

RIVER PARISHES COMMUNITY COLLEGE

MATH 1100: COLLEGE ALGEBRA

EQUATIONS AND INEQUALITIES

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## 2.1 The Rectangular Coordinate System

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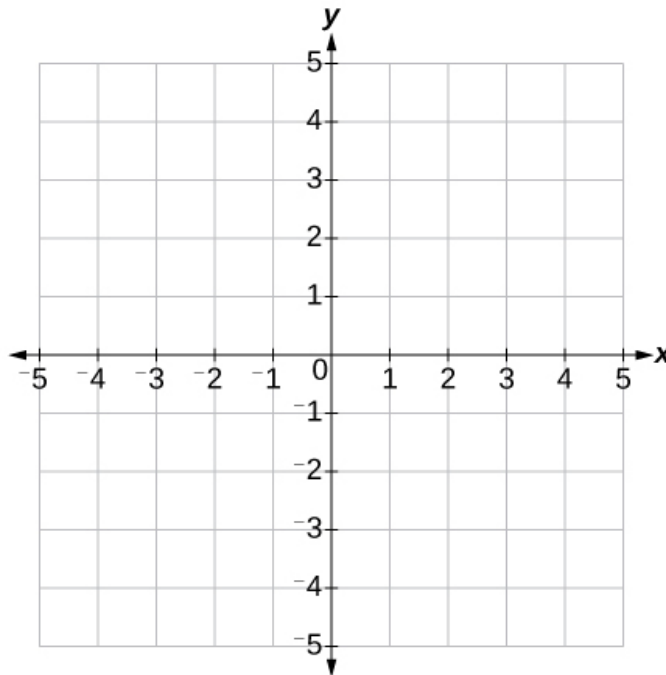
## Learning Objectives

In this section, you will learn:

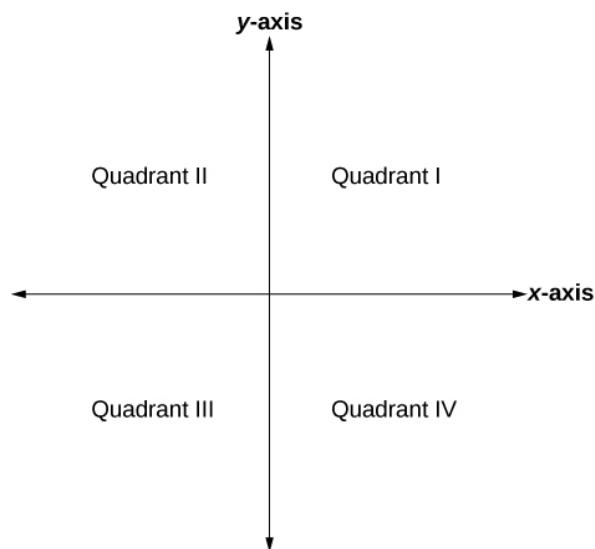
- ♣ Plot ordered pairs in a Cartesian coordinate system.
- ♣ Graph equations by plotting points.
- ♣ Find  $x$ -intercepts and  $y$ -intercepts.
- ♣ Use the distance formula.
- ♣ Use the midpoint formula.

## 1 Introduction : Cartesian Co-ordinate System

Imagine you have a sheet of paper in your hand. Now draw equally spaced horizontal lines and vertically lines perpendicular to each other all over the paper. Once you fix a point where the vertical and horizontal line intersect, you can go to any other intersecting points simply by moving left/right and up/down. This is an idea of tremendous importance first thought by the Great Mathematician **Rene Descartes**. This allows us to mathematically go from one point to another point in a sheet of paper or any geometrical plane.



The point we fixed is called origin. The horizontal line passing through the origin is called  $x$  - *axis*. The vertical line passing through the origin is called  $y$  - *axis*. Since the plane is divided into rectangles or squares, this is called *Rectangular Coordinate System* also known as *Cartesian Coordinate System*. Each squares or rectangles are called units. Each section is called quadrants.

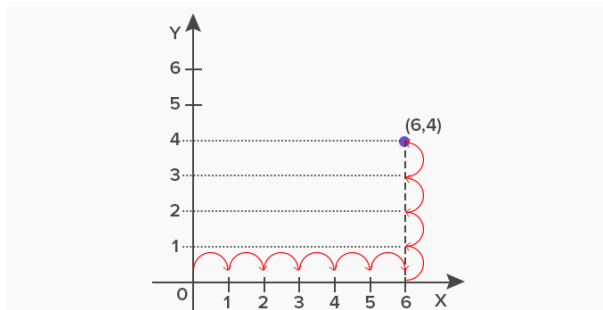


## 2 Order Pairs

There are several ways to reach to any point in the plane from the origin. You can first go up/down and left/right. You can also reach the same point by going left/right first , then up/down. So, by convention, we move horizontal (left or right) first , then we move vertically (up or down). This will establish that each point in the plane will be reached only one way.

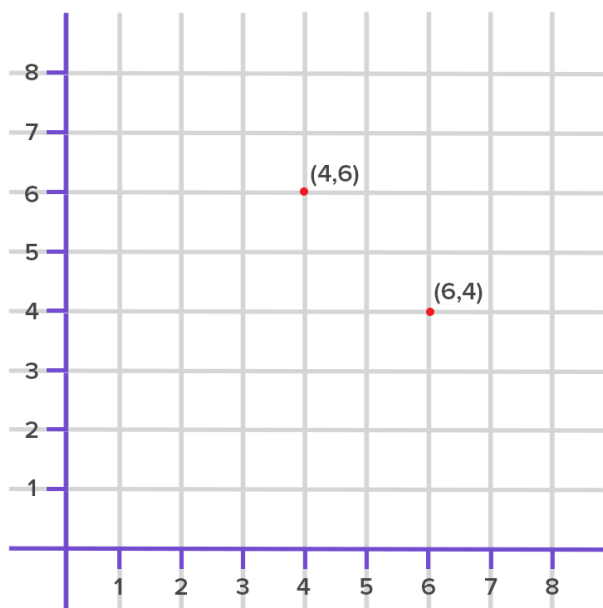
### Example

The point  $(6, 4)$  can be reached first by moving 6 units to the right first and then 4 units up.  
 The point  $(-6, 4)$  can be reached first by moving 6 units to the left first and then 4 units up.  
 The point  $(6, -4)$  can be reached first by moving 6 units to the right first and then 4 units down.  
 The point  $(-6, -4)$  can be reached first by moving 6 units to the left first and then 4 units down.



The point  $(x, y)$  is called *order pair* emphasizing that one moves  $x$  distance horizontally first, and then moves  $y$  distance vertically.  $x$  and  $y$  are called co-ordinates. So, each point in the plane is uniquely (only one way) identified by an order pair.

As the name “ordered pair” suggests, the order in which values are written in a pair is very important. The ordered pair  $(6, 4)$  is different from the pair  $(4, 6)$ . Both represent two different points as shown below.



Note that when either coordinate is zero, the point must be on an axis. If the x-coordinate is zero, the point is on the y-axis. If the y-coordinate is zero, the point is on the x-axis.

### 3 Graphing Equations by Plotting Points

Plotting Points is one of the important way to see how the graph looks like or its behaviour around a region. Since  $(x, y)$  consists of two variables, so we can fix one and find the other one. This means we can randomly choose some values for  $x$  and solve for  $y$  or vice versa.

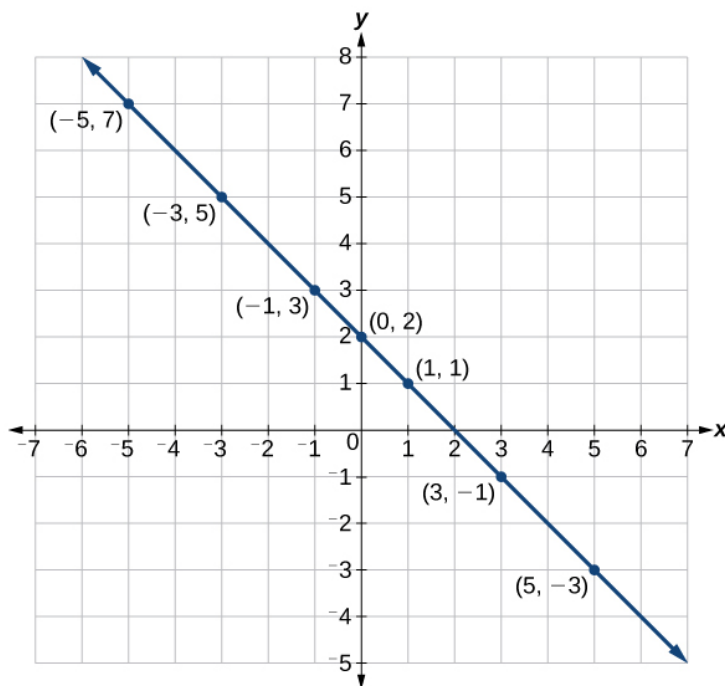
How to graph by plotting points given the equation ?

1. Make a table with one column labeled  $x$ , a second column labeled with the equation, and a third column listing the resulting ordered pairs.
2. Enter  $x$ -values down the first column using positive and negative values. Selecting the  $x$ -values in numerical order will make the graphing simpler.
3. Select  $x$ -values that will yield  $y$ -values with little effort, preferably ones that can be calculated mentally.
4. Plot the ordered pairs.
5. Connect the points if they form a line.

### Example

Graph the equation  $y = -x + 2$  by plotting points.

$x$	$y = -x + 2$	$(x, y)$
-1	$y = -(-1) + 2 = 3$	$(-1, 3)$
0	$y = -(0) + 2 = 2$	$(0, 2)$
1	$y = -1 + 2 = 1$	$(1, 1)$
2	$y = -(2) + 2 = 0$	$(2, 0)$



## 4 $x$ -intercepts and $y$ -intercepts

The intercepts of a graph are points at which the graph crosses the axes. The  $x$ -intercept is the point at which the graph crosses the  $x$ -axis. At this point, the  $y$ -coordinate is zero. The  $y$ -intercept is the point at which the graph crosses the  $y$ -axis. At this point, the  $x$ -coordinate is zero. To determine the  $x$ -intercept, we set  $y$  equal to zero and solve for  $x$ . Similarly, to determine the  $y$ -intercept, we set  $x$  equal to zero and solve for  $y$ .

Eg. Find the  $x$  and  $y$  intercepts of  $y = 2x + 6$

To find  $x$  intercepts set  $y = 0$ .

$$0 = 2x + 6$$

$$-6 = 2x$$

$$-3 = x$$

The graph cross the  $x$ -axis at  $(-3, 0)$ .

To find the  $y$ -intercept set  $x = 0$ .

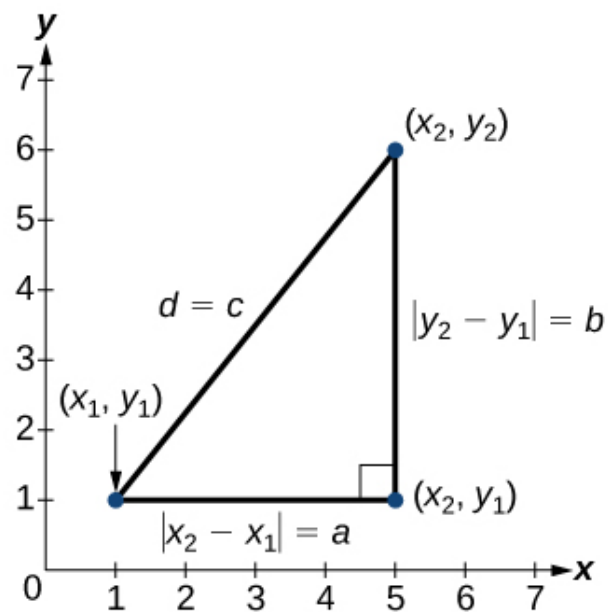
$$y = 2(0) + 6$$

$$y = 6$$

The graph cross the  $y$  axis at  $(0, 6)$ .

## 5 Distance Formula and Midpoint Formula

The distance formula is actually a property of a right angled triangle where  $c^2 = a^2 + b^2$  as shown in the figure below:



### Distance Formula

Given endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  the distance between two points is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

### Midpoint Formula

Given endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  the midpoint between two points is given by

$$M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Eg. Determine the distance and Midpoint between two points  $(3, -3)$  and  $(-9, 2)$

$$\begin{aligned} D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-9 - 3)^2 + (2 - -3)^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$\begin{aligned} M &= \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left( \frac{3 - 9}{2}, \frac{-3 + 2}{2} \right) \\ &= \left( \frac{-6}{2}, \frac{-1}{2} \right) \\ &= \left( -3, \frac{-1}{2} \right) \end{aligned}$$