

Linear overlap to volume overlap

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Chapter 1

Intro

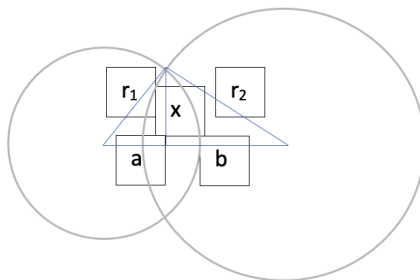
The aim of this document is to explain the formulae behind function `getOverlapVolume`. This function is part of the interaction and returns the effect volume overlap (at the moment assuming spheres). In the next section is the maths behind this formulae and also in the directory is the mathematica notebook to do the calculation (`OverlapVolume.nb`).

Chapter 2

The Math

Given overlap δ and particle radii r_1, r_2 , we want to the overlap volume V .

First we compute the thickness of the spherical caps, $t_1 = r_1 - a$ and $t_2 = r_2 - b$, in the diagram below.



We know

$$x^2 = r_1^2 - a^2, \quad x^2 = r_2^2 - b^2, \quad a + b = r_1 + r_2 - \delta$$

Thus

$$r_1^2 - a^2 = r_2^2 - (r_1 + r_2 - \delta - a)^2$$

Mathematica yields with the following commands:

```
Simplify[r1 - a /. Solve[r1^2 - a^2 == r2^2 - (r1 + r2 - d - a)^2, a]]
```

$$t_1 = r_1 - a = \frac{r_2 - \delta/2}{r_1 + r_2 - \delta} \delta \quad (2.1)$$

$$t_2 = r_2 - b = \frac{r_1 - \delta/2}{r_1 + r_2 - \delta} \delta \quad (2.2)$$

Thus, the volume of the two spherical caps is

$$V = \pi \left\{ \int_0^{t_1} r_1^2 - r^2 dr + \int_0^{t_2} r_2^2 - r^2 dr \right\}$$

$$V = \pi \{ r_1 t_1^2 - t_1^3/3 + r_2 t_2^2 - t_2^3/3 \} \quad (2.3)$$

It seems, this cannot be simplified further. The full expression in terms of r_1 , r_2 , δ is

$$V = \pi \left\{ \frac{\delta^2(4\delta r_1 + 4\delta r_2 - 12r_1 r_2 - \delta^2)}{12(\delta - r_1 - r_2)} \right\}$$

For another clear explanation see: <https://mathworld.wolfram.com/Sphere-SphereIntersection.html>