Modeling Interference Color

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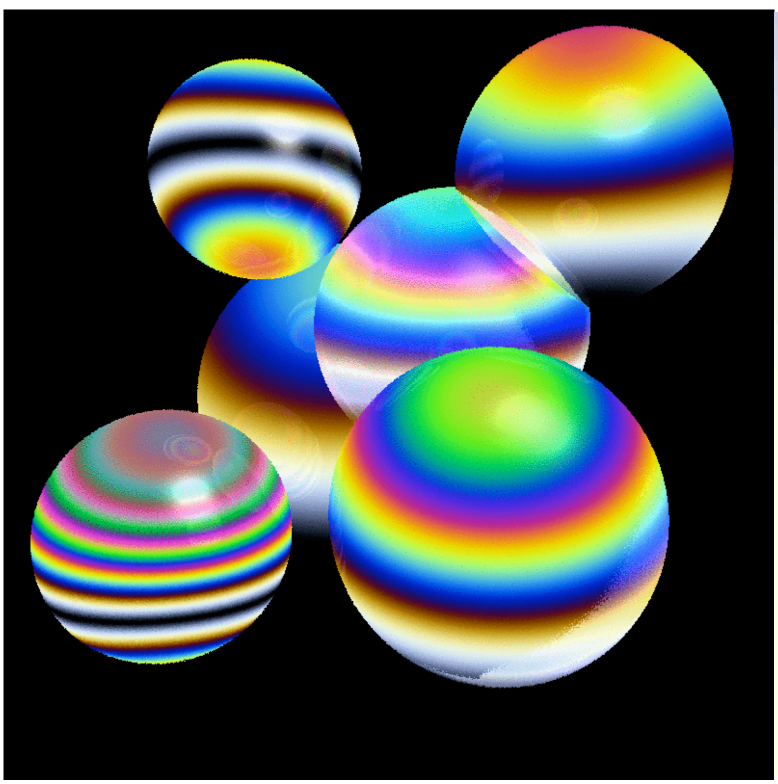

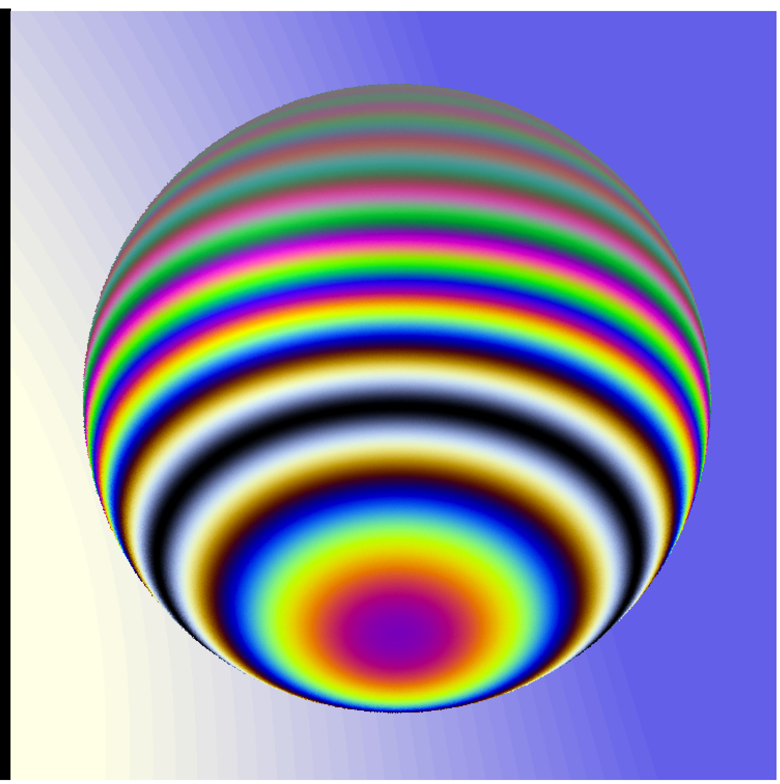

Modeling the phenomenon of interference color is the interesting problem of trying to represent multiple reflections of light between two surfaces. Maria Lurdes Dias in her article "Ray Tracing Interference Color", uses both her knowledge of physics and graphics to derive a formula for reflectance of the film (a thin layer, such as in a soap bubble). The formula makes it possible to model interference color by ray tracing.

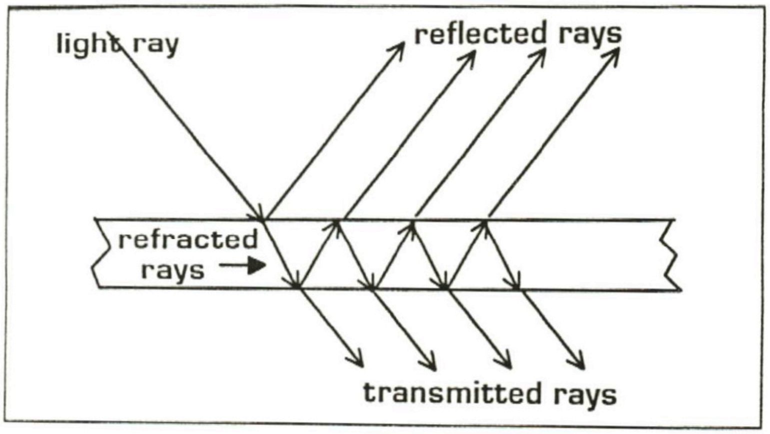

Figure 1. Interference color involving multiple reflections in a thin film

The Phenomenon

Interference color can be observed when sunlight rays come in contact with a thin layer, such as a soap bubble (see figure 1). The color seen by the eye (see figure 2) is due to perceiving the multiple reflected rays as one ray. The superposition of these rays may increase or decrease the intensity of the light. *Constructive interference* (the resulting intensity is greater than the intensity of any of the separate rays) and *destructive interference* (the resulting intensity is less than the intensity of any of the rays or equal to zero) are produced from the phase difference between the rays.

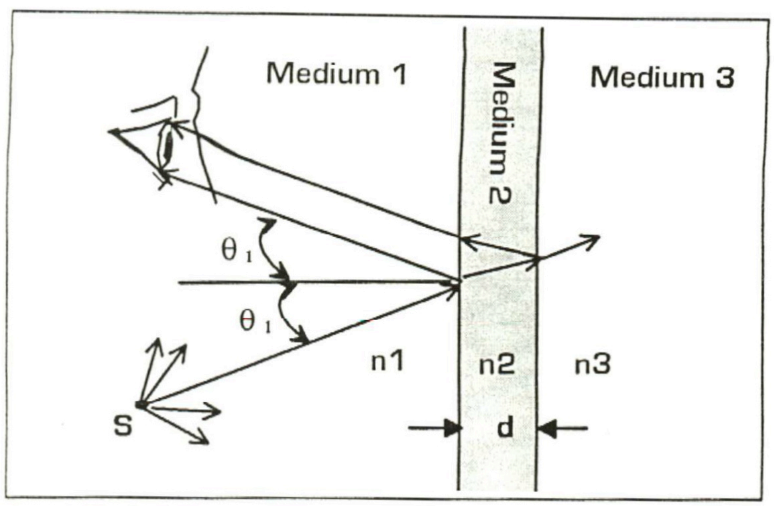

Figure 2. A thin film is viewed by light reflected from source S.
Source: David Halliday and Robert Resnick, "Fundamentals of Physics" (Wiley, NY, 1988).

Importance of Modeling

Maria explains that importance of interference phenomenon, is to provide the physical basis for optics of thin films and their applications. I feel that being able to model interference color and produce a visual, gives the added benefit of seeing how changes in variables affects the visual. An example from the article, showed how various thickness of the film changed position within the CIE diagram. As the thickness increases the interference color approaches very close to the point corresponding to the position of the white light source. Thus a connection between thin and thick films was found.

The Solution

Maria states it is possible to represent any type of material by using just one equation for reflectance:

$$R(\lambda) = \frac{R_{1-2}^2 + R_{2-3}^2 + 2R_{1-2}R_{2-3}\cos\delta}{1 + R_{1-2}^2 R_{2-3}^2 + 2R_{1-2}R_{2-3}\cos\delta} \qquad \delta = \frac{4\pi}{\lambda} n_2 d \cos\theta_2$$

The formula was derived based on a generalization of Fresnel's formula from physics. The formula below is a simplified version of the formula above. This formula was used to represent soap bubbles and Newton's rings, and is for layers with low index refraction where we can neglect multiple refraction.

$R = 4R_{1-2}R_{2-3}(\varepsilon + \cos^2(\delta/2))$ $\longrightarrow$ $R \approx \varepsilon + \cos^2(\delta/2)$

Where: $\varepsilon = \frac{(R_{1-2} - R_{2-3})^2}{4R_{1-2}R_{2-3}}$

$$R_{1-2} = \frac{n_1 - n_2}{n_1 + n_2}$$

$$R_{2-3} = \frac{n_2 - n_3}{n_2 + n_3}$$

$$\delta = \frac{4\pi}{\lambda} n_2 d \cos\theta_2$$

d - thickness.
n_1 - index of refraction for medium 1.
n_2 - index of refraction for medium 2 (the film).
n_3 - index of refraction for medium 3.
λ - wavelength in vacuum of incident light.
θ_1 - angle of incidence.
θ_2 - angle of refraction.

The term ε relates to the purity of the color. The color is most pure when $\varepsilon = 0$ or -1. As $|\varepsilon|$ becomes greater, the less pure the interference color becomes. The way in which Maria does not give important values for variables or an example calculation, I think is a strong weakness. Variables for which I thought she could have mentioned some values for or some range of values for, include n_2 , d, and λ. It is not clear from reading this article how the variables affect phenomenon. For example I found out that the thickness changes in different spots in the bubble, causing the rings that can be seen in Maria's results. Due to gravity, the top of the bubble has the least thickness, while at the bottom the thickness is the greatest. Another situation is with the index of refraction when using various wavelengths, it would seem that the index of refraction does depend on the wavelength of the incident ray. I have been unsuccessful in finding tables for the index of refraction of different materials at different wavelengths. Not being able to find this information will mean that ε will be constant with a value of -1, affecting the purity of the colors. Maria says this equation can be used to represent any material, I disagree. The equation fails to take in account turbulence within a fluid. In Maria's results, the bubbles contain rings of various colors, but in nature you might see swirls of colors.

Implementation

I chose this article because of the stunning graphics produced by Maria's ray tracing program and wondering how to get such colors from using the CIE colorimetric system. Upon trying to implement Maria's formulas for the CIE colorimetric system, I found it very interesting. The CIE XYZ tristimulus values from the paper are given by the following equations:

Maria made a correction 1994 article by

$$X = \varepsilon \int S\bar{x}\,d\lambda + \int S\bar{x}\cos^2(\delta/2)\,d\lambda \quad : X = 4R_{1-2}R_{2-3}\left(\varepsilon \int S\bar{x}\,d\lambda + \int S\bar{x}\cos^2(\delta/2)\,d\lambda\right)$$
$$Y = \varepsilon \int S\bar{y}\,d\lambda + \int S\bar{y}\cos^2(\delta/2)\,d\lambda \quad : Y = 4R_{1-2}R_{2-3}\left(\varepsilon \int S\bar{y}\,d\lambda + \int S\bar{y}\cos^2(\delta/2)\,d\lambda\right)$$
$$Z = \varepsilon \int S\bar{z}\,d\lambda + \int S\bar{z}\cos^2(\delta/2)\,d\lambda \quad : Z = 4R_{1-2}R_{2-3}\left(\varepsilon \int S\bar{z}\,d\lambda + \int S\bar{z}\cos^2(\delta/2)\,d\lambda\right)$$

Where: S - is the relative spectral irradiance of the light source.
$\bar{x}, \bar{y}, \bar{z}$ - the tristimulus values of the equal-energy spectrum.

As one can see the equations are based on the formula for reflectance. Having no idea what S was, I turned to another source to find out more about the CIE colorimetric system. The book by G. Wyszecki and W. Stiles called "Color Science: Concepts and Methods, Quantitative Data and Formulae" was very helpful in explaining how to use and find the CIE XYZ values. The book contains many tables from which I derived one table for what I needed to calculate the tristimulus values (see table 1). The book also explained that integration can be replaced by summation. The new equations are:

$$X = k\Sigma\, \beta(\lambda) S(\lambda) \bar{x}(\lambda) \Delta\lambda$$
$$Y = k\Sigma\, \beta(\lambda) S(\lambda) \bar{y}(\lambda) \Delta\lambda$$
$$Z = k\Sigma\, \beta(\lambda) S(\lambda) \bar{z}(\lambda) \Delta\lambda$$

Where: $k = \frac{100}{\sum S(\lambda)\bar{y}(\lambda)\Delta\lambda}$
β - Spectral reflectance factor.

The reason I think that Maria used the CIE colorimetric system was because the three equations above take in account a light source (S(λ) - Standard Illuminance C) and the reflectance (β(λ)) for an object. Another nice feature of the CIE XYZ system, is that information can be plotted on the CIE chromatic chart, as was done for optical thickness. I think it would be considerably more tedious to use the RGB system and figure out the intensities for various light sources, to arrive at the same results. The CIE XYZ system uses a standrd illuminate C which is intended to represent direct sunlight. I think the standard illuminance C is a better representation than using intensities with the RGB system, because it is more common to see brilliantly colored bubbles in the sunlight. Maria takes the CIE XYZ tristimulus values and converts them to the RGB system to display on a monitor. The Book "Computer Graphics: Principles and Practice" by J. D. Foley,

λ	Standard Illuminant C $S(\lambda)$	$\bar{x}(\lambda)$	$\bar{y}(\lambda)$	$\bar{z}(\lambda)$
380	33.0	0.0014	0.0000	0.0065
390	47.40	0.0042	0.0001	0.0201
400	63.30	0.0143	0.0004	0.0679
410	80.60	0.0435	0.0012	0.2074
420	98.10	0.1344	0.0040	0.6456
430	112.40	0.2839	0.0116	1.3856
440	121.50	0.3483	0.0230	1.7471
450	124.00	0.3362	0.0380	1.7721
460	123.10	0.2908	0.0600	1.6692
470	123.80	0.1954	0.0910	1.2876
480	123.90	0.0956	0.1390	0.8130
490	120.70	0.0320	0.2080	0.4652
500	112.10	0.0049	0.3230	0.2720
510	102.30	0.0093	0.5030	0.1582
520	96.90	0.0633	0.7100	0.0782
530	98.00	0.1655	0.8620	0.0422
540	102.10	0.2904	0.9540	0.0203
550	105.2	0.4334	0.9950	0.0087
560	105.30	0.5945	0.9950	0.0039
570	102.30	0.7621	0.9520	0.0021
580	97.80	0.9163	0.8700	0.0017
590	93.20	1.0263	0.7570	0.0011
600	89.70	1.0622	0.6310	0.0008
610	88.40	1.0026	0.5030	0.0003
620	88.10	0.8544	0.3810	0.0002
630	88.00	0.6424	0.2650	0.0000
640	87.80	0.4479	0.1750	0.0000
650	88.20	0.2835	0.1070	0.0000
660	87.90	0.1649	0.0610	0.0000
670	86.30	0.0874	0.0320	0.0000
680	84.00	0.0468	0.0170	0.0000
690	80.20	0.0227	0.0082	0.0000
700	76.30	0.0114	0.0041	0.0000
710	72.40	0.0058	0.0021	0.0000
720	68.30	0.0029	0.0010	0.0000
730	64.40	0.0014	0.0005	0.0000
740	61.50	0.0007	0.0002	0.0000
750	59.20	0.0003	0.0001	0.0000
760	58.10	0.0002	0.0001	0.0000
770	58.20	0.0001	0.0000	0.0000

Table 1, Used to find the CIE XYZ tristimulus values.

Source: Wyszecki, G., and W. Stiles, *Color Science: Concepts and Methods, Quantitative Data and formulae*, second edition, Wiley, New York, 1982.

A. van Dam, S. K. Feiner, and J. F. Hughes, describes a conversion matrix that converts the RGB system to the CIE XYZ system. To do a CIE XYZ to a RGB I took the inverse of the conversion matrix. The equation below is the one I derived for a RGB color specification that is based on the standard NTSC RGB phosphor, with a white point that is the standard illuminant C.

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.0191 & -0.0053 & -0.0029 \\ -0.0098 & 0.0200 & -0.0003 \\ 0.0006 & -0.0012 & 0.0090 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Although Maria says she uses a ray tracing program, she does not mention any details of how ray tracing was used. She probably did not metion it because it is very straightforward. A conventional ray tracing program can be used with only a few modifications.

Conclusion

Maria does a nice job in getting readers interested. She does this through her remarkable ray traced images. She does not give enough information in the article for a student of graphics to implement this phenomenon. Maria does give hints in what directions she took, which is very helpful. Through other books like the ones I made reference to in the report, one can implement this phenomenon. The only information that I could not find and might cause discrepencies in the implementation is the index of refraction of materials at various wavelengths.

Kevin:

- your revision has improved the paper. I like the additional detail & observations, although I would have liked to see further analysis & observations, & would have liked to see the observations you added discussed in a little more detail. Fascinating topic. I look forward to seeing your project.

Phil Lay

Grade: B