# Матан 3 сем

## ✓ - точно будет на экзамене

## №3952 - нет

## №3955

№3955

$$\iint \sin\sqrt{x^2+y^2}\,dx\,dy$$

$$\pi^2 \le x^2+y^2 \le 4\pi^2$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$x^2+y^2 = r^2\cos^2\varphi + r^2\sin^2\varphi = r^2$

$\pi^2 \le r^2 \le 4\pi^2$

$r^2 \ge \pi^2 \qquad r^2 \le 4\pi^2$

$r \ge \pi \qquad r \le 4\pi$

$$\underbrace{\int_0^{2\pi} d\varphi \int_\pi^{2\pi} r\sin r\,dr}\ (=)$$

$r = u \qquad du = dr$

$\sin r\,dr = dV \qquad V = -\cos r$

$-\cos r \cdot r + \int \cos r\,dr$

$-r\cos r + \sin r$

$$-r\cos r + \sin r\,\Big|_\pi^{2\pi} =$$

$= -2\pi\cos 2\pi + \sin 2\pi + \pi\cos\pi - \sin\pi =$

$= -2\pi - \pi = -3\pi$

$$(=)\ \int_0^{2\pi} -3\pi\,d\varphi = -3\pi\int_0^{2\pi} d\varphi = \qquad \varphi\Big|_0^{2\pi} = 2\pi$$

$$= -3\pi \cdot 2\pi = \underline{-6\pi^2}$$

# №3964

№3964

$$\iint\limits_{\Omega} f(xy)\,dx\,dy \qquad xy=1 \qquad xy=2 \qquad y=x \qquad y=4x$$

$$u = xy \qquad V = \frac{y}{x}$$

$xy = 1 \Rightarrow u = 1$

$xy = 2 \Rightarrow u = 2$

$\frac{y}{x} = 1 \Rightarrow V = 1$

$\frac{y}{x} = 4 \Rightarrow V = 4$

$1 \le u \le 2$

$1 \le V \le 4$

$$\frac{\partial(u,V)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} \end{vmatrix}^{-1} = \begin{vmatrix} \frac{\partial(xy)}{\partial x} & \frac{\partial(xy)}{\partial y} \\ \frac{\partial\left(\frac{y}{x}\right)}{\partial x} & \frac{\partial\left(\frac{y}{x}\right)}{\partial y} \end{vmatrix}^{-1} =$$

$$= \begin{vmatrix} y & x \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix}^{-1} = \left|\frac{y}{x} + \frac{y}{x}\right|^{-1} = \left|2\frac{y}{x}\right|^{-1} = |2V|^{-1} =$$

$$= \frac{1}{2V}$$

$$\iint\limits_{\Omega} f(xy)\,dx\,dy = \cancel{\ldots} = \cancel{\int_1^4 \ldots \int_1^2 \ldots}\,du$$

$$= \iint \frac{f(u)}{2V}\,du\,dV = \int_1^4 \frac{dV}{2V} \int_1^2 f(u) = \left(\frac{1}{2}\ln 4 - \frac{1}{2}\ln 1\right) \int_1^2 f(u) =$$

$$= \underline{\ln 2 \int_1^2 f(u)}$$

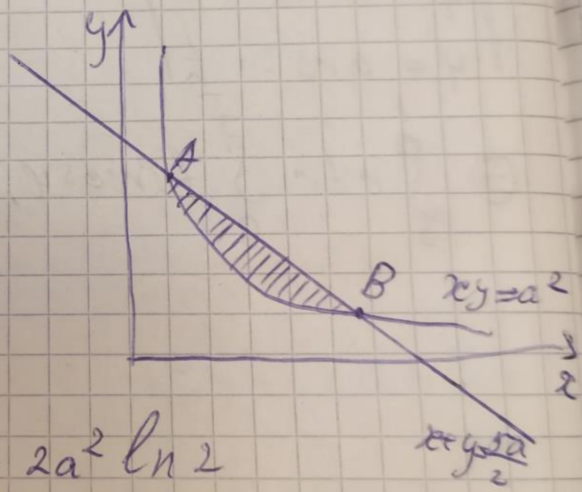
## №3984

№ 3984

$xy = a^2$

$x + y = \frac{5a}{2} \quad (a>0)$

$A\left(\frac{a}{2}, 2a\right), B\left(2a, \frac{a}{2}\right)$




$$S = \int_{\frac{a}{2}}^{2a} dx \int_{\frac{a^2}{x}}^{\frac{5a}{2}-x} dy = \frac{15a^2}{8} - 2a^2 \ln 2$$

## №3987 ✓

$\boxed{№ 3987}$

$x^2+y^2 = a\sqrt{2(x^2-y^2)}$

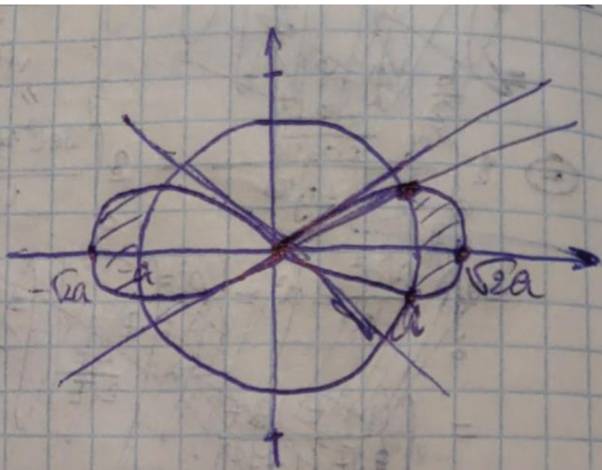
$x^2+y^2 \le a^2$

$r^4 = 2a^2 r^2 \cos 2\varphi$

$r^2 = 2a^2 \cos 2\varphi$

$r^2 \geq a^2 \Rightarrow r \geq a$

$r = \pm\sqrt{2}\, a \sqrt{\cos 2\varphi} = a\sqrt{2\cos 2\varphi}$




$$4\int_0^{\pi/6} d\varphi \int_a^{\sqrt{2a^2\cos 2\varphi}} r\,dr = 4\int_0^{\pi/6} \frac{r^2}{2}\Big|_a^{\sqrt{2a^2\cos 2\varphi}} d\varphi = 2\int_0^{\pi/6} (2a^2\cos 2\varphi - a^2)\,d\varphi =$$

$$= 2\int_0^{\pi/6} 2a^2 \cos^2\varphi\, d\varphi - 2\int_0 a^2\, d\varphi = a^2 2\sin 2\varphi\Big|_0^{\pi/6} - 2a^2\varphi\Big|_0^{\pi/6} =$$

$$= 2a^2\frac{\sqrt{3}}{2} - 2a^2\frac{\pi}{6} = \frac{2(3\sqrt{3}-\pi)a^2}{6} = \frac{a^2(3\sqrt{3}-\pi)}{3}$$

# №3996

№3996

$x+y=a \qquad x+y=b \qquad y=\alpha x \qquad y=\beta x$

$0<a<b \qquad 0<\alpha<\beta$

$x+y=u \qquad \frac{y}{x}=v$

$u=a \qquad v=\alpha$

$u=b \qquad v=\beta$

$a\le u\le b \qquad \alpha\le v\le\beta$

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}^{-1} = \begin{vmatrix} 1 & 1 \\ -\frac{y}{x^2} & \frac{1}{x} \end{vmatrix}^{-1} = \left(\frac{1}{x}+\frac{y}{x^2}\right)^{-1} = \left(\frac{y+x}{x^2}\right)^{-1} = \frac{x^2}{y+x} =$$

$$= \dots ? \dots = \frac{u}{(1+v)^2}$$

какие-то ш-ги

в антидемид.

$$S=\underbrace{\int_a^b u\,du}\;\underbrace{\int_\alpha^\beta \frac{dv}{(1+v)^2}}$$

$\left.\frac{u^2}{2}\right|_a^b = \qquad t=1+v$

$t'=1$

$=\frac{b^2}{2}-\frac{a^2}{2}= \qquad \int\frac{dv}{(1+v)^2}=\int\frac{dt}{t^2}=-\frac{1}{t}=-\frac{1}{1+v}$

$=\frac{b^2-a^2}{2}$

$$\left.-\frac{1}{1+v}\right|_\alpha^\beta = -\frac{1}{1+\beta}+\frac{1}{1+\alpha}$$

$$\frac{b^2-a^2}{2}\cdot\left(\frac{1}{1+\alpha}-\frac{1}{1+\beta}\right)=\frac{1}{2}\left(\frac{b^2-a^2}{1+\alpha}-\frac{b^2-a^2}{1+\beta}\right)=\frac{1}{2}\,\frac{(\beta-\alpha)\,(b^2-a^2)}{(1+\alpha)\,(1+\beta)}$$

**№4008**

5408

$x+y+z=a$ $\quad x^2+y^2=R^2$ $\quad x=0, \; y=0, \; z=0$

$a \geq R\sqrt{2}$

$y=\sqrt{R^2-x^2}$

$z=a-x-y$

$x=\sqrt{R^2-y^2} \Rightarrow x=\sqrt{R^2}=R$

$$\int_0^R dx \int_0^{\sqrt{R^2-x^2}} \underbrace{(a-x-y)\,dy}_{} =$$

$$= \int_0^R \quad ay - xy - \frac{y^2}{2}$$

$$(a-x)y - \frac{y^2}{2}$$

$$\|$$

$$(a-x)y - \frac{y^2}{2}\Big|_0^{\sqrt{R^2-x^2}} = (a-x)\sqrt{R^2-x^2} - \frac{R^2-x^2}{2}$$

$$= \int_0^R \left((a-x)\sqrt{R^2-x^2} - \frac{R^2-x^2}{2}\right)dx = \underbrace{\int_0^R a\sqrt{R^2-x^2}\,dx}_{1} - \underbrace{\int_0^R x\sqrt{R^2-x^2}}_{2} -$$

$$- \underbrace{\int \frac{R^2-x^2}{2}\,dx}_{3} =$$

1) $\int_0^R a\sqrt{R^2-x^2}\,dx = a\int_0^R \sqrt{R^2-x^2}\,dx = a\int_0^R \sqrt{R^2-R^2\sin^2 t}\cdot R\cos t\,dt$

$x=R\sin t \quad dx = x'dt$

$x' = r\cos t$

$$= a\int_0^R R\sqrt{1-\sin^2 t}\cdot R\cos t\,dt = a\int_0^R R\sqrt{\cos^2 t}\cdot R\cos t\,dt =$$

$$= a\int_0^R R^2\cos^2 t\,dt = aR^2\int_0^R \cos^2 t\,dt = aR^2\int_0^R \frac{1+\cos 2t}{2}\,dt =$$

$$= \frac{aR^2}{2}\int_0^R 1+\cos 2t\,dt = \frac{aR^2}{2}\left(\int_0^R dt + \int_0^R \cos 2t\,dt\right) =$$

$$= \cancel{\frac{aR^2}{2}\int_0 \left(t + \frac{\sin 2t}{2}\right) dt}$$

$\sin(2t) = 2\sin t\cos t$

$$\cancel{\frac{aR^2}{2}\left(\int_0 dt + \int \cos 2t\, dt\right.}$$

$$\frac{aR^2}{2}\left(t + \frac{\sin 2t}{2}\right) = \frac{aR^2}{2}\left(\arcsin\frac{x}{R} + \frac{\sin\left(2\arcsin\frac{x}{R}\right)}{2}\right)$$

$$= \frac{aR^2}{2}\arcsin\frac{x}{R} + \frac{\cancel{2}\sin\left(\arcsin\frac{x}{R}\right)\cos\left(\arcsin\frac{x}{R}\right)aR^2}{\cancel{4}\,2} =$$

$$= \frac{aR^2\arcsin\frac{x}{R}}{2} + \frac{\frac{x}{R}\cdot\sqrt{1-\left(\frac{x}{R}\right)^2}\cdot aR^2}{2} =$$

$$= \frac{aR^2\arcsin\frac{x}{R}}{2} + \frac{xaR\sqrt{1-\left(\frac{x}{R}\right)^2}}{2} =$$

$$\sqrt{1-\left(\frac{x}{R}\right)^2} = \sqrt{1-\frac{x^2}{R^2}} = \sqrt{\frac{R^2-x^2}{R^2}} = \frac{\sqrt{R^2-x^2}}{R}$$

$$= \frac{aR^2\arcsin\frac{x}{R}}{2} + \frac{xa\sqrt{R^2-x^2}}{2} = \left.\frac{aR^2\arcsin\frac{x}{R} + xa\sqrt{R^2-x^2}}{2}\right|_0^R =$$

$$= \frac{aR^2\arcsin\frac{R}{R} + Ra\sqrt{R^2-R^2}}{2} - \frac{aR^2\arcsin\frac{0}{R} + 0a\sqrt{R^2}}{2} =$$

$$= \frac{aR^2\frac{\pi}{2} + 0}{2} - \frac{0+0}{2} = \frac{aR^2\pi}{4} = 1$$

2 $$\int_0^R x\sqrt{R^2-x^2}\,dx =$$

$t = R^2 - x^2$

$t' = -2x$

$$\int \cancel{x}\sqrt{R^2-x^2}\left(-\frac{dt}{2\cancel{x}}\right) = \int -\frac{1}{2}\sqrt{R^2-x^2}\,dt =$$

$$= -\frac{1}{2}\int\sqrt{t}\,dt = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}t\sqrt{t}}{3} = -\frac{t\sqrt{t}}{3} =$$

$$= -\left.\frac{(R^2-x^2)\sqrt{R^2-x^2}}{3}\right|_0^R = \frac{R^3}{3} = 2$$

(3) $\int_0^R \frac{R^2 - x^2}{2}\,dx = \frac{1}{2}\left(\int_0^R R^2\,dx - \int_0^R x^2\,dx\right) =$

$= \frac{1}{2}\left(xR^2\Big|_0^R - \frac{x^3}{3}\Big|_0^R\right) = \frac{1}{2}\left(R^3 - \frac{R^3}{3}\right) = \frac{2R^3}{6} = \frac{R^3}{3} = (3)$

$= \frac{aR^2\pi}{4} - \frac{R^3}{3} - \frac{R^3}{3} = \frac{aR^2\pi}{4} - \frac{2R^3}{3}$

# №4009

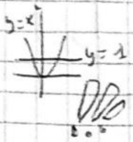
$\sqrt{4009}$

$Z = x^2 + y^2 \qquad y = x^2 \qquad y = 1 \qquad z = 0 \qquad x^2 = y$

$x = \pm\sqrt{y}$

$x = \pm 1$

$$\int_{-1}^{1} dx \int_{x^2}^{1} (x^2 + y^2)\, dy$$





$$\int (x^2 + y^2)\, dy = \int x^2 dy + \int y^2 dy = yx^2 + \frac{y^3}{3}$$

$$yx^2 + \frac{y^3}{3} \Big|_{x^2}^{1} = x^2 + \frac{1}{3} - x^4 - \frac{x^6}{3}$$

$$\int_{-1}^{1} \left(x^2 + \frac{1}{3} - x^4 - \frac{x^6}{3}\right) dx$$

$$\frac{x^3}{3} + \frac{x}{3} - \frac{x^5}{5} - \frac{x^7}{21} \Big|_{-1}^{1} = \frac{1}{3} + \frac{1}{3} - \frac{1}{5} - \frac{1}{21} - \left(-\frac{1}{3} - \frac{1}{3} + \frac{1}{5} + \frac{1}{21}\right) =$$

$$= \frac{4}{3} - \frac{2}{5} - \frac{2}{21} = \frac{20 - 6}{15} - \frac{2}{21} = \frac{14}{15} - \frac{2}{21} = \frac{14 \cdot 21 - 2 \cdot 15}{21 \cdot 15} =$$

$$= \frac{264}{21 \cdot 15} = \frac{88}{105} \qquad \frac{88}{105}$$

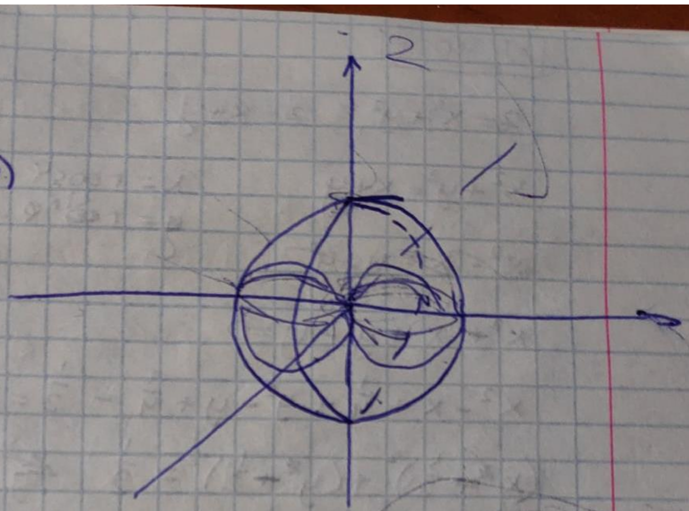
# №4016 ✓

$\boxed{N\ 4016}$

$$x^2+y^2+z^2=a^2$$

$$x^2+y^2 \geq a|x| \quad (a>0)$$

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$



$$z=\sqrt{a^2-x^2-y^2}$$

$$r=\frac{x}{\cos\varphi}=\frac{a}{\cos\varphi}$$

$$r^2 \geq a|r\cos\varphi|, \quad r^2 \geq \pm a r\cos\varphi$$

$$r \geq \pm a\cos\varphi$$

# №4020 ✓

N 4020

$z = x^2 + y^2, \quad z = x + y$

$x^2 + y^2 = x + y$ $\qquad x = r\cos\varphi + \frac{1}{2}$ $\quad \Rightarrow r = \frac{x - \frac{1}{2}}{\cos\varphi}$

$\qquad\qquad\qquad\qquad y = r\sin\varphi + \frac{1}{2}$

$\cancel{x^2 - x = y - y^2}$

$x^2 - x + y^2 - y = 0$

$x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} - \frac{1}{2} = 0$

$(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{2}$

$0 \le \varphi \le 2\pi$

$z = r\cos\varphi + r\sin\varphi + 1$

$r = \frac{z - 1}{\cos\varphi + \sin\varphi}$

$z = r^2\cos^2\varphi + \frac{1}{4} + r\cos\varphi + r^2\sin^2\varphi + \frac{1}{4} + r\sin\varphi =$

$= \frac{1}{2} + r^2 + r\cos\varphi + r\sin\varphi$

$r^2\cos^2\varphi + r^2\sin^2\varphi = \frac{1}{2}$

$r^2 = \frac{1}{2} \Rightarrow r = \pm\frac{1}{\sqrt{2}}$ $\qquad$ т.к. $r \ge 0 \Rightarrow 0 \le r \le \frac{1}{\sqrt{2}}$

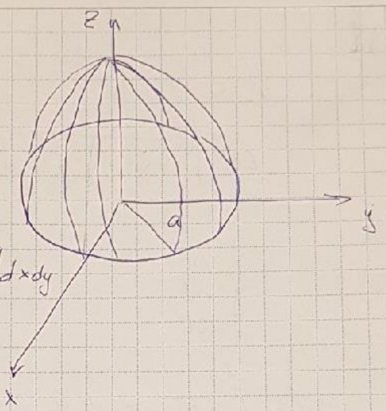
# №4036

N 4036

$az = xy$

$x^2 + y^2 = a^2$ (цилиндр)

$S_{поверх}$ - ?




$$S_{поверх} = \iint\limits_{x^2+y^2 \le a^2} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy$$

$$\frac{\partial z}{\partial x} = \frac{y}{a} \qquad \frac{\partial z}{\partial y} = \frac{x}{a}$$

$$S_{поверх} = \int dy \int\limits_{x^2+y^2 \le a^2} \sqrt{1 + \frac{y^2}{a^2} + \frac{x^2}{a^2}}\, dx = \int d\varphi \int\limits_{r \le a} \sqrt{1+r^2}\, a^2 r\, dr =$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$= 2\pi a^2 \int \sqrt{1+r^2}\, r\, dr =$$

$$= \pi a^2 \int \sqrt{1+r^2}\, d(1+r^2) = \pi a^2 \cdot \frac{2}{3}(1+r^2)^{3/2} =$$

$$= \frac{2\pi a^2}{3} \sqrt{(1+r^2)^3}\,\Big|_0^1 = \frac{2\pi a^2}{3}\sqrt{8} = \frac{2\pi a^2}{3}(2\sqrt{2} - 1)$$

## №4038

№4038 $x^2+y^2+z^2=a^2$ $z=\sqrt{a^2-x^2-y^2}$

$\underbrace{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$ $(b\le a)$

$y=\pm\sqrt{b^2-\frac{b^2x^2}{a^2}}$

$y=\pm\frac{b}{a}\sqrt{a^2-x^2}$

$$S=\iint\limits_{\Omega}\sqrt{1+\left(\frac{\partial\sqrt{a^2-x^2-y^2}}{\partial x}\right)^2+\left(\frac{\partial\sqrt{a^2-x^2-y^2}}{\partial y}\right)^2}\,dxdy=$$

$$=\iint\limits_{\Omega}\sqrt{1+\frac{x^2}{(\sqrt{a^2-x^2-y^2})^2}+\frac{y^2}{\sqrt{a^2-x^2-y^2}^2}}\,dxdy$$

$$\frac{z^2+x^2+y^2}{a^2-x^2-y^2}=\frac{a^2}{a^2-x^2-y^2}=\frac{a}{\sqrt{a^2-x^2-y^2}}$$

Взял условие

$0\le x\le a$

$0\le y\le\frac{b}{a}\sqrt{a^2-x^2}$

$$8\int_0^a dx\underbrace{\int_0^{\frac{b}{a}\sqrt{a^2-x^2}}\frac{a}{\sqrt{a^2-x^2-y^2}}dy}=$$

?!!

$$a\int\frac{dy}{\sqrt{\sqrt{a^2-x^2}^2-y^2}}=a\cdot\arcsin\left(\frac{y}{\sqrt{a^2-x^2}}\right)\Bigg|_0^{\frac{b}{a}\sqrt{a^2-x^2}}=a\cdot\arcsin\frac{b}{a}$$

$$=8\int_0^a a\cdot\arcsin\frac{b}{a}\,dx=\underline{8a^2\arcsin\frac{b}{a}}$$

## №4041 - нет

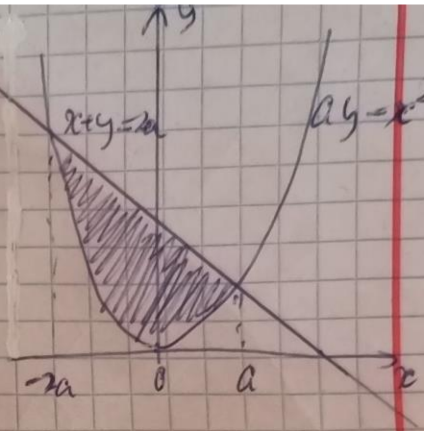
## №4042 - нет

# №4052

№ 4052

$ay = x^2, \quad x + y = 2a$

$(a > 0)$




$$M = \rho \int_{-2a}^{a} dx \int_{\frac{x^2}{a}}^{2a-x} dy =$$

$$= \rho \int_{-2a}^{a} \left(2a - x - \frac{x^2}{a}\right) dx =$$

$$= \rho \, \frac{9a^2}{2}$$

$$x_0 = \frac{1}{M} \iint_{\Omega} \rho x \, dx \, dy = \frac{\rho \int_{-2a}^{a} x \, dx \int_{\frac{x^2}{a}}^{2a-x} dy}{M} =$$

$$= \frac{\cancel{\rho} \, \left(-\frac{9}{4} a^3\right)}{\cancel{\rho} \, \frac{9}{2} a^2} = -\frac{a}{2}$$

$$y_0 = \frac{1}{M} \iint_{\Omega} \rho y \, dx \, dy = \frac{\rho \int_{-2a}^{a} dx \int_{\frac{x^2}{a}}^{2a-x} y \, dy}{M} =$$

$$= \frac{\rho \, \frac{36}{5} a^3}{\rho \, \frac{9}{2} a^2} = \frac{8}{5} a$$

№4066

№ 4066 $\quad I_0 = \iint_S (x^2+y^2)\,dx\,dy$

$$(x^2+y^2)^2 = a^2(x^2-y^2)$$

$x = r\cos\varphi$

$y = r\sin\varphi$

$$(r^2\cos^2\varphi + r^2\sin^2\varphi)^2 = a^2(r^2\cos^2\varphi - r^2\sin^2\varphi)$$

$r^4 = a^2 \cancel{r^2}\cos 2\varphi \qquad 0 \le \cos 2\varphi \le 1$

$r^2 = a^2\cos 2\varphi \qquad 0 \le 2\varphi \le \frac{\pi}{2}$

$r = a\sqrt{\cos 2\varphi} \qquad 0 \le \varphi \le \frac{\pi}{4}$

$$4\int_0^{\frac{\pi}{4}} d\varphi \int_0^{a\sqrt{\cos 2\varphi}} r^3\,dr$$

$$\frac{r^4}{4}\Bigg|_0^{a\sqrt{\cos 2\varphi}} = \frac{(a\sqrt{\cos 2\varphi})^4}{4} = \frac{a^4\cos^2 2\varphi}{4}$$

$$\cancel{4}\int_0^{\frac{\pi}{4}} \frac{a^4\cos^2 2\varphi}{\cancel{4}}\,d\varphi = a^4\int_0^{\frac{\pi}{4}}\cos^2 2\varphi\,d\varphi = a^4\cdot\left(\frac{\pi}{8}\right) =$$

$t = 2\varphi$

$$\int \frac{\cos^2 t}{2}\,dt$$

$$\frac{1}{2}\int \cos^2 t\,dt$$

$$\frac{1}{2}\int \frac{1+\cos 2t}{2}\,dt$$

$$\frac{1}{4}\int 1+\cos 2t\,dt$$

$$\frac{1}{4}\left(\int dt + \int \cos 2t\,dt\right)$$

$$\frac{1}{4}\left(t + \frac{\sin 2t}{2}\right)$$

$$\frac{1}{4}\left(2\varphi + \frac{\sin 4\varphi}{2}\right)$$

$$\left(\frac{\varphi}{2} + \frac{\sin 4\varphi}{8}\right)\Bigg|_0^{\frac{\pi}{4}}$$

$$\frac{\pi}{8}$$

$$= a^4\cdot\frac{\pi}{8} = \frac{\pi a^4}{8}$$

# №4066.1

№4066.1

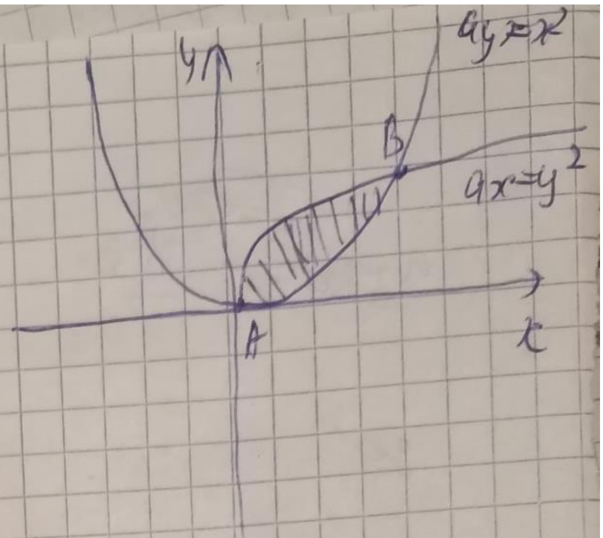
$ay = x^2$

$ax = y^2$

$a > 0$

$A(0, 0)$

$B(a, a)$




$$I_{xy} = \rho \iint\limits_{\Omega} xy\,dx\,dy$$

$$I_{xy} = \rho \int_0^a dx \int_{\frac{x^2}{a}}^{\sqrt{ax}} xy\,dy = \rho \int_0^a \left(\frac{ax^2}{2} - \frac{x^5}{2a^2}\right) dx =$$

$$= \rho\left(\frac{a^4}{6} - \frac{a^4}{12}\right) = \rho\,\frac{a^4}{12}$$

**№4076 ✓**

№4076

$$\iiint xy^2z^3\,dx\,dy\,dz$$

$V:\ z=xy$

$y=x$

$x=1$

$z=0$

$$\iint\limits_V dx\,dy\int_0^{xy} xy^2z^3\,dz=\iint\limits_V dx\,dy\,\frac{xy^2\cdot z^4}{4}\Big|_0^{xy}=$$

$$=\iint\limits_V dx\,dy\,\frac{xy^2\cdot x^4y^4}{4}=\iint\limits_V \frac{x^5y^6}{4}\,dx\,dy=$$

$$=\int_0^1 dx\int_0^x \frac{x^5y^6}{4}\,dy=\int_0^1 dx\,\frac{x^5y^7}{28}\Big|_0^x=$$

$$=\int_0^1 \frac{x^{12}}{28}\,dx=\frac{1}{28}\int_0^1 x^{12}=\frac{1}{28}\cdot\frac{1}{13}=\frac{1}{364}$$

## №4077 ✓

$\cos^2\varphi+\sin^2\varphi$

№4077.

$\iiint\limits_V \frac{dx\,dy\,dz}{(1+x+y+z)^3}$, где область $V$ ограничена $x+y+z=1$, $x=0$, $y=0$, $z=0$

$$\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} \frac{dz}{(1+x+y+z)^3} = -\frac{1}{2}\int_0^1 dx \int_0^{1-x} dy \left(\frac{1}{(1+x+y+z)^2}\right)\Big|_0^{1-x-y} =$$

$$= -\frac{1}{2}\int_0^1 dx \int_0^{1-x} \left(\frac{1}{4} - \frac{1}{(1+x+y)^2}\right) dy = -\frac{1}{2}\int_0^1 dx \left(\frac{y}{4} + \frac{1}{1+x+y}\right)\Big|_0^{1-x} =$$

$$= -\frac{1}{2}\int_0^1 \left(\frac{1-x}{4} + \frac{1}{2} - \frac{1}{1+x}\right) dx = -\frac{1}{2}\left(\frac{x}{4} - \frac{x^2}{8} + \frac{x}{2} - \ln|1+x|\right)\Big|_0^1 =$$

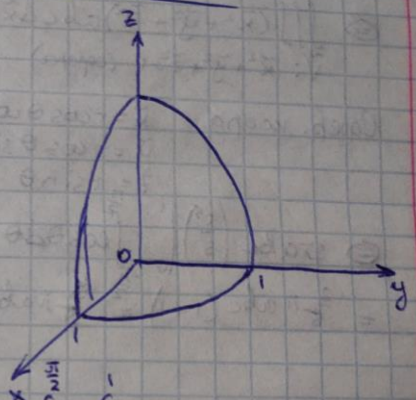
$$= -\frac{1}{2}\left(\frac{1}{4} - \frac{1}{8} + \frac{1}{2} - \ln 2\right) = \frac{1}{2}\ln 2 - \frac{5}{16}$$

## 4078 ✓

№4078

$$\iiint xyz\,dx\,dy\,dz =$$

$$V: \begin{cases} x^2+y^2+z^2=1 \\ x=0 \\ y=0 \\ z=0 \end{cases}$$




$$= \iint dx\,dy \int_0 xyz\,dz =$$

$$= \iint dx\,dy \frac{xy(1-x^2-y^2)}{2} = \begin{bmatrix} x = r\cos\varphi \\ y = r\sin\varphi \end{bmatrix} = \int_0^{\frac{\pi}{2}} d\varphi \int_0^1 \frac{r^2\cos\varphi\sin\varphi(1-r^2)}{2}\cdot r\,dr =$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin 2\varphi\,d\varphi}{4} \int_0^1 r^3(1-r^2)\,dr = \int_0^{\frac{\pi}{2}} \frac{\sin 2\varphi}{8}\,d2\varphi \left(\frac{1}{4} - \frac{1}{6}\right) =$$

$$= -\frac{\cos 2\varphi}{2}\Big|_0^{\frac{\pi}{2}} \cdot \frac{1}{12} = -\frac{1}{8}\cdot\frac{1}{12}(-1-1) = \frac{1}{48}$$

**4087**

№ 4087

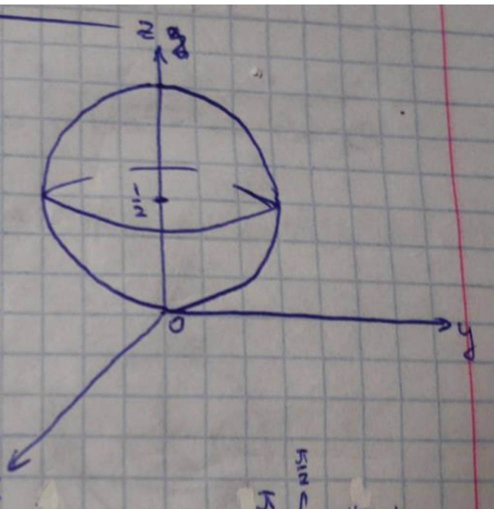
$$\iiint \sqrt{x^2+y^2+z^2}\,dx\,dy\,dz =$$

$V: x^2+y^2+z^2=z$

$x^2+y^2+z^2=z$

$x^2+y^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}$ (сфера)

$R=\frac{1}{2}$ $\quad V: r^2=r\sin\theta \Rightarrow r=\sin\theta$



$$= \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\theta \int_0^{\sin\theta} r^3\cos\theta\,dr =$$

$$= 2\pi \int_0^{\frac{\pi}{2}} d\theta \left(\frac{r^4\cos\theta}{4}\right)\Bigg|_0^{\sin\theta} = \frac{2\pi}{4}\int_0^{\frac{\pi}{2}} d\theta\,(\sin^4\theta\cos\theta) = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \sin^4\theta\,d\sin\theta =$$

$$= \frac{\pi}{2}\,\frac{\sin^5\theta}{5}\Bigg|_0^{\frac{\pi}{2}} = \frac{\pi}{10}$$

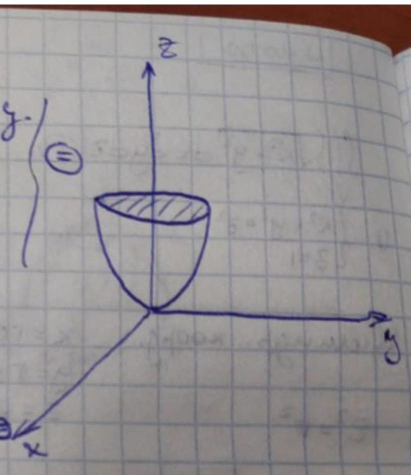
**4088**

**4091**

№ 4091

$$\iiint (x^2+y^2)\,dx\,dy\,dz = \left|\begin{array}{l}\text{Цилиндр. коорд.}\\ x=r\cos\varphi\\ y=r\sin\varphi\\ z=z\\ J=r\end{array}\right| =$$

$V: \begin{cases} x^2+y^2=2z \\ z=2 \end{cases}$ $\quad r^2=2z,\ z=\frac{r^2}{2}$



$$= \int_0^{2\pi} d\varphi \int_0^2 r^3 dr \int_{\frac{r^2}{2}}^2 dz =$$

$$= 2\pi\int_0^2 r^3 dr\left(2-\frac{r^2}{2}\right) = 2\pi\int_0^2 2r - \frac{r^5}{2}\,dr =$$

$$\cancel{= 2\pi\left(r^3 - \frac{r^4}{8}\right)\Bigg|_0^2 = 2\pi\cdot 2 = 4\pi}$$

$$= 2\pi\left(\frac{r^4}{2} - \frac{r^4}{12}\right)\Bigg|_0^2 = 2\pi\left(8-\frac{16}{3}\right) = \frac{16\pi}{3}$$

## 4101

№4101

$z = x^2 + y^2 \qquad z = 2x^2 + 2y^2 \qquad y = x, \; y = x^2$

$x^2 + y^2 \le z \le 2x^2 + 2y^2 \qquad x^2 \le y \le x$ ← по постр

$0 \le x \le 1$

$$V = \int_0^1 dx \int_{x^2}^{x} dy \int_{x^2+y^2}^{2x^2+2y^2} dz =$$

$$z \Big|_{x^2+y^2}^{2x^2+2y^2} = 2x^2 + 2y^2 - x^2 - y^2 = x^2 + y^2$$

$$= \int_0^1 dx \int_{x^2}^{x} (x^2 + y^2)\, dy =$$

$$yx^2 + \frac{y^3}{3} \Big|_{x^2}^{x} = x^3 + \frac{x^3}{3} - x^4 - \frac{x^6}{3}$$

$$= \int_0^1 \left(x^3 + \frac{x^3}{3} - x^4 - \frac{x^6}{3}\right) dx =$$

$$= \left(\frac{x^4}{4} + \frac{x^4}{12} - \frac{x^5}{5} - \frac{x^7}{21}\right) \Big| = \frac{x^4}{3} - \frac{x^5}{5} - \frac{x^7}{21} \Big|_0^1 = \frac{1}{3} - \frac{1}{5} - \frac{1}{21} -$$

$$-(0) = \frac{1}{3} - \frac{1}{5} - \frac{1}{21} = \frac{2}{15} - \frac{1}{21} = \frac{42-15}{315} = \frac{27}{315} = \frac{3}{35}$$

## 4102 - нет

## 4103 - нет

## 4133 - нет

**4134**

№ 4134

$x_0, y_0, z_0 - ?$

$z = x^2+y^2$

$x+y=a$

$x=0=y=z$

$y = a-x$

$0 = a-x$

$$M = \iiint_V \rho\,dx\,dy\,dz$$

$$x_0 = \frac{1}{M}\iiint_V \rho x\,dx\,dy\,dz$$

$$y_0 = \frac{1}{M}\iiint_V \rho y\,dx\,dy\,dz$$

$$z_0 = \frac{1}{M}\iiint_V \rho z\,dx\,dy\,dz$$

$$M = \int_0^a dx \int_0^{a-x} dy \int_0^{x^2+y^2} dz = \int_0^a dx \int_0^{a-x} (x^2+y^2)\,dy = \int_0^a \left(x^2 y + \frac{y^3}{3}\right)\Big|_0^{a-x} dx =$$

$$= \int_0^a \left(x^2(a-x) + \frac{(a-x)^3}{3}\right)dx = \int_0^a ax^2\,dx - \int_0^a x^3\,dx - \int_0^a \frac{(a-x)^3}{3}\,d(a-x) =$$

$$= \frac{a^4}{3} - \frac{a^4}{4} + \frac{(a-x)^4}{12}\Big|_0^a = \frac{4a^4 - 3a^4 + a^4}{12} = \frac{2a^4}{12} = \frac{a^4}{6}$$

$$x_0 = \frac{1}{\cancel{M}} \int_0^a x\,dx \int_0^{a-x} dy \int_0^{x^2+y^2} dz$$

$\frac{a^4}{6}$

$$y_0 = \frac{1}{\cancel{M}}\frac{a^4}{6}\int dx \int y\,dy \int dz$$

$$z_0 = \frac{a^4}{6}\int dx \int dy \int z\,dz$$

**4143 - нет**

**4145 - нет**

## 4222

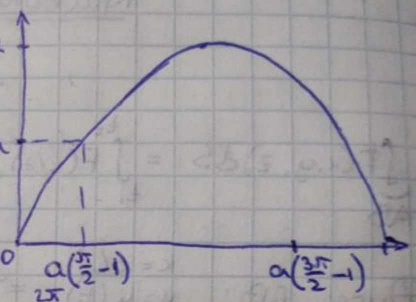
N 4222

$\int y^2\,dS = \cancel{\int_0^{2\pi} a^2(1-\cos t)^2 a\sqrt{(1-\cos t)^2+}}$

$x = a(t-\sin t)$ $\quad \dot{x} = a(1-\cos t)$

$y = a(1-\cos t)$ $\quad \dot{y} = a\sin t$

$t \in [0;\ 2\pi]$




$= \int_0^{2\pi} a^2(1-\cos t)^2 a\sqrt{(1-\cos t)^2+\sin^2 t}\,dt = a^3\int_0^{2\pi}(1-\cos t)^2\sqrt{1-2\cos t+\cos^2 t+\sin^2 t} =$

$= a^3\int_0^{2\pi}\left(2\sin^2\frac{t}{2}\right)^2\sqrt{2(1-\cos t)}\,dt = a^3\int_0^{2\pi} 4\sin^4\frac{t}{2}\cdot 2\sin\frac{t}{2}\,dt =$

$= -16a^3\int_0^{2\pi}\left(1-\cos^2\frac{t}{2}\right)^2 d\left(\cos\frac{t}{2}\right) = -16a^3\int_0^{2\pi}\left(1-2\cos^2\frac{t}{2}+\cos^4\frac{t}{2}\right)d\left(\cos\frac{t}{2}\right) =$

$= -16a^3\left(\cos\frac{t}{2} - \frac{2\cos^3\frac{t}{2}}{3} + \frac{\cos^5\frac{t}{2}}{5}\right)\Big|_0^{2\pi} = -16a^3\left(-1+\frac{2}{3}-\frac{1}{5}-\left(1-\frac{2}{3}+\frac{1}{5}\right)\right) =$

$= -16a^3\left(-2+\frac{4}{3}-\frac{2}{5}\right) = \frac{256a^3}{15}$

## 4223 - нет

## 4224

N 4224

$\int_C xy\,dS = a^3\int_0^{t_0}\operatorname{ch}t\operatorname{sh}t\sqrt{\operatorname{ch}2t}\,dt = \frac{a^3}{2}\int_0^{t_0}\operatorname{sh}2t\sqrt{\operatorname{ch}2t}\,dt =$

$x = a\operatorname{ch}t$

$y = a\operatorname{sh}t$

$0 \le t \le t_0$

$dS = \sqrt{a^2\operatorname{sh}^2 t + a^2\operatorname{ch}^2 t}\,dt = a\sqrt{\operatorname{ch}2t}\,dt$

$x' = a\operatorname{sh}t$

$y' = a\operatorname{ch}t$

$= \frac{a^3}{4}\int_0^{t_0}\sqrt{\operatorname{ch}2t}\,d(\operatorname{ch}2t) = \frac{a^3}{4}\cdot\left(\frac{2}{3}(\operatorname{ch}2t)^{\frac{3}{2}}\right)\Big|_0^{t_0} =$

$= \frac{a^3}{6}\left((\operatorname{ch}2t_0)^{\frac{3}{2}} - (\cancel{\operatorname{ch}0}^{\,1})^{\frac{3}{2}}\right)$

## 4231

$\boxed{№\ 4231}$

$x = 3t \quad \Rightarrow \quad t_1 = 0 \qquad \dot{x} = 3$

$y = 3t^2 \qquad t_2 = 1 \qquad \dot{y} = 6t$

$z = 2t^3 \qquad \dot{z} = 6t^2$

$O(0,0,0)$

$A(3,3,2)$

$$l = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt = \int_0^1 \sqrt{9 + 36t^2 + 36t^4}\, dt =$$

$$= 3\int_0^1 \sqrt{1 + 4t^2 + 4t^4}\, dt = 3\int_0^1 (1 + 2t^2)\, dt = 3\left(t + \frac{2t^3}{3}\right)\Big|_0^1 = 5$$

## 4234? - нет

## говно

## 4237

№4237

$$\int_C (x^2 + y^2 + z^2)\, ds = \int_C (a^2\cos^2 t + a^2 \sin^2 t + b^2 t^2) \cdot \sqrt{a^2 + b^2}\, dt \ominus$$

$C:\ x = a\cos t,\quad y = a\sin t,\quad z = bt \quad (0 \le t \le 2\pi)$

$\dot{x} = -a\sin t,\quad \dot{y} = a\cos t,\quad \dot{z} = b$

$$\ominus \int_C a^2 + b^2 t^2 \sqrt{a^2 + b^2}\, dt = \sqrt{a^2 + b^2} \int_C (a^2 + b^2 t^2)\, dt =$$

$$= \sqrt{a^2 + b^2}\left(a^2 t + \frac{b^2 t^3}{3}\right)\Big|_0^{2\pi} = 2\pi\sqrt{a^2 + b^2}\left(a^2 + \frac{b^2 4\pi^2}{3}\right) =$$

$$= \frac{2\pi}{3}\sqrt{a^2 + b^2}\,(3a^2 + 4\pi^2 b^2)$$

## 4239 - нет

## 4241 - нет

**4242**

$\boxed{N4242}$

$M - ?$

$x = at$

$y = \frac{a}{2}t^2$

$z = \frac{a}{3}t^3$

$(0 \le t \le 1)$

$\rho = \sqrt{\frac{2y}{a}}$

$x' = a$

$y' = at$

$z' = at^2$

$$M = \int_C \rho(x,y,z)\,dS = \int_0^1 at\sqrt{1+t^2+t^4}\,dt =$$

$$dS = \sqrt{x'^2+y'^2+z'^2} = \sqrt{a^2 + a^2t^2 + a^2t^4}\,dt$$

$$= a\sqrt{1+t^2+t^4}\,dt$$

$$\rho = \sqrt{\frac{2y}{a}} = \sqrt{t^2} = t$$

$$= a\int_0^1 \frac{1}{2}\sqrt{1+t^2+t^4}\,d(t^2) =$$

$$= \frac{a}{2}\int_0^1 \sqrt{1+b+b^2}\,db = \frac{a}{2}\int_0^1 \sqrt{\left(b+\frac{1}{2}\right)^2+\frac{3}{4}}\,d\left(b+\frac{1}{2}\right) \cancel{= \frac{a}{2}\int_0^1 \left(\left(b+\frac{1}{2}\right)^2+\frac{3}{4}\right)^{\frac{1}{2}} d\left(b+\frac{1}{2}\right)} =$$

$$= \frac{a}{2}\int_0^1 \sqrt{u^2+\frac{3}{4}}\,du = \frac{a}{2}\int_0^1 \frac{\left(u^2+\frac{3}{4}\right)du}{\sqrt{u^2+\frac{3}{4}}} = \frac{a}{2}\int_0^1 \frac{u^2\,du}{\sqrt{u^2+\frac{3}{4}}} + \frac{3a}{8}\int_0^1 \frac{du}{\sqrt{u^2+\frac{3}{4}}} =$$

$$= \frac{3a}{8}\ln\left|u^2 + \sqrt{u^2+\frac{3}{4}}\right| + \frac{a}{2}\int_0^1 d$$

**4244 - нет**

**4244.1 - нет**

**4244.2 - нет**

## 4250

№4250.

$\int\limits_C (x^2-2xy)\,dx + (y^2-2xy)\,dy =$, где $C$ - парабола

$C: y = x^2 \quad (-1 \le x \le 1)$

$x = t \qquad \dot{x} = 1$

$y = t^2 \qquad \dot{y} = 2t$

$= \int\limits_{-1}^{1} (t^2-2t^3)\,dt + (t^4-2t^3)\cdot 2t\,dt =$

$\int\limits_{-1}^{1} (2t^5-4t^4-2t^3+t^2)\,dt = \frac{2t^6}{6} - \frac{4t^5}{5} - \frac{2t^4}{4} + \frac{t^3}{3}\Big|_{-1}^{1} =$

$\frac{1}{3} - \frac{4}{5} - \frac{1}{2} + \frac{1}{3} - \left(\frac{1}{3} + \frac{4}{5} - \frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3} - \frac{4}{5} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{3}}$

$-\frac{4}{5} + \cancel{\frac{1}{2}} + \frac{1}{3} = \frac{2}{3} - \frac{8}{5} = \frac{10-24}{15} = -\frac{14}{15}$

## 4253 - нет

## 4256

№4256

$\int\limits_{AB} \sin y\,dx + \sin x\,dy = \int\limits_{0}^{\pi} \sin(\pi - t)\,dt - \sin(t)\,dt =$

$dx = dt \quad dy = -dt$

$A(0, \pi)$

$B(\pi, 0)$

$y = \pi - x \quad \begin{cases} x = t \\ y = \pi - t \end{cases} \qquad t \in (0, \pi)$

$= \cos(\pi - t) + \cos t \Big|_0^{\pi} = +1 - 1 + 1 - 1 = 0$

## 4279 - нет

**4308**

N 4308

Эллипс:

$x = a\cos t$

$y = b\sin t$

$0 \le t \le 2\pi$

$$S = \frac{1}{2}\oint_C (x\,dy - y\,dx) = \frac{1}{2}\int_0^{2\pi} (ab\cos^2 t + ab\sin^2 t)\,dt =$$

$$= \frac{ab}{2}\int_0^{2\pi} dt = \cancel{2}\, ab\pi$$

**4346 ✓**

N 4346

$$\iint_S |xyz|\,dS = \iint_\sigma |xyz|\sqrt{1+4x^2+4y^2}\,dx\,dy =$$

$z = x^2+y^2$

$z = 1$

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = r^2 \Rightarrow r = 1$

$\cancel{r^4\cos\varphi\sin\varphi \ge 0}$

$\frac{\partial z}{\partial x} = 2x \qquad \frac{\partial z}{\partial y} = 2y \quad \Rightarrow \sqrt{1+4x^2+4y^2}$

$\cancel{\frac{r^4}{2}\sin 2\varphi \ge 0}$

$$= \iint |xy(x^2+y^2)|\sqrt{1+4x^2+4y^2}\,dx\,dy =$$

$$= 4\int_0^{\pi/2} d\varphi \int_0^1 |r^2\cos\varphi\sin\varphi\,(r^2)|\sqrt{1+4r^2}\,r\,dr =$$

$x^2+y^2=1$

$r^2(\cos^2\varphi+\sin^2\varphi)=1$

$$= 2\int_0^{\pi/2} d\varphi \int_0^1 r^5 \sin 2\varphi\,\sqrt{1+4r^2}\,dr =$$

$=$

**4349 - нет**

**4352.1 - нет**

**4355 ✓**

N 4355

$z=\sqrt{x^2+y^2}$ $\quad z'_x=\frac{x}{\sqrt{x^2+y^2}}$ $\quad G = x^2+y^2 \le ax$

$x^2+y^2=ax$ $\quad z'_y=\frac{y}{\sqrt{x^2+y^2}}$ $\quad \rho = 1$

$$M=\iint\limits_G \sqrt{1+\cancel{\frac{x^2+y^2}{x^2+y^2}}}\,\rho\,dx\,dy = \sqrt{2}\rho\iint\limits_G dx\,dy =$$

$$= \sqrt{2}\int_0 dx\int_{-\sqrt{ax-x^2}}^{\sqrt{ax-x^2}} dy = \sqrt{2}\int_0^a\left(\sqrt{ax-x^2}+\sqrt{ax-x^2}\right)dx =$$

$z^2=x^2+y^2 \Rightarrow y^2=z^2-x^2$ $\quad z^2-\cancel{x^2}=ax-\cancel{x^2}$

$ax=x^2+y^2$ $\quad y^2=ax-x^2$ $\quad z=\sqrt{ax}$

$x=a-\frac{y^2}{x} = \frac{xa-y^2}{x}$

$$= \sqrt{2}\cdot 2\int_0^a\sqrt{ax-x^2}\,dx =$$

**4368 - нет**

**4370**

N 4370

$$\oint\limits_{C\text{ – эллипс}} \underbrace{(y+z)}_{A}dx+\underbrace{(z+x)}_{B}dy+\underbrace{(x+y)}_{C}dz =$$

$x=a\sin^2 t$

$y=2a\sin t\cos t$

$z=a\cos t$

$$= \iint\limits_S \begin{vmatrix}\bar i & \bar j & \bar k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A & B & C\end{vmatrix}\bar n\,dS =$$

$$= \iint\left(i\left(\frac{\partial C}{\partial y}-\frac{\partial B}{\partial z}\right)-j\left(\frac{\partial C}{\partial x}-\frac{\partial A}{\partial z}\right)+k\left(\frac{\partial B}{\partial x}-\frac{\partial A}{\partial y}\right)\right)\cdot$$

$$\cdot(n_x, n_y, n_z)\,dS = \iint 0\cdot\bar n\,dS = 0$$

**4373 - нет**

**говно**

**4376**

$\boxed{N\ 4376}$

$$\cancel{\iint_S yz\,dydz + zx\,dzdx + xy\,dxdy}$$

$$\iint_S x^3 dydz + y^3 dzdx + z^3 dxdy \;=$$

$$\operatorname{div} V = 3x^2 + 3y^2 + 3z^2$$

$$= 3\iiint_V (x^2+y^2+z^2)\,dxdydz$$

**4378**

$\boxed{N\ 4378}$

$$\iint_S \frac{x\cos\alpha + y\cos\beta + z\cos\gamma}{\sqrt{x^2+y^2+z^2}}\,dS = 2\iiint_V \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$$

$\cancel{\operatorname{div} V}$ $P = \dfrac{x}{\sqrt{x^2+y^2+z^2}}$ $\quad Q = \dfrac{y}{\sqrt{x^2+y^2+z^2}}$ $\quad S = \dfrac{z}{\sqrt{x^2+y^2+z^2}}$

$$\operatorname{div} V = P'_x + Q'_y + S'_z \;=$$

$$= \frac{y^2+z^2+x^2+z^2+x^2+y^2}{(x^2+y^2+z^2)^{3/2}} = 2\,\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{3/2}} =$$

$$= \frac{2}{\sqrt{x^2+y^2+z^2}}$$

$$P' = \frac{\sqrt{x^2+y^2+z^2} - x^2/\sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2} = \frac{y^2+z^2}{(x^2+y^2+z^2)^{3/2}}$$

$$S' = \frac{\sqrt{x^2+y^2+z^2} - y^2/\sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2} = \frac{x^2+z^2}{-\shortparallel-}$$

$$Q' = \frac{\sqrt{x^2+y^2+z^2} - z^2/\sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2} = \frac{x^2+y^2}{-\shortparallel-}$$

**4387 - нет**

## 4388

N 4388

$$\iint_S x^3\,dydz + y^3\,dzdx + z^3\,dxdy = \iiint_{V_{сферы}} (3x^2+3y^2+3z^2)\,dx\ldots$$

S : вн. стор. сферы

$$x^2+y^2+z^2=a^2$$

$$F=(x^3, y^3, z^3)$$

$$\operatorname{div}\bar F = 3x^2+3y^2+3z^2$$

$$dydz = \left\{\begin{array}{l} \text{сф. коорд.} \\ x = r\cos\theta\cos\varphi \\ y = r\cos\theta\sin\varphi \\ z = r\sin\theta \\ J = r^2\cos\theta \end{array}\right\} =$$

$$= \int_0^{2\pi} d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^a 3r^2\cos\theta \cdot r^2\,dr = 2\pi \sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \, 3\frac{r^5}{5}\Big|_0^a =$$

$$= 2\pi \cdot 2 \cdot \frac{3 \cdot r^5}{5} = \frac{12\pi a^5}{5}$$

## 4390

N 4390

$$\iint_S (x^2\cos\alpha + y^2\cos\beta + z^2\cos\gamma)\,dS =$$

$$x^2+y^2=z^2$$

$$z=h,\quad x^2+y^2\le h^2 \qquad 0\le z\le h$$

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

$$r^2 \le h^2 \Rightarrow r = \pm h$$

$$\operatorname{div} V = 2x + 2y + 2z$$

$$= \iiint_V (2x+2y+2z)\,dxdydz =$$

$$= 2\int_0^{2\pi} d\varphi \int_0^h r\,dr \int_0^h (r\cos\varphi + r\sin\varphi + z)\,dz =$$

$$= 2\int_0^{2\pi} d\varphi \int_0^h r\left(rz\cos\varphi + rz\sin\varphi + \frac{z^2}{2}\right)\Big|_0^h dr =$$

$$= 2\int_0^{2\pi} d\varphi \int_0^h \left(r^2h\cos\varphi + r^2h\sin\varphi + \frac{h^2}{2}\right) dr =$$