7.3 – The Unit Circle

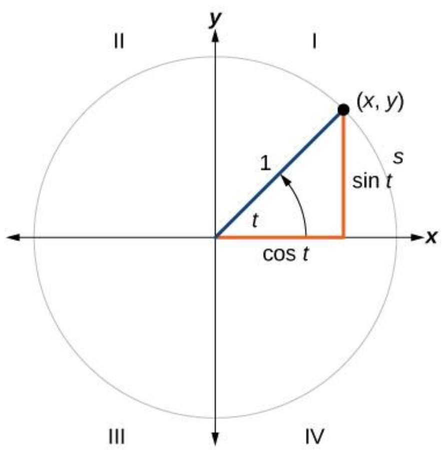
Learning Objectives

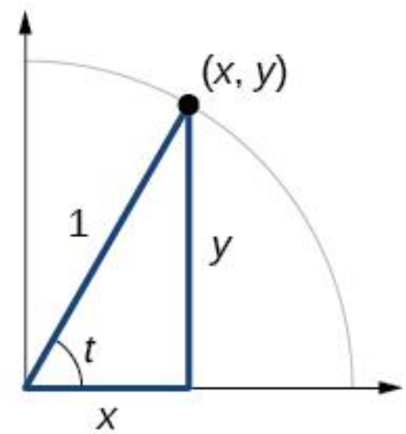
In this section you will:

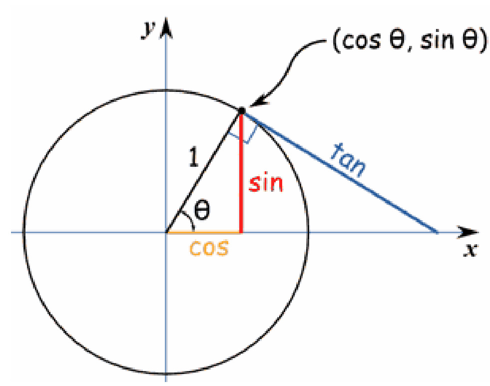
- Find function values for the sine and cosine of
$$30° \text{ or } \left(\frac{\pi}{6}\right), 45° \text{ or } \left(\frac{\pi}{4}\right),\ and\ 60° \text{ or } \left(\frac{\pi}{3}\right).$$
- Identify the domain and range of sine and cosine functions.
- Find reference angles.
- Use reference angles to evaluate trigonometric functions.

Finding Trig Functions Using The Unit Circle

For any angle t, we can label the intersection of the terminal side and the unit circle as by its coordinates, (x, y).The coordinates x and y will be the outputs of the trigonometric functions $f(t)$=cos t and $f(t)$=sin t, respectively. This means x=cos t and y=sin t.







UNIT CIRCLE

A **unit circle** has a center at (0,0) and radius 1. In a unit circle, the length of the intercepted arc is equal to the radian measure of the central angle t.

Let (x , y) be the endpoint on the unit circle of an arc of **arc length** s. The (x , y) coordinates of this point can be described as functions of the angle.

Defining Sine and Cosine Functions from the Unit Circle

Like all functions, the sine function has an input and an output. Its input is the measure of **an angle(Degree or Radians)** , its output is **y-cordinate** the -coordinate of the corresponding point on the unit circle.

The cosine function of an angle t equals the x-value of the endpoint on the unit circle of an arc of t. In the figure below, cosine is equal to **x.**

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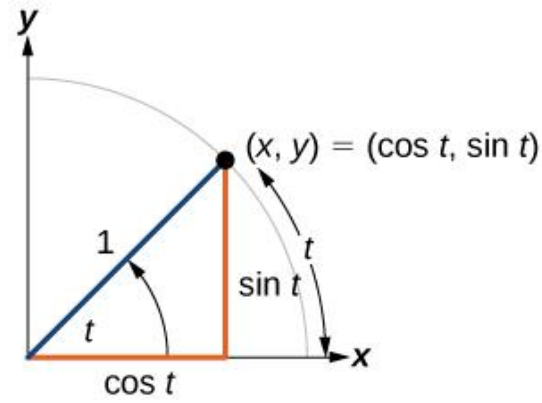

Because it is understood that sine and cosine are functions, we do not always need to write them with parentheses: sin *t* is the same as sin(*t*) and cos *t* is the same as cos(*t*). Likewise, $\cos^2 t$ is a commonly used shorthand notation for$(\cos(t))^2$.Be aware that many calculators and computers do not recognize the shorthand notation. When in doubt, use the extra parentheses when entering calculations into a calculator or computer.

A GENERAL NOTE: SINE AND COSINE FUNCTIONS

If t is a real number and a point (x, y) on the unit circle corresponds to a central angle t, then

$$\cos t = x$$
$$\sin t = y$$

How To:

Given a point *P* (x, y) on the unit circle corresponding to an angle of t, **find the sine and cosine.**

1. The sine of t is equal to the *y*-coordinate of point P: $\sin t = y$.
2. The cosine of t is equal to the *x*-coordinate of point P: $\cos t = x$.

Example **Finding Function Values for Sine and Cosine**

Point *P* is a point on the unit circle corresponding to an angle of *t*, as shown in **Figure 4.** Find $\cos(t)$ and $\sin(t)$.

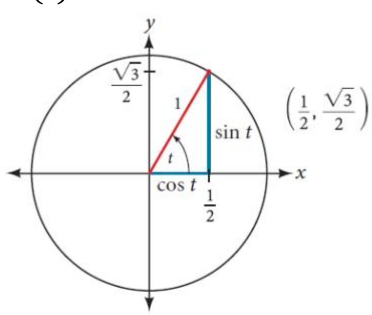

Figure 4

Answer:

Sin t = $\frac{\sqrt{3}}{2}$

Cos t = $\frac{1}{2}$

Try It: A certain angle t corresponds to a point on the unit circle at $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ as shown in Figure. Find $\cos t$ and $\sin t$.

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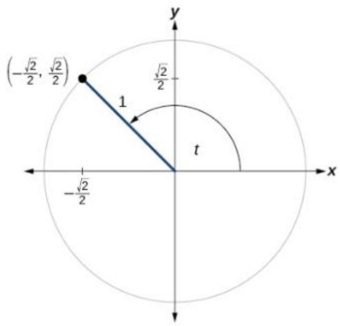
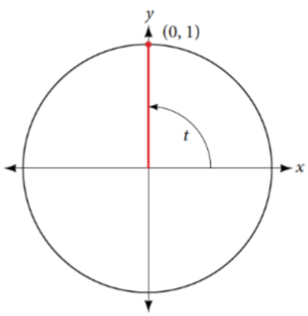


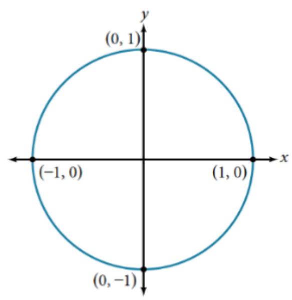
Answer:

y=Sin t = $\frac{\sqrt{2}}{2}$

x=Cos t = - $\frac{\sqrt{2}}{2}$

For quadrantral angles, the corresponding point on the unit circle falls on the *x*- or *y*-axis. In that case, we can easily calculate cosine and sine from the values of *x* and *y*.





Example Calculating Sines and Cosines along an Axis
Find cos(90°) and sin(90°).

Sin 90° = 1
Cos 90°= 0

Try It: Find cosine and sine of the angle π.

Sin π = 0
Cos π = -1

From the given information, find the quadrant in which the terminal point determined by t lies. Input I, II, III or IV.

a. Sin (t) < 0 and Cos (t) < 0, Quadrant <u>III</u>
b. Sin (t) > 0 and Cos (t) < 0, Quadrant <u>II</u>
c. Sin (t) > 0 and Cos (t) > 0, Quadrant <u>I</u>
d. Sin (t) < 0 and Cos (t) > 0, Quadrant <u>I</u>

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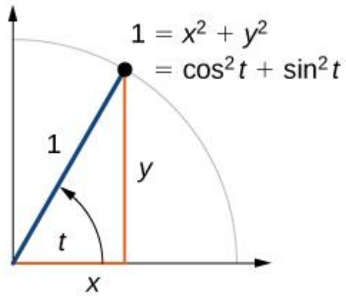
PYTHAGOREAN IDENTITY

The **Pythagorean Identity** states that, for any real number t,

$$cos^2\ t\ +\ sin^2\ t\ =\ 1$$



How To:

Given the sine of some angle t **and its quadrant location, find the cosine of** t.

1. Substitute the known value of sin t into the Pythagorean Identity.
2. Solve for cos t.
3. Choose the solution with the appropriate sign for the *x*-values in the quadrant where t is located

Example **Finding a Cosine from a Sine or a Sine from a Cosine**

If $sin(t)\ =\ 3/7$ and t is in the second quadrant, find $cos(t)$.

Sin *t* = $\frac{opposite}{hypotenuse} = \frac{3}{7}$,

So your opposite side is 3, hypotenuse is 7. To find adjacent side "b", we use Pythagorean theorem,

$3^2 + b^2 = 7^2$

$9 + b^2 = 49$

$b^2 = 49 - 9$

$b^2 = 40$

b= $\sqrt{40}$=2$\sqrt{10}$

Cos *t* = $\frac{adjacent}{hypotenuse} = \frac{2\sqrt{10}}{7}$

Since this is in second quadrant , Cost (x) must be negative so Cos *t*=-$\frac{2\sqrt{10}}{7}$

Try It: If cos(t) = $\frac{24}{25}$ and t is in the fourth quadrant, find sin(t).

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Cos $t = \frac{adjacent}{hypotenuse} = \frac{24}{25}$,
So your adjacent side is 3, hypotenuse is 7. To find opposite side "a", we use Pythagorean theorem,

$24^2 + a^2 = 25^2$

$576 + \text{a} = 625$

$a^2 = 625 - 576$

$a^2 = 49$

a= $\sqrt{49}$=7

Sin $t = \frac{opposite}{hypotenuse} = \frac{7}{25}$

Since this is in 4th quadrant , y is negative so, Sin(t) $= -\frac{7}{25}$

Example: The point P is on the unit circle. If y-cordinate of P is -4/5 , and P is in quadrant III, then find x.
Ans: Remember y coordinate is the sine function, x coorindate is cosine function. To find x, we need to find cosine function.

Sin $t = \frac{opposite}{hypotenuse} = \frac{4}{5}$, (you can simply drop negative sign as long as you remember this is in quadrant III)
So your opposite side is 4, hypotenuse is 5. To find adjacent side "b", we use Pythagorean theorem,

$4^2 + b^2 = 5^2$

$16 + b^2 = 25$

$b^2 = 25 - 16$

$b^2 = 9$

b= $\sqrt{9}$=3

Cos $t = \frac{adjacent}{hypotenuse} = \frac{3}{5}$

Since this is in third quadrant , so both coordinate must be negative so Cos t=-$\frac{3}{5}$

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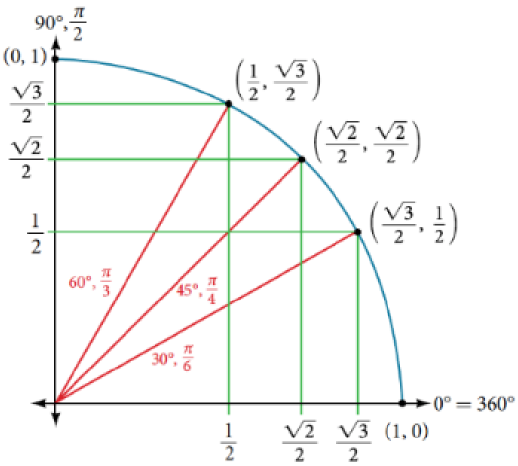
Finding Sine and Cosine of Special Angles

The image shows the ordered pairs of the cosine and sine values for all of the most commonly encountered angles in the first quadrant of the unit circle.
Table 1 summarizes these values.



Angle	0	$\frac{\pi}{6}$, or 30°	$\frac{\pi}{4}$, or 45°	$\frac{\pi}{3}$, or 60°	$\frac{\pi}{2}$, or 90°
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Note: Sine and cosine of the quadrantal and special angles will require an exact answer.
Using your calculator for these angles is not the proper approach.
Use the side length relationship with s = 1.

Example **Evaluate with Sine and Cosine of Special Angles**
Evaluate cos(45°).

Sin 45°= $\frac{\sqrt{2}}{2}$

Cos 45°= $\frac{\sqrt{2}}{2}$

Example **Evaluate with Sine and Cosine of Special Angles**
Evaluate sin($\pi/6$).

Sin $\pi/6$ = $\frac{1}{2}$

Cos $\pi/6$ = $\frac{\sqrt{3}}{2}$

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Example **Evaluate with Sine and Cosine of Special Angles**
Evaluate cos(60°).

Sin 60° $=\frac{\sqrt{3}}{2}$ Cos 60°$=\frac{1}{2}$

Try It: Evaluate $\sin\left(\frac{\pi}{3}\right)$

Sin $\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$ Cos $\left(\frac{\pi}{3}\right)=\frac{1}{2}$

Example **Evaluate with Sine and Cosine of Special Angles**
Evaluate $cos(\pi/4)sin(\pi/6)$.

$cos\left(\frac{\pi}{4}\right)sin\left(\frac{\pi}{6}\right)=\frac{\sqrt{2}}{2}*\frac{1}{2}=\frac{\sqrt{2}}{4}$

Most calculators can be set into “degree” or “radian” mode, which tells the calculator the units for the input value.

How To:
Given an angle in radians, use a graphing calculator to find the cosine.

1. If the calculator has degree mode and radian mode, set it to radian mode.
2. Press the COS key.
3. Enter the radian value of the angle and press the close-parentheses key ")".
4. Press ENTER.

Be aware: We only use a calculator to find the cosine and sine of angles ***other than*** quadrantal and the special angles.

Example **Using a Graphing Calculator to Find Sine and Cosine**
Evaluate cos(7π/5)= -0.3090 Evaluate sin(114°)=0.9135 Evaluate sin(π/5)=0.5877

Identifying Domain and Range of Sine and Cosine Functions

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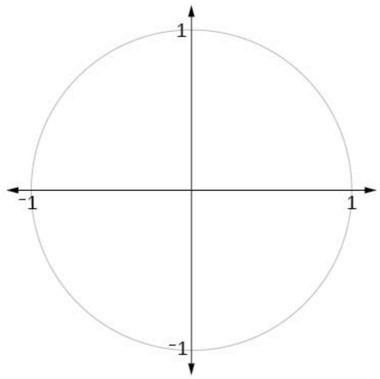


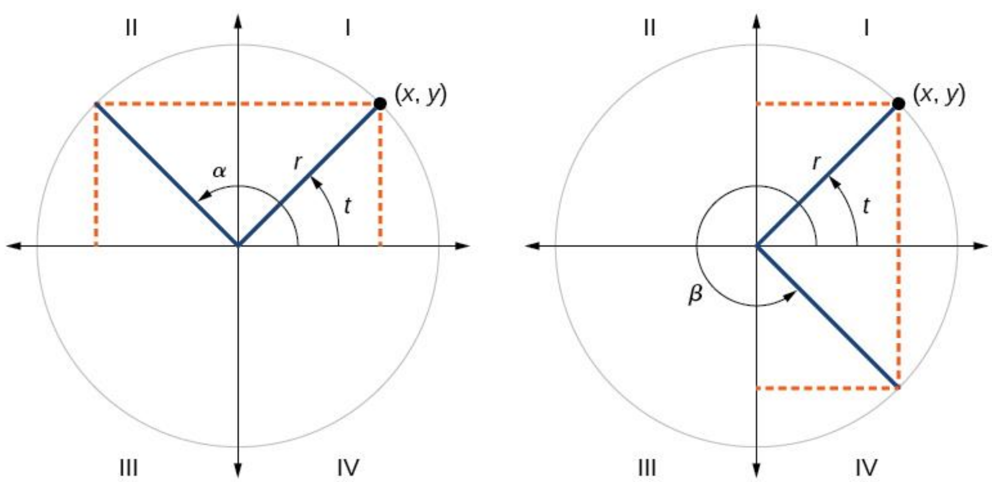
What are the domains of the sine and cosine functions? That is, what are the smallest and largest numbers that can be inputs of the functions? Because angles smaller than 0 and angles larger than 2π can still be graphed on the unit circle and have real values of x, y, and r, there is no lower or upper limit to the angles that can be inputs to the sine and cosine functions. The **input** to the sine and cosine functions is the rotation from the positive x-axis, and that **<u>may be any real number.</u>**

What are the ranges of the sine and cosine functions? What are the least and greatest possible values for their output? We can see the answers by examining the unit circle, as shown above. The bounds of the x-coordinate are **[-1,1].**The bounds of the y-coordinate are also **[-1,1].**Therefore, the range of both the sine and cosine functions is$[-1,1]$.

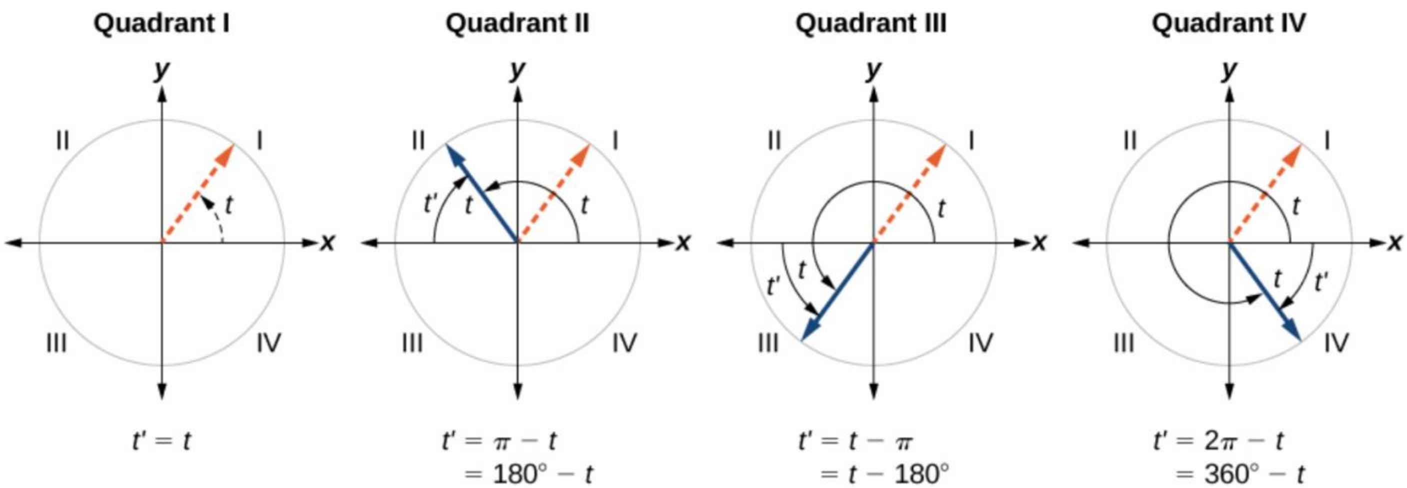
Finding Reference Angles

For any given angle in the first quadrant, there is an angle in the second quadrant with the same sine value. Because the sine value is the ***<u>Y-</u>***coordinate on the unit circle, the other angle with the same **<u>Sine</u>** will share the same y-value, but have the **<u>opposite</u>** x-value. Therefore, its **<u>Cosine</u>** value will be the opposite of the first angle's cosine value. Likewise, there will be an angle in the fourth quadrant with the **<u>Same</u>** cosine as the original angle. The angle with the same cosine will share the **<u>Same</u>** x-value but will have the opposite y-value. Therefore, its sine value will be the opposite of the original angle's sine value.

$$sin(t) = sin(\alpha) \text{ and } cos(t) = -cos(\alpha)$$
$$sin(t) = -sin(\beta) \text{ and } cos(t) = cos(\beta)$$



Recall that an angle's reference angle is the acute angle, t, formed by the terminal side of the angle t and the horizontal axis. A reference angle is always an angle between 0 and 90°,or 0 and $\frac{\pi}{2}$ radians.



How To:

Given an angle between 0 and 2π, find its reference angle.

1. An angle in the first quadrant is its own reference angle.
2. For an angle in the second or third quadrant, the reference angle is $|\pi-t|$ or $|180°-t|$.
3. For an angle in the fourth quadrant, the reference angle is $2\pi-t$ or $360°-t$.
4. If an angle is less than 0 or greater than 2π, add or subtract 2π as many times as needed to find an equivalent angle between 0 and 2π.

Example Finding a Reference Angle

Find the reference angle of 120°.

120° lies in second quadrant, so we subtract it from 180.
180-120=60°

Find the reference angle of 225°

225° lies in third quadrant so we subtract 180 from it,
225-180 =225°

Try It: Find the reference angle of $\frac{5\pi}{3}$.

$\frac{5\pi}{3}$ lies in third quadrant, so we subtract it from 2 π.

$$2\pi-\frac{5\pi}{3}=\frac{6\pi}{3}-\frac{5\pi}{3}=\frac{6\pi-5\pi}{3}=\frac{\pi}{3}$$

Using Reference Angles to Evaluate Trigonometric Functions

A GENERAL NOTE: USING REFERENCE ANGLES TO FIND COSINE AND SINE

Angles have cosines and sines with the same absolute value as their reference angles. The sign (positive or negative) can be determined from the quadrant of the angle.

How To:

Given an angle in standard position, find the reference angle, and the cosine and sine of the original angle.

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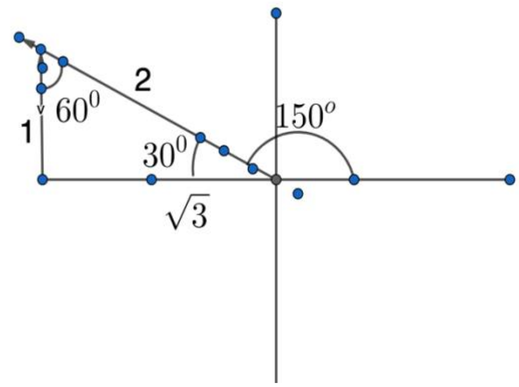
1. Measure the angle between the terminal side of the given angle and the horizontal axis. That is the reference angle.
2. Determine the values of the cosine and sine of the reference angle.
3. Give the cosine the same sign as the *x*-values in the quadrant of the original angle.
4. Give the sine the same sign as the *y*-values in the quadrant of the original angle.

Example **Using Reference Angles to Find Sine and Cosine**
Without using a calculator, compute the sine and cosine 150° by using the reference angle.
What is reference angle ? 180-150= 30°.
Using a reference angle, find the exact value of cos(150°) and sin(150°).



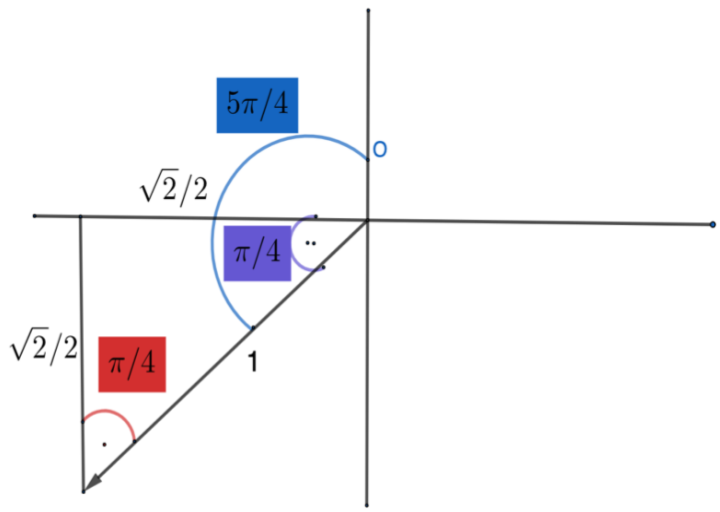
Once we find the reference angle, we construct a right angled triangle, then use 30-60-90 triangle to find the values.

Sin (150)=Sin(30)= $\frac{opposite}{hypotenuse} = \frac{1}{2}$,

Cos (150)=-cos(30)= $\frac{adjacent}{hypotenuse} = \frac{-\sqrt{3}}{2}$, (The negative sign in front is because x co-ordinate is negative in 2nd quadrant.

Using the reference angle, find cos 5π/4 and sin 5π/4.

5π/4 lies in third quadrant, so you will subtract π from 5π/4 , so Reference angle = 5π/4-π=π/4.
Now construct a 45-45-90 triangle.



Once we find the reference angle, we construct a right angled triangle, then use 45-45-90 triangle to find the values.

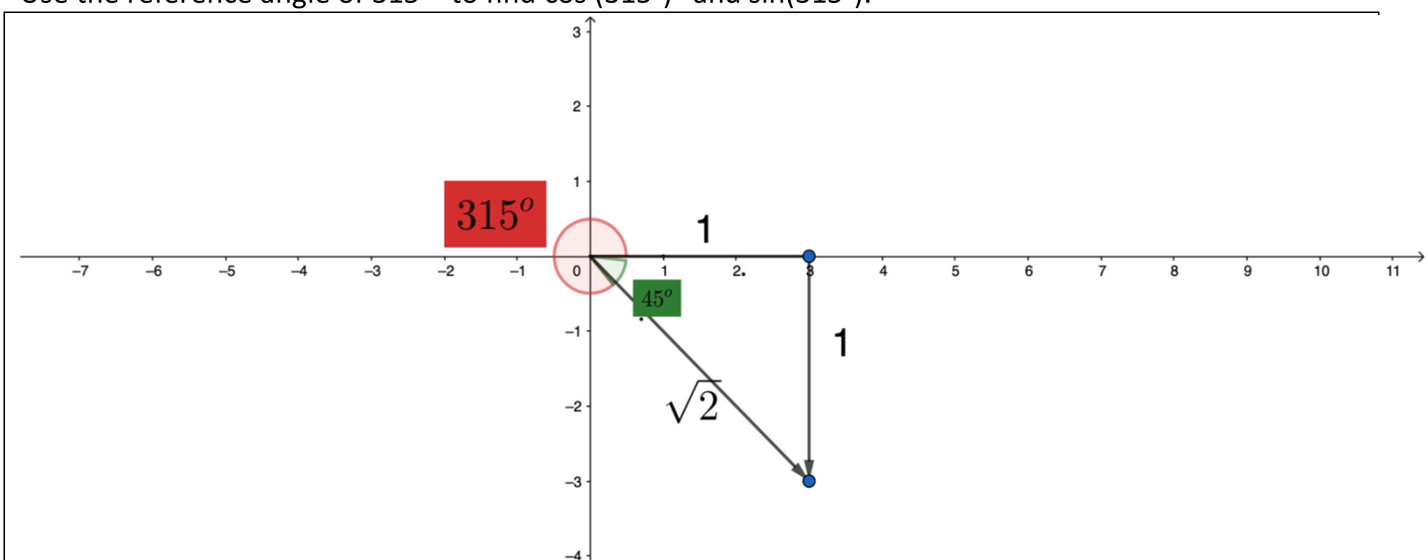
Sin ($5\pi/4$)=-Sin($\pi/4$)= $\dfrac{opposite}{hypotenuse} = \dfrac{-\sqrt{2}}{2}$,

Cos ($5\pi/4$)=- Cos($\pi/4$)= $\dfrac{adjacent}{hypotenuse} = \dfrac{-\sqrt{2}}{2}$

The negative sign in front is because they are co-ordinates in 3rd quadrant.

Try It:

a. Use the reference angle of 315° to find cos (315°) and sin(315°).



315° lies in 4th quadrant so, to find the reference angle, we subtract 315° from 360°. So Reference angle =360°-315°=45°. Then we construct, 45-45-90 triangle.

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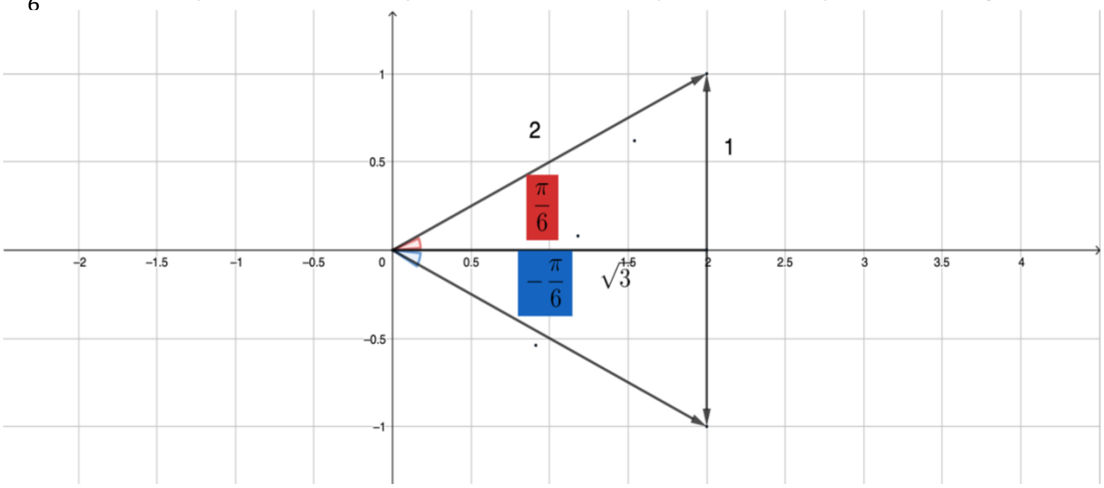
Sin (315)=-Sin(45)= $\frac{opposite}{hypotenuse} = -\frac{1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$, (negative sign here because y value is negative in 4th quadrant.

Cos (315)=cos(45)= $\frac{adjacent}{hypotenuse} = \frac{\sqrt{2}}{2}$,

b. Use the reference angle of $-\frac{\pi}{6}$ to find cos($-\frac{\pi}{6}$) and sin($-\frac{\pi}{6}$).

$-\frac{\pi}{6}$ lies in 4th quadrant. In 4th quadrant, x value is positive and y-value is negative.



Note that they share the same x coordinate. However, y is negative in 4th quadrant, so we have

cos($-\frac{\pi}{6}$) = cos($\frac{\pi}{6}$) = $\frac{\sqrt{3}}{2}$,

sin($-\frac{\pi}{6}$)= -sin($\frac{\pi}{6}$) = -$\frac{1}{2}$,

Example **Evaluate with Sine and Cosine of Special Angles**

Evaluate cos($7\pi/6$)sin($3\pi/2$).

Reference angle: *$7\pi/6$ lies in third quadrant, $7\pi/6-\pi=\pi/6$,*

So cos($7\pi/6$) = -cos($\pi/6$)= -$\frac{\sqrt{3}}{2}$

Sin($3\pi/2$)= -1.

cos($7\pi/6$)sin($3\pi/2$)= -$\frac{\sqrt{3}}{2} * -1 = \frac{\sqrt{3}}{2}$

How To:

Given the angle of a point on a circle and the radius of the circle, find the (x , y) **coordinates of the point.**

1. Find the reference angle by measuring the smallest angle to the *x*-axis.

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2. Find the cosine and sine of the reference angle.
3. Determine the appropriate signs for x and y in the given quadrant.

Example **Using the Unit Circle to Find Coordinates**

Find the coordinates of the point on the unit circle at an angle of $7\pi/6$.

Reference angle: *$7\pi/6$ lies in third quadrant, $7\pi/6-\pi=\pi/6$,* now draw 30-60-90 triangle

x-coordinate = $\cos(7\pi/6) = -\cos(\pi/6) = -\frac{\sqrt{3}}{2}$

y-coordinate = $\sin(7\pi/6) = -\sin(\pi/6) = -\frac{1}{2}$

Try It: Find the coordinates of the point on the unit circle at an angle of $\frac{5\pi}{3}$.

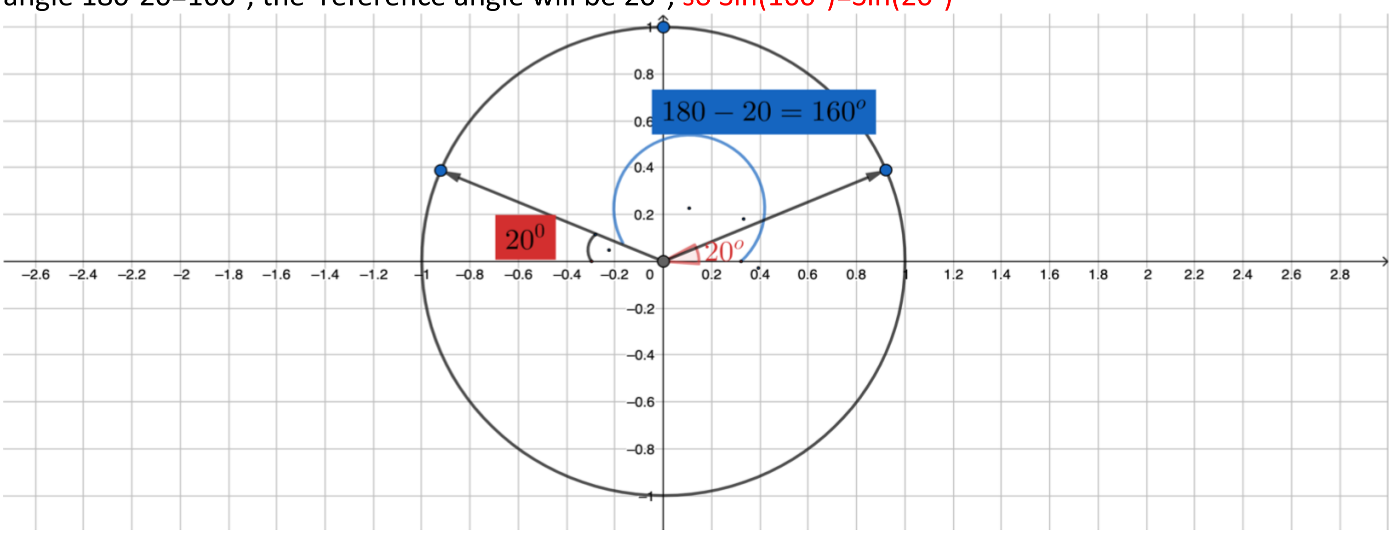
Reference angle: $5\pi/3$ lies in 4th quadrant, $2\pi-\frac{5\pi}{3}=\frac{\pi}{3}$. now draw 30-60-90 triangle

x-coordinate = $\cos(5\pi/3) = \cos(\pi/3) = \frac{1}{2}$.

y-coordinate = $\sin(5\pi/3) = -\sin(\pi/3) = -\frac{\sqrt{3}}{2}$.

Find an angle θ with $0° < \theta < 360°$ that has the same : Sin(20°) and Cos(20°).

Remember, sine function is "Y" coordinate of a point in a unit circle. That means you need to find which angle has same "Y" co-ordinate that the point in a unit circle at 20° has. Lets draw it out first. Note that 20° lies in first quadrant, so y value is positive. The only other place where y is positive is in 2nd quadrant. So , if you have angle 180-20=160°, the reference angle will be 20°, so Sin(160°)=Sin(20°)



We will do the same thing for x-coordinate as well. Again, remember that cosine is the x value of a point in a unit circle. We already know 20° lies in first quadrant, so we know that Cos(20°) or x value is positive. The only other place where x coordinate is positive is 4th quadrant. Lets draw it out.

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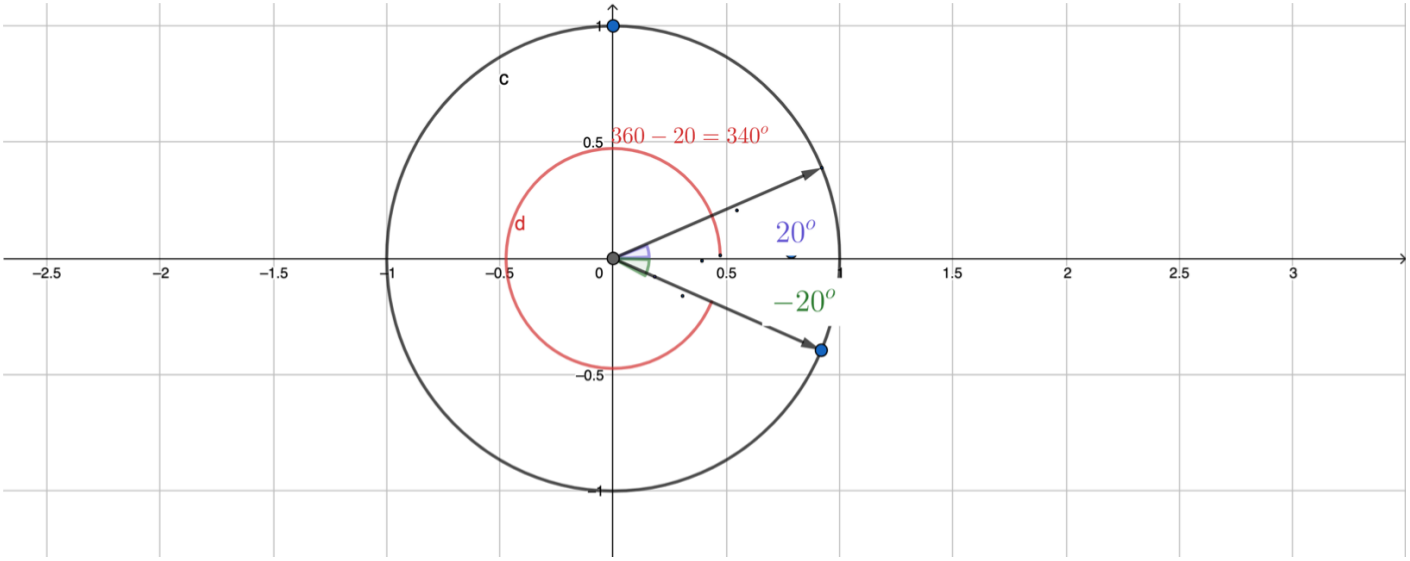


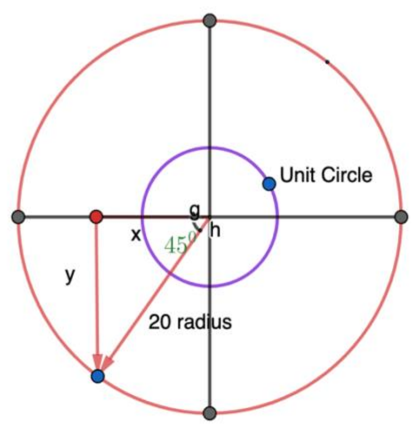
Cos(20°)=Cos(360-20)=Cos(340°).

Find the coordinates of a point on circle with radius 30 corresponding to an angle of 225°.

Note that this question has a circle with radius 30, so this is not a unit circle. Finding Sin(225°) and Cos(225°) is not enough here. But, lets first find them.
To find Sin(225°) and Cos(225°), we need a reference angle. Again, note that this is in 3rd quadrant, so refrence angle = 270-225= 45°. Now we need to draw 45-45-90 triangle. Also note, that in 3rd quadrant, both sine and cosine are negative.

Sin(225°) = -Sin(45°) = $\frac{-\sqrt{2}}{2}$.

Cos(225°) = -Cos(45°) =- $\frac{-\sqrt{2}}{2}$.



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Sin (225)= $\frac{opposite}{hypotenuse} = \frac{y}{20}$

$\frac{-\sqrt{2}}{2} = \frac{y}{20}$

$20\frac{-\sqrt{2}}{2} = y$

$-10\sqrt{2} = y$

This is a isosceles triangle, so you get same value for x as well, $-10\sqrt{2} = x$.

Or, you can also use cosine,

Cos (225)= $\frac{opposite}{hypotenuse} = \frac{x}{20}$

$\frac{-\sqrt{2}}{2} = \frac{x}{20}$

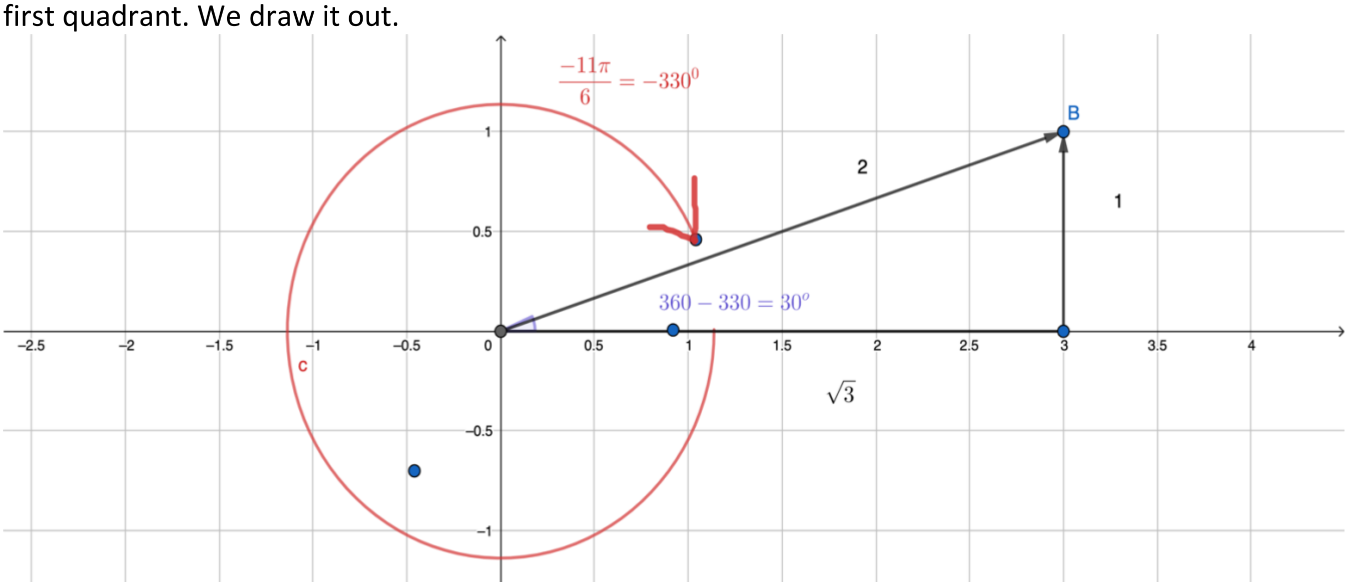
$20\frac{-\sqrt{2}}{2} = x$

$-10\sqrt{2} = x$

If t = $-\frac{11\pi}{6}$, find the terminal point P(x,y) on the unit circle, x---------------, y-------------------.

There are two ways to do. First one is simply using: Sin($-\frac{11\pi}{6}$)= -sin($\frac{11\pi}{6}$), Cos($-\frac{11\pi}{6}$)= Cos($\frac{11\pi}{6}$). Here we find sine and cosine for positive angle $\frac{11\pi}{6}$, and then use their properties.

The next method is to find reference angle first. Again Draw $-\frac{11\pi}{6}$ (i.e counter-clockwise direction). It will lie in first quadrant. We draw it out.



Whether the angle is positive or negative, the process of finding the reference angle is same.

Y=Sin ($-\frac{11\pi}{6}$)= Sin($\frac{\pi}{6}$)= $\frac{1}{2}$.

X=Cos($-\frac{11\pi}{6}$)= Cos($\frac{\pi}{6}$)= $\frac{\sqrt{3}}{2}$.

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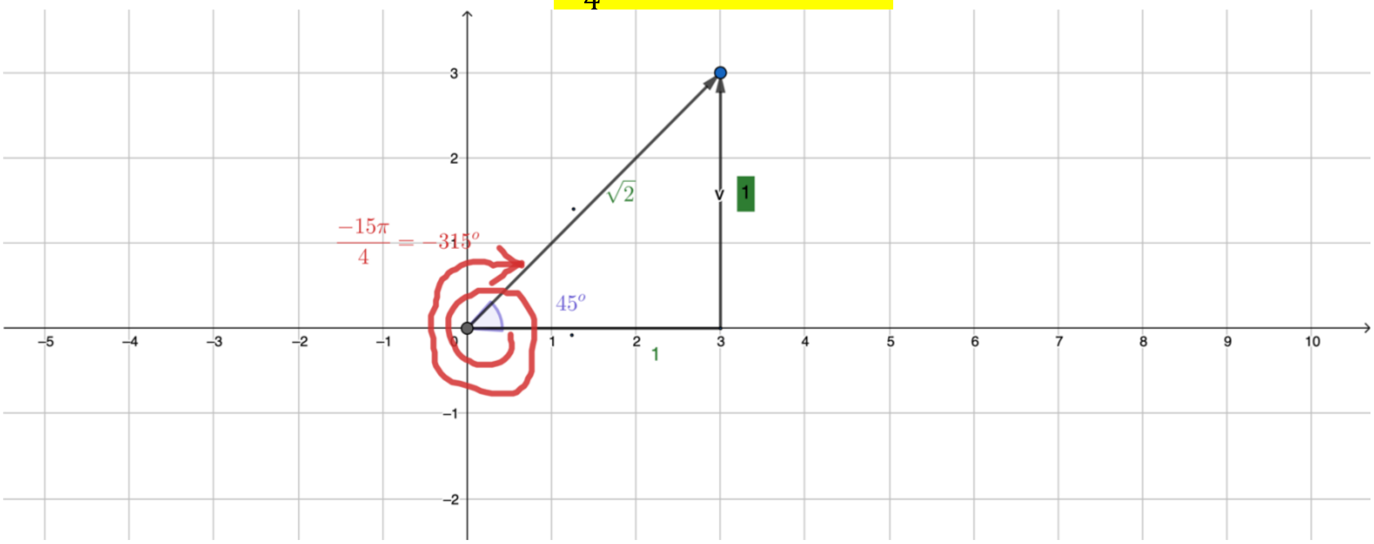
Compute the exact value of each of the following:

Sin ($-\frac{15\pi}{4}$)=-----------------------   Cos ($-\frac{15\pi}{4}$)=-----------------------   Tan ($-\frac{15\pi}{4}$)=-----------------------

Again, we can proceed with positive angles and then use their properties. Or you can proceed with negative angles.

Here, I will proceed with negative angles. I will draw it out first.

$$\frac{-15\pi}{4} = -675° = -315°$$



$-\frac{15\pi}{4}$ lies in 1st quadrant so the reference angle is 4 π $-\frac{15\pi}{4}$= $\frac{\pi}{4}$. Both coordinates are positive here.

So Sin($-\frac{15\pi}{4}$)= Sin ($\frac{\pi}{4}$)= $\frac{\sqrt{2}}{2}$

Cos($-\frac{15\pi}{4}$)= Cos ($\frac{\pi}{4}$)= $\frac{\sqrt{2}}{2}$

Tan($-\frac{15\pi}{4}$)= Tan ($\frac{\pi}{4}$)= 1

Find the terminal point P(x,y) on the unit circle determined by the given value of t:

$\frac{\pi}{2}$ and $\frac{\pi}{3}$ lies in first quadrant, so their negative angles lies in 4th quadrant. $\frac{3\pi}{4}$ lies in 2nd quadrant, so its negative angle lies in 3rd quadrant.

a. If t = $\frac{\pi}{2}$, then x = 0, y = 1
b. If t = $-\frac{\pi}{2}$, then x = 0, y = -1
c. If t = $\frac{\pi}{3}$, then x = $\frac{\sqrt{3}}{2}$, y = $\frac{1}{2}$
d. If t = $-\frac{\pi}{3}$, then x = $\frac{\sqrt{3}}{2}$, y = $-\frac{1}{2}$
e. If t = $\frac{3\pi}{4}$, then x = $-\frac{\sqrt{2}}{2}$, y = $\frac{\sqrt{2}}{2}$
f. If t = $-\frac{3\pi}{4}$, then x = $-\frac{\sqrt{2}}{2}$, y = $\frac{-\sqrt{2}}{2}$

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For each value of t below, determine the quadrant in which the terminal point is found and find the corresponding reference number t.

a. t= $\frac{2\pi}{3}$ is found in quadrant II, and t'= $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

b. t= $\frac{5\pi}{4}$ is found in quadrant III, and t'= $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

c. t= $\frac{19\pi}{6}$ is found in quadrant III , and t'= $\frac{19\pi}{6} - 2\pi = 7\pi/6$. Again, $7\pi/6 - \pi = \pi/6$.

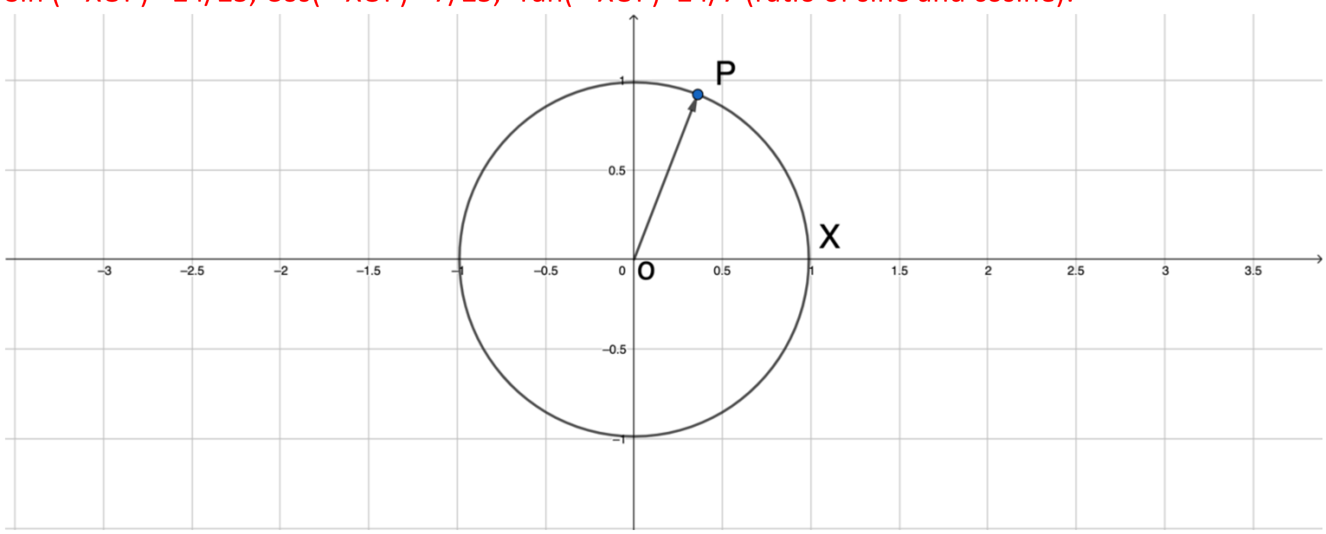
d. t= 3 is found in quadrant II, and t' = π-3.

The graph shows a unit circle with point P at $(\frac{7}{25}, \frac{24}{25})$

Enter the exact values or DNE if the value is undefined.

Again, x represents cosine of that angle, and y is the sine of that angle

Sin (<XOP)= 24/25, Cos(<XOP)= 7/25, Tan(<XOP)=24/7 (ratio of sine and cosine).



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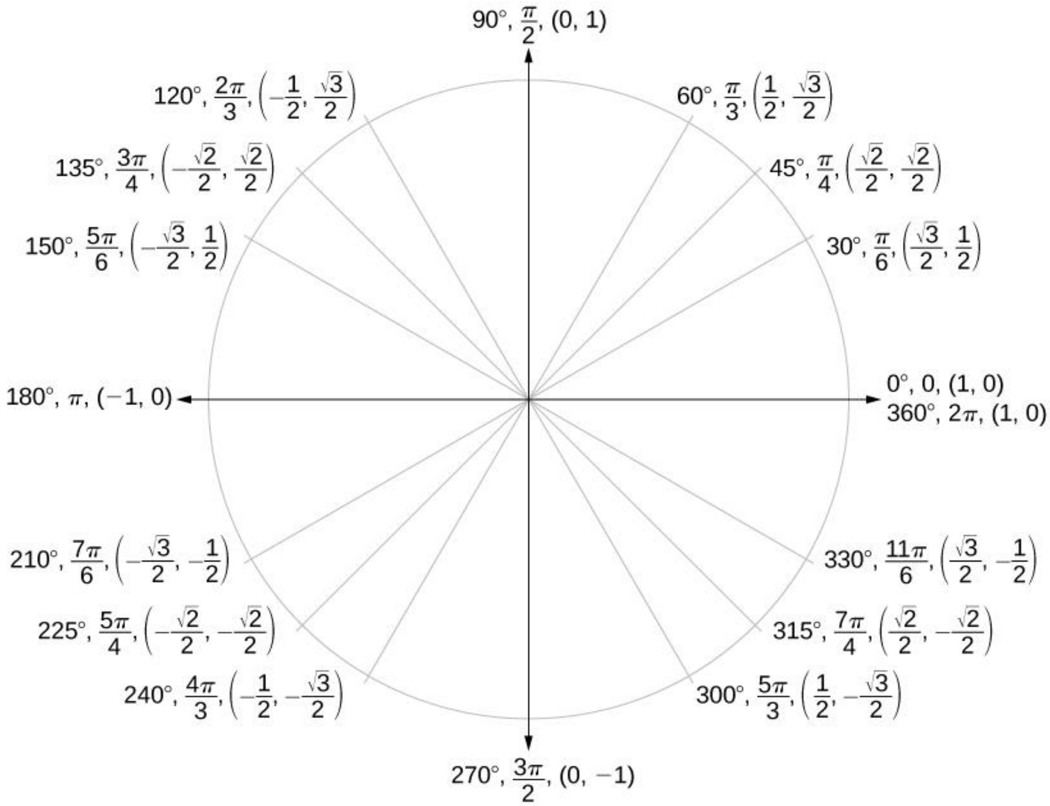

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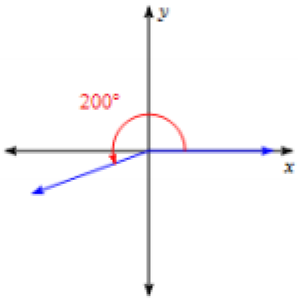
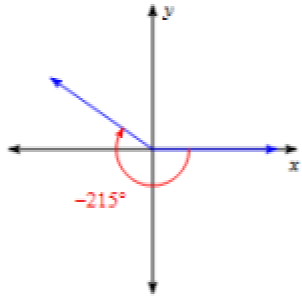
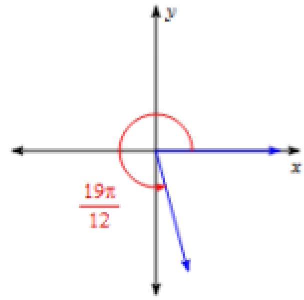
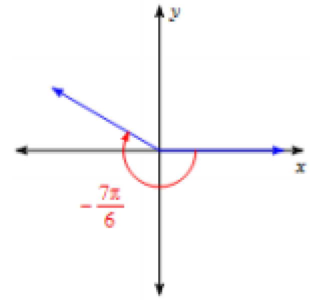
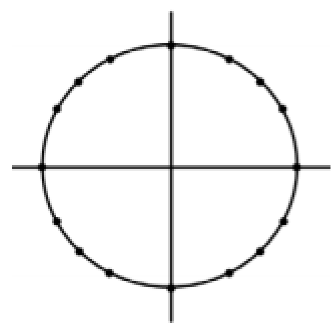
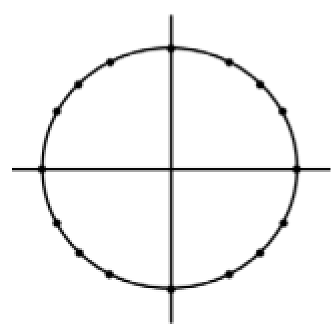
<table>
<tr><th colspan="4">Find the reference angle.</th></tr>
<tr>
<td>1.
</td>
<td>2.
</td>
<td>3.
</td>
<td>4.
</td>
</tr>
<tr>
<td>5. $-130°$</td>
<td>6. $230°$</td>
<td>7. $-\frac{13\pi}{9}$</td>
<td>8. $\frac{3\pi}{4}$</td>
</tr>
</table>

<table>
<tr><th colspan="4">Find the exact value.</th></tr>
<tr><td>9. $\sin 90° =$</td><td>10. $\cos 120° =$</td><td>11. $\tan 45° =$</td><td rowspan="5"></td></tr>
<tr><td>12. $\tan 120° =$</td><td>13. $\cos 225° =$</td><td>14. $\sin 135° =$</td></tr>
<tr><td>15. $\sin 330° =$</td><td>16. $\tan 315° =$</td><td>17. $\cos 240° =$</td></tr>
<tr><td>18. $\sin(-225°) =$</td><td>19. $\cos(-240°) =$</td><td>20. $\tan(-300°) =$</td></tr>
<tr><td>21. $\sec(180°) =$</td><td>22. $\csc(-270°) =$</td><td>23. $\cot(-315°) =$</td></tr>
</table>

<table>
<tr><th colspan="4">Find the exact value.</th></tr>
<tr><td>24. $\sin\frac{\pi}{2} =$</td><td>25. $\tan\frac{\pi}{4} =$</td><td>26. $\cos\frac{3\pi}{2} =$</td><td rowspan="5"></td></tr>
<tr><td>27. $\cos\frac{4\pi}{3} =$</td><td>28. $\cos\frac{\pi}{6} =$</td><td>29. $\tan\pi =$</td></tr>
<tr><td>30. $\sin\frac{5\pi}{4} =$</td><td>31. $\cos\frac{5\pi}{3} =$</td><td>32. $\sin\frac{5\pi}{6} =$</td></tr>
<tr><td>33. $\tan\frac{7\pi}{4} =$</td><td>34. $\sin(-\pi) =$</td><td>35. $\tan\left(-\frac{3\pi}{2}\right) =$</td></tr>
<tr><td>36. $\cos\left(-\frac{\pi}{3}\right) =$</td><td>37. $\sec\left(-\frac{\pi}{2}\right) =$</td><td>38. $\sin\left(-\frac{5\pi}{4}\right) =$</td></tr>
</table>

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