

Binary Search Recursion:
calling the same function again inside the method definition.

Ex: $(10, 20, 30, 40, 50, 60)$ $x = 50$

binarySearch (arr, 0, 5):

$$\text{mid} = (i+j)/2 = 2 \text{ (lower bound)}$$

while $i \leq j$:

if $\text{arr}[\text{mid}] == x$; $\leftarrow 50 \neq 50 \times$
return mid;

elif $\text{arr}[\text{mid}] < x$:

$50 < 50 : \checkmark$

binarySearch (arr, 3, 5) : right move
 $\text{mid} = 4$

$\text{arr}[\text{mid}] == x = \checkmark$

$\text{arr}[\text{mid}] > x$:

$\leftarrow \text{else}$

binarySearch (arr, 0, 1)
left move.

* for mid, we can use

$$\text{mid} = i + (j-i)/2 \quad \text{mid} = i + j/2$$

most preferable.

code Binary Search:

def binarySearch (arr, x, left, right):

if left > right:

return -1

$$\text{mid} = \lfloor \text{left} + (\text{right}-\text{left})/2 \rfloor$$

if $\text{arr}[\text{mid}] == x$:

return mid

elif $\text{arr}[\text{mid}] > x$:

return binarySearch (arr, x, mid+1, right)

else

return binarySearch (arr, x, left, mid-1)

arr = [10, 20, 30, 40, 50]

$T(\frac{n}{2})$

left = 0

$\text{right} = \text{len}(\text{arr}) - 1$
 $x = \text{int}(\text{input}(\text{"enter no."}))$, result = binary search(arr , x , left , right)
 if result != -1:
 print(result)
 else:
 print("not found")

Recurrence Relation:

$$T(n) = T\left(\frac{n}{2}\right) + c \quad \begin{array}{l} \text{Substitution} \\ \text{master's theorem} \end{array}$$

(or)

$$T(n) \neq 2T\left(\frac{n}{2}\right) + c \quad [\text{wrong}]$$

① master's Theorem;

$$\begin{array}{ll} a=1 & k=0 \\ b=2 & p=0 \end{array} \quad \log_b^a = \log_2^1 = 0.$$

$$\log_b^a = k. \quad [\text{case-2}]$$

$$\therefore p > -1 \Rightarrow \Theta(n^k \log^{p+1} n) \\ \underline{\Theta(n \log n)}$$

② By Substitution,

$$T(n) = T\left(\frac{n}{2}\right) + c$$

$$= T\left(\frac{n}{2^2}\right) + c + c$$

$$= T\left(\frac{n}{2^3}\right) + 2c$$

$$= T\left(\frac{n}{2^3}\right) + 3c$$

$\underbrace{\quad}_{k \text{ times}}$

$$= T\left(\frac{n}{2^K}\right) + kc$$

$$\frac{n}{2^K} = 1$$

$$n = 2^K$$

$$= T\left(\frac{n}{2^{\log_2 n}}\right) + c \cdot k \quad \boxed{\log_2 n = k}$$

$$\underline{\underline{\Theta(\log_2 n)}}$$

Interview Questions on Binary Search:

- ① Face book (Technical Round): $n \rightarrow$ size of an array.
 ↳ Not sorted manner.

(-23 40 55 1 2 57 -89 ∞ ∞ ∞ ∞ ∞)
 0 1 2 3 4 [5] 6 7 8 9 10 11

Index of first infinite.
 ↓

output = 7.

- ② since, this is not sorted, \Rightarrow Binary Search is not applicable.

Going for Linear Search.

for $i=0$ to $n-1$
 if $\text{arr}[i] == \text{'inf'}$ } $O(n)$
 return i

- ② modified Binary Search:

$$\text{mid} = 0 + (11-0)/2 = 0+5=5$$

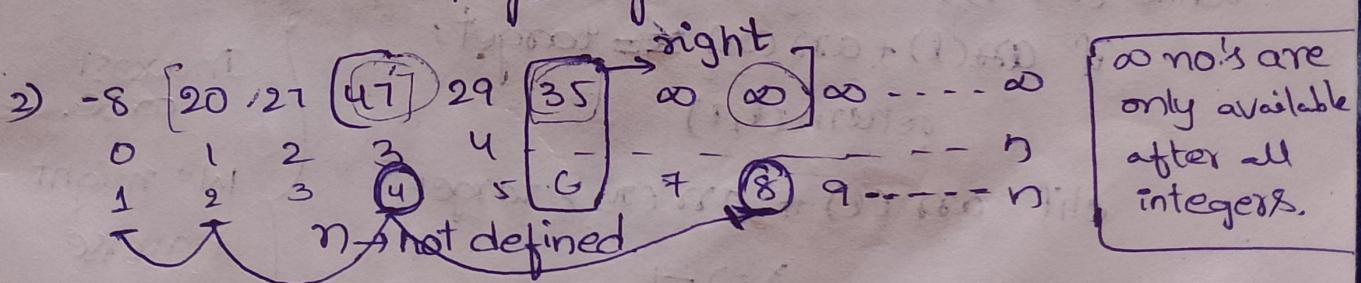
if $\text{arr}(\text{mid}) == \text{int}$:

$\text{left} = \text{mid}+1;$ \rightarrow move to right side

$\text{arr}(\text{mid}) <= \text{'inf'}$

$\text{right} = \text{mid}-1$ \rightarrow Search Space

Note: 1. Don't judge by looking whether the array is physically sorted or not.



Sol: Do exponential increase \rightarrow range of numbers.

↑
 Binary Search.

[47 29 35 ∞ ∞]
 4 5 6 7 8

mid = 6, move right
 $\text{mid} = \frac{7+8}{2} = \frac{15}{2} = 7$

mid = 7, move right

check
 if $\text{arr}[i-1] == \text{int}$
 return $\text{arr}[i]$

③. Array is sorted.

$$20, 40, 60, 80, 90, 120, 240 \quad | 130 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

$a+b = 210 \rightarrow (a, b)$.

Approach-1: for $i=0$ to $n-1$:

for $j=0$ to $n-1$:

$\text{arr}[i] + \text{arr}[j] == \text{target}$

return i, j

$O(n^2)$

Brute force

Approach-2: one loop for fixing key value.

say key = 20.

$210 - 20 = 190$

and search for this 190.

using Linear Search

$\downarrow n$

Approach-3:

one loop for fixing key value.

Search the remaining

Target Value Using

Binary Search

$\Rightarrow O(n \log n)$

Approach-4: optimized approach is Greedy (Two-pointers).

Two-Sum.

20, 40, 60, 80, 90, 120, 240

\uparrow
①

while ($l <= r$):

if $\text{arr}(l) + \text{arr}(r) \leq \text{target}$:

return (l, r)

elif $\text{arr}(l) + \text{arr}(r) > \text{target}$:

$r = r - 1$

else:

$l = l + 1$

$\text{arr}(l) + \text{arr}(r) < \text{sum-value}$

In worst case.

We might traverse whole array i.e. $O(n)$.

optimized