Middle East Technical University
Electrical-Electronics Engineering Department

EE 583 Pattern Recognition
Homework 2

Due Date: 01.11.2020, 23:55 via odtuclass.metu.edu.tr

Using MATLAB, attempt the questions below:

1) Maximum Likelihood Estimation from sample data: Let $N(\mu_1,\Sigma_1)$ be equal to a the class-conditional density for a particular feature for class-1, ω_1. It is given that μ_1=[-0.75 0.5]^T, while $\Sigma_1 = \begin{bmatrix} 0.5 & 0.3 \\ 0.3 & 0.8 \end{bmatrix}$. Generate and plot 10 samples from the distribution above. Using these samples, find the Maximum Likelihood (ML) estimates for μ_1 and Σ_1. Repeat this step by using 1000 samples. Comment on the results.

2) Bayesian Parameter Estimation: Assume a scalar class-conditional Gaussian density, $N(\mu,\sigma)$, while only the mean vector is to be estimated (variance is known and fixed, as σ=0.7).

i) Generate and plot 25 samples from the distribution above, if μ=3. Find the Maximum Likelihood (ML) estimate of μ for these 25 samples.

ii) Calculate the Maximum A Posteriori (MAP) estimate of the parameter, μ, analytically, if it is assumed that μ is distributed normally whose mean value is equal to 2.8 and variance 0.8. Compare these results with part-(i).

iii) Repeat parts (i) & (ii) by using 1000 samples. Comment on the results of increasing samples vs a priori info.

3) Minimum error-rate classifier: Assume a 2-class problem, while the observed features are also 2-dimensional. For both classes, ω_1 and ω_2, the features are Gaussian distributed by $N(\mu_1,\Sigma)$ and $N(\mu_2,\Sigma)$, respectively. Let the a priori class probabilities be equal to each other. Let μ_1=[-1 -1]^T and μ_1=[1 1]^T, whereas $\Sigma = \begin{bmatrix} 1.4 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$. Generate and plot four sets (train sets and test sets for two classes, ω_1 and ω_2), each with 250 samples, from the distributions above.

i) Determine and plot the decision boundary analytically from the given parameters for a minimum error-rate classifier. Apply your test data set to this decision boundary and calculate the probability of error for your classifier. Determine the ratio of wrong classifications to total number of test samples.

iii) Find the Maximum Likelihood (ML) estimates for μ_1, μ_2 and Σ from the available training set. Determine the ratio of wrong classifications to total number of test samples for a minimum error-rate classifier that is obtained directly from the estimated parameters.

4) Non-parametric Density Estimation: For a normal distributed feature vector, x, whose mean is μ_i=[-1.5 1.5]^T and the covariance matrix is $\Sigma = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.6 \end{bmatrix}$. Plot this distribution in 3D. Generate 50 samples from this given distribution. Estimate this density non-parametrically using Parzen window estimation from the generated samples. Choose the initial volume and volume shrinking formula appropriately. Plot the estimated distribution.

5) Execute the **Fitting an Orthogonal Regression Using PCA** example at https://www.mathworks.com/help/stats/fitting-an-orthogonal-regression-using-principal-components-analysis.html *step-by-step* via "Try This Example" option in your internet browser (or you may execute it in your local MATLAB software by using "View MATLAB Command"). Perform the same steps of this example for `fisheriris` data (@HW1), while decreasing the dimension of all 3-classes from 2D to 1D. Plot 1D histograms for these 3 classes in order to observe their discrimination capability.