*"Scalars" α, β are elements of fields, rings, etc.

Set

{Katze, bobr, sphere, 4, ω}

Axioms

0. Closure of a binary operation
$x, y \in M \Rightarrow \; x * y \in M$

Addition

1. Associativity of addition
$(x+y)+z = x+(y+z)$

2. Additive identity
$x+0 = 0+x = x$

3. Additive inverse
$x+x' = x'+x = 0$

4. Commutativity of addition
$x+y = y+x$

Multiplication by "scalar"*

5. Associativity of scalar multiplication
$\alpha(\beta x) = (\alpha\beta)x$

6. Distributivity of scalar multiplication over scalars
$(\alpha+\beta)x = \alpha x + \beta x$

7. Distributivity of scalar multiplication over vectors
$\alpha(x+y) = \alpha x + \alpha y$

8. Unitarity
$1x = x$

9. Compatibility with scalar multiplication
$(\alpha x)(\beta y) = (\alpha\beta)(xy)$

10. Right distributivity
$(x+y)z = xz + yz$

11. Left distributivity
$z(x+y) = zx + zy$

Multiplication

12. Associativity of multiplication
$(xy)z = x(yz)$

13. Commutativity of multiplication
$xy = yx$

14. Multiplicative identity
$xe = ex = x$

15. Multiplicative inverse
$xx' = x'x = e$

https://www.youtube.com/@vektorfeld

Structures

Magma
$\mathbb{R}$ with "xy-x" as a binary operation

Semigroup
The set of positive integers with addition.
Ideal of a ring with the multiplication of the ring. (e.g. $2\mathbb{Z}$ of $\mathbb{Z}$)

Monoid
The set of non-negative integers with addition.

Group
Symmetry group of a triangle
Permutation group
Free group

Abelian group
Integers with addition
Non-negative integers with multiplication
Cyclic group
Free abelian group

Vector/Linear space (over a field)
$\mathbb{R}^n$ Field: $\mathbb{R}$ | very common: just vectors
Polynomial vector space $\mathbb{F}[x]$ Field: $\mathbb{F}$
Matrices $M_{n\times m}(\mathbb{F})$ Field: $\mathbb{F}$
Solution space of a linear system $Ax = 0$

Module (over a ring)
Abelian group may be considered as a module over $\mathbb{Z}$

Module (over an algebra)
Group representations as modules over group algebras

Algebra (over a field)
Lie algebra:
instead of associativity it satisfies the Jacobi identity
$[x,[y,z]] + [y,[z,x]] + [z,[x,y]] = 0$
and also Anticommutativity
$[x,y] = -[y,x]$
Vectors in 3D space with cross product as a product and vector addition as addition (also Lie)

Algebra (over a ring)
Polynomial ring $\mathbb{Z}[x]$ is $\mathbb{Z}$-algebra

Associative Algebra
Algebra of Square Matrices

Commutative Algebra
Jordan algebra

Unital Algebra
Split-Octonions

Division Algebra
Octonions

Field
Rational numbers
Real numbers
Complex numbers

Division ring/ skew field
Quaternions

Unital Ring/ Ring
The set of all square n-by-n matrices with entries from $\mathbb{R}$

Commutative Ring/Rng
Even integers with the ordinary addition and multiplication of integers

Ring/Rng
Set of 3-by-3 real matrices whose bottom row is zero