1 + 6d_1 = 0}$

$\Rightarrow a_0 = 6,\ c_0 = 0,\ a_1 = 2$

$b_0 + c_0 + d_0 = -4 \Rightarrow b_0 + d_0 = -4 \rightarrow ①$

$b_1 + c_1 + d_1 = 2 \rightarrow ②$

$b_0 + 2c_0 + 3d_0 = b_1 \Rightarrow b_0 + 3d_0 - b_1 = 0 \rightarrow ③$

$2c_0 + 6d_0 = 2c_1 \Rightarrow 6d_0 - 2c_1 = 0 \rightarrow ④$

$2c_1 + 6d_1 = 0 \rightarrow ⑤$

In $Ax = b$ form

$$\begin{array}{ccccc} b_0 & d_0 & b_1 & c_1 & d_1 \end{array}$$
$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 3 & -1 & 0 & 0 \\ 0 & 6 & 0 & -2 & 0 \\ 0 & 0 & 0 & 2 & 6 \end{bmatrix} \begin{bmatrix} b_0 \\ d_0 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

from ④ $6d_0 - 2c_1 = 0 \Rightarrow c_1 = 3d_0$

from ⑤ $2c_1 + 6d_1 = 0 \Rightarrow 2(3d_0) + 6d_1 = 0 \Rightarrow d_1 = -d_0$

from ①, ③ $b_0 + 3d_0 - b_1 = 0 \Rightarrow b_0 + d_0 + 2d_0 - b_1 = 0$

$\Rightarrow -4 + 2d_0 = b_1 \Rightarrow b_1 = 2d_0 - 4$

from ② $b_1 + c_1 + d_1 = 2 \Rightarrow 2d_0 - 4 + 3d_0 - d_0 = 2$

$4d_0 - 4 = 2$

$d_0 = 6/4 = 3/2$

$d_1 = -3/2$

① $b_0 + d_0 = -4 \Rightarrow b_0 = -4 - 3/2 = -11/2$

④ $6d_0 - 2c_1 = 0 \Rightarrow c_1 = 3 \cdot 3/2 = 9/2$

② $b_1 = 2 - c_1 - d_1 = 2 - 9/2 + 3/2 = -1$

$\Rightarrow$

$a_0 = 6$	$a_1 = 2$
$b_0 = -11/2$	$b_1 = -1$
$c_0 = 0$	$c_1 = 9/2$
$d_0 = 3/2$	$d_1 = -3/2$

$$S_0(x) = 6 - 11/2\,(x+1) + 0(x+1)^2 + 3/2\,(x+1)^3$$
$$= 6 - 11/2\,x - 11/2 + 3/2\,x^3 + 9/2\,x^2 + 9/2\,x + 3/2$$
$$S_0(x) = 3/2\,x^3 + 9/2\,x^2 - x + 2$$

$$S_1(x) = 2 - 1(x-0) + 9/2\,(x-0)^2 - 3/2\,(x-0)^3$$
$$= 2 - x + 9/2\,x^2 - 3/2\,x^3$$
$$S_1(x) = -3/2\,x^3 + 9/2\,x^2 - x + 2$$

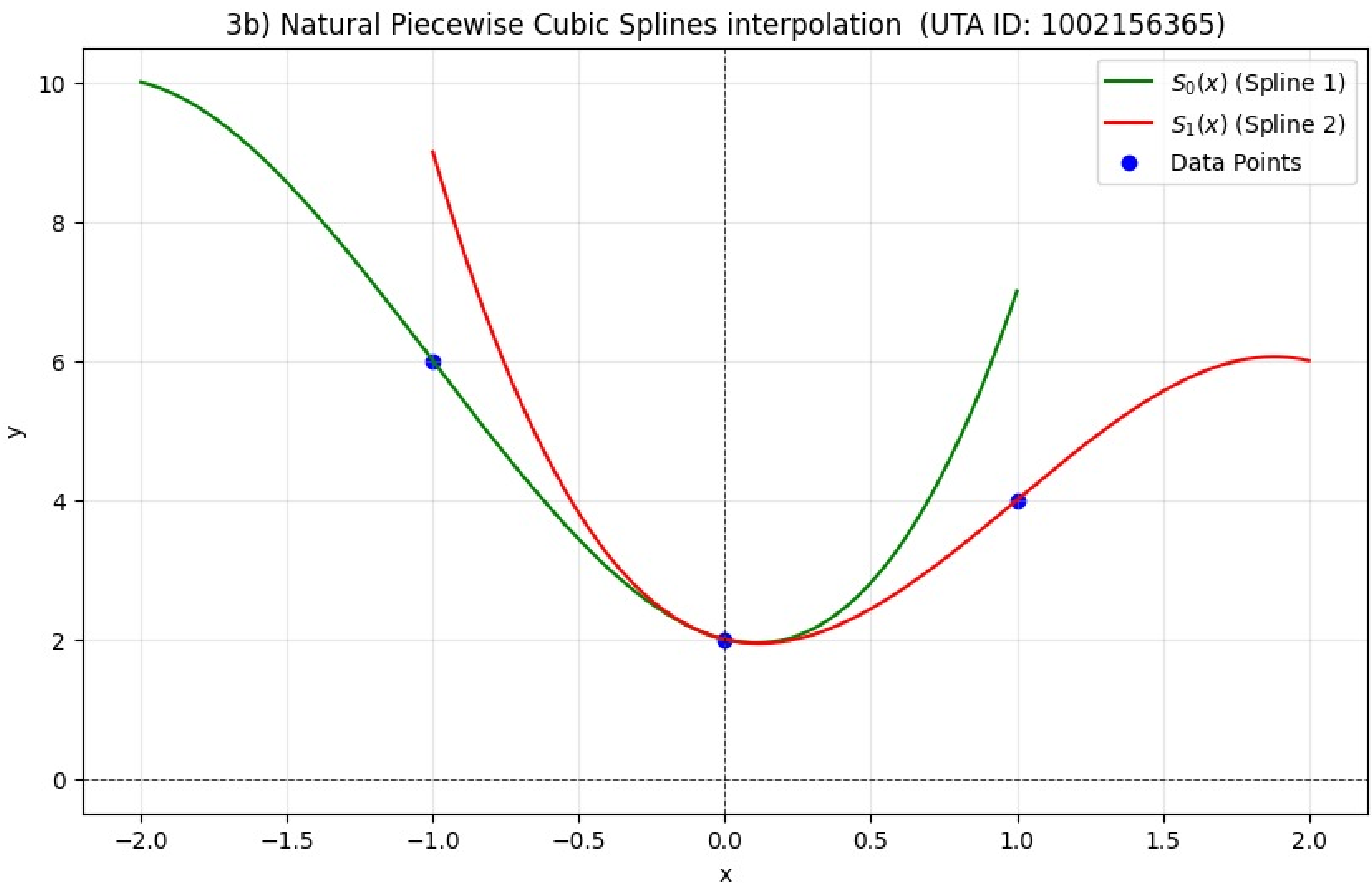
$\therefore$ $\boxed{S_0(x) = 3/2\,x^3 + 9/2\,x^2 - x + 2}$ $\boxed{S_1(x) = -3/2\,x^3 + 9/2\,x^2 - x + 2}$

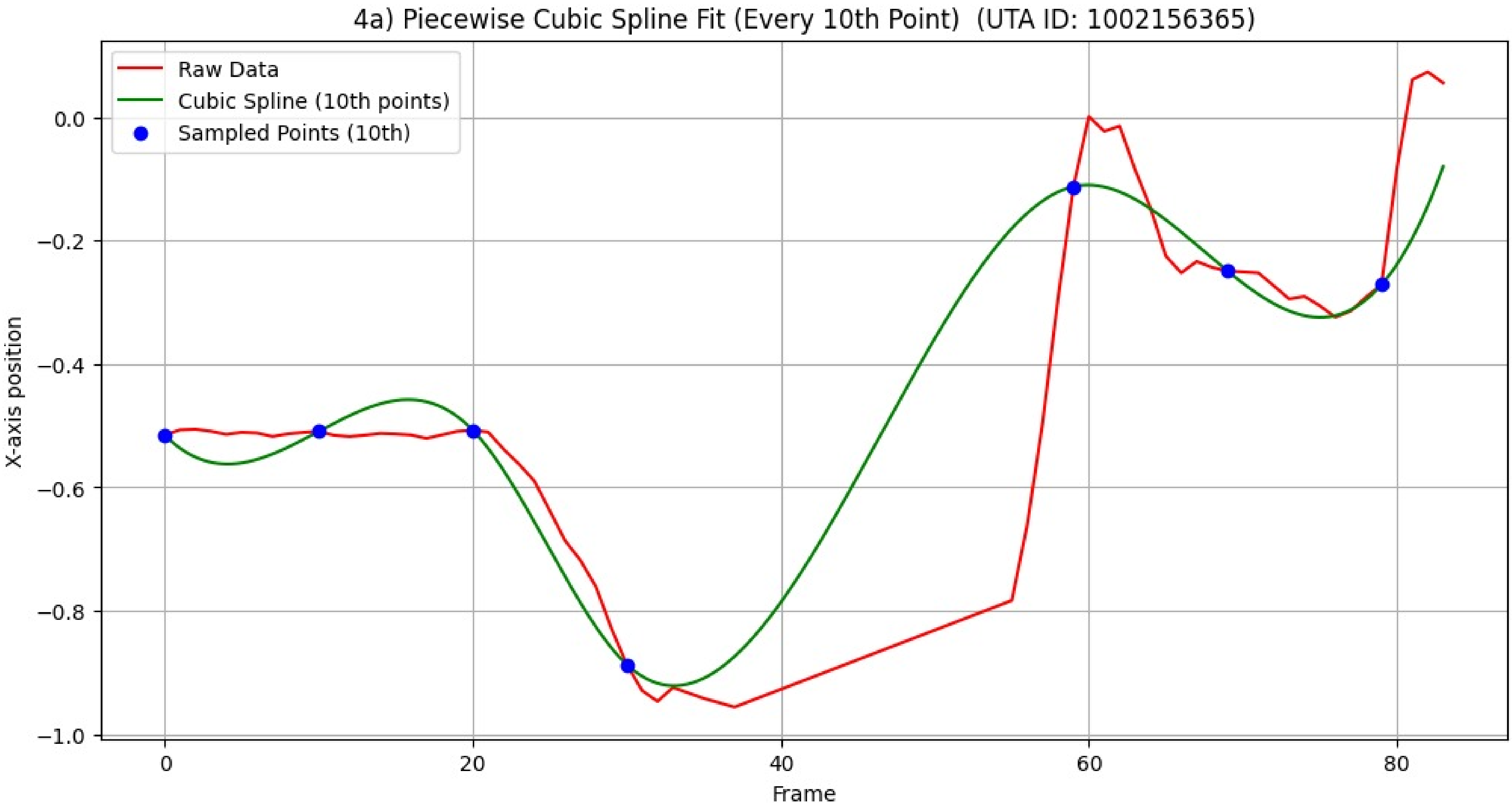
3.c) 1) Newton interpolation preferable, because of its simplicity and direct fitting properties, when the data is smooth and evenly distributed.

2) piecewise Cubic splines are preferable as they provide robust interpolation when the data is noisy and has irregular intervals.

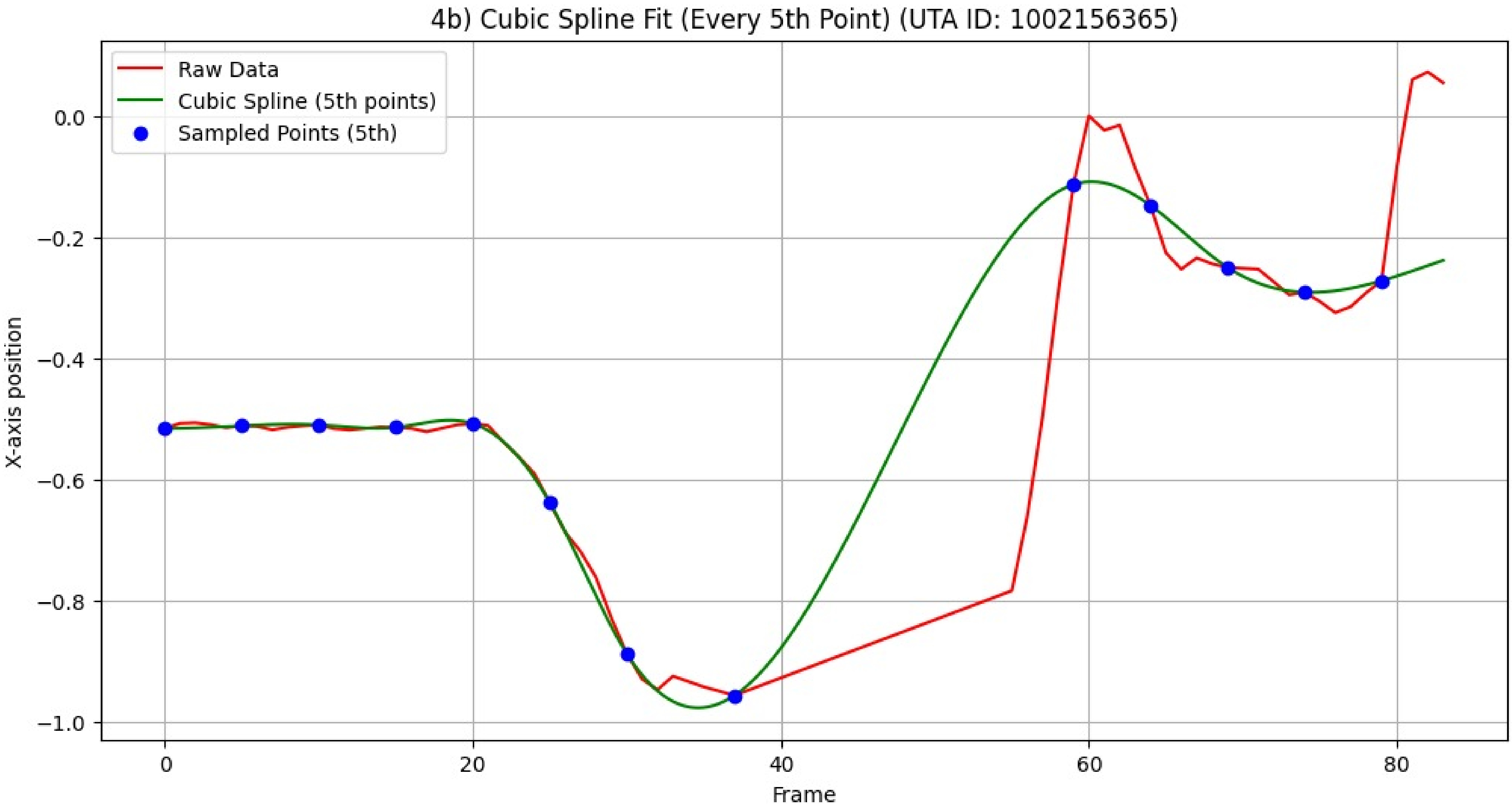
In The given problems given data is smooth and has no noise & missing values.

$\therefore$ we would prefer Newton polynomial interpolation


3b) Natural Piecewise Cubic Splines interpolation (UTA ID: 1002156365)
$S_0(x)$ (Spline 1)
$S_1(x)$ (Spline 2)
Data Points
y
x
10
8
6
4
2
0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0



4a) Piecewise Cubic Spline Fit (Every 10th Point) (UTA ID: 1002156365)
Raw Data
Cubic Spline (10th points)
Sampled Points (10th)
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
X-axis position
0
20
40
60
80
Frame


4b) Cubic Spline Fit (Every 5th Point) (UTA ID: 1002156365)
Raw Data
Cubic Spline (5th points)
Sampled Points (5th)
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
X-axis position
0
20
40
60
80
Frame