

# **Progress Presentation**

**7.2. - 27.2.**

- self-organised criticality
- updates to the model
- representing games

### **Research question:**

What kind of strategies do civilisations in the universe employ to ensure their survival, and how do these strategies change over time and space?

# Self-organised Criticality

## Evolution of Cooperation in sudden jumps

### REVIEW

## Five Rules for the Evolution of Cooperation

Martin A. Nowak

Cooperation is needed for evolution to construct new levels of organization. Genomes, cells, multicellular organisms, social insects, and human society are all based on cooperation. Cooperation means that selfish replicators forgo some of their reproductive potential to help one another. But natural selection implies competition and therefore opposes cooperation unless a specific mechanism is at work. Here I discuss five mechanisms for the evolution of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. For each mechanism, a simple rule is derived that specifies whether natural selection can lead to cooperation.

Evolution is based on a fierce competition between individuals and should therefore reward only selfish behavior. Every gene, every cell, and every organism should be designed to promote its own evolutionary success at the expense of its competitors. Yet we observe cooperation on many levels of biology.

well-mixed populations needs help for establishing cooperation.

### Kin Selection

When J. B. S. Haldane remarked, “I will jump into the river to save two brothers or eight cousins” he anticipated what becomes later known

observe cooperation between unrelated individuals or even between members of different species. Such considerations led Trivers (10) to propose another mechanism for the evolution of cooperation, direct reciprocity. Assume that there are repeated encounters between the same two individuals. In every round, each player has a choice between cooperation and defection. If I cooperate now, you may cooperate later. Hence, it might pay off to cooperate. This game theoretic framework is known as the repeated Prisoner’s Dilemma.

But what is a good strategy for playing this game? In two computer tournaments, Axelrod (11) discovered that the “winning strategy” was the simplest of all, tit-for-tat. This strategy always starts with a cooperation, then it does whatever the other player has done in the previous round: a cooperation for a cooperation, a defection for a defection. This simple concept captured the fascination of all enthusiasts of the repeated Prisoner’s Dilemma. Many empirical and theoretical studies were inspired by Axelrod’s groundbreaking work.

Journal of the Korean Physical Society, Vol. 60, No. 6, February 2012, pp. 311~316

## Emergence of Cooperation with Self-organized Criticality

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(Received 18 August 2011)

Cooperation and self-organized criticality are two main keywords in current studies of evolution. We propose a generalized Bak-Sneppen model and provide a natural mechanism which accounts for both phenomena simultaneously. We use the prisoner’s dilemma games to mimic the interactions among the members in the population. Each member is identified by its cooperation probability, and its fitness is given by the payoffs from neighbors. The least fit member with the minimum payoff is replaced by a new member with a random cooperation probability. When the neighbors of the least fit one are also replaced with a non-zero probability, a strong cooperation emerges. The Bak-Sneppen process builds a self-organized structure so that the cooperation can emerge even in the parameter region where a uniform or random population decreases the number of cooperators. The emergence of cooperation is due to the same dynamical correlation that leads to self-organized criticality in replacement activities.

PACS numbers: 02.50.Le, 87.23.-n, 87.23.Kg

Keywords: Evolutionary-game theory, Cooperation, Self-organized criticality

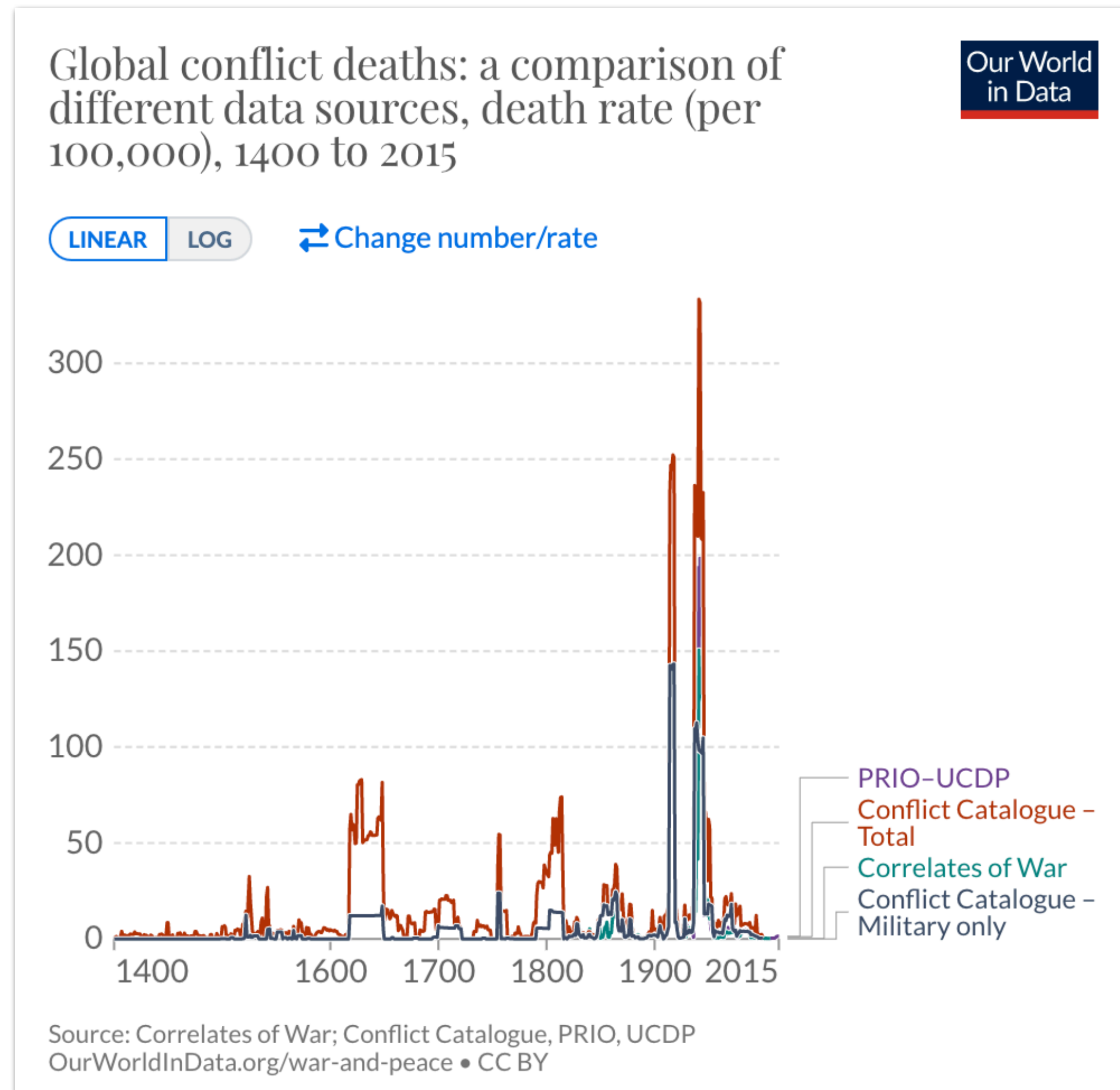
DOI: 10.3938/jkps.60.311

### 1. INTRODUCTION

Evolutionary game theory involves the spatio-temporal correlation between the least

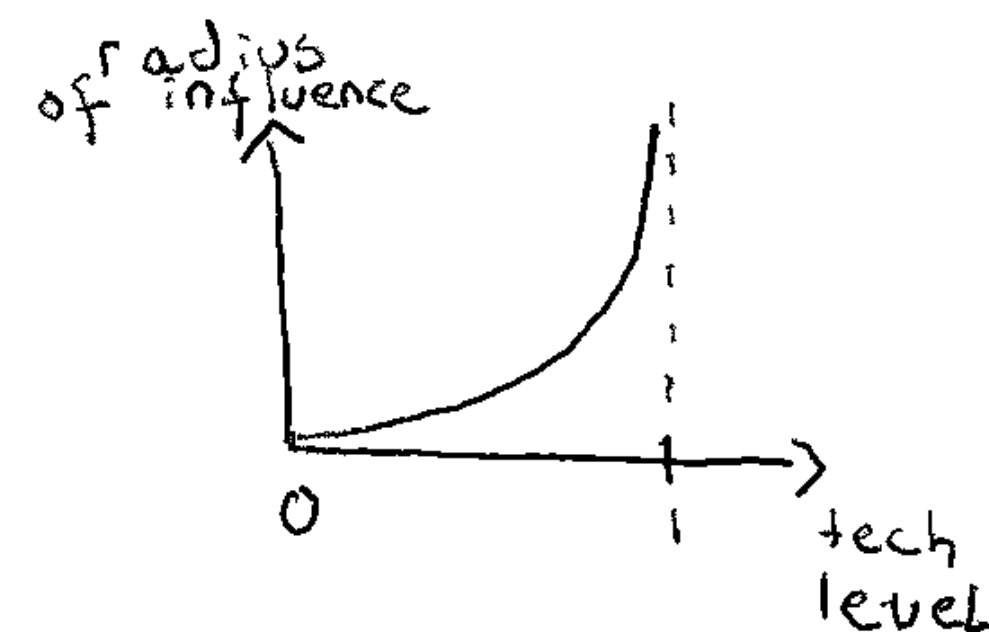
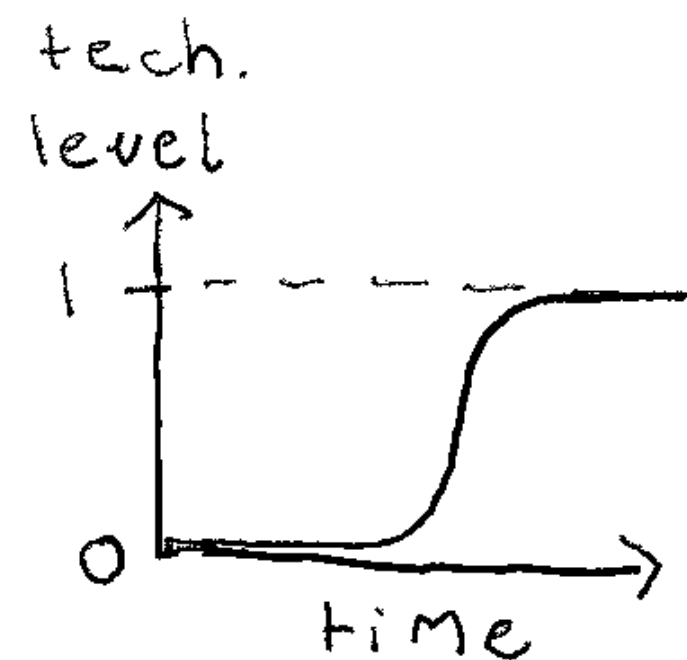
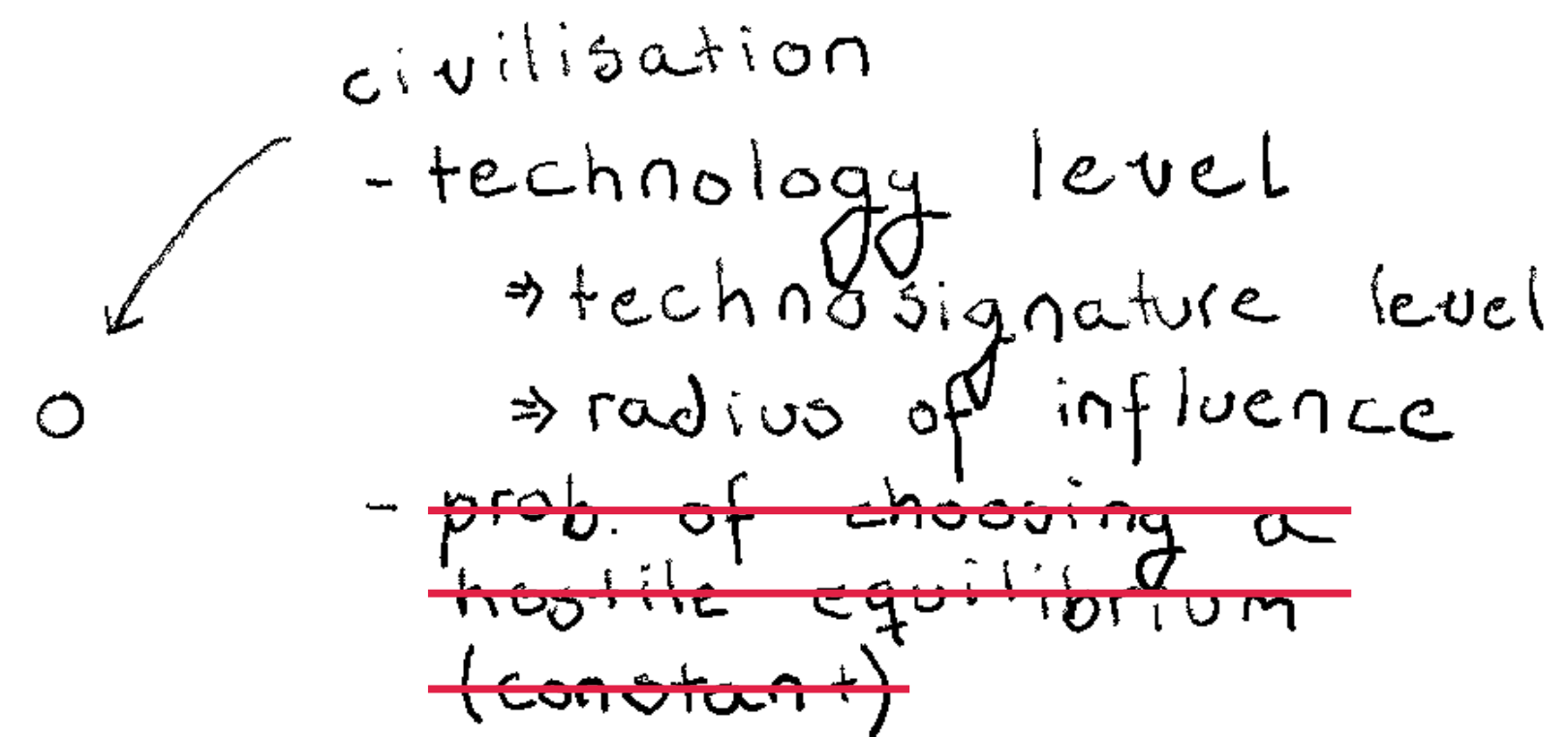
# Self-organised Criticality

## Evolution of Cooperation in sudden jumps



# Updates to the Model

## 1. Removed the prescribed tendency for hostility

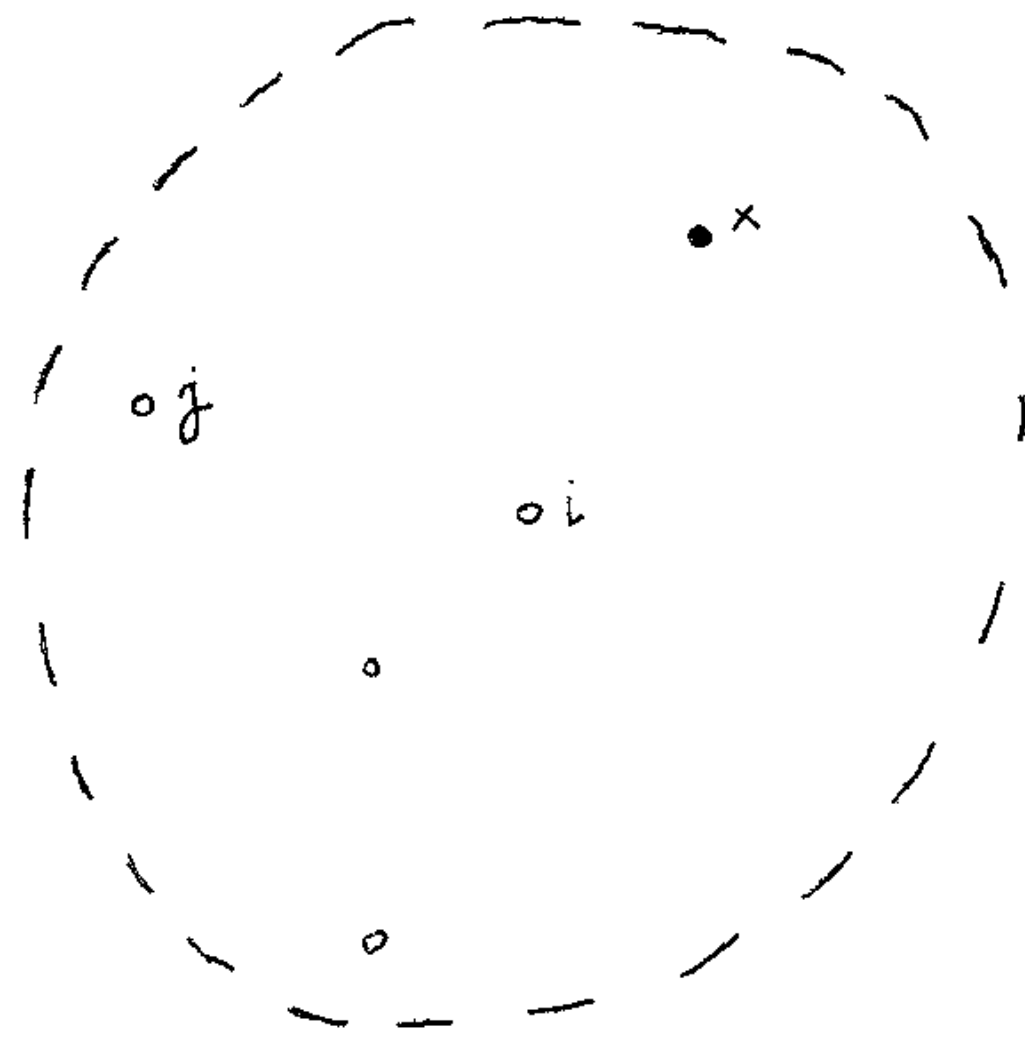




# Updates to the Model

## 2. Updating beliefs: hostility

*Def.* A civilisation is hostile if it has attacked another at any point in time.



*i* assumes that one of its neighbours destroyed *x*.

$h_j^{(i)}$  = probability that *j* has attacked someone at any point in the past (according to *i*)

attacked  $\neq$  destroyed

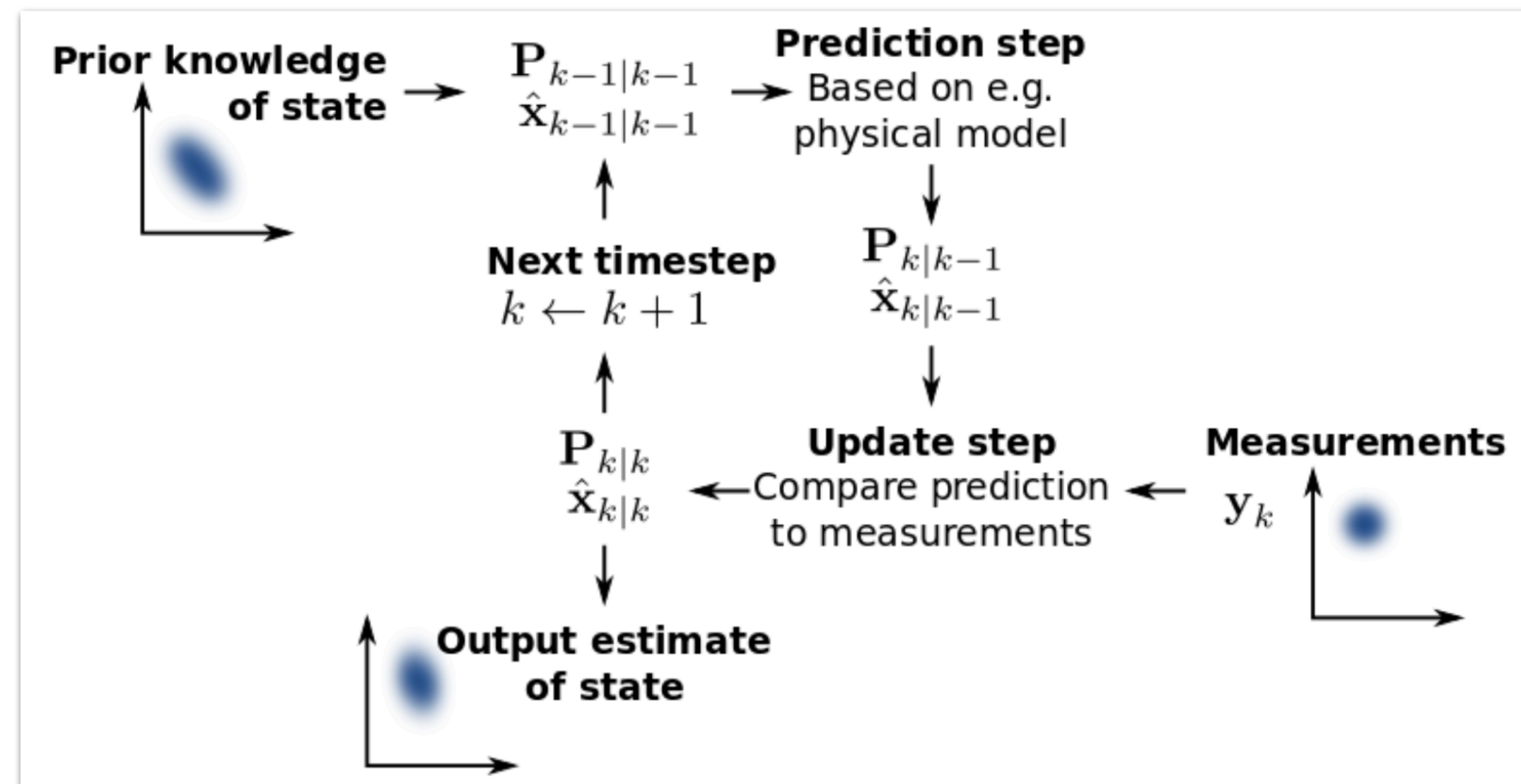
$$c_j^{(i)} = \mathbb{P}[t_j^{(i)} > \max(t_x^{(i)}, t_r)] \quad \text{where} \quad t_r = r^{-1}(d(x, j))$$

$$\hat{h}_j^{(i)} := \frac{c_j^{(i)} h_j^{(i)}}{\sum_k c_k^{(i)} h_k^{(i)}} \quad (\text{summation is over } i\text{'s neighbours})$$

# Updates to the Model

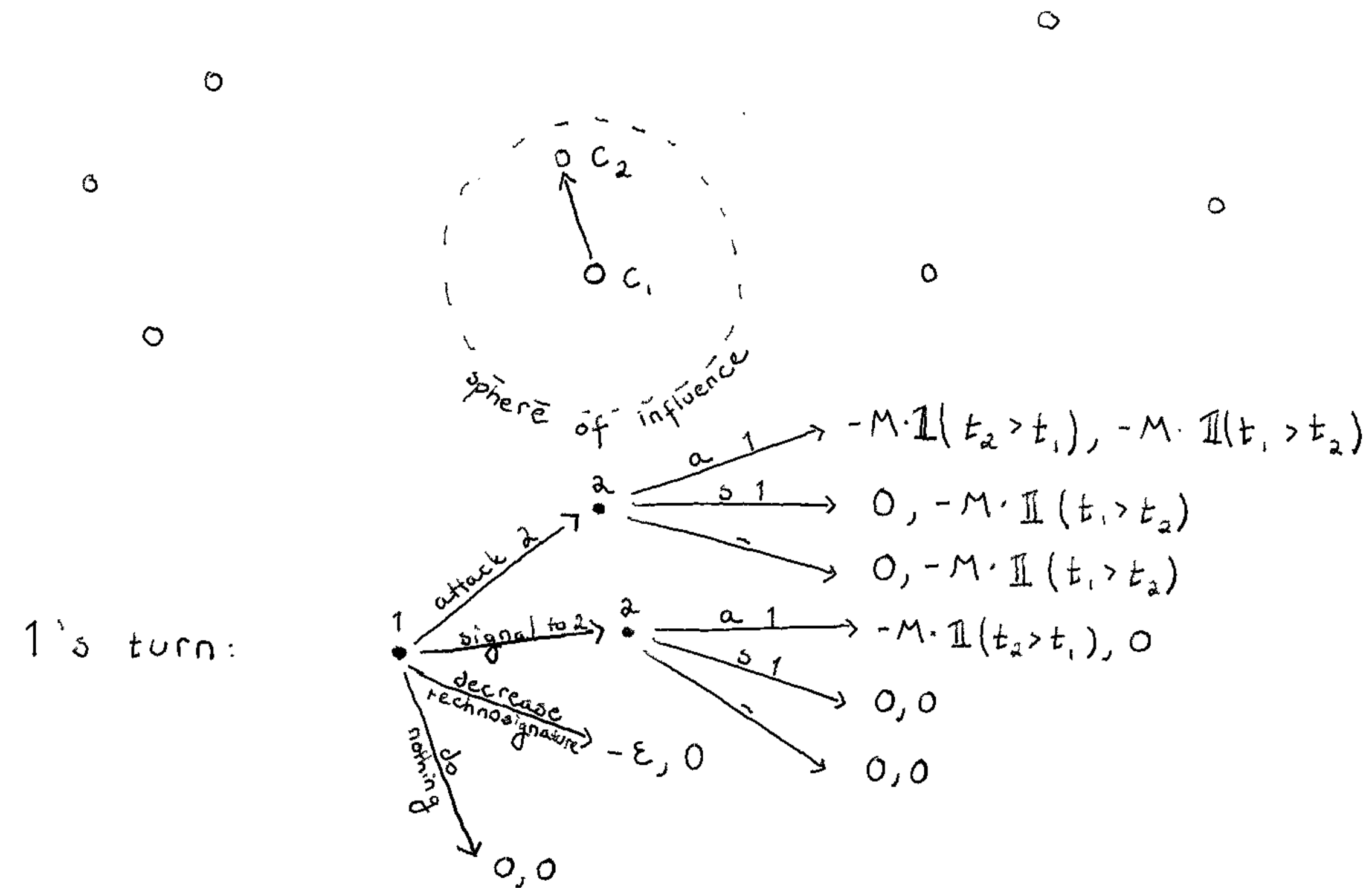
## 2. Updating beliefs: technology level

recursive Bayesian filtering / Kalman filter?



# Updates to the Model

## 3. Removed the "revenge" stage. Signalling?





# Updates to the Model

## 4. Some progress on decision-making

### Two ideas:

- randomly pick  $m$  neighbours  $c_1$  can observe
- risk-averse
  - $\alpha_1$  is the neighbour  $c_1$  thinks is the most dangerous to it
  - $\alpha_2$  is the neighbour ( $c_1$  thinks)  $\alpha_1$  thinks is most dangerous to it, etc.

sequence of interactions involving  $c_1$

$(c_1) \dots (c_j \rightarrow c_1) \dots (c_k \rightarrow c_1) \dots (c_1) \dots$  actual (unpredictable)

$(c_1) (\alpha_1) \dots (\alpha_m)$

hypothesised by  $c_1$  to  
decide what to do

# Representing Games

- Agents have lots of information (beliefs about others' technology levels, knowledge of own technology level, beliefs of others' hostility) that needs to be incorporated into the games
  - With extensive form games we would have to have splits (with chance nodes) on every one of these, which becomes difficult to visualise and think about

# Representing Games

## Multi-Agent Influence Diagrams



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Games and Economic Behavior 45 (2003) 181–221

[www.elsevier.com/locate/geb](http://www.elsevier.com/locate/geb)

GAMES and  
Economic  
Behavior

### Multi-agent influence diagrams for representing and solving games <sup>☆</sup>

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Received 26 April 2001

#### Abstract

The traditional representations of games using the extensive form or the strategic form obscure much of the structure of real-world games. In this paper, we propose a graphical representation for noncooperative games—*multi-agent influence diagrams* (MAIDs). The basic elements in the MAID representation are *variables*, allowing an explicit representation of dependence, or relevance, relationships among variables. We define a decision variable  $D'$  as *strategically relevant* to  $D$  if, to optimize the decision rule at  $D$ , the decision maker needs to consider the decision rule at  $D'$ . We provide a sound and complete graphical criterion for determining strategic relevance. We then show how strategic relevance can be used to decompose large games into a set of interacting smaller games, which can be solved in sequence. We show that this decomposition can lead to substantial savings in the computational cost of finding Nash equilibria in these games.

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JEL classification: C72; D82

### Equilibrium Refinements for Multi-Agent Influence Diagrams: Theory and Practice

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#### ABSTRACT

Multi-agent influence diagrams (MAIDs) are a popular form of graphical model that, for certain classes of games, have been shown to offer key complexity and explainability advantages over traditional extensive form game (EFG) representations. In this paper, we extend previous work on MAIDs by introducing the concept of a MAID subgame, as well as subgame perfect and trembling hand perfect equilibrium refinements. We then prove several equivalence results between MAIDs and EFGs. Finally, we describe an open source implementation for reasoning about MAIDs and computing their equilibria.

#### KEYWORDS

multi-agent influence diagrams; equilibrium refinements; extensive form games; probabilistic graphical models

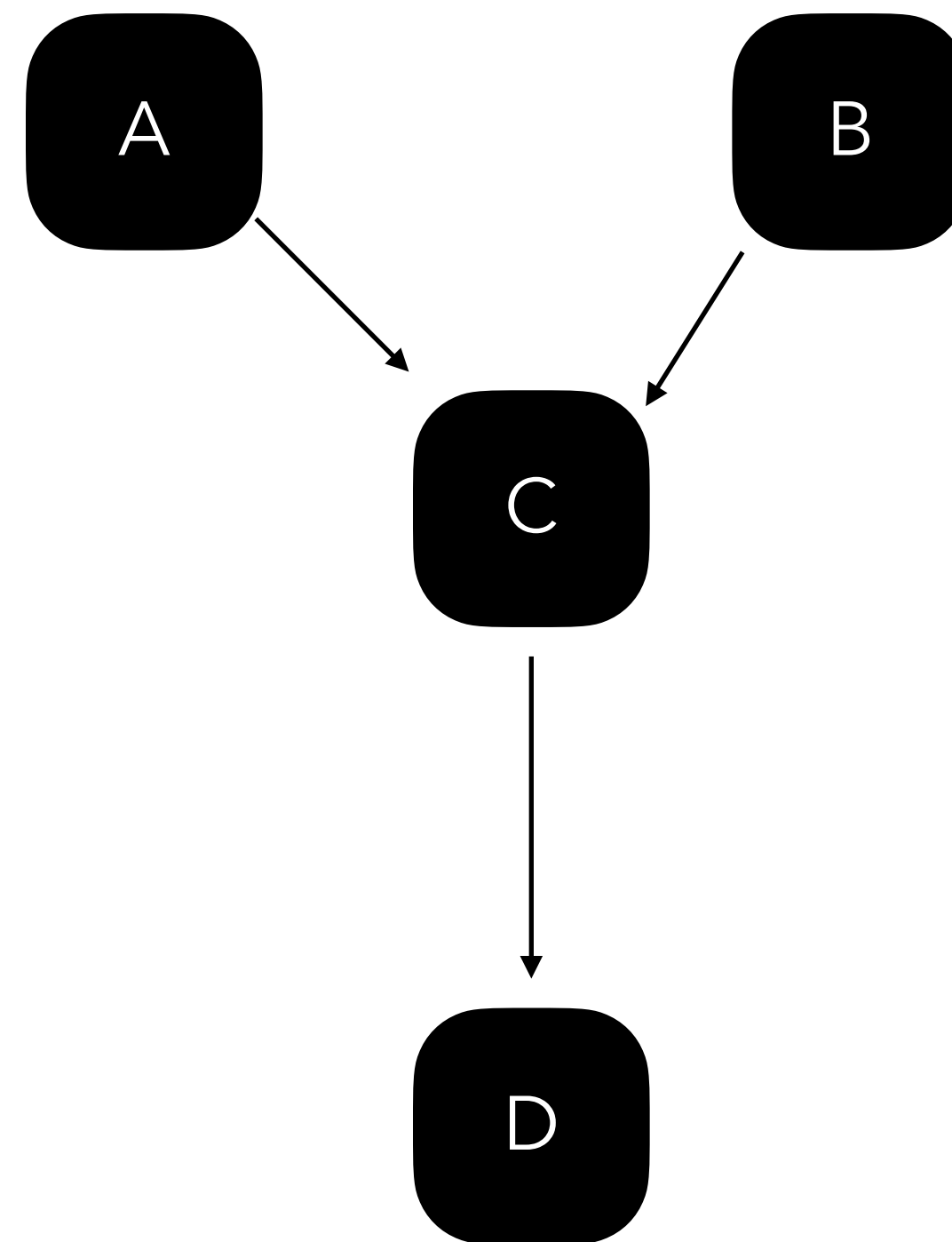
the theory and practical tools for MAIDs in order to allow both researchers and practitioners to make the most of their strengths.

Previous work on MAIDs has focussed on Nash equilibria as the core solution concept [20]. Whilst this is arguably the most important solution concept in non-cooperative game theory, if there are many Nash equilibria we often wish to remove some of those that are less ‘rational’. Many refinements to the Nash equilibrium have been proposed [17], with two of the most important being subgame perfect Nash equilibria [26] and trembling hand perfect equilibria [27]. The first rules out ‘non-credible’ threats and the second requires that each player is still playing a best-response when other players make small mistakes. On the practical side, while much software exists for normal or extensive form games, there is no such implementation for reasoning about games expressed as MAIDs, despite their computational advantages.

1 [cs.MA] 9 Feb 2021

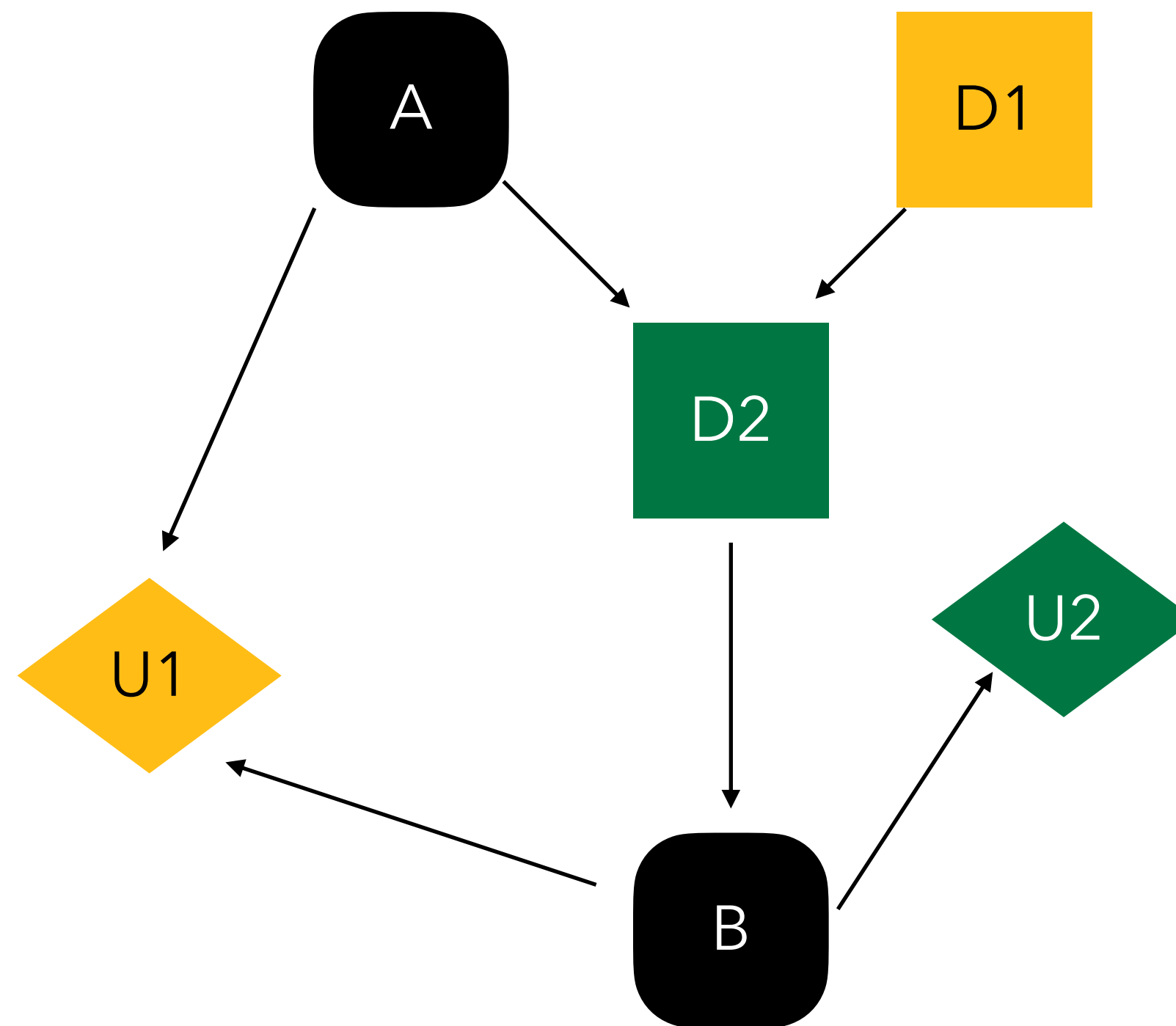
# Multi-Agent Influence Diagrams

## Bayesian Networks



$$P(a, b, c, d) = P(a)P(b)P(c \mid a, b)P(d \mid c)$$

# Multi-Agent Influence Diagrams



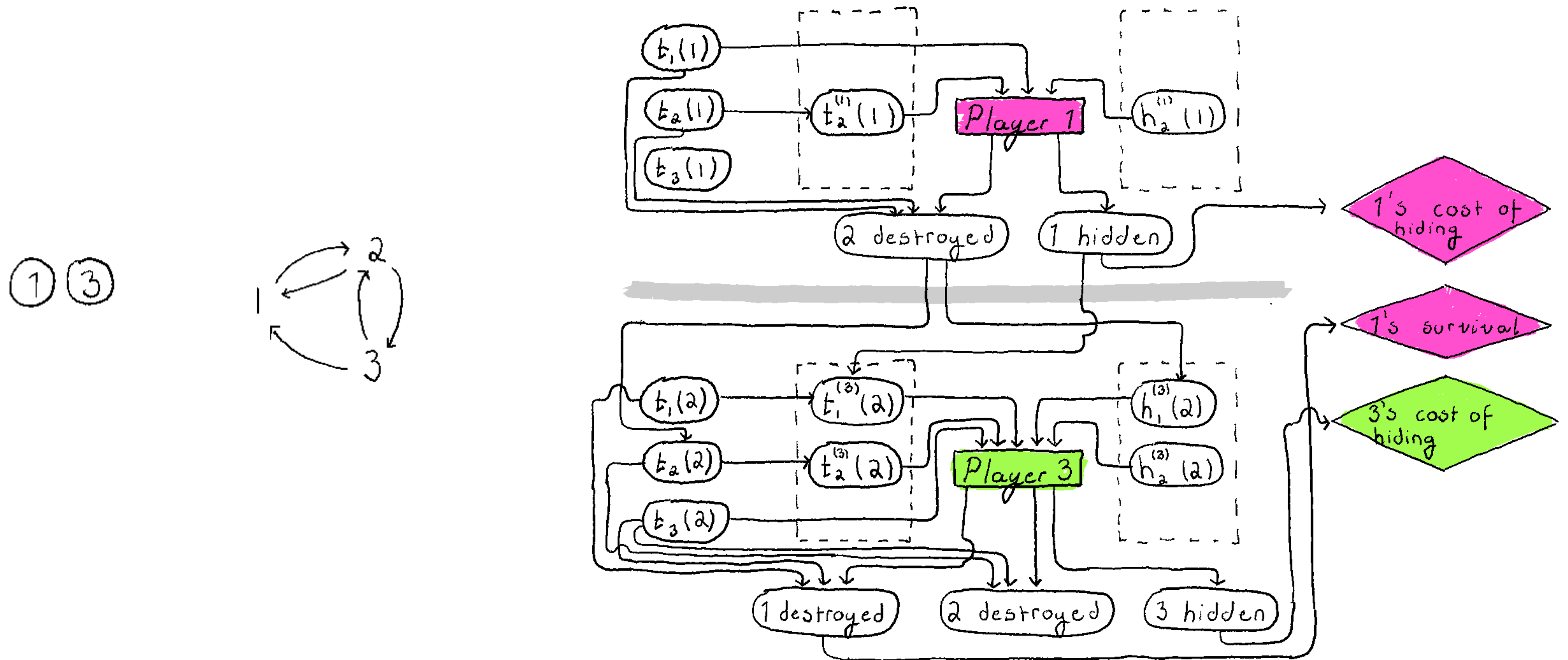
**Decision rule:** a conditional probability distribution (conditional on the values of the parents) for a decision variable

**Strategy of agent  $i$ :** decision rules for all decision variables of  $i$

**Strategy profile:** decision rules for all decision variables

# Multi-Agent Influence Diagrams

## Example Scenario





# Multi-Agent Influence Diagrams

## Tools

setup.cfg

remove unnecessary nashpy package from re...

5 months ago

setup.py

Configure version number from git tag

2 years ago

README.md

### PyCID: Causal Influence Diagrams library



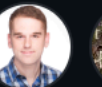



This package implements causal influence diagrams and methods to analyze them, and is part of the [Causal Incentives](#) project.

Building on [pgmpy](#) and [NetworkX](#), pycid provides methods for defining CBNs, CIDs and MACIDs, computing optimal policies in CIDs, pure and mixed Nash equilibria in multi-agent CIDs, studying the effects of interventions, and checking graphical criteria for various types of incentives.

### News

Version 0.8 breaks backwards compatibility by removing the get\_all\_pure\_ne

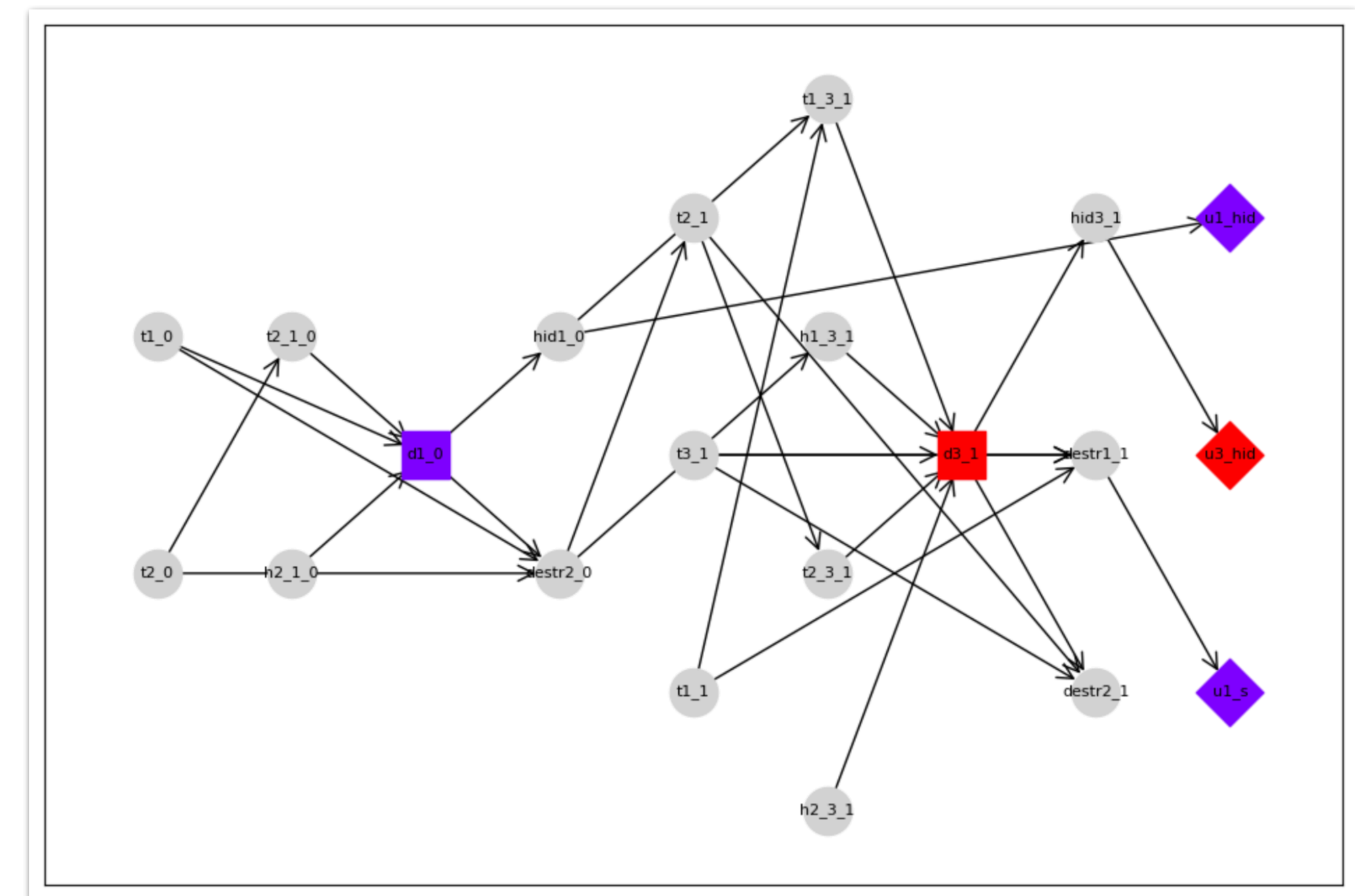
Contributors 6



Languages

Jupyter Notebook 81.8%

Python 18.2%



# Problems

- how should civilisations use their hostility beliefs in the hypothetical games?
  - maybe  $i$  assumes that the probability that  $j$  attacks is  $h_i^{(j)}$ ? But they can attack any neighbour, which one?
- so far we have assumed that the model is run in steps, and at each step a random civilisation gets to act. But is this too artificial?
  - if a civilisation feels unsafe and wants to e.g. hide, it would still have to wait for its turn in the model whereas in reality it would do so immediately