

**The University of Yonsei**  
**Faculty of Industrial Engineering**  
Tsoding MachineLearning In C

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## 1 Introduction

안녕하세요 한국말이 처음 입니다.

## 2 Gradient Descent

$$C'(w) = \lim_{\epsilon \rightarrow 0} \frac{C(w + \epsilon) - C(w)}{\epsilon} \quad (1)$$

### 2.1 Twice

sequence of derivating  $C(w)$  with respect to  $w$ .

$$C(w) = \frac{1}{n} \sum_{i=1}^n (x_i w - y_i)^2 \quad (2)$$

$$C'(w) = \left( \frac{1}{n} \sum_{i=1}^n (x_i w - y_i)^2 \right)' \quad (3)$$

$$= \frac{1}{n} \left( \sum_{i=1}^n (x_i w - y_i)^2 \right)' \quad (4)$$

$$= \frac{2}{n} \sum_{i=1}^n (x_i w - y_i)(x_i)' \quad (5)$$

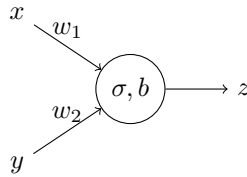
Cost funttion

$$C(w) = \frac{1}{n} \sum_{i=1}^n (x_i w - y_i)^2 \quad (6)$$

Derviative of Cost function

$$C'(w) = \frac{2}{n} \sum_{i=1}^n (x_i w - y_i)(x_i)' \quad (7)$$

## 2.2 One Neuron Model with 2 inputs



$$y = \sigma(xw_1 + yw_2 + b) \quad (8)$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \quad (9)$$

$$\sigma'(x) = \sigma(x)(1 - \sigma(x)) \quad (10)$$

### 2.2.1 Cost

subscript i is referring to a sample number

$$a_i = \sigma(x_i w_1 + y w_2 + b) \quad (11)$$

$$\partial_{w_1} a_i = \partial_{w_1} (\sigma(x_i w_1 + y w_2 + b)) \quad (12)$$

$$= a_i(1 - a_i) \partial_{w_1} (x_i w_1 + y w_2 + b) \quad (13)$$

$$= a_i(1 - a_i) x_i \quad (14)$$

$$\partial_{w_2} a_i = a_i(1 - a_i) y_i \quad (15)$$

$$\partial_b a_i = a_i(1 - a_i) \quad (16)$$

$$C = \frac{1}{n} \sum_{i=1}^n (a_i - y_i)^2 \quad (17)$$

$$\partial_{w_1} C = \frac{1}{n} \sum_{i=1}^n \partial_{w_1} ((a_i - z_i)^2) \quad (18)$$

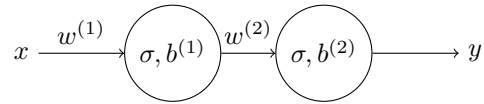
$$= \frac{1}{n} \sum_{i=1}^n 2(a_i - z_i) \partial_{w_1} a_i \quad (19)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i - z_i) a_i(1 - a_i) x_i \quad (20)$$

$$\partial_{w_2} C = \frac{1}{n} \sum_{i=1}^n 2(a_i - z_i) a_i(1 - a_i) y_i \quad (21)$$

$$\partial_b C = \frac{1}{n} \sum_{i=1}^n 2(a_i - z_i) a_i(1 - a_i) \quad (22)$$

### 2.3 Two Neuron Model with 1 inputs



superscript  $i$  is referring to a layer number

$$a_i^{(1)} = \sigma(x_i w^{(1)} + b^{(1)}) \quad (23)$$

$$\partial_{w^{(1)}} a_i^{(1)} = a_i^{(1)} (1 - a_i^{(1)}) x_i \quad (24)$$

$$\partial_{b^{(1)}} a_i^{(1)} = a_i^{(1)} (1 - a_i^{(1)}) \quad (25)$$

$$a_i^{(2)} = \sigma(a_i^{(1)} w^{(2)} + b^{(2)}) \quad (26)$$

$$\partial_{w^{(2)}} a_i^{(2)} = a_i^{(2)} (1 - a_i^{(2)}) a_i^{(1)} \quad (27)$$

$$\partial_{b^{(2)}} a_i^{(2)} = a_i^{(2)} (1 - a_i^{(2)}) \quad (28)$$

$$\partial_{a_i^{(1)}} a_i^{(2)} = a_i^{(2)} (1 - a_i^{(2)}) w^{(2)} \quad (29)$$

$$C^{(2)} = \frac{1}{n} \sum_{i=1}^n (a_i^{(2)} - y_i)^2 \quad (30)$$

$$\partial_{w^{(2)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n \partial_{w^{(2)}} ((a_i^{(2)} - y_i)^2) \quad (31)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) \partial_{w^{(2)}} (a_i^{(2)}) \quad (32)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) a_i^{(1)} \quad (33)$$

$$\partial_{b^{(2)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) \quad (34)$$

$$\partial_{a_i^{(1)}} C^{(2)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(2)} - y_i) a_i^{(2)} (1 - a_i^{(2)}) w^{(2)} \quad (35)$$

$$e_i = a_i^{(1)} - \partial_{a_i^{(1)}} C^{(2)} \quad (36)$$

$$C^{(1)} = \frac{1}{n} \sum_{i=1}^n (a_i^{(1)} - e_i)^2 \quad (37)$$

$$\partial_{w^{(1)}} C^{(1)} = \partial_{w^{(1)}} \left( \frac{1}{n} \sum_{i=1}^n (a_i^{(1)} - e_i)^2 \right) \quad (38)$$

$$= \frac{1}{n} \sum_{i=1}^n \partial_{w^{(1)}} ((a_i^{(1)} - e_i)^2) \quad (39)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(1)} - e_i) \partial_{w^{(1)}} a_i^{(1)} \quad (40)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(a_i^{(1)} - (a_i^{(1)} - \partial_{a_i^{(1)}} C^{(2)})) \partial_{w^{(1)}} a_i^{(1)} \quad (41)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) \partial_{w^{(1)}} a_i^{(1)} \quad (42)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) a_i^{(1)} (1 - a_i^{(1)}) x_i \quad (43)$$

$$\partial_{b^{(1)}} C^{(1)} = \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) \partial_{b^{(1)}} a_i^{(1)} \quad (44)$$

$$= \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(1)}} C^{(2)}) a_i^{(1)} (1 - a_i^{(1)}) \quad (45)$$

## 2.4 Arbitrary Neurons Model with 1 inputs

Let's assume that we have  $m$  layers.

### 2.4.1 Feed-Forward

Let's assume that  $a_i^{(0)}$  is  $x_i$

$$a_i^{(l)} = \sigma(a_i^{(l-1)}w^{(l)} + b^{(l)}) \quad (46)$$

$$\partial_{w^{(l)}} a_i^{(l)} = a_i^{(l)}(1 - a_i^{(l)})a_i^{(l-1)} \quad (47)$$

$$\partial_{b^{(l)}} a_i^{(l)} = a_i^{(l)}(1 - a_i^{(l)}) \quad (48)$$

$$\partial_{a_i^{(l-1)}} a_i^{(l)} = a_i^{(l)}(1 - a_i^{(l)})w^{(l)} \quad (49)$$

$$(50)$$

### 2.4.2 Back-Propagation

Let's denote  $a_i^{(m)} - y_i$  as  $\partial_{a_i^{(m)}} C^{(m+1)}$

$$C^{(l)} = \frac{1}{n} \sum_{i=1}^n (\partial_{a_i^{(l)}} C^{(l+1)})^2 \quad (51)$$

$$\partial_{w^{(l)}} C^{(l)} = \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(l)}} C^{(l+1)})a_i^{(l)}(1 - a_i^{(l)})x_i \quad (52)$$

$$\partial_{b^{(l)}} C^{(l)} = \frac{1}{n} \sum_{i=1}^n 2(\partial_{a_i^{(l)}} C^{(l+1)})a_i^{(l)}(1 - a_i^{(l)}) \quad (53)$$

$$\partial_{a_i^{(l-1)}} C^{(l)} = \frac{1}{n} \sum_{i=1}^n 2(a_i^{(l)} - y_i)a_i^{(l)}(1 - a_i^{(l)})w^{(l)} \quad (54)$$