Exploration and Analysis of the Evolution of Strategies for Mancala Variants

Colin Divilly, Colm O'Riordan and Seamus Hill

Abstract—**This paper describes approaches to evolving strategies for Mancala variants. The results are compared and the robustness of both the strategies and heuristics across variants of Mancala is analysed. The aim of this research is to evaluate the performance of a collection of heuristics across a selection of Mancala games. The performance of the individual heuristics can be evaluated on games with varying rules regarding capture rules, varying number of pits per row and for different seeds per pit at the start of the game.**

I. INTRODUCTION

Board games and strategy games have been the focus of much research in computer science. Games such as Checkers, Chess and Go have been studied in a number of works with the aim of solving the game, i.e. is there an optimal strategy for players [9]. Mancala games refer to a large family of 'seed-sowing' games. These have been less studied in the literature.

Previous research into some variants of Mancala has looked at exploring good heuristics, good strategies and in some work, solving the game. However, it is still unknown as to how applicable certain heuristics are across related variants. Certain heuristics are not applicable due to rule changes; other heuristics may be weakened or strengthened across variants due to rule changes.

We attempt to develop a collection of heuristics that fit within the general rules of the games in the Mancala family and that can be applied to a wide range of these games. The aim of this research is to evaluate the performance of these of heuristics across a selection of mancala games and to bring these heuristics together into strong combinations. It is hoped to explore whether any strong combinations of heuristics are robust across a selection of mancala variants. It is hoped that the development of these robust heuristics combinations will improve our understanding of the complexity with the family of mancala games.

The paper is structured as follows. Section II aims to give the reader an insight into the Mancala family and the variety of games within it. In Section III, we outline research that has all ready been conducted in the area of mancala games. Section IV covers how we plan to achieve our research aims. In Section V we outline the results of our experiments and Section VI presents conclusions that can be drawn from them. And finally, in Section VII we outline some potential future work within the Mancala family of games.

Colin Divilly, Colm O'Riordan and Seamus Hill are with the discipline Information Technology in the National University of Ireland, Galway, Ireland; emails:colindivilly@gmail.com,colm.oriordan@nuigalway.ie, seamus.hill@nuigalway.ie

II. MANCALA GAMES

Mancala is the name given to a family of board games which date back several thousand years. There are many variants of the game played in disparate geographical regions. Variants of the game number in the hundreds. The game is generally a two-player game where the players take turns to move pieces on a wooden board. In the literature on Mancala, the two players are referred to as South and North because the players sit each side of the board facing each other. South takes the first move in the game. The board contains a number of pits across two or more rows. These pits contain the playing pieces of the game. All the playing pieces are distributed across the pits at the beginning of the game. There is normally the same number of seeds in each pit. In addition, some boards have larger pits at each side of the board. These two pits are referred to as stores and are used to store the pieces that each player has captured in the game. One of the issues when researching Mancala is the naming of certain games. It is quite common that a certain set of game rules can be known by more than one name.

The family of Mancala games are often referred to as a 'count and capture' games [4] or as a 'sowing' games [4]. These names derive from how seeds are moved across the board; this movement of seeds is referred to as sowing. There are two main methods of sowing found in the Mancala family, single lap sowing and multiple lap sowing. Single lap sowing is found in the games of Kalah and Awari, while multiple lap sowing is used in, among others, the game of Dakon [8].

Variations can occur in the board configuration, with changes in the number of pits in a row and the initial number of seeds per pit. The notation (x, y) is commonly used to refer to a game where there are x amount of holes per row and y amount of seeds per pit. For example, the game of Wari, as outlined by Russ [8], which has 6 pits per row with 4 seeds in every pit at the start of the game is a (6,4) game. This game has a total of 48 seeds in a game. Compare this with the game of Torguz Xorgol is a (9,9) game [8]. Despite the large number of variants there are certain features which are commonly found across the variants; these include [4]:

- The game is played on a board with pits arranged in two or more rows.
- The playing pieces are counters such as stones, seeds, coins or shells.
- Players own pits rather than the seeds in the pits.
- Moves are made by sowing the contents of a pit along the board in some direction. After sowing, captures may

978-1-4673-5311-3/13/$31.00 ©2013 IEEE

occur if certain conditions are met.
- The winner is the player who has captured the majority of seeds.

A player may capture pieces while sowing or upon completion of sowing. Capturing refers to the removal of pieces from the board and placing them into the player's store/scoring pit. Once all the seeds are sown and all captured pieces are moved to the store, the opposition player can now take their turn. In some games a player's own store (but not their opponents store) are included in the pits into which they can sow seeds. One of the most common and most important variations in the game rules is how seeds are captured. Donkers, Uiterwijk and de Voogt [4] outline the four types of captures that have been recorded:

- Number capture: after a player has sown all of their seeds and the last seed sown is placed in one of their opponent's pits with that pit now containing a specific number of seeds (for example, 2 or 3 seeds), then these seeds may then be captured.
- Place capture: after a player has sown all of their seeds and the last seed sown is in one of their own pits with that pit now containing, for example, 1 seed, the seeds in this pit and in the pit on the opposite side of the board (the opponents pit) are captured.
- En-passant capture: while a player is sowing their seeds, a capture can occur if any of their own pits now contain a specific number of seeds, for example, 4 seeds.
- Store capture: while a player is sowing their seeds, if they pass over their own store, they capture one seed.

The *number* and *place* captures can also be augmented by checking the pits preceding the captured pit. If the preceding pits also fulfil the same criteria for a capture in an unbroken sequence of pits, they too can also be captured by the sowing player.

III. Related Work

One of the main aims when researching into AI and games is the solving of games by verifying the game-theoretic value of the game. Only two games in the mancala family have been solved so far. The first game to be solved was the game of Kalah (solved by Irving et al [5]). Small versions (in terms of seed amount) were strongly solved while larger versions were only weakly solved. With larger versions, a simple heuristic function was used in helping the search process; the number of seeds captured minus the number of seeds captured by the opponent. The game of Awari was then strongly solved by Roemin and Bal [7]. The solving of Awari was a tougher task than the solving of Kalah. It is the opinion of the researchers this occurs because of the difference in rules between the two games.

Despite the success of exhaustive search, heuristics still had to be used in the weakly solving of Kalah. With the amount of Mancala games in existence and only a small fraction of games solved through the use of exhaustive search, it is unknown if all the variants have search spaces and game-tree complexities that are low enough for exhaustive search to be of practical use. There may still be the need for the use of heuristics to guide the search in Mancala variants. Researchers must make use of these because the state space of a problem is too large for exhaustive search algorithms to be used due to time and resource limitations. Heuristics can vary in complexity to simple rules of thumb to more advanced rules that require a substantial look ahead.

Numerous papers have looked into the use of heuristics in the games of Awari and Kalah. Kendall and Davis [3] evolved an Awari player that can play the game at a reasonably high level. The Awari player developed uses a search tree with a depth of seven moves. A mini-max search algorithm is then used to decide which move the Awari player should take. The value put on the nodes in the search tree is calculated via an evaluation function. This evaluation function is based on a set of six heuristics. In the evaluation function, each heuristic has a weight associated with it. These weights [w1...w6] can range from -1 to +1. The heuristics and their weights are used in the evaluation function. A co-evolutionary approach is used to discover the weights to be assigned to each heuristic. The higher the weight ,the bigger the potential contribution of that heuristics to the evaluation function.

Another approach adopting heuristics by Daoud et al [2] attempts to improve the evaluation heuristics used by Davis et al [3]. They demonstrate that good knowledge representation of a problem with a small look-ahead is superior to a poor knowledge representation with a large look-ahead. They used the six heuristics used by Kendall et al [3] and added six more heuristics with a smaller look-ahead (three and five compared to seven move look-ahead) in the evaluation function. The heuristics and weights [w1...w12] with a range between 0 and 1 are used in the evaluation function.

Jordan and O'Riordan [6] conducted research into the use of strategies into the game of Kalah. The researchers call the game of Kalah by the name of Bantumi. They test these heuristics with Kalah played with 3, 4, 5 and 6 seeds per pit at the start of the game. The heuristics in the research require a look-ahead of just one move or two moves. The first test was to identify which single heuristic had the strongest performances from the set of heuristics designed. A round-robin tournament was used to fulfill this aim. Secondly, a genetic algorithm was used to identify the optimal linear ordering of these heuristics.

Gifford et al [1] also have researched the use of heuristics in the game of Kalah (6,4). Six heuristics were designed and used in an evaluation function. To decide which move to take, a search tree is built with a look-ahead of six and a mini-max search method used with Alpha-Beta pruning. An evaluation function is used to assign values to the leaves in the bounded search tree. This evaluation function uses one, or a combination of, heuristics to decide which move to make. The aim of the research was to discover both the strongest single heuristic and the strongest combination of heuristics. A round-robin tournament was again used to judge the strength of the heuristics and heuristic combinations.

From the above research, the strongest heuristics are ones that deal with the number of seeds that have been captured in a game. In both Kalah and Awari there is some consistency across the performance of the heuristics. The strongest heuristic was the number of seeds that a player has captured in a game. This heuristic was identified as the strongest of the round-robin tournament [1]. While in Awari, this heuristic had the highest weights returned on both runs of the experiments [2]. Similar heuristics that can be classified as attacking strategies were shown to be the stronger heuristics in other research into Kalah. Jordan and O'Riordan [6] showed that picking another pit that would lead to a player having another turn was the strongest heuristic found. Making this move will lead to a capture of one seed.

In Mancala, *hoarding* refers to keeping as many seeds in your own pits; this has the effect of limiting our opponents moves and increasing the number of seeds in your pits which in many variants are added to the number of seeds captured during the games. *Hoarding* type heuristics have been shown to be beneficial. Gifford et al [1] showed the benefit of having large amounts seeds in certain pits in a players own side of the board. Hoarding tactics were also identified in [6] to be the cause of games losses of the evolved linear order.

Some interesting strategic insights were made into the game of Awari when solved by Romein and Bal [7]. When a player has an opportunity to make a capture in a game, it is not always the best move that a player can make. When a player is in a position with a choice between a move that leads to a capture and a move that doesn't lead to a capture, for 22% of these positions it is better to take a position that doesn't lead to a capture. This indicates that there is a need for heuristics beyond the heuristics that deal with the number of seeds that have been captured in a game. Also, the best opening move in the game that a player can make in the game is to make a move from the rightmost pit on a player's side. All other opening moves lead to a player losing the game.

Some of the strongest heuristics discovered in the game of Kalah are specific to that game. In Kalah, a player is allowed to sow into their store and if the last seed is won into this store, then a player is allowed to take another turn at sowing seeds. Research [6] showed that strongest performing heuristic in this included picking a pit that will lead to a player taking another turn in a game. A heuristic such as this doesn't translate to the game of Awari as that option doesn't exist as part of the games rules.

Overall, it appears that there are some heuristics that can be applied from game to game that can lead a strong player of mancala. However, it is not known how robust or applicable these heuristics are in other mancala game variants. Identifying robust heuristics across variants would be a useful step in identifying general approaches to these games but to also allow further classification of these games in terms of relatedness or complexity.

IV. Methodology

The first task that needed to be accomplished was the selection of a sample of games in the Mancala family. Awari was picked as a base game. Even though the game of Awari being strongly solved [7], this game was picked due to the amount of previous research into the game and the research into heuristics [2], [3]. It will allow for the comparison of the results from our own research with research done previously. We then selected a set of related games with shared rule sets. These included the games of Oware, Érhérhé and Vai Lung Thlan. After some initial runs of the game simulator, a cap of 250 moves in a game was applied. All the games have the exact same rules as Awari except for the features described below:

- **Oware**: Captures can be made if the last pit sown is on the opponents side and if there are 2, 3 or 4 seeds in the pit. The seeds in any preceding pits that satisfy the same condition (having 2, 3 or 4 seeds) are also captured [8].
- **Érhérhé**: Captures can be made if the last pit sown is on the opponent's side and has 2 or 4 seeds. The seeds in any preceding pits that satisfy the same condition are also captured. This game typically has multiple rounds; we do not implement rounds and deem the player with the most seeds following one round to be the winner [8].
- **Vai Lung Thlan**: The game begins with 5 seeds per pit at the start of the game. Seeds are sown in a clockwise direction across the board. Captures are made if the final seed sown on a move is into a pit with 1 seed; seeds in preceding pits with the same condition are also captured [8]. One of the consequences of the capture rule is that the pieces are removed at a slower rate than the games of Awari, Oware and Érhérhé.

A set of heuristics that satisfied certain criteria were chosen to use in the experiments. Firstly, we wish to explore heuristics without a large look-ahead. Our goal is not to solve any variants of the game, but rather to explore robust heuristics and strategies. Secondly, we wish to select heuristics that are transferable between games[1]. Some of the strongest heuristics from the previous were picked along with heuristics that haven't been investigated before. The heuristics chosen are as follows:

- H1: *Hoard as many seeds as possible in one pit.* At the end of the game, all of these seeds in this hoarding pit will be moved into a players own store. This heuristic, with a look ahead of one move works by attempting to keep as many seeds as possible in the right-most pit on the board (given clockwise sowing). There is some evidence in the literature that this is a safer pit in which hoard seeds [1].
- H2: *Keep as many seeds on the players own side.* This heuristic is a generalised version of H1 and is included to investigate the benefit of hoarding seeds across all of a players pits.
- H3: *Have as many moves as possible from which to choose*. This, with a look ahead of one, is included

[1]One of the best performing heuristics in Kalah is to pick a move that will lead to another turn for a player. But this heuristic can only be used in games where a player can sow into their own store. This rule is not found in a game like Awari, so we exclude from our set

to explore whether there is a benefit to be gained by maintaining a diverse range of moves for a player to choose from.

- H4: *Maximise the amount of seeds in a players own store.* This heuristic aims to pick a move that will maximise the amount of seeds that a player has captured in a game. Previous research relating to maximising a players number of seeds a player has shown this form of heuristic performed well. It has a look ahead of one move.
- H5: *Move the seeds from the pit closest to the opponents side.* This heuristic, with a look ahead of one, aims to make a move from the pit closest to the opponents side of the board. If this pit is empty, then the next pit is checked if it can be played from It was chosen because of its good performance in the game of Kalah [6]. Further, in strongly solving the game of Awari, the only opening move that will lead to a player not losing a game, is to play the right most pit as the opening move.
- H6: *Keep the opponents score to a minimum*. This heuristic, with a look ahead of two moves, attempts to minimise the number of seeds an opponent can win on their next move.

The heuristics can be roughly categorised as follows: H1 and H2 are forms of a hoarding strategy that can be played in a game. H3 attempts to maximise the number of moves a player can make. H4 and H5 can be grouped as attacking heuristics, while H6 is a defensive heuristic. The range of potential return values for each heuristic function will vary from game to game because of the change in seed numbers in a game. The heuristic function returns for H5 will return a 1 for the first pit that seeds can be moved from and a 0 for the rest of the pits. Games with sowing direction of clockwise had alternative implementations for some of the heuristics. For example, H1 will aim to keep as many seeds a possible in the left most.

From the literature, a couple of competitive mechanisms and algorithms have been used to measure a heuristic performance. A round-robin tournament will be used to identify the strongest stand alone single heuristic. From the previous research [6], [1] a round-robin tournament provides a mechanism to evaluate a heuristic's strengths and weaknesses against the other heuristics. Each heuristic will be compare against the other heuristics, against itself and against a random strategy across the mancala games chosen. A random strategy was included as a baseline comparison to investigate if the heuristics are better than a random search through the state space. Each heuristic will take turns going both first and second so as to remove any bias in going first in a game. These round robin tournaments will be run across all the variants of mancala developed. If two or more pits return the same heuristic value, then one of said pits was picked at random.

In previous research, a weighted model was used to create strong combinations of heuristics [2], [3]. This experiment has the aim to discover the level of contribution each heuristic should make when all the heuristics are used together to develop an overall strong strategy. This is achieved by creating an evaluation function. In this case, the heuristics will all be considered at once with each heuristic having its own weight in the function. The higher an heuristic's weight, the higher the potential contribution that the heuristic can make in evaluating a position in the game. The values of these weights decide how well a player preforms in the game. In the evaluation function, each heuristic has a weight assigned to it. These weights [w1...w6] are in the range from 0 to 1. The following function is used to evaluate what value should be placed on a potential move:

$$f = H1w1 + H2w2 + H3w3 + H4w4 + H5w5 - H6w6$$

H6 and its weight will be subtracted in the function. H6 aims to estimate the most seeds an opponent can score after a player has sown their seeds.

The genetic algorithm uses a real number representation. The genetic algorithm runs for 250 generations with a population size of 50. The mutation rate is set to 0.1 and tournament selection is used to help prevent premature convergence on local optima. A Gaussian mutator is used. Uniform crossover is applied with a rate of 0.5.

The fitness of a candidate is based on how they compete against the rest of the population of weights. The use of co-evolutionary algorithms have been used with some success in previous research [2], [3]. The candidate will play five games going first and five games going second against the entire population including itself. One point is received for a win, 0.5 for a draw and zero for a loss. The fitness value returned is the percentage of points received out of all the points that were available to be won. The genetic algorithm library GAlib [10] was used in our research. This algorithm will be undertaken only in the game environment of Érhérhé. The genetic algorithm is run for twenty independent runs.

A series of experiments were then undertaken to discover which weighted player was the strongest one evolved. The set of weights from each run will be compared to a linear model of selecting heuristics used in previous research [6]. This model works by placing each heuristic in a linear order. If the heuristic can't be applied or can't make an improvement to the current position in a game, the algorithm moves onto the next heuristic in that linear order. A genetic algorithm will be run to discover the strong orders of heuristics in Érhérhé. The strongest weight from this experiment will labeled as our evolved strategy.

In the final experiment, we aim to test the robustness of the weighted evolved strategy in other mancala game environments. The weighted evolved strategy will play the single heuristics in the variants of Mancala that were developed. If the evolved player's performance remains strong throughout the alternative games then a robust strategy has been developed across a selection of mancala variants. This will be done first in the game of Érhérhé to display the strength of the evolved strategy in the game environment in which it was evolved. This will allow for the comparison of results with other game environments. This is then tested against the individual heuristics in the game environments of Oware, Awari and Vai Lung Thlan.

The next section will outline the results of our experiments. In summary, the experiments that were undertaken are as follows:

- Round-robin tournament involving all the heuristics across the four variants of mancala developed.
- A genetic algorithm will attempt develop an robust evolved strategy in the game of 'Erhérhé using the heuristics and a set of weights.
- Testing this evolved strategy strength in Erhérhé.
- Testing the evolved strategy for robustness in the other game environments (Oware, Awar and then Vai Lung Thlan).

V. Results

The first experiments that were undertaken were the round-robin tournaments. Certain trends emerged regarding the heuristic's performance. Across all of the mancala variants the heuristic H3 is by far the worst-performing heuristic that has been developed. It doesn't win the majority of games against any heuristic or even against a randomly selected strategy. The rest of the heuristics are all far superior to a random search. Of the two hoarding heuristics, H1 is stronger across all the games tested. In the games of Awari, Érhérhé and Oware, heuristics H6, H5 and H1 were the strongest. While in the game of Vai Lung Thlan, the hoarding heuristics (H1, H2) are strongest in this game with H1 is easily the best heuristic in this game.

TABLE I
Round-Robin Tournament Results

Game	Strongest Heuristics	Weakest Heuristic
Erhérhé	H6, H5 , H1	H3
Awari	H6, H5, H1	H3
Oware	H6, H5, H1	H3
Vai Lung Thlan	H1,H2	H3

The weighted genetic algorithm was run twenty times. The best performing solution was selected from the final generation of the algorithm in each of the twenty runs. The results are summarized in Table II, with all of the weight values rounded to 3 decimal places. Although the algorithm doesn't converge upon the same set of weights during the twenty runs, there are some trends observable in the data:

- The weight for H4 (w4) is consistently the highest weight or joint highest weight in the set. For all instances bar one the weight is evolved to the highest value it possibly can be.
- The performance of H3 on its own made it the worst of all the heuristics. It failed even against a random strategy. But the weight value returned from nine out of twenty runs returned a weight value that was over 0.5. It was frequently the fourth highest weight value. The weights for H1 and H2 never go over the value of 0.4.
- The fitness values are high. This may show that there are some weak solutions in the final generation of the genetic algorithm.
- The weight for H5 varies from one extreme to another, on four occasions it is the maximum value allowed and on one occasion it is the smallest value allowed.
- In comparing the weights for H4 (attacking) and H6 (defensive), it seems that there is more emphasis on attack than defensive in the game of Érhérhé.

TABLE II
Weighted Genetic Algorithm Results

Run	W1	W2	W3	W4	W5	W6	Fitness
1	0.386	0.107	0.359	1	0.813	1	58.8
2	0.232	0.179	0.469	1	0.639	0.629	59.8
3	0.329	0.316	0.382	1	1	0.665	64
4	0.199	0.190	0.371	1	0.419	0.566	60
5	0.272	0.258	0.587	1	1	0.751	59.8
6	0.308	0.170	0.717	1	1	0.924	57.5
7	0.214	0.148	0.489	1	0.561	0.575	61.4
8	0.233	0.09	0.58	1	0.687	0.65v	61.1
9	0.0436	0.107	0.496	1	0.523	0.59	65.1
10	0.241	0.09	0.681	1	0.641	0.741	61.6
11	0.320	0.227	0.596	1	0.911	0.830	60.3
12	0.375	0.332	0.374	1	0	0.450	64
13	0.044	0	0.373	1	0.173	0.761	62.7
14	0.351	0.253	0.592	1	0.765	0.842	61
15	0.373	0.21	0.389	1	0.468	0.999	60.1
16	0.301	0.3	0.607	1	0.818	0.821	64
17	0.244	0.24	0.525	1	1	0.62	66.4
18	0.254	0.159	0.439	0.862	0.651	0.573	59.9
19	0.237	0.0286	0.492	1	0.67	0.596	62.3
20	0.126	0.104	0.564	1	0.51	0.605	63.2

The next experiment was designed to discover the strongest solution from the twenty runs of the genetic algorithm. The evolved weights were played against the linear order in a thousand games of Érhérhé. The weighted model was far superior, with it winning more than 87% of the games. The set of weights with the highest win rate will be tested for its robustness across the other mancala variants developed. The following subsections outline the results of the evolved strategy across the mancala variants. The percentage values in the table represent the how the evolved strategy performed against the individual heuristics. A thousand games of the evolved player going first per heuristic and a thousand games of the evolved player going second per heuristic are undertaken in order to judge the performance of the evolved strategy. The strongest solution from this experiment is outlined in Table III.

Evolved Strategy in Érhérhé: The evolved player is very strong in this environment. This is as expected as the evolved player was evolved using this games rules. Against H3, H4, H5, H6 and a random strategy it wins between 97% and 100% of the games played. It performs worst against the hoarding

TABLE III
EVOLVED STRATEGY

W1	W2	W3	W4	W5	W6
0.198649	0.190084	0.370793	1	0.418841	0.565937

heuristic H1 but, the evolved player still wins 81.5% of games going first and 77.6% of games going second.

TABLE IV
EVOLVED STRATEGY IN ÉRHÉRHÉ

	Evolved Strategy Going 1st			Evolved Strategy Going 2nd		
	Wins	Losses	Draws	Wins	Losses	Draws
H1	81.5%	16.5%	2%	77.6%	20%	2.4%
H2	91%	7%	2%	88.8%	10%	1.2%
H3	99.6%	0.1%	0.3%	100%	0%	0%
H4	97.7%	2%	0.3%	97.7%	2.1%	0.2%
H5	99.5%	0.5%	0%	99.9%	0.1%	0%
H6	98.9%	1.1%	0%	99%	0.9%	0.1%
Random	99.8%	0.1%	0.1%	99.9%	0%	0.1%

Evolved Strategy in Oware: The evolved player remains strong in the environment of Oware. The evolved player actually has higher win rates in this game than in the game of Érhérhé. The evolved player wins at least 84.9% of games against all the heuristics. And against H3 and random, it wins over 99% of the games going first and second. Going second against H5, it wins 100% of all games.

TABLE V
EVOLVED STRATEGY IN OWARE

	Evolved Strategy Going 1st			Evolved Strategy Going 2nd		
	Wins	Losses	Draws	Wins	Losses	Draws
H1	84.9%	13.7%	1.4%	85.7%	12.6%	1.7%
H2	89.2%	10%	0.8%	90%	9.1%	0.9%
H3	99.1%	0.5%	0.4%	99.8%	0.1%	0.1%
H4	96.2%	3.1%	0.7%	97.1%	2.5%	0.4%
H5	81.2%	18.8%	0%	100%	0%	0%
H6	95.4%	4.3%	0.3%	95.3%	4.5%	0.2%
Random	99.9%	0.1%	0%	99.8%	0.2%	0%

Evolved Strategy in Awari: The evolved player also performs strongly in this game environment. Again, the evolved player wins at least 84% of games against all the heuristics. And against H3 and random, it wins over 99% of the games going first and second.

Evolved Strategy in Vai Lung Thlan: The performance of the evolved player doesn't remain high in this game. Against H1 and H2, the evolved player fails to win the majority of games. Even against the random strategy, the performance of the evolved player isn't as strong as it is against the random strategy in the other games.

The following is a summary of the findings from the results of the experiments:

TABLE VI
EVOLVED STRATEGY IN AWARI

	Evolved Strategy Going 1st			Evolved Strategy Going 2nd		
	Wins	Losses	Draws	Wins	Losses	Draws
H1	84.1%	14.9%	1%	87.2%	11.6%	1.2%
H2	89.2%	10.2%	0.6%	89.1%	10.1%	0.8%
H3	99%	0.9%	0.1%	99.1%	0.6%	0.3%
H4	95.1%	4%	0.9%	97.2%	2.5%	0.3%
H5	93.1%	6.9%	0%	99.3%	0%	0.7%
H6	97.3%	2.7%	0%	96.8%	2.8%	0.4%
Random	100%	0%	0%	99.7%	0.3%	0%

TABLE VII
EVOLVED STRATEGY IN VAI LUNG THLAN

	Evolved Strategy Going 1st			Evolved Strategy Going 2nd		
	Wins	Losses	Draws	Wins	Losses	Draws
H1	21.3%	77.3%	1.4%	22%	75.8%	2.2%
H2	0%	100%	0%	0%	100%	0%
H3	96.9%	2.8%	0.3%	98.4%	1.1%	0.5%
H4	85.5%	13.1%	1.4%	86.8%	11.1%	2.1%
H5	100%	0%	0%	86.9%	11.9%	1.2%
H6	97.8%	2%	0.2%	98.9%	0.7%	0.4%
Random	97.6%	2%	0.4%	98.7%	1.1%	0.2%

- The evolved player performs very strongly across the games of Érhérhé, Awari and Oware. It comprehensively defeats all the heuristics across all of these games. But the evolved player fails to remain robust in the games of Vai Lung Thlan.
- The result of the evolved player confirms earlier round-robin tournament results that the Oware, Érhérhé and Awari are similar games. The performance of the evolved strategy in the game of Vai Lung Thlan also demonstrates that there is a difference in what constitutes a good combination of heuristics in Vai Lung Thlan.
- In the round-robin tournament H4 had an above average performance, but never came out as the strongest performing heuristic in any game. But with the best performing solutions from the weighted genetic algorithm results, this heuristic always had one of the highest weights. Having to pick a random pit a portion of turns during a game reveals a limitation to this heuristic when looked at in isolation. This reveals that availability of captures in a game is not too frequent but is the most important aspect of the game.

VI. CONCLUSIONS

With our research, we were able to identify a set of six heuristics that were valid across a variety of games in the mancala family. From our round-robin tournament, we showed that five of the six were superior to a random search across the four variants of Mancala developed. Some interesting insights can be made when comparing the performance of heuristics in the round-robin tournament to the weights returned from the evolutionary algorithm. H4 has limited use as a heuristic on its own, but when used with others, it always contributes to a strong combination of heuristics. Even H3 which was easily the worst heuristic, returns a relatively large weight from the genetic algorithm.

We also demonstrated the limitations of evolving a robust strategy that will remain consistently strong across a variety of mancala games. The evolved strategy from Érhérhé only remained strong in the game environments of Awari and Oware while in Vai Lung Thlan there was a considerable reduction in performance. It appears from our research that a variation in a mancala game that may appear minimum, can have a large effect on a strategy's effect in a game.

Returning to the solving of mancala games, heuristics still are being used by researchers. When large versions of Kalah were weakly solved [5], a basic heuristic which counted the number of seeds a player had captured minus the number of seeds an opponent had captured was used. With our research, we have demonstrated the limitations of only counting the number of seeds captured in a game as a suitable heuristic when reducing the search space.

VII. Future Work

With the wide variety of games in the mancala family, there still a vast amount of games that almost no research has been done on. With Kalah and Awari being solved, it is unknown which mancala game which would be worth solving next. And which game, within reasonable resources, is possible to be solve next. Identifying robust heuristics across variants would be a useful step in identifying general approaches to these games but to also allow further classification. Our results have shown that the games of Awari, Érhérhé and Oware can possibly be grouped together due to their game playing strategy compatibility. Can we group some of the mancala games by complexity relatedness by examining a games rules and varying game strategies? Questions like this have been brought up throughout the mancala literature [4]. Answering this question may allow researchers to concentrate on games 'worth' solving rather than waste precious time and resources on games all ready within our bounds of solvability.

References

[1] Dayo Ajayi Chris Gifford, James Bley and Zach Thompson. Searching and game playing: An artificial intelligence approach to mancala, technical report. Technical Report ITTC-FY2009-TR-03050-03, Information Telecommunication and Technology Center, Universityof Kansas, Lawrence, KS, 2008.

[2] M. Daoud, N. Kharma, A. Haidar, and J. Popoola. Ayo, the awari player, or how better representation trumps deeper search. In *Evolutionary Computation, 2004. CEC2004. Congress on*, volume 1, pages 1001 – 1006 Vol.1, june 2004.

[3] J.E. Davis and G. Kendall. An investigation, using co-evolution, to evolve an awari player. In *Evolutionary Computation, 2002. CEC '02. Proceedings of the 2002 Congress on*, volume 2, pages 1408 –1413, 2002.

[4] H. H. L. M. Donkers, J. W. H. M. Uiterwijk, and A. de Voogt. Mancala games: Topics in mathematics and artificial intelligence. page 133 146. Edition Universitaire, 2001.

[5] Geoffrey Irving, Jeroen Donkers, and Jos Uiterwijk. Solving kalah. *ICGA Journal*, 2000.

[6] Damien Jordan and Colm O'Riordan. Evolution and analysis of strategies for mancala games. In *GAMEON*, 2011.

[7] John W. Romein and Henri E. Bal. Solving the game of awari using parallel retrograde analysis. *IEEE Computer*, Vol.36:26 33, 2003.

[8] Laurence Russ. *Mancala Games (The Folk Games Series, No.1)*. Reference Publications, 1984.

[9] H. Jaap van den Herik, Jos W. H. M. Uiterwijk, and Jack van Rijswijck. Games solved: now and in the future. *Artif. Intell.*, 134(1-2):277–311, January 2002.

[10] Matthew Wall. Galib: A c++ library of genetic algorithm components. http://lancet.mit.edu/ga/, December 2012.