HW2

May 22, 2022

1 EE798 HW2

```
[ ]: from scipy.linalg import toeplitz, dft, inv, eigvals, eig, kron, eigh
     from scipy.signal import convolve2d, convolve
     import numpy as np
     from numpy.fft import fft, ifft
     import matplotlib.pyplot as plt
```

1.1 Q4 a

```
[ ]: def linear_conv_mat(h:np.array,output_size):
         L = h.size
         h_ex = np.zeros(output_size)
         h_ex[0:L] = h
         first_row = np.roll(np.flip(h_ex),1)
         return toeplitz(h_ex.T,first_row[0:(output_size-L+1)]) # H + X - 1 =␣
      ↪output_size
```

1.1.1 Demonstration

```
[ ]: h = np.array([1,2,3,4,5])
     H = linear_conv_mat(h,8)
     H
```

```
[ ]: array([[1., 0., 0., 0.],
            [2., 1., 0., 0.],
            [3., 2., 1., 0.],
            [4., 3., 2., 1.],
            [5., 4., 3., 2.],
            [0., 5., 4., 3.],
            [0., 0., 5., 4.],
            [0., 0., 0., 5.]])
```

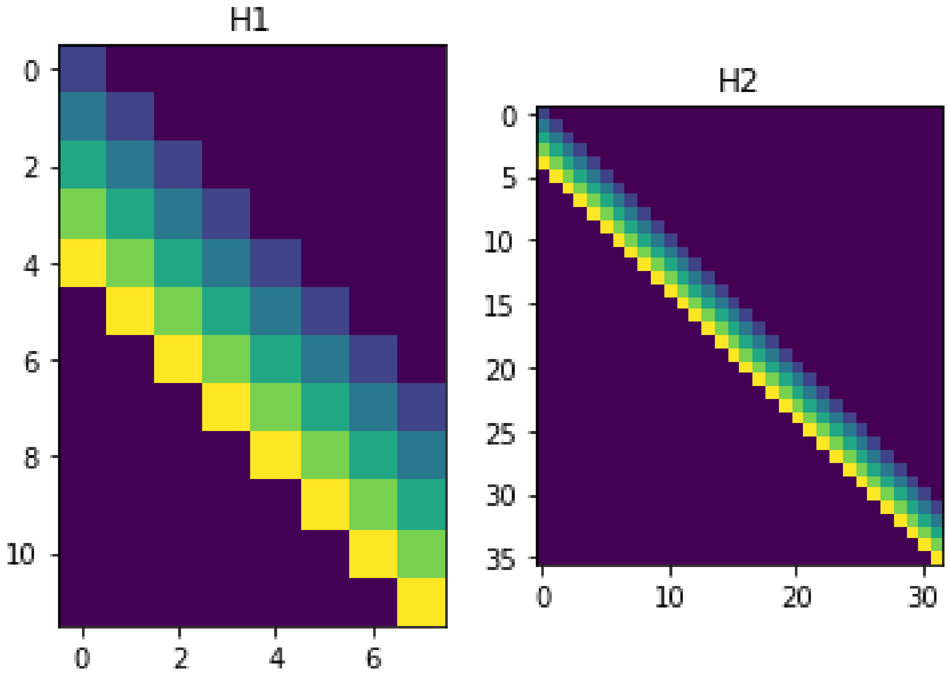
1.1.2 Verification

```
[ ]: x = np.random.randint(0,10,(8))
     res = np.convolve(x,h)
     H = linear_conv_mat(h,res.size)
     res2 = np.matmul(H,x.T)
     assert np.array_equal(res,res2)

     x = np.random.randint(0,150,(32))
     res = np.convolve(x,h)
     H2 = linear_conv_mat(h,res.size)
     res2 = np.matmul(H2,x.T)
     assert np.array_equal(res,res2)

     print('All tests are success!')
```

```
All tests are success!
```

```
[ ]: plt.subplot(1,2,1)
     plt.imshow(H)
     plt.title("H1")
     plt.subplot(1,2,2)
     plt.title("H2")
     plt.imshow(H2)
```

```
[ ]: <matplotlib.image.AxesImage at 0x214362a2400>
```



1.2 Q4 b

1.2.1 The circular convolution should be of length

$$N = L_x + L_y - 1$$

to obtain linear convolution.

1.2.2 The sequences $\hat{h}$ and $\hat{x}$ are zero-padded version of sequences h and x to the length of the linear convolution. This relationship is hidden inside the FFT implementation, which automatically pads zero to the sequence if the FFT points N is larger than length of the sequence L.

1.2.3 If N is smaller than the length of any of the input vectors, then the vectors are "rolled" on to itself and the circular convolution is conducted with those rolled versions:

$$x_{Rolled}[n] = \sum_{k=-\infty}^{+\infty} x[n+kN]$$

```
[ ]: def cconv(x:np.array, y:np.array, N:int):
         return np.round(np.real(ifft(np.multiply(fft(x,N),fft(y,N)))),3)
```

```
[ ]: x = [0,1,2]
     y = [4,5,6]
     print(np.convolve(x,y))
     cconv(x,y,5)
```

```
[ 0  4 13 16 12]
```

```
[ ]: array([-0.,  4., 13., 16., 12.])
```

Verification

```
[ ]: x = np.random.randint(0,5,(5))
     y = np.random.randint(0,5,(4))
     N = 8
     res = cconv(x,y,N)
     print("X = ",x,"\nY = ",y)
     print("Linear convolution: ", np.convolve(x,y))
     print("Circular convolution with N =", N, ":", res)
     N = 6
     res = cconv(x,y,N)
     print("Circular convolution with N =", N, ":", np.round(res,2))
```

```
X =  [1 1 2 2 2]
Y =  [0 4 0 3]
Linear convolution:  [ 0  4  4 11 11 14  6  6]
```

```
Circular convolution with N = 8 : [-0.  4.  4. 11. 11. 14.  6.  6.]
Circular convolution with N = 6 : [ 6. 10.  4. 11. 11. 14.]
```

1.3 Q4 c

1.3.1 The implementation is based on the transformation of basis vectors of $\mathbb{R}^N$ where transform is the circular convolution.

[ ]:

```
def  cconvmtx(h:np.array, N:int):
    I = np.eye(N)
    for i in range(N):
        I[:,i] = np.round(cconv(h,I[:,i].T,N).T,3)
    return I
```

[ ]:

```
N = 1024
Max = 10
for trial in range(10): # Number of test cases
    h = np.random.randint(0,Max,N)
    x = np.random.randint(0,Max,N)
    res = cconv(x,h,N)
    H = cconvmtx(h,N)
    res2 = np.matmul(H,x)
    assert np.sum(np.isclose(res,res2)) == res.size # np.array_equal sometimes␣
 ↪outputs False due to precision errors
print("All tests are success!")
```

```
All tests are success!
```

1.4 Q4 d

1.4.1 Verification

- N values are swept from 4 to 64 and every case is asserted.

[ ]:

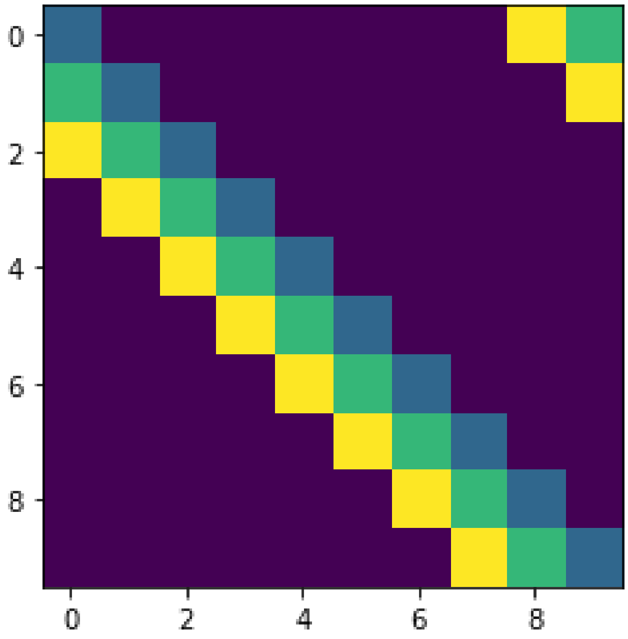
```
for N in range(4,64):
    h = np.random.randint(0,12,N)
    x = np.random.randint(0,12,N)
    F = dft(N)
    X = np.matmul(F,x)
    H = np.matmul(F,h)
    res = np.matmul(inv(F),np.multiply(X,H))
    res2 = cconv(x,h,N)
    assert np.sum(np.isclose(res,res2)) == res.size # np.array_equal may output␣
 ↪False due to precision errors
print("All tests are success!")
```

```
All tests are success!
```

1.5 Q4 e

```
[ ]: N = 10
     H = cconvmtx([1,2,3],N)
     plt.imshow(H)
```

```
[ ]: <matplotlib.image.AxesImage at 0x2143632efd0>
```

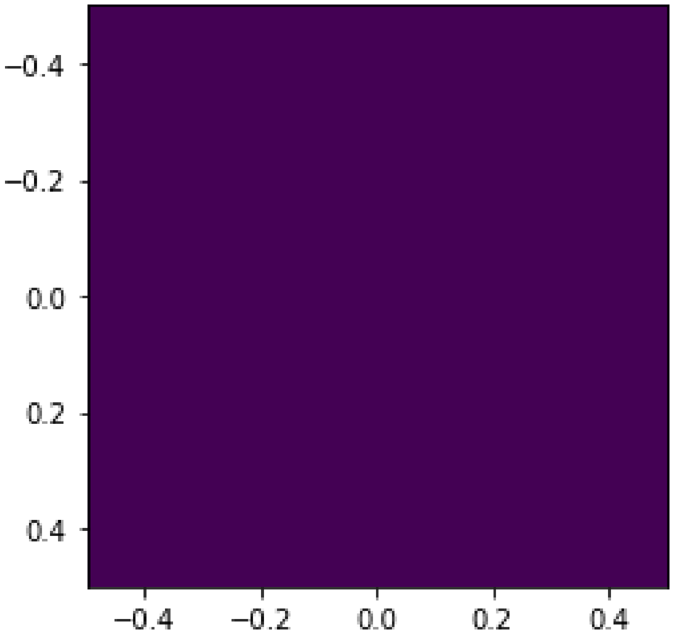


1.5.1 Verification

```
[ ]: for trial in range(20):
         N = np.random.randint(1,20,1).item()
         h = np.random.randint(0,100,N)
         H = cconvmtx(h,N)
         F = dft(N)
         f_H = np.matmul(F,h)
         FCFH = F.dot(H).dot(F.conj().transpose())/N
         assert np.sum(np.isclose(FCFH, np.diag(f_H))) == FCFH.size
     print("All tests are success!")
     plt.imshow(np.abs(FCFH))
```

```
All tests are success!
```

```
[ ]: <matplotlib.image.AxesImage at 0x21438397fa0>
```



1.5.2 First column experiment

```
[ ]: first_col = H[:,0]
     FIRST_COL = np.matmul(F,first_col)
     assert np.sum(np.isclose(FIRST_COL, f_H)) == FIRST_COL.size
     print("All tests are success!")
```

```
All tests are success!
```

1.5.3 Eigen Decomposition

```
[ ]: N = np.random.randint(1,5,1).item()
     h = np.random.randint(0,100,N)
     H = cconvmtx(h,N)
     P = (dft(N)) # Diagonalizing matrix
     E = np.diag(P.dot(H).dot(inv(P)))
```

```
[ ]: container = []
     M = H.dot(P)
     for i in range(N):
         container.append(np.true_divide(M[:,i],P[:,i])[0])
     container = np.array(container)
```

1.5.4 Due to ordering issue, real and imaginary parts are sorted and compared separately

[ ]:

```
assert (np.sum(np.isclose(np.sort_complex(np.real(container)),np.
 ↪sort_complex((np.real(E))))) == E.size and
        np.sum(np.isclose(np.sort_complex((np.imag(container))),np.
 ↪sort_complex(np.imag(E)))) == E.size)
print("All tests are success!")
```

```
All tests are success!
```

2 Q5 a

[ ]:

```
def sepconv2(X:np.array, h1:np.array, h2:np.array):
    X = convolve(X,h1)
    return convolve(X,h2)
```

2.0.1 Verification

[ ]:

```
from skimage import data
X = data.camera()
N1 = 5
N2 = 9

l1 = np.ones((N1,1)) / N1
l2 = np.ones((N2,1)) / N2
h1 = np.array(l1)
h2 = np.array(l2)
h1 = np.reshape(h1,(len(h1),1))
h2 = np.reshape(h2,(len(h2),1)).T
res = sepconv2(X,h1,h2)
res2 = convolve2d(X, np.matmul(h1,h2))
assert np.sum(np.isclose(res,res2)) == res.size # np.array_equal may output␣
 ↪False due to precision errors

for trial in range(20):
    l1 = np.random.randint(-100,100,N1)
    l2 = np.random.randint(-100,100,N2)
    h1 = np.array(l1)
    h2 = np.array(l2)
    h1 = np.reshape(h1,(len(h1),1))
    h2 = np.reshape(h2,(len(h2),1)).T
    res = sepconv2(X,h1,h2)
    res2 = convolve2d(X, np.matmul(h1,h2))
    assert np.sum(np.isclose(res,res2)) == res.size # np.array_equal may output␣
 ↪False due to precision errors
```

```
print("All tests are success!")
```

```
All tests are success!
```

3 Q5 b

[ ]:

```
def sepconv2mtx(L1:int, L2:int, h1:np.array, h2:np.array):
    Lh1 = h1.shape[0] # column vector
    Lh2 = h2.shape[1] # row vector
    output1 = Lh1 + L1 - 1
    output2 = Lh2 + L2 - 1
    H1 = linear_conv_mat(h1.T, output_size=output1)
    H2 = linear_conv_mat(h2, output_size=output2)
    return kron(H2,H1)
```

3.0.1 Verification

[ ]:

```
N = 32
X = np.random.randint(0,100,(N,N))
vecX = X.flatten(order="F").T
N1 = 5
N2 = 9
L1, L2 = X.shape[0], X.shape[1]
for trial in range(20):
    l1 = np.random.randint(-100,100,N1) / N1
    l2 = np.random.randint(-100,100,N2) / N2
    h1 = np.array(l1)
    h2 = np.array(l2)
    h1 = np.reshape(h1,(len(h1),1))
    h2 = np.reshape(h2,(len(h2),1)).T
    sepConv = sepconv2mtx(L1, L2, h1, h2)
    res = np.reshape(np.matmul(sepConv,vecX),(N1+L1-1,N2+L2-1), order="F")
    res2 = convolve2d(X,np.matmul(h1,h2))
    assert np.sum(np.isclose(res,res2)) == res.size # np.array_equal may output␣
 ↪False due to precision errors
print("All tests are success!")
```

```
All tests are success!
```