HW#2

Due 23:59 on 11th of December, Sunday

In the homework, your task is to obtain some of the results illustrated in the lectures. In each part, you are referred to one or more slides used in the lectures and asked to generate the results shown in the slides in Matlab (or Phyton). Slide numbers given are the exact page numbers in the documents (not the written slide numbers).

In your solution, you should briefly explain how your implementation works, the mathematical background behind it, and provide figures that show your results. **All Matlab codes should also be provided** to get credit. You should not use any built-in Matlab/Phyton function to directly perform the required task unless stated. General-purpose functions like fft2, ifft2, conv2, imagesc, image, etc. can be used if needed.

Q1) (Matrix representation of transforms, basis functions, separable 2D transforms)

a) Generate the matrix to compute the 1D Hadamard transform of a 8-length signal and plot the basis functions used in this transform
(slide 2 in “Image transforms_conted_week7”)
b) Using the 1D basis functions in part (a), compute and plot the basis images of 2D Hadamard transform for an 8x8 image.
(slide 4 in “Image transforms_conted_week7”)
c) Randomly generate an 8x8 image and decompose this image using Hadamard bases. For this, you should compute all the representation coefficients using the 2D basis images in part (b).
(slide 5 in “Image transforms_conted_week7” & slide 11 in “Image transforms_conted_week6")
d) Alternatively, obtain the representation (transform) coefficients using the 1D transform matrix in part (a). Verify that same result is obtained as part (c).
(slide 17 in “Image transforms_conted_week6")

Q2) Image to be used: 256x256 grayscale Lena image
a) By treating each column of the image as the realization of a 1D random signal of length 256, compute and plot the covariance matrix. Also compute the fraction of coefficient values in its diagonal.
(slide 16 in “Image transforms_conted_week6")
b) Apply 1D DFT, DCT, and KLT to the columns of the image and re-compute the covariance matrix. Plot again the covariance matrices and compare the fraction of coefficient values in their diagonals. Comment on your results.
(slide 16 in “Image transforms_conted_week6")
c) Partition the 256x256 Lena image to non-overlapping 16x16 patches. Convert these 16x16 image patches to vectors (i.e. 1D signals) by concatenating their columns. Repeat part (a) for these vectors of length 256.
d) Apply 2D DFT and DCT to image patches of size 16x16. Convert the transformed 16x16 images to vectors and compute the covariance matrix. Plot again the covariance matrices and compare the fraction of coefficient values in their diagonals.
(slide 20 in “Image transforms_conted_week6")
e) Apply 2D KLT to image patches of size 16x16. Plot the covariance matrix and compare the fraction of coefficient values in the diagonal. Comment on your results.
(slide 17 & 18 in “Image transforms_conted_week6")

Q3) Image to be used: 256x256 grayscale Lena image
a) Plot the frequency response for the ideal low-pass filter, Gaussian LPF, and Butterworth LPF with the same Do parameter

(slide 9, “Image transforms applications_week8”)

b) Low-pass filter the Lena image using the ideal LPF, Gaussian LPF, Butterworth LPF for different values of Do parameter
(slide 10, “Image transforms applications_week8”)

Q4) Image to be used: 256x256 grayscale Lena image

a) Compute the single-level discrete 2-D wavelet transform of an image using Haar basis (you can use the built-in Matlab function wavedec2) and plot the result
(slide 32, “wavelet transform_week8”)
b) Compute the two-level discrete 2-D wavelet transform of an image using Haar basis (you can use the built-in Matlab function wavedec2) and plot the result
c) Generate a noisy image and then denoise it by hard thresholding its Wavelet coefficients both using a 1-level and 2-level Wavelet transform. To perform a fair comparison, keep same number of transform coefficients for each case and use PSNR&SSIM to compare the filtered image to the original image. Comment on your results.
(slide 33-35, “wavelet transform_week8”)