Exercise 1:

(a) $\Rightarrow$ discrete time convolution:

$$(f*h)(n) = f(n) * h(n) = \sum_{k=0}^{n} f(k)\cdot h(n-k) = \sum_{k=0}^{n} f(n-k)h(k)$$

$\Rightarrow$ proof: distributive property

$$f(n) * (g(n) + h(n))$$

$$= (f * (g+h))(n)$$

$$= \sum_{k=0}^{n} f(n-k)\,(g+h)(n)$$

$$= \sum_{k=0}^{n} f(n-k)\,(g(n) + h(n))$$

$$= \sum_{k=0}^{n} f(n-k)g(n) + f(n-k)h(n)$$

$$= \sum_{k=0}^{n} f(n-k)g(n) + \sum_{k=0}^{n} f(n-k)h(n)$$

$$= (f*g)(n) + (f*h)(n) \quad \square$$

(b) $\Rightarrow$ continuous time convolution:

$$(f*g)(t) = \int_{u=0}^{t} f(u)\,g(t-u)\,du = \int_{u=0}^{t} f(t-u)\,g(u)\,du$$

$\Rightarrow$ proof: associative property

$$((f*g)*h)(t)$$

$$= \int_{s=0}^{t} (f*g)(s)\,h(t-s)\,ds$$

$$= \int_{s=0}^{t} \int_{u=0}^{s} f(u)\,g(s-u)\,du\;h(t-s)\,ds$$

$$= \iint_{0<u<s<t} f(u)\,g(s-u)\,h(t-s)\,du\,ds$$

$$= \int_{u=0}^{t} f(u) \int_{s=u}^{t} g(s-u)\,h(t-s)\,ds\,du$$

$$= \int_{u=0}^{t} f(u) \int_{s=0}^{t-u} g(s)\,h(t-s-u)\,ds\,du$$

$$= \int_{u=0}^{t} f(u)\,(g*h)(t-u)\,du = (f*(g*h))(t) \quad \square$$