

EE583 Pattern Recognition HW2

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Question 1

Maximum Likelihood Estimate of Multivariate Gaussian random variable are calculated as:

- $\hat{\mu} = \frac{1}{m} \sum_{i=1}^m X^{(i)}$
- $\hat{\Sigma} = \frac{1}{m} \sum_{i=1}^m (X^{(i)} - \hat{\mu})(X^{(i)} - \hat{\mu})^T$

where $X^{(i)}$ is the i^{th} observation.

```

1 % Q1
2 m = 1000;
3 sigma_1 = [0.5 0.3; 0.3 0.8];
4 mu_1 = [-0.75; 0.5];
5 X = mvnrnd(mu_1, sigma_1, m);
6 mu_1_hat = mean(X);
7
8 sum = zeros(size(sigma_1));
9 for i = 1:m
10     sum = sum + (X(i, :)-mu_1_hat)'*(X(i, :)-mu_1_hat);
11 end
12 sigma_1_hat = sum/m;
13 mu_1_hat
14 sigma_1_hat

```

The given code produces the following results:

$$\begin{array}{lll}
 m = 10 & \hat{\mu} = [-0.4519 & 1.0984] & \hat{\Sigma} = \begin{bmatrix} 0.3191 & 0.1799 \\ 0.1799 & 0.6506 \end{bmatrix} \\
 m = 1000 & \hat{\mu} = [-0.7367 & 0.5176] & \hat{\Sigma} = \begin{bmatrix} 0.4730 & 0.2925 \\ 0.2925 & 0.8079 \end{bmatrix}
 \end{array}$$

It is observed that the estimations get more accurate with increasing number of samples.

Question 2

MAP Estimation formula:

$$p(\mu|N) = \frac{p(N|\mu)p(\mu)}{p(N)}$$

$$x_{MAP} = \underset{x}{\operatorname{argmax}} \log(p(\mu|N))$$