Исследование устотнивости и инпроженнации схем.

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$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0$$

$$u(x, 0) = V(x)$$

Необходино иссяд, уст, и апучоко. след. скан

Crema:

$$\frac{u_i^{\kappa+1} - u_i^{\kappa}}{T} + a \frac{u_{i+1}^{\kappa} - u_i^{\kappa}}{h} = 0$$

Антроксинация:

Vargue ulbezeg-ocmanon om nogomanoben mornore penema; $V = \frac{U(x; t^{k}+7) - U(x; t^{k})}{T} + \alpha \frac{U(x; t^{k}, t^{k}) - U(x; t^{k})}{h}$

Toznonemen l'perg Mensopa:

$$u(x_i,t^k) + \tau \cdot u_t(x_i,t^k) + \frac{\tau^2}{2} \cdot u_{tt}(x_i,t^k) - u(x_i,t^k) + \frac{\tau^2}{2} \cdot u_{tt}(x_i,t^k) + \frac{\tau^2}{2} \cdot u_{tt}($$

$$\alpha \frac{U(x_i, t^k) + h \cdot u_x(x_i, t^k) + \frac{h^2}{2} \cdot u_{xx}(x_i, t^k) - u(x_i, t^k)}{h} = n$$

$$\frac{\partial u(x_i,t^k)}{\partial t} + \frac{\sigma}{2} \cdot \frac{\partial^2 u(x_i,t^k)}{\partial t^2} + a \cdot \frac{\partial u(x_i,t^k)}{\partial x} + \frac{ah}{2} \cdot \frac{\partial^2 u(x_i,t^k)}{\partial x^2} = n$$

$$\frac{\partial u(x_i,t^k)}{\partial t} + \alpha \frac{u(x_i,t^k)}{\partial x} = 0 \implies r = O(\frac{\sqrt{2}}{2} + \frac{\alpha h}{2}) = O(h+\tau) \rightarrow 0$$

Genoiruboems; Trogomabien pemenne b buge 1k, einx $\frac{\lambda^{k+1}, e^{i\omega x_i} - \lambda^k, e^{i\omega x_i}}{\tau} + a, \frac{\lambda^k, e^{i\omega(x_i + h)} - \lambda^k, e^{i\omega x_i}}{h} = 0$ $\frac{\lambda-1}{T} + \frac{a}{h} \cdot (e^{i\alpha h} - 1) = 0 \Rightarrow \lambda = 1 - \frac{aT}{h} \cdot (e^{i\alpha h} - 1)$ Hangein mogyro λ , cruma, mo $\frac{\alpha J}{h} = \delta$, cv. h = 0[N]= 1+5- 5cos0)2+ 52 sin20 >> \Rightarrow $1+2\delta^2(1-\cos\theta)+2\delta(1-\cos\theta)$ Die you reodxoguno bornoun. yer. 121=1 (1-cos0). (0+1) €0 Reprobenento bonomiema 40. Uneen: 501-1;03 U; - U; + a u; - u; - = 0 Anyrekennayur: Полуши певерку-остаток от подстоловки тогного решения; $r = \frac{\mathcal{U}(x_i, t^k + J) - \mathcal{U}(x_i, t^k)}{J} + a \frac{\mathcal{U}(x_i, t^k) - \mathcal{U}(x_i - h, t^k)}{h}$ Jagronemen lyng Meinopa; $u(x_i,t^k) + \overline{\tau} \cdot ut(x_i;t^k) + \frac{\overline{\tau}^2}{2} \cdot utt(x_i,t^k) - u(x_i,t^k) + \overline{\tau}^2$

 $\alpha = \frac{u(x_i,t^k) + h \cdot u_{\mathcal{R}}(x_i,t^k) - \frac{h^2}{2} \cdot u_{\mathcal{R}}(x_i,t^k) - u(x_i,t^k)}{h} = n$

$$\frac{\partial u(x_i,t^k)}{\partial t} + \frac{\sigma}{2} \cdot \frac{\partial^2 u(x_i,t^k)}{\partial t^2} + \alpha \cdot \frac{\partial u(x_i,t^k)}{\partial x} - \frac{\alpha h}{2} \cdot \frac{\partial^2 u(x_i,t^k)}{\partial x^2} = 1$$

$$\frac{\partial u(x_i,t^k)}{\partial t} + a \cdot \frac{u(x_i,t^k)}{\partial x} = 0$$

$$r = O(\frac{\sqrt{2} - \frac{ah}{2}}) = O(h+7) \rightarrow 0$$

Gemournbooms:

Mogematium permenne b bruge
$$\lambda^{k}$$
, $e^{i\omega x}$

$$\frac{\lambda^{k+1}e^{i\omega x_{i}} - \lambda^{k}e^{i\omega x_{i}}}{\lambda} + \alpha \frac{\lambda^{k}, e^{i\omega x_{i}} - \lambda^{k}, e^{i\omega}(x_{i} - h)}{h} = 0$$

$$\frac{\lambda-1}{\sqrt{2}} + \frac{\alpha}{h} \cdot (-e^{i\omega h} + 1) = 0$$

$$\lambda = 1 - \frac{\alpha \sqrt{2}}{h} (e^{-i\omega h} + 1)$$

Aparoumo creme 1:

Crema:

$$\frac{Q}{i-1} = 0$$

$$\frac{U_i^{K+1} - U_i^{K}}{\sqrt{2}} + \alpha \frac{U_i^{K+1} - U_i^{K+1}}{h} = 0$$

Anproxumayus:

Franzisch pelazieg-ocmaniek om rogemenobien mornoro penienie: $P = \frac{u(x_i, t^k + J) - u(x_i, t^k)}{J} + a \frac{u(x_i, t^k + J) - u(x_i - h, t^k + J)}{h}$

Jaguraneene 6 pug Themopa!

$$\frac{l((x_i, t^k + \sigma) + \sigma \cdot ht(x_i, t^k + \sigma) - \frac{J^2}{2} \cdot utt(x_i, t^k + \sigma) - u(x_i, t^k + \sigma)}{\tau} + \frac{l((x_i, t^k + \sigma) + h \cdot ux(x_i, t^k + \sigma) - \frac{h^2}{2} \cdot uxx(x_i, t^k + \sigma) - u(x_i, t^k + \sigma)}{\tau} + \frac{h}{2} \cdot uxx(x_i, t^k + \sigma) - u(x_i, t^k + \sigma)}{\tau} + \frac{h}{2} \cdot uxx(x_i, t^k + \sigma) - u(x_i, t^k + \sigma)}{\tau} + \frac{h}{2} \cdot uxx(x_i, t^k + \sigma) - u(x_i, t^k + \sigma)}{\tau} - \frac{h}{2} \cdot \frac{h^2 ux_i, t^k}{\tau}$$

$$\frac{h}{h} + \frac{h}{h} \cdot \frac$$

$$\frac{u_i^{k+1}-u_i^k}{T}+\alpha\frac{u_{i+1}^{k+1}-u_i^{k+1}}{h}=0$$

Anyroxamazua:

Tayrun nebezky-ocmamok om nogemanobku mornoro pemerme:
$$r = \frac{U(x; t^{k} + 5) - U(x; t^{k})}{J} + a \frac{-U(x; t^{k} + 7) + U(x; t^{k}, t^{k} + 5)}{h}$$

Toznonem l pag Themopa:

$$\frac{u(x_i,t^k+\sigma)+\sigma\cdot ut(x_i,t^k+\sigma)-\frac{\tau^2}{2}\cdot utt(x_i,t^k+\sigma)-u(x_i,t^k+\sigma)}{\tau}+$$

+ a
$$\frac{u(x; t^{k}+J) + h \cdot ux(x; t^{k}+J) + \frac{h^{2}}{2} \cdot uxx(x; t^{k}+J) - u(x; t^{k}+J)}{h} = h$$

$$\frac{\partial u(x;t^{k}+5)}{\partial t} = \frac{\tau}{2} \cdot \frac{\partial^{2} u(x;t^{k}+5)}{\partial t^{2}} + \alpha \cdot \frac{\partial^{2} u(x;t^{k}+5)}{\partial x} = \frac{\partial^{2} u(x;t^{k}+5)}{\partial x^{2}} = \frac{\partial^{2} u(x;t^{k}+5)}{\partial x^{2}}$$

$$\frac{\partial u(x;,t^{x}+y)}{\partial t} + a \cdot \frac{u(x;,t^{x}+y)}{\partial x} = 0$$

Genowubocno:

Megemakun pemenne bluge 2k. eier &

$$\frac{1-\lambda^{-1}}{5} + \frac{\alpha}{h} \cdot (e^{iesh} - 1) = 0$$

$$S = \left(1 - \frac{as}{h}, \left(-e^{i\omega h} + 1\right)\right)$$

$$J = \frac{1}{1 + \sigma(1 - e^{\frac{1}{2}}\theta)}, \text{ age } \delta = -\frac{a\delta}{h} \text{ u } \theta = a_0 h$$

$$-\frac{a\delta}{h} \epsilon \left(-c_0; -1\right] \text{ u } [c_0; +c_0]$$

$$\frac{a\delta}{h} \epsilon \left(-c_0; 0\right) \text{ u } [c_0; +c_0]$$

Carra!

$$\frac{U_{i}^{K+1} - U_{i}^{K}}{T} + a \frac{U_{i+1}^{K} - U_{i-1}^{K}}{2h} = 0$$

Annyokamaisus:

Frangum nebogny-ocmanok om nogemenoken momero penenna: $V = \frac{U(x_i, t^k + J) - U(x_i, t^k)}{J} + a \frac{U(x_i + h, t^k) - U(x_i - h, t^k)}{Jh}$

Tagnomen & pag Tuesturopa

$$u(x;,t^{k}) + \tau \cdot ut(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) - u(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) - u(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) - u(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) - u(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) - u(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,t^{k}) + \frac{5^{2}}{2} \cdot utt(x;,$$

+ a
$$\frac{u(x_i,t^k) + h \cdot u_{\infty}(x_i,t^k) + \frac{h^2}{2} \cdot u_{\infty}(x_i,t^k) + \frac{h^3}{6} u_{\infty}(x_i,t^k) - u(x_i,t^k)}{2h}$$

- h (x:, tx) + h? · l(xx(x), tx) - h3 (xxx (x; tx))

$$\frac{\partial u(x;t^k)}{\partial t} + \frac{\tau}{2} \frac{\partial^2 u(x;t^k)}{\partial t^2} + \alpha \cdot \frac{\partial u(x;t^k)}{\partial x} + \frac{\alpha h^2}{6} \cdot \frac{\partial^3 u(x;t^k)}{\partial x^3} = h$$

$$\frac{\partial u(x;t^k)}{\partial t} + \alpha \cdot \frac{u(x;t^k)}{\partial x} = 0$$

$$r = O\left(\frac{\tau}{2} + \frac{\alpha h^2}{6}\right) = O(h^2 + 5) \rightarrow 0$$

Jonnature β διης
$$x^{k}$$
, $e^{i\alpha x}$
 $\frac{1}{1}$
 $\frac{1}$
 $\frac{1}{1}$
 $\frac{1}{1}$
 $\frac{1}{1$

$$\frac{\partial u(x;t^k)}{\partial t} + \frac{\pi}{2} \cdot \frac{\partial^2 u(x;t^k)}{\partial t^2} + \alpha \frac{\partial u(x;t^k)}{\partial x} \cdot \frac{h^2}{2^7} \cdot \frac{\partial^2 u(x;t^k)}{\partial x^2} + \frac{(h^2 + 3u(x;t^k))}{\partial x^3} + \frac{\partial^2 u(x;t^k)}{\partial x^3} = 0$$

$$N = 0 \cdot \left(\frac{\sqrt{2} + \alpha h^2}{6} - \frac{h^2}{2^7} \right) = 0 \cdot \left(\frac{h^2 + \tau + h^2}{2^7} \right) = 0$$

Genomulocm;

Fragamabius perneme l'buge xk, e'es x

 $\frac{\int_{0}^{k+1} e^{i\alpha x} x^{2} - \frac{\int_{0}^{k} e^{i\alpha x} x^{2} + \int_{0}^{k} e^{i\alpha x} (x^{2} + h)}{2} + \frac{\int_{0}^{k} e^{i\alpha x} x^{2} + \int_{0}^{k} e^{i\alpha x} (x^{2} + h)}{2h} = \frac{\int_{0}^{k} e^{i\alpha x} (x^{2} + h)}{2h} =$

$$\frac{2\lambda - e^{-i\alpha h} - e^{-i\alpha h}}{\tau} + \frac{a}{h} \cdot (e^{-i\alpha h} - e^{-i\alpha h}) = 0 \Rightarrow$$

 $2\lambda = -i\frac{a\tau}{h} + 2\cos(exh) \implies \lambda = \cos\theta - i\delta\sin\theta$ $2ge \delta = \frac{a\tau}{2h}, \theta = \omega. \lambda$

$$\cos^{2}\theta + \sin^{2}\theta + (\delta^{2}-1) \cdot \sin^{2}\theta \leq 1$$

$$(\delta^{2}-1) \sin^{2}\theta \leq 0$$

$$-\frac{aT}{2h} \in \Gamma - 1; 1 \Rightarrow \frac{aS}{2h} \in \Gamma - 1; 1$$