

The University of Yonsei

Faculty of Industrial Engineering

Tsoding MachineLearning In C

Lim Dohyun

June 10, 2023

1 Introduction

안녕하세요 한국말이 처음 입니다.

2 Gradient Descent

$$C'(w) = \lim_{\epsilon \rightarrow 0} \frac{C(w + \epsilon) - C(w)}{\epsilon} \quad (1)$$

2.1 Twice

sequence of derivating $C(w)$ with respect to w .

$$C(w) = \frac{1}{n} \sum_{i=1}^n (x_i w - y_i)^2 \quad (2)$$

$$C'(w) = \left(\frac{1}{n} \sum_{i=1}^n (x_i w - y_i)^2 \right)' \quad (3)$$

$$= \frac{1}{n} \left(\sum_{i=1}^n (x_i w - y_i)^2 \right)' \quad (4)$$

$$= \frac{2}{n} \sum_{i=1}^n (x_i w - y_i)(x_i)' \quad (5)$$

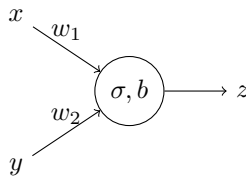
Cost function

$$C(w) = \frac{1}{n} \sum_{i=1}^n (x_i w - y_i)^2 \quad (6)$$

Derviative of Cost function

$$C'(w) = \frac{2}{n} \sum_{i=1}^n (x_i w - y_i)(x_i)' \quad (7)$$

2.2 One Neuron Model with 2 inputs



$$y = \sigma(xw_1 + yw_2 + b) \quad (8)$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (9)$$

$$\sigma'(x) = \sigma(x)(1 - \sigma(x)) \quad (10)$$

2.2.1 Cost

subscript i is referring to a sample number

$$a_i = \sigma(x_i w_1 + y w_2 + b) \quad (11)$$

$$\partial_{w_1} a_i = \partial_{w_1} (\sigma(x_i w_1 + y w_2 + b)) \quad (12)$$

$$= a_i(1 - a_i) \partial_{w_1} (x_i w_1 + y w_2 + b) \quad (13)$$

$$= a_i(1 - a_i) x_i \quad (14)$$

$$\partial_{w_2} a_i = a_i(1 - a_i) y_i \quad (15)$$

$$\partial_b a_i = a_i(1 - a_i) \quad (16)$$

$$C = \frac{1}{n} \sum_{i=1}^n (a_i - y_i)^2 \quad (17)$$

$$\partial_{w_1} C = \frac{1}{n} \sum_{i=1}^n \partial_{w_1} ((a_i - y_i)^2) \quad (18)$$

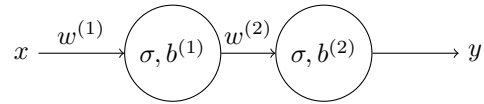
$$= \frac{1}{n} \sum_{i=1}^n 2(a_i - y_i) \partial_{w_1} a_i \quad (19)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i - y_i) a_i(1 - a_i) x_i \quad (20)$$

$$\partial_{w_2} C = \frac{1}{n} \sum_{i=1}^n 2(a_i - y_i) a_i(1 - a_i) y_i \quad (21)$$

$$\partial_b C = \frac{1}{n} \sum_{i=1}^n 2(a_i - y_i) a_i(1 - a_i) \quad (22)$$

2.3 Two Neuron Model with 1 inputs



superscript i is referring to a layer number

$$a_i^{(1)} = \sigma(x_i w^{(1)} + b^{(1)}) \quad (23)$$

$$\partial_{w^{(1)}} a_i^{(1)} = a_i^{(1)}(1 - a_i^{(1)})x_i \quad (24)$$

$$\partial_{b^{(1)}} a_i^{(1)} = a_i^{(1)}(1 - a_i^{(1)}) \quad (25)$$

$$a_i^{(2)} = \sigma(a_i^{(1)} w^{(2)} + b^{(2)}) \quad (26)$$

$$\partial_{w^{(2)}} a_i^{(2)} = a_i^{(2)}(1 - a_i^{(2)})a_i^{(1)} \quad (27)$$

$$\partial_{b^{(2)}} a_i^{(2)} = a_i^{(2)}(1 - a_i^{(2)}) \quad (28)$$

$$\partial_{a_i^{(1)}} a_i^{(2)} = a_i^{(2)}(1 - a_i^{(2)})w^{(2)} \quad (29)$$

$$C^{(2)} = \frac{1}{n} \sum_{i=1}^n (a_i^{(2)} - y_i)^2 \quad (30)$$

$$\partial_{w^{(2)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n \partial_{w^{(2)}} ((a_i^{(2)} - y_i)^2) \quad (31)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) \partial_{w^{(2)}} (a_i^{(2)}) \quad (32)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) a_i^{(1)} \quad (33)$$

$$\partial_{b^{(2)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) \quad (34)$$

$$\partial_{a_i^{(1)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) w^{(2)} \quad (35)$$

$$e_i = a_i^{(1)} - \partial_{a_i^{(1)}} C^{(2)} \quad (36)$$

$$C^{(1)} = \frac{1}{n} \sum_{i=1}^n (a_i^{(1)} - e_i)^2 \quad (37)$$

$$\partial_{w^{(1)}} C^{(1)} = \partial_{w^{(1)}} \left(\frac{1}{n} \sum_{i=1}^n (a_i^{(1)} - e_i)^2 \right) \quad (38)$$

$$= \frac{1}{n} \sum_{i=1}^n \partial_{w^{(1)}} \left((a_i^{(1)} - e_i)^2 \right) \quad (39)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(1)} - e_i) \partial_{w^{(1)}} a_i^{(1)} \quad (40)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(1)} - (a_i^{(1)} - \partial_{a_i^{(1)}} C^{(2)})) \partial_{w^{(1)}} a_i^{(1)} \quad (41)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) \partial_{w^{(1)}} a_i^{(1)} \quad (42)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) a_i^{(1)} (1 - a_i^{(1)}) x_i \quad (43)$$

$$\partial_{b^{(1)}} C^{(1)} = \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) \partial_{b^{(1)}} a_i^{(1)} \quad (44)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) a_i^{(1)} (1 - a_i^{(1)}) \quad (45)$$

2.4 Arbitrary Neurons Model with 1 inputs

2.4.1 Feed-Forward

$$a_i^{(1)} = \sigma(x_i w^{(1)} + b^{(1)}) \quad (46)$$

$$\partial_{w^{(1)}} a_i^{(1)} = a_i^{(1)} (1 - a_i^{(1)}) x_i \quad (47)$$

$$\partial_{b^{(1)}} a_i^{(1)} = a_i^{(1)} (1 - a_i^{(1)}) \quad (48)$$

$$a_i^{(2)} = \sigma(a_i^{(1)} w^{(2)} + b^{(2)}) \quad (49)$$

$$\partial_{w^{(2)}} a_i^{(2)} = a_i^{(2)} (1 - a_i^{(2)}) a_i^{(1)} \quad (50)$$

$$\partial_{b^{(2)}} a_i^{(2)} = a_i^{(2)} (1 - a_i^{(2)}) \quad (51)$$

$$\partial_{a_i^{(1)}} a_i^{(2)} = a_i^{(2)} (1 - a_i^{(2)}) w^{(2)} \quad (52)$$

$$(53)$$

2.4.2 Back-Propagation

$$C^{(2)} = \frac{1}{n} \sum_{i=1}^n (a_i^{(2)} - y_i)^2 \quad (54)$$

$$\partial_{w^{(2)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) a_i^{(1)} \quad (55)$$

$$\partial_{b^{(2)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) \quad (56)$$

$$\partial_{a_i^{(1)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) w^{(2)} \quad (57)$$

$$e_i = a_i^{(1)} - \partial_{a_i^{(1)}} C^{(2)} \quad (58)$$

$$C^{(1)} = \frac{1}{n} \sum_{i=1}^n (a_i^{(1)} - e_i)^2 \quad (59)$$

$$\partial_{w^{(1)}} C^{(1)} = \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) a_i^{(1)} (1 - a_i^{(1)}) x_i \quad (60)$$

$$\partial_{b^{(1)}} C^{(1)} = \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) a_i^{(1)} (1 - a_i^{(1)}) \quad (61)$$