EE583 Pattern Recognition HW2

Kutay Uğurlu 2232841

November 17, 2021

Question 1

Maximum Likelihood Estimate of Multivariate Gaussian random variable are calculated as:

- $\hat{\mu} = \frac{1}{m} \sum_{i=1}^{m} X^{(i)}$

- $\hat{\Sigma} = \frac{1}{m} \sum_{i=1}^{m} (X^{(i)} - \hat{\mu})(X^{(i)} - \hat{\mu})^T$

where $X^{(i)}$ is the ith observation.

```matlab
1   % Q1
2   m = 1000;
3   sigma_1 = [0.5  0.3;0.3  0.8];
4   mu_1 = [-0.75;0.5];
5   X = mvnrnd(mu_1,sigma_1,m);
6   mu_1_hat = mean(X);
7
8   sum = zeros(size(sigma_1));
9   for i = 1:m
10      sum = sum + (X(i,:)-mu_1_hat)'*(X(i,:)-mu_1_hat);
11  end
12  sigma_1_hat = sum/m;
13  mu_1_hat
14  sigma_1_hat
```

The given MATLAB code produces the following results:

$m = 10$	$\hat{\mu} = \begin{bmatrix} -0.4519 & 1.0984 \end{bmatrix}$	$\hat{\Sigma} = \begin{bmatrix} 0.3191 & 0.1799 \\ 0.1799 & 0.6506 \end{bmatrix}$
$m = 1000$	$\hat{\mu} = \begin{bmatrix} -0.7367 & 0.5176 \end{bmatrix}$	$\hat{\Sigma} = \begin{bmatrix} 0.4730 & 0.2925 \\ 0.2925 & 0.8079 \end{bmatrix}$

It is observed that the estimations get more accurate with increasing number of samples.

Question 2

25 Samples

$\hat{x}_{ML} = 2.7456$ $\hat{x}_{MAP} = 2.7475$

1000 samples

$\hat{x}_{ML} = 2.7456$ $\hat{x}_{MAP} = 2.7475$