$$p_\theta(t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(xcos(\theta)+ysin(\theta)-t)dxdy \tag{1}$$

$$S_\theta(f) = \int_{-\infty}^{+\infty} p_\theta(t)e^{-j2\pi ft}dt \tag{2}$$

$$S_\theta(f) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(xcos(\theta)+ysin(\theta)-t)e^{-j2\pi ft}dxdydt \tag{3}$$

$$S_\theta(f) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-j2\pi(xcos(\theta)+ysin(\theta))f}dxdy \tag{4}$$

$$S_\theta(f) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-j2\pi fcos(\theta)x}e^{-j2\pi fsin(\theta)y}dxdy \tag{5}$$

$$u = f\cos\theta \tag{6}$$

$$v = f\sin\theta \tag{7}$$

$$J = \begin{vmatrix} \frac{\partial u}{\partial f} & \frac{\partial v}{\partial f} \\ \frac{\partial u}{\partial \theta} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -f\sin\theta & f\cos\theta \end{vmatrix} \tag{8}$$

$$= f(\sin^2\theta + \cos^2\theta) = f \tag{9}$$

$$S_\theta(f) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{-j2\pi ux}e^{-j2\pi vy}dxdy = F(u,v) = F(fcos\theta, fsin\theta) \tag{10}$$