

# Accurate Prediction of Worst Case Eye Diagrams for Non-Linear Signaling Systems

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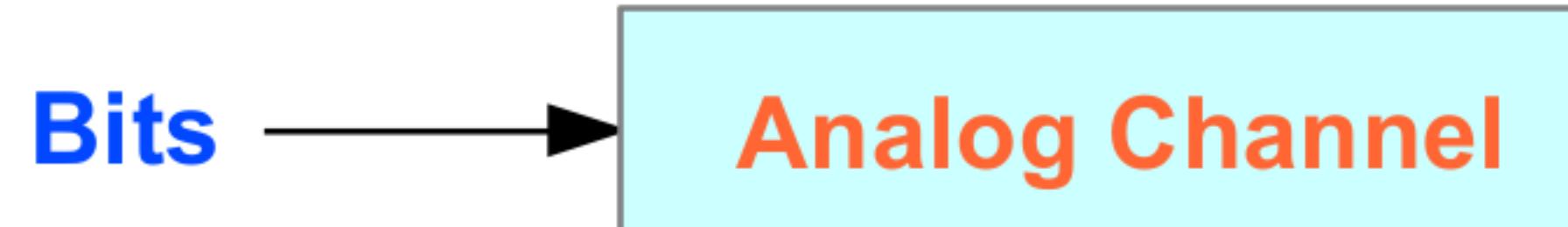
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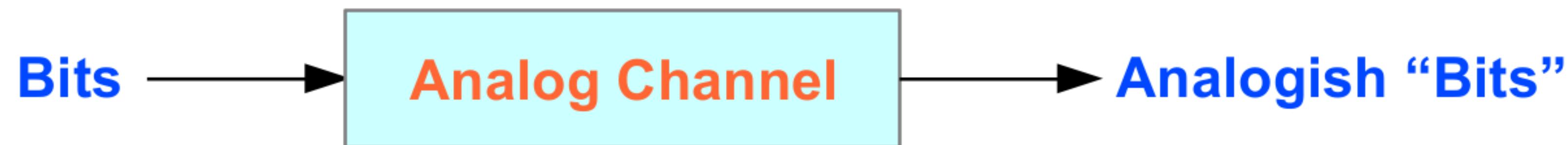
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  - *Our technique is much less pessimistic than PDA*

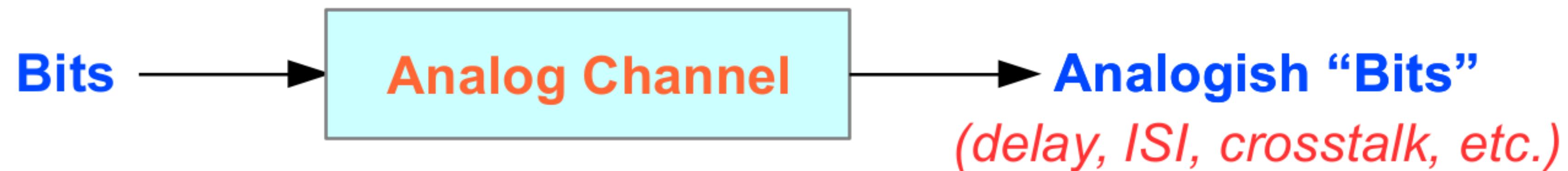
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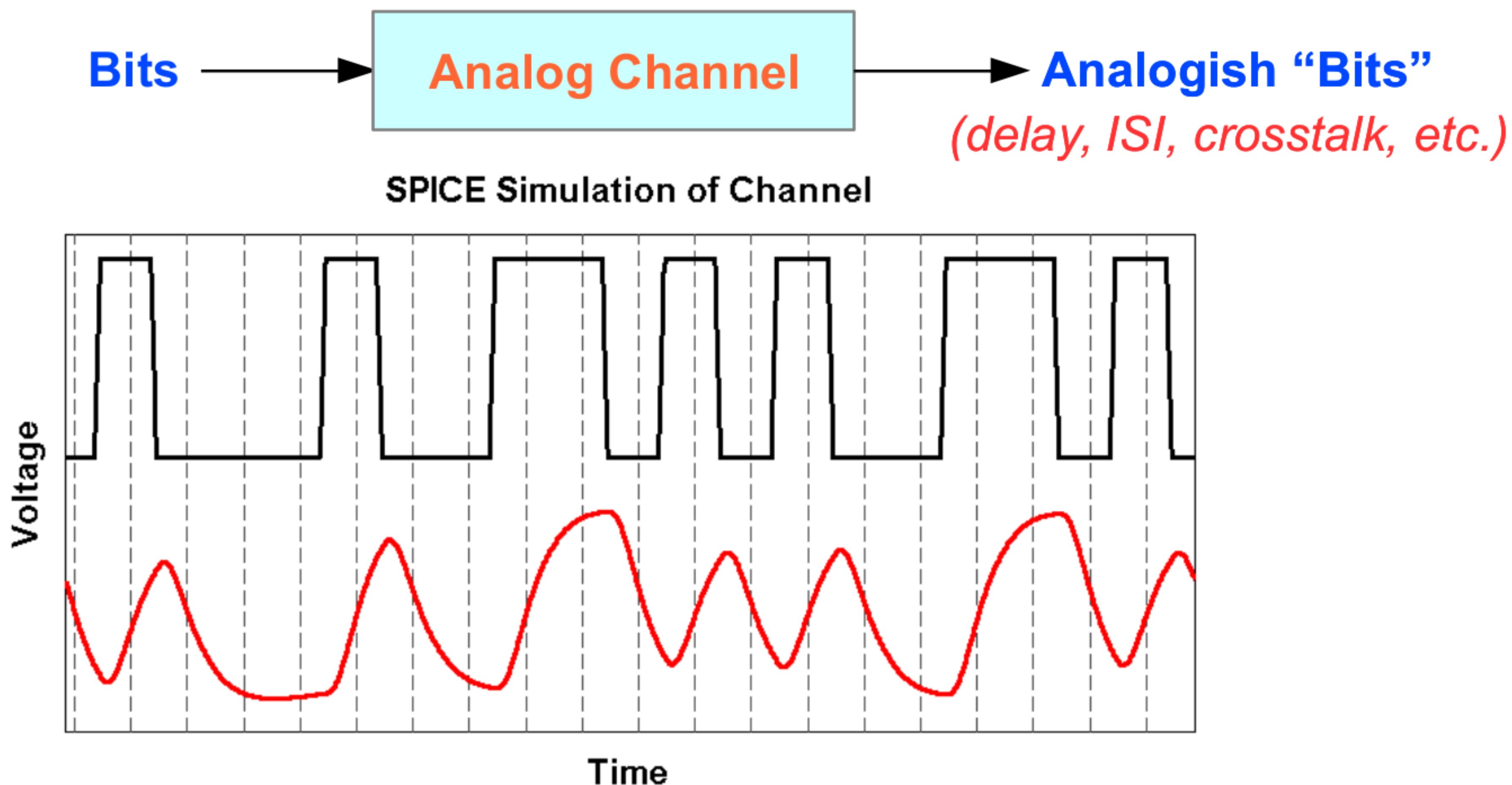
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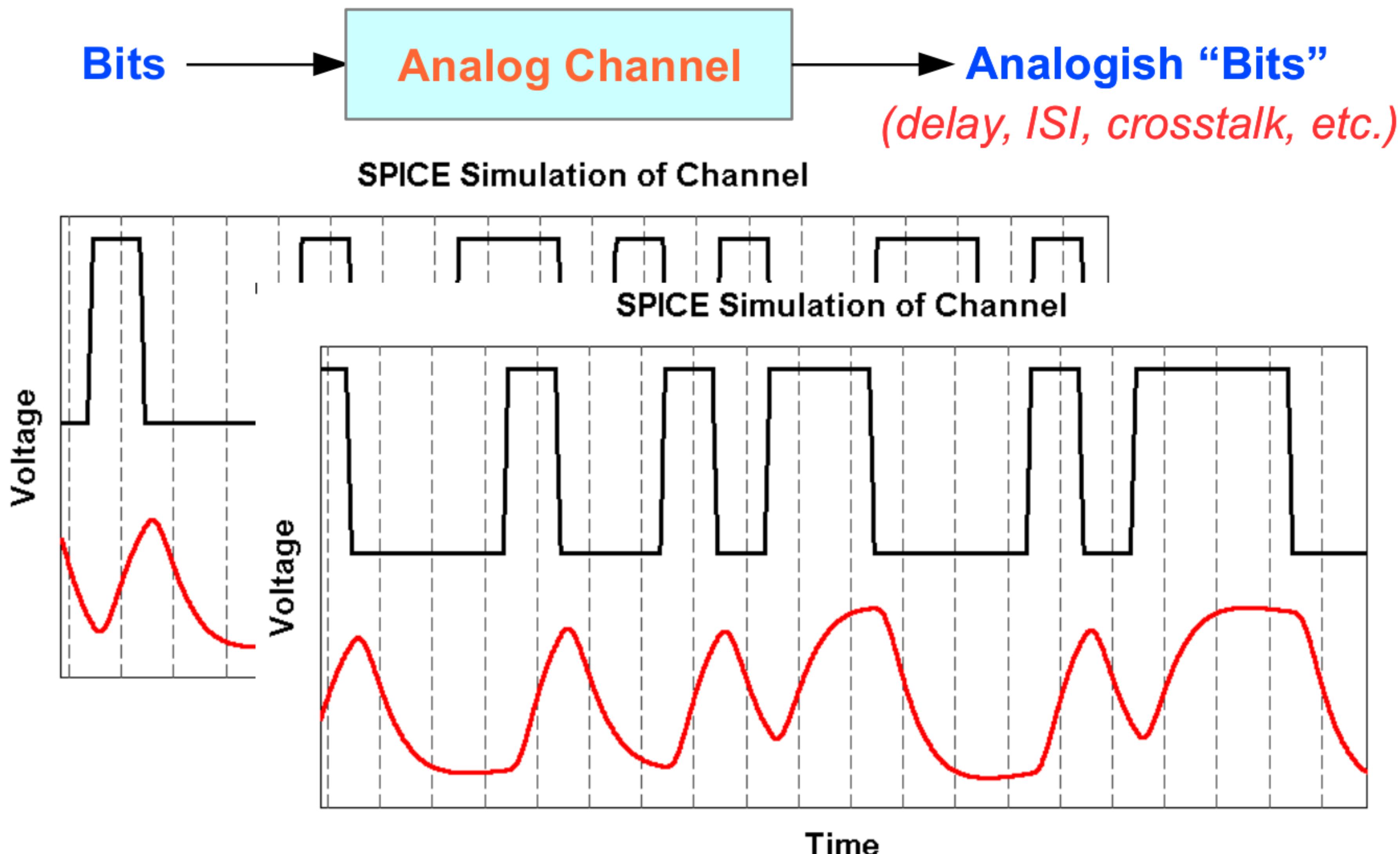
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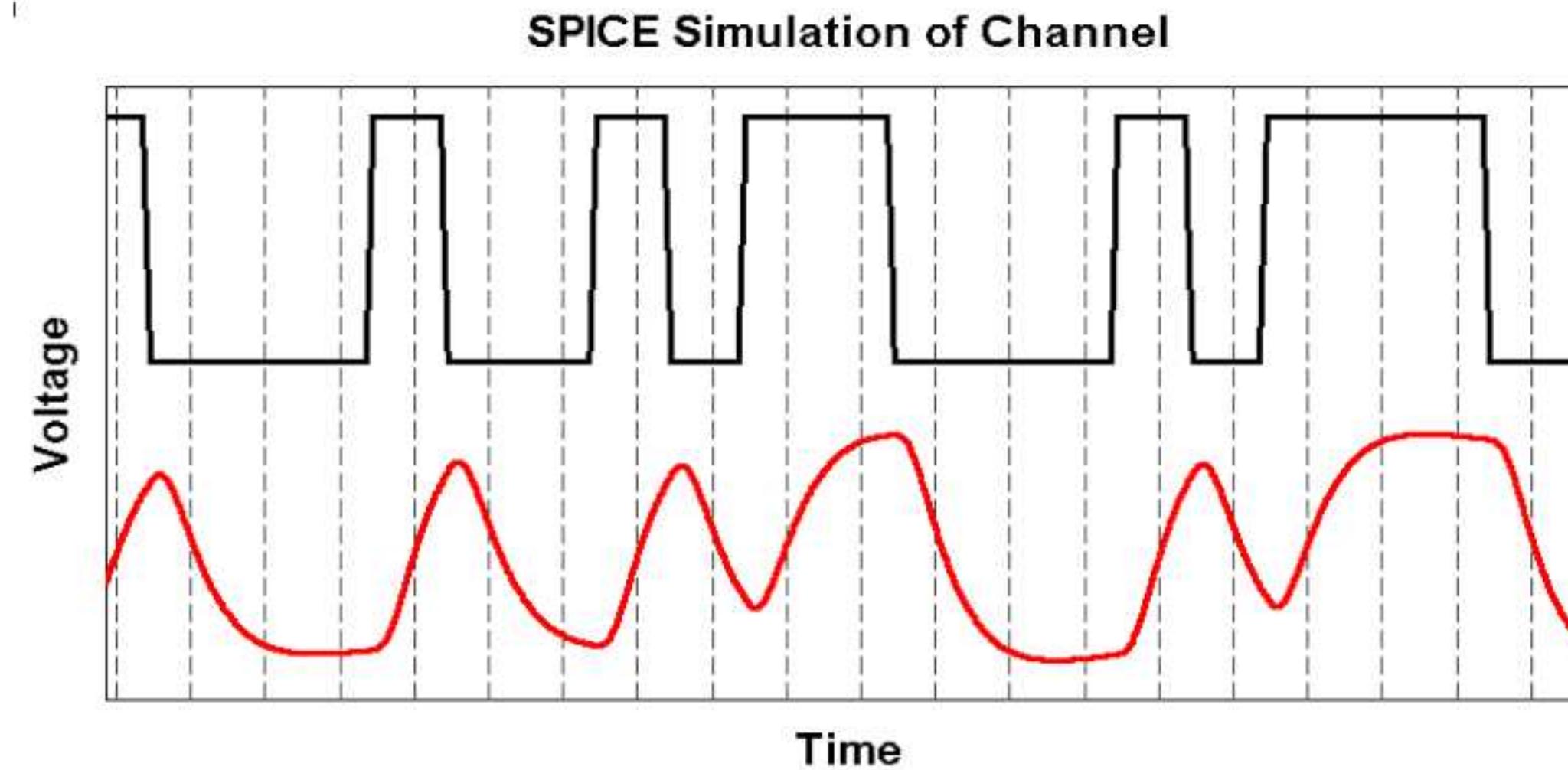
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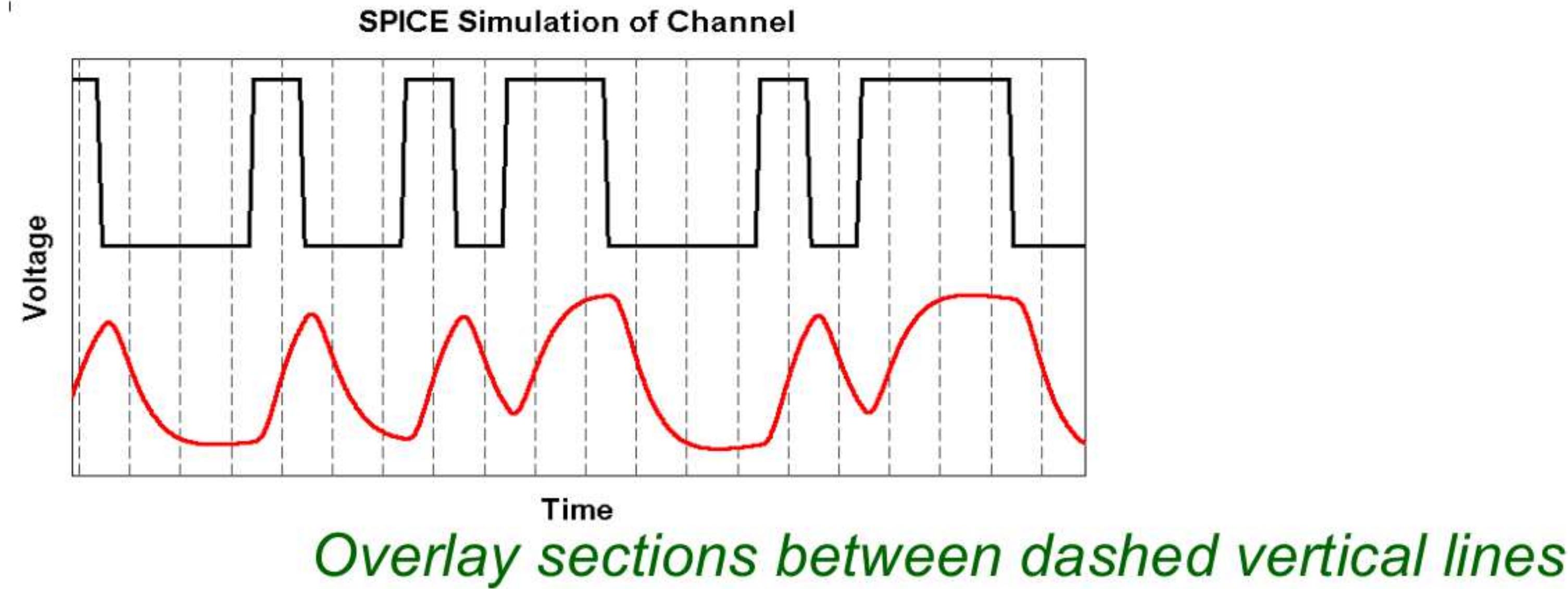
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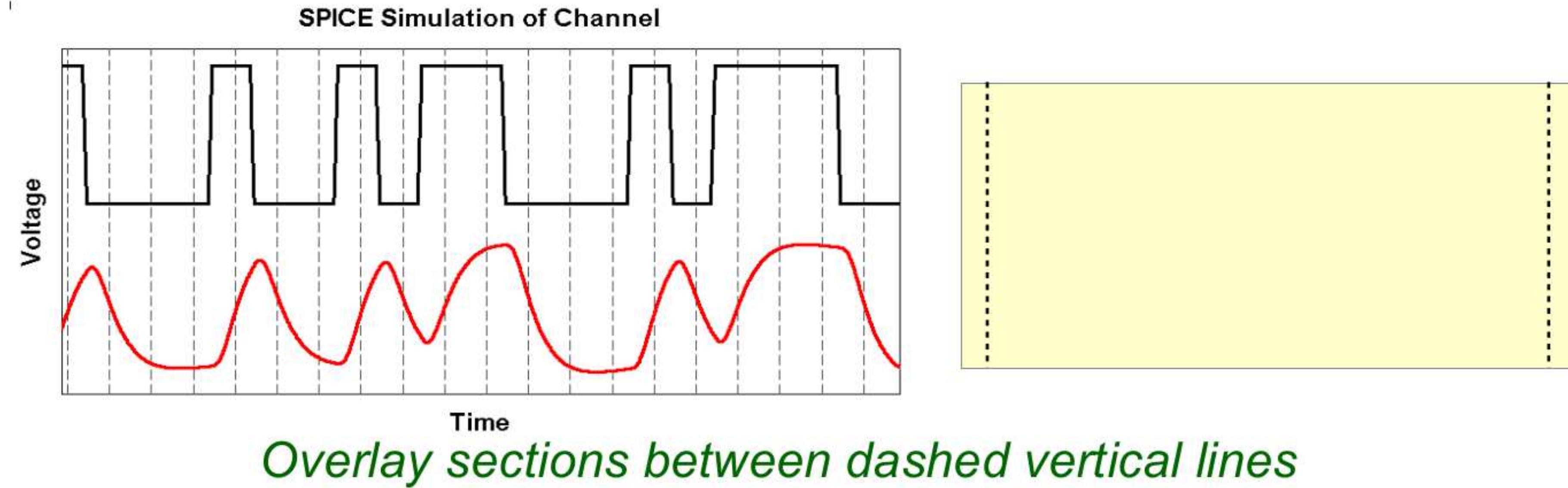
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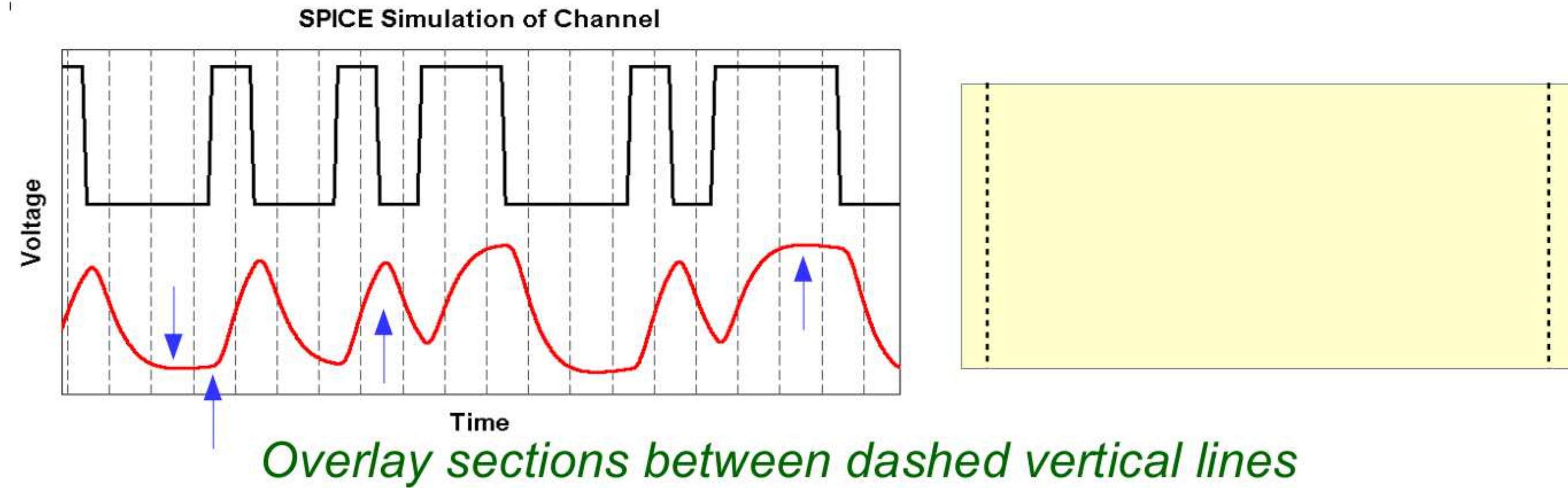
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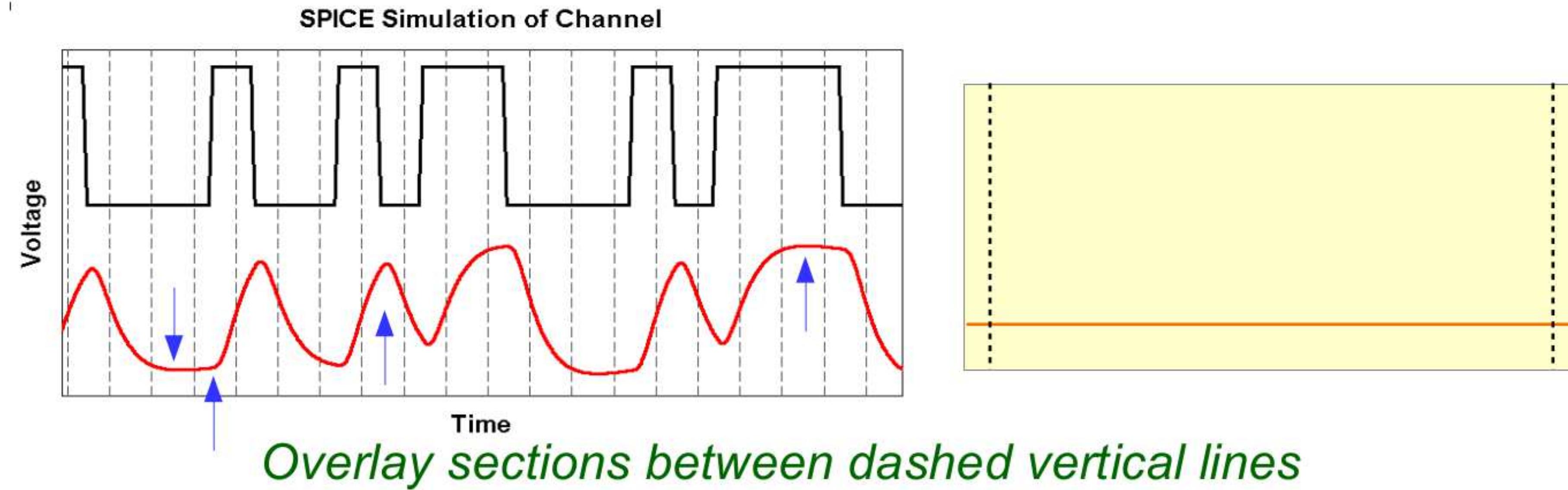
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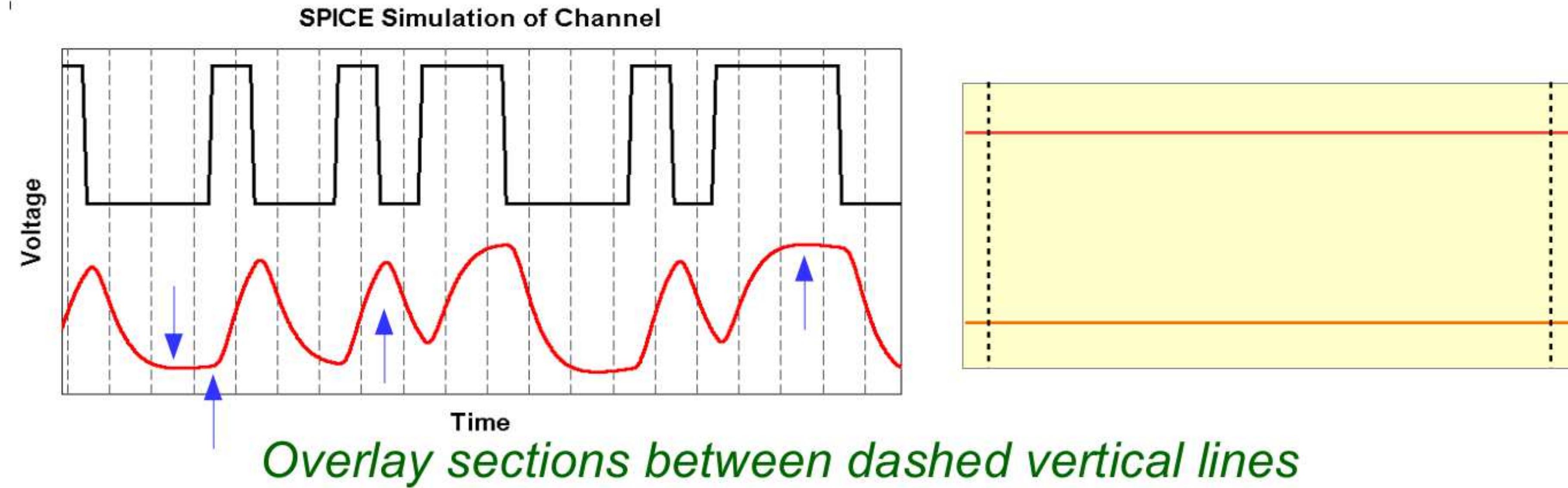
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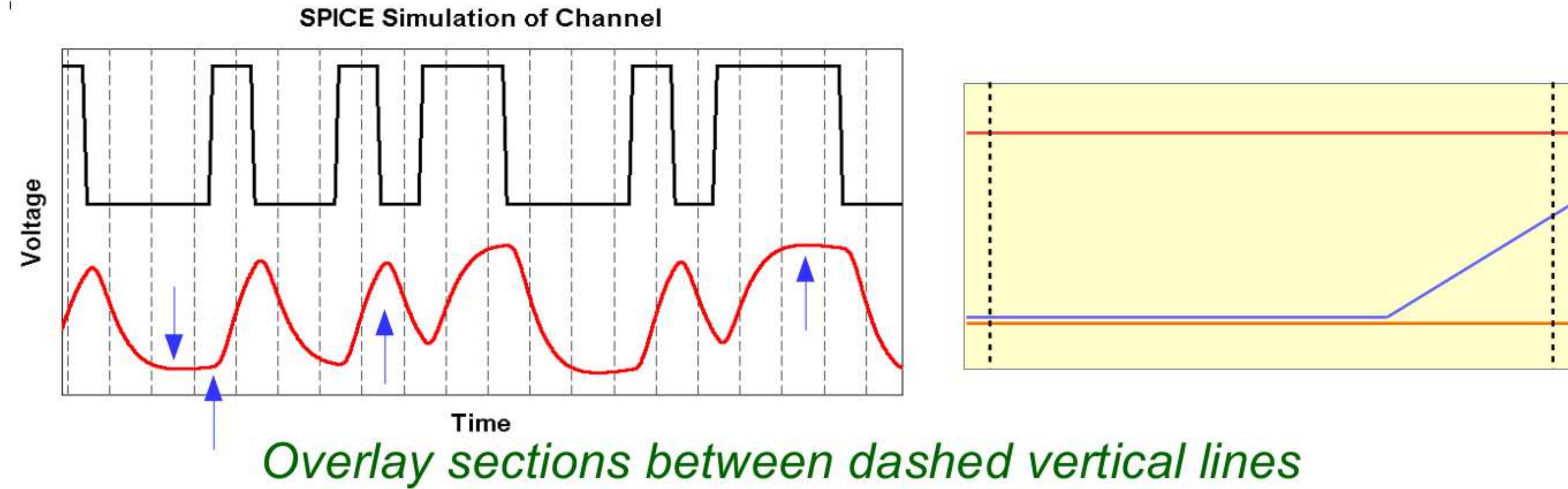
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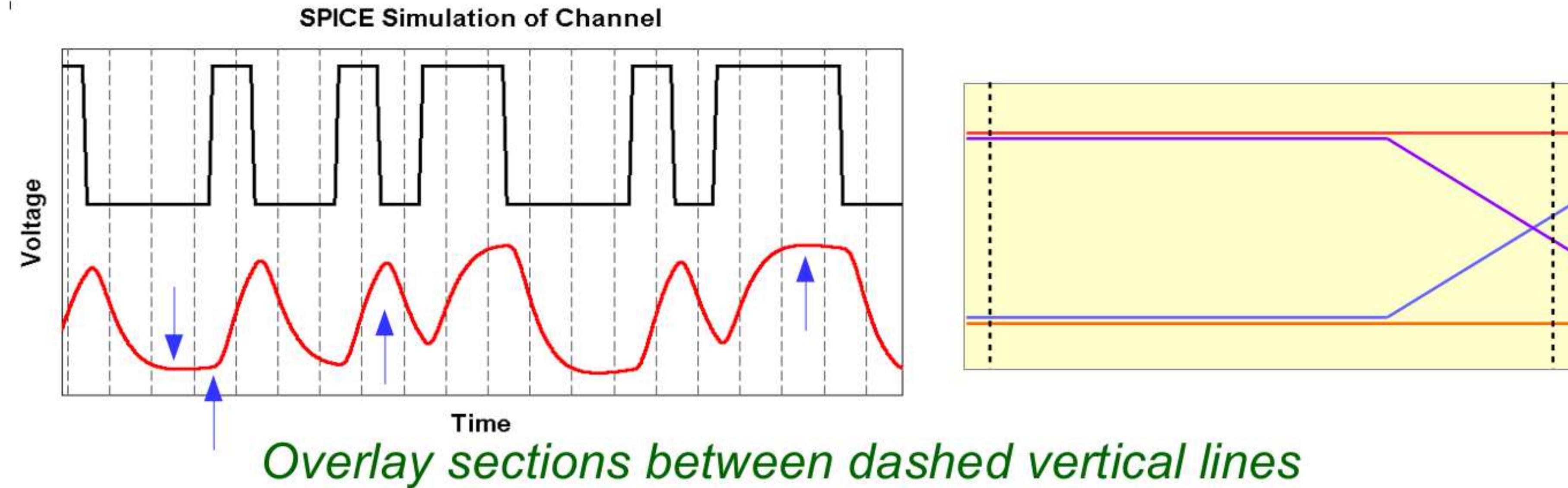
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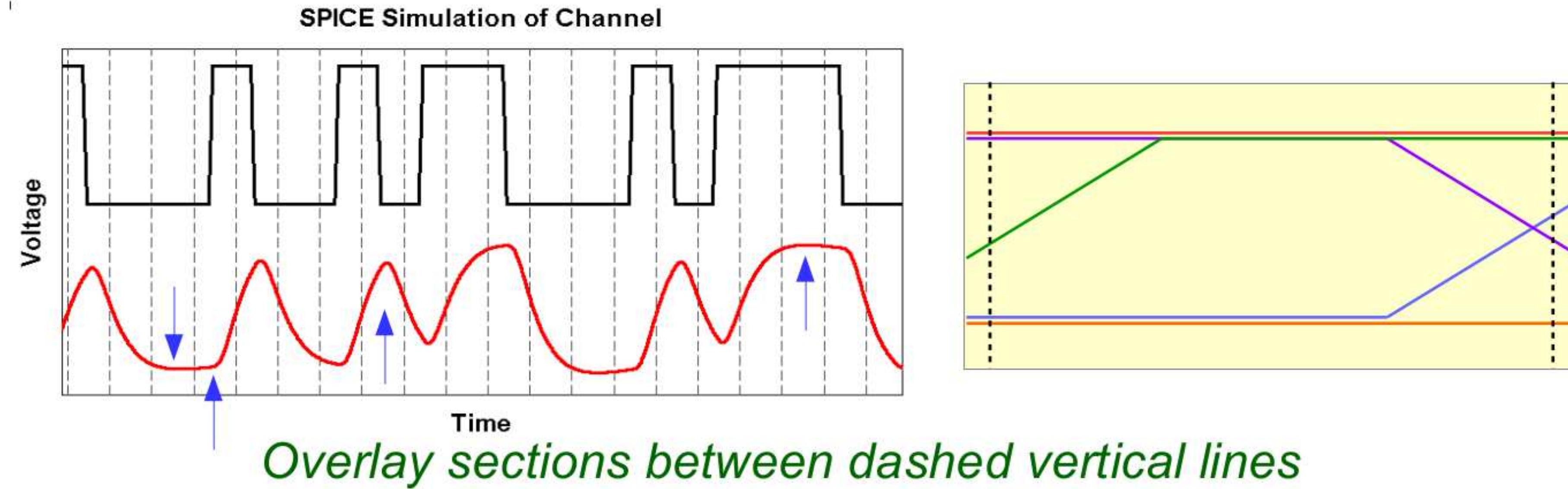
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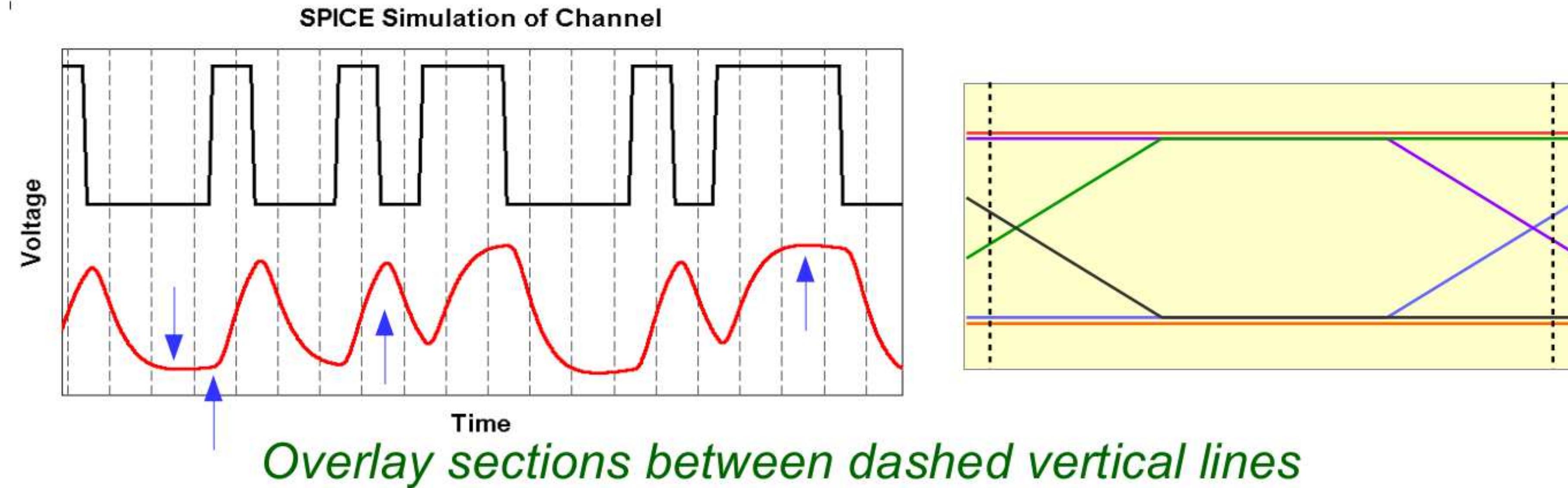
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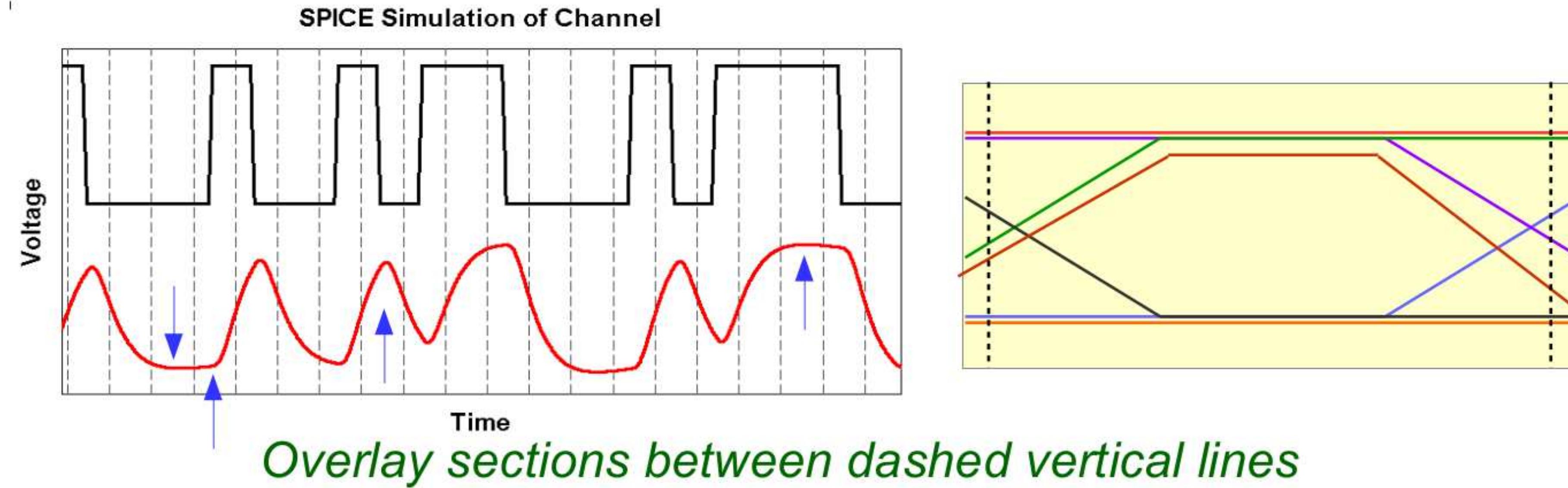
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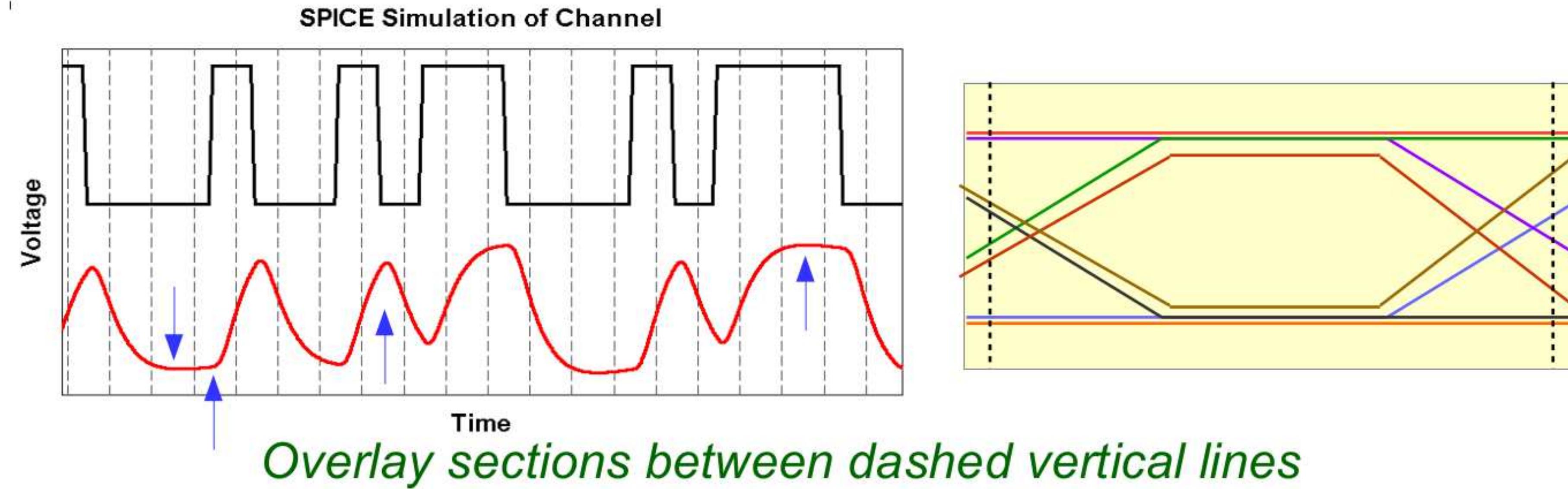
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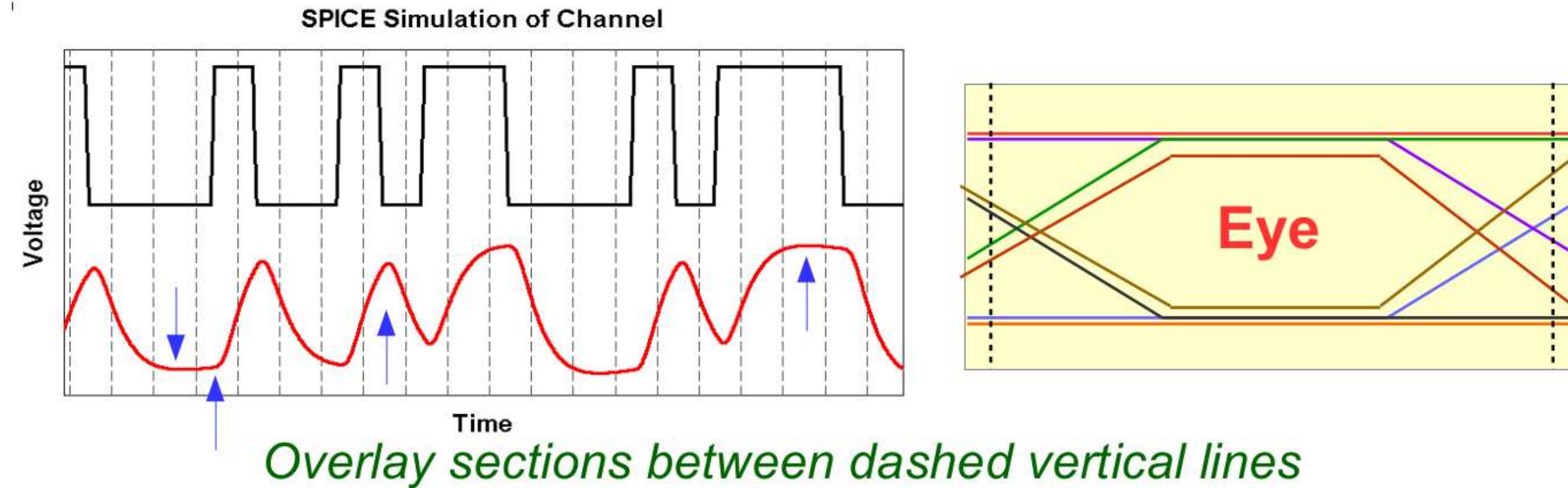
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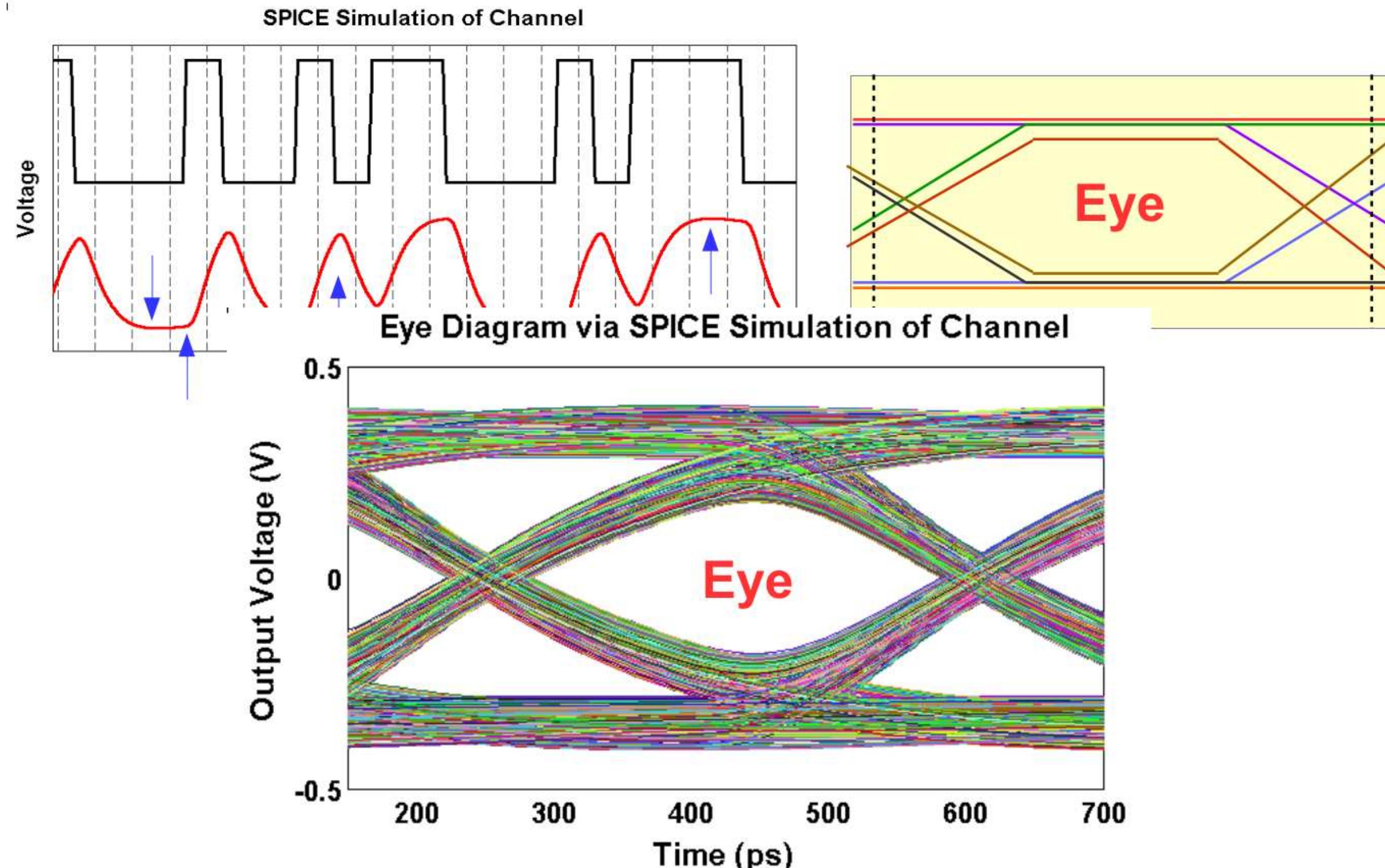
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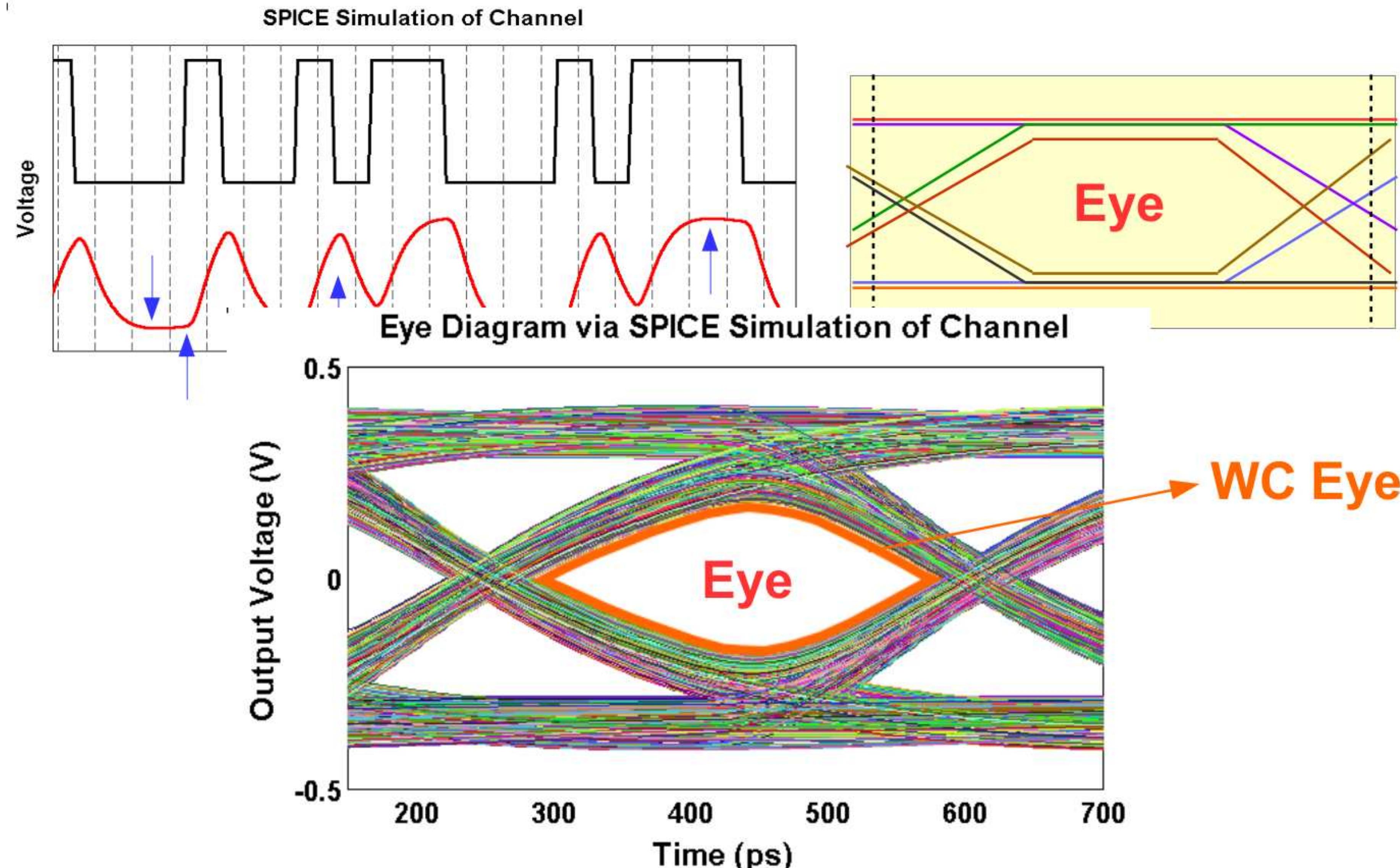
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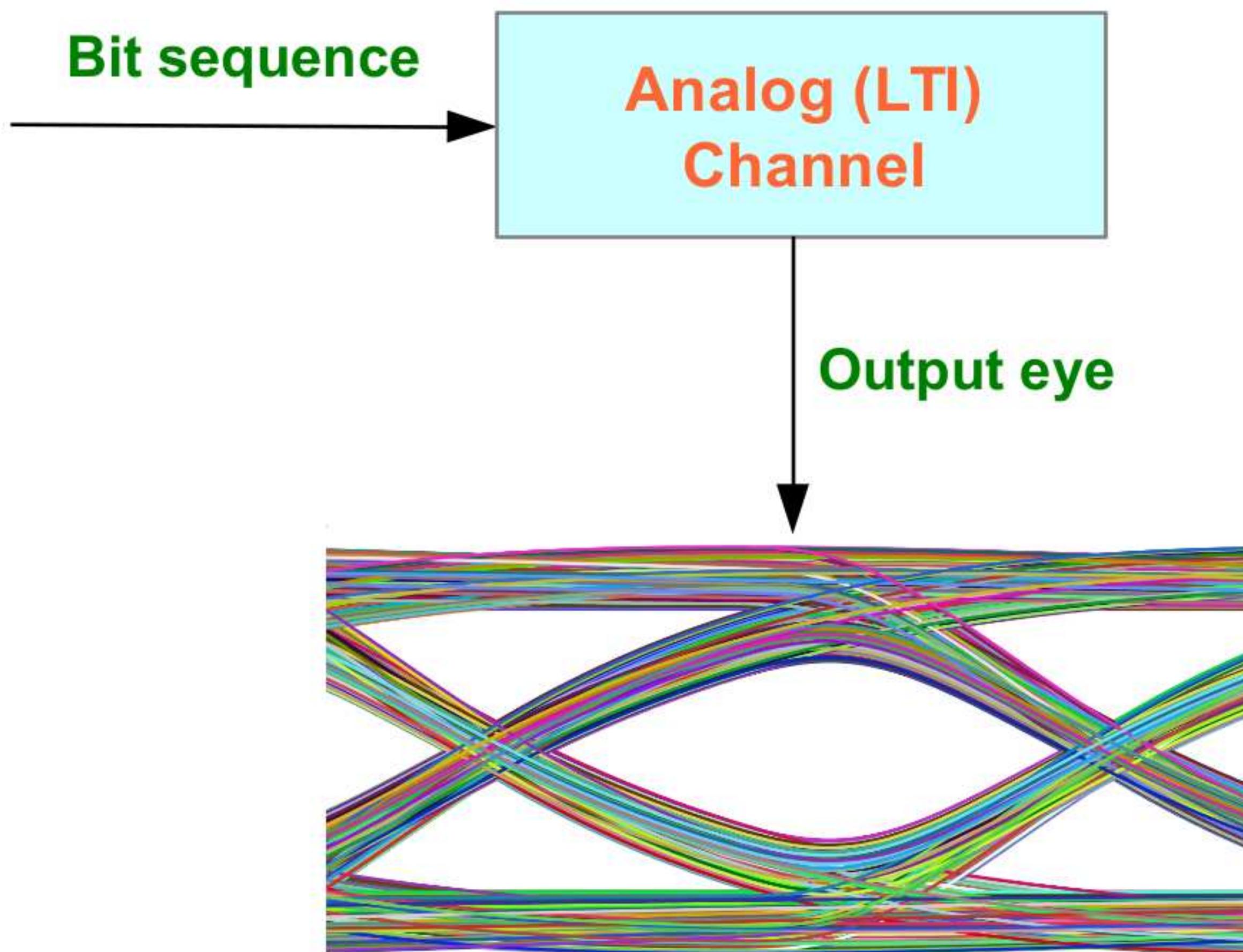
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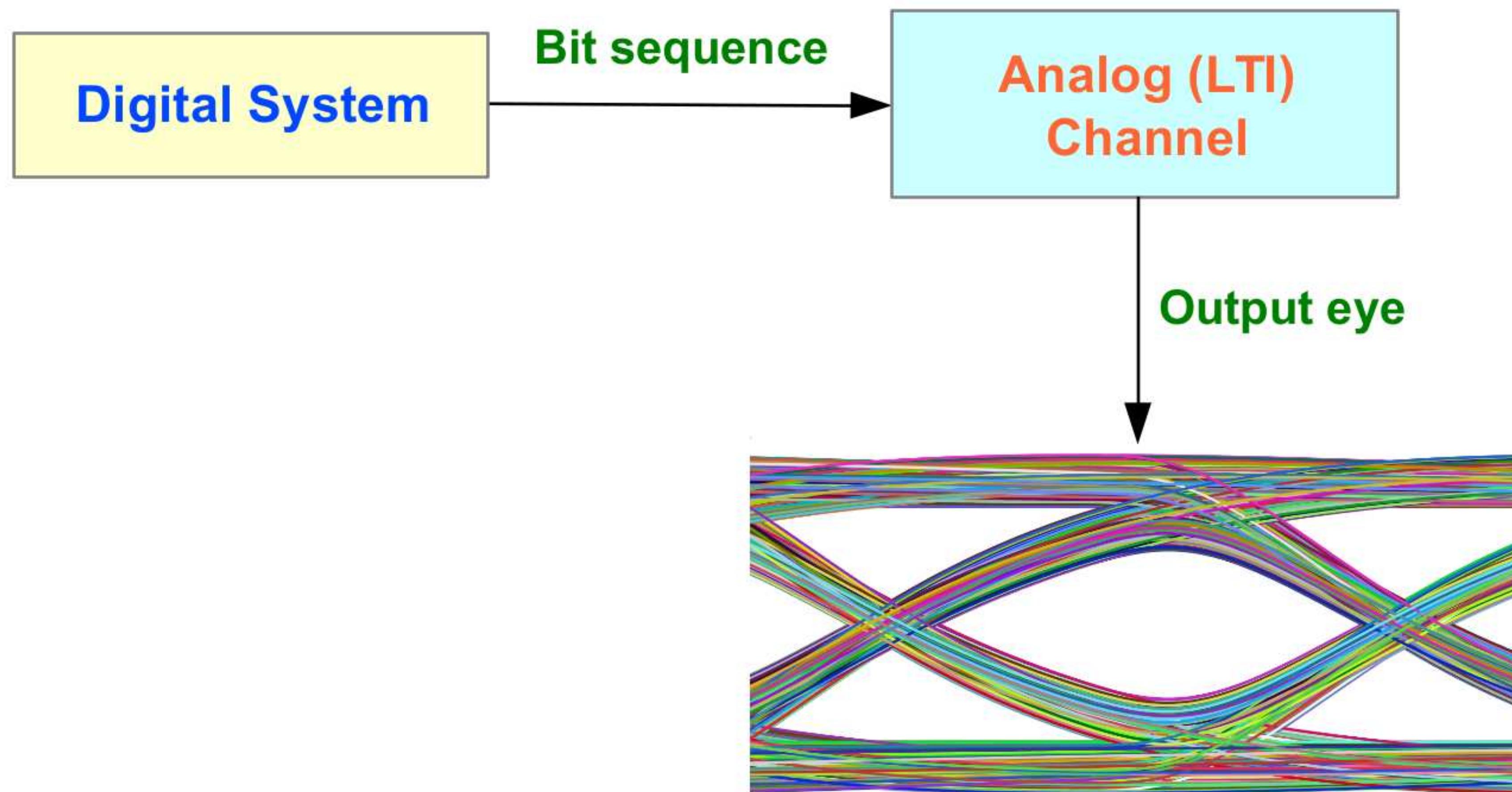
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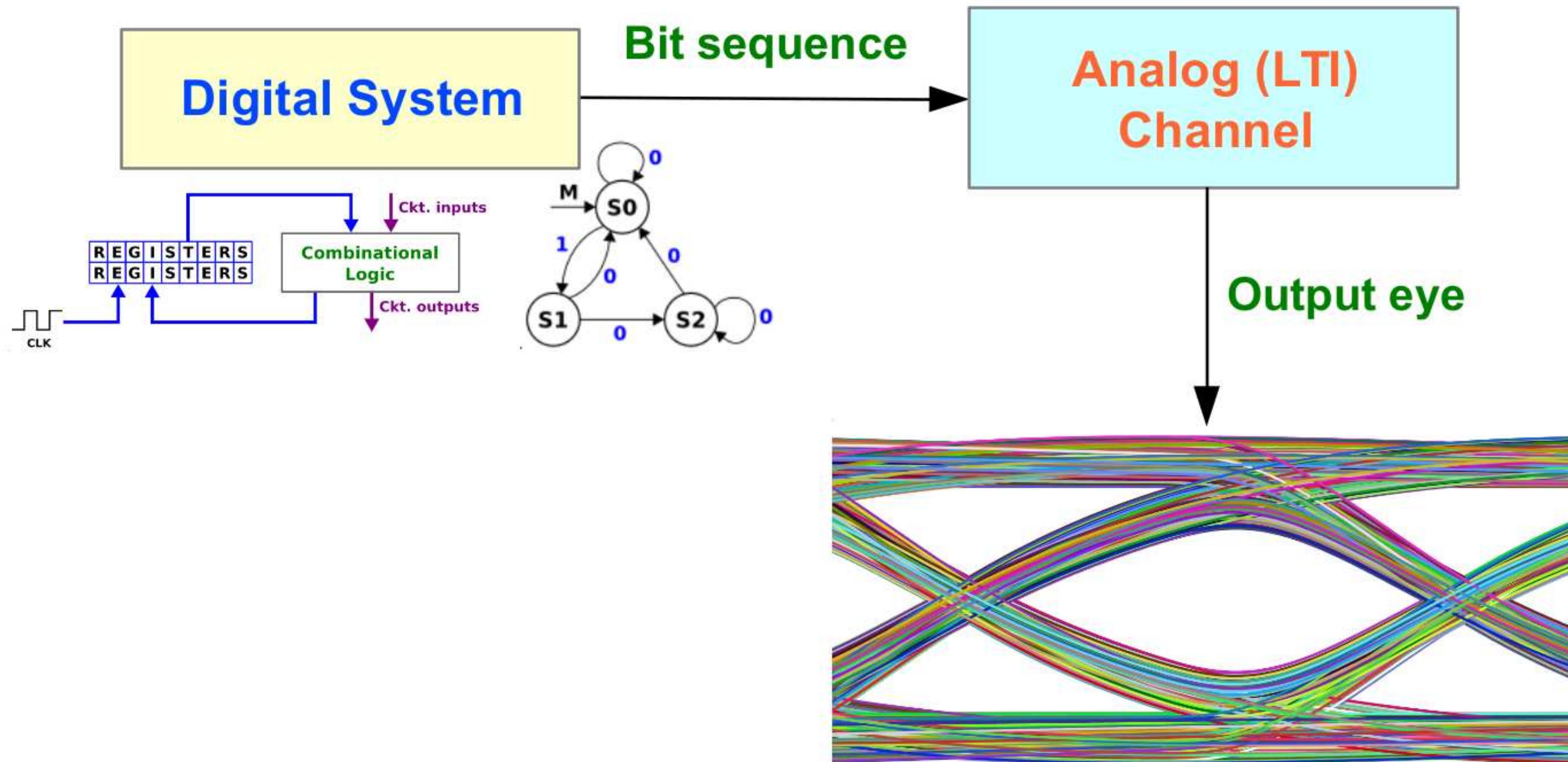
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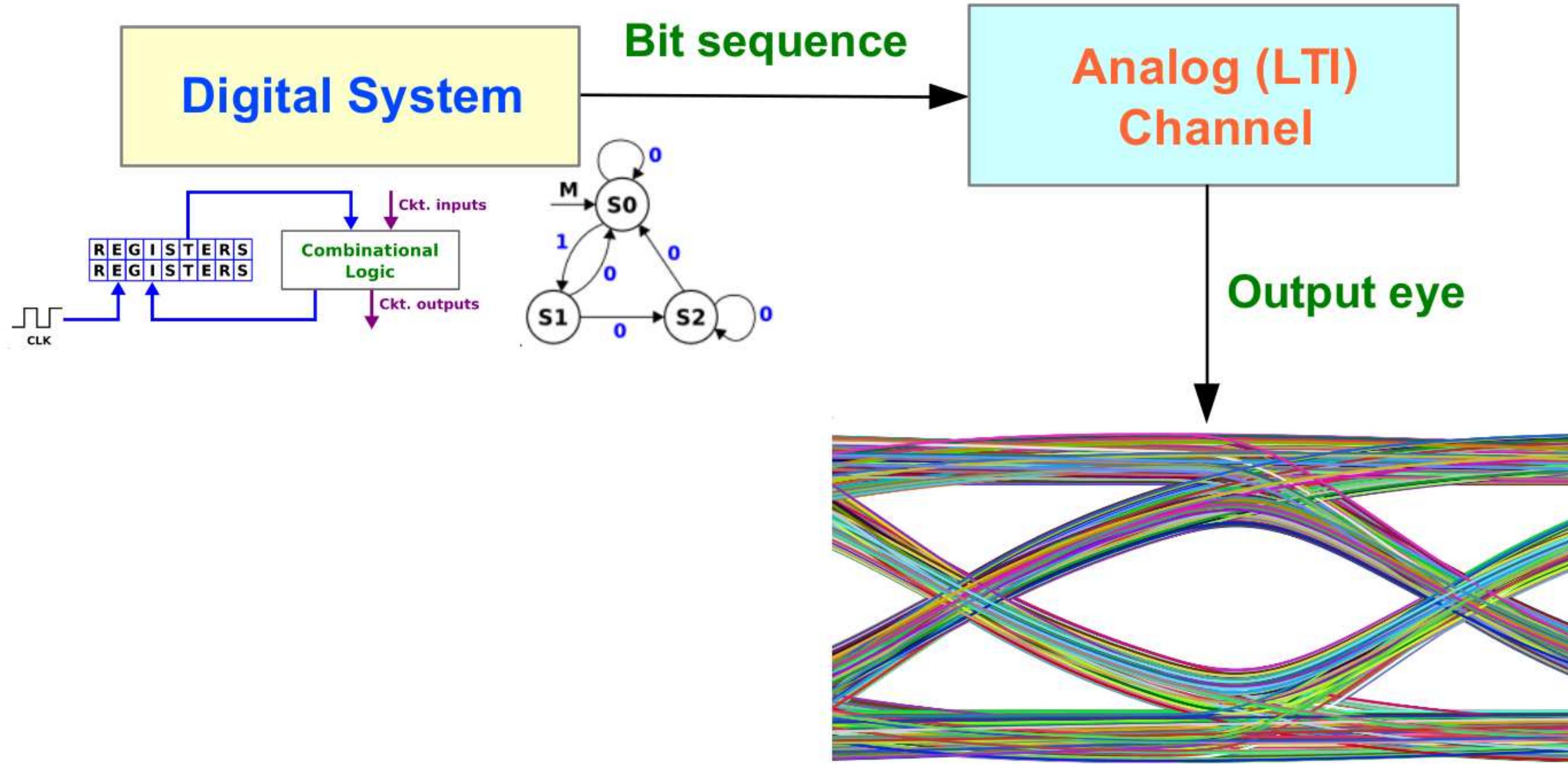
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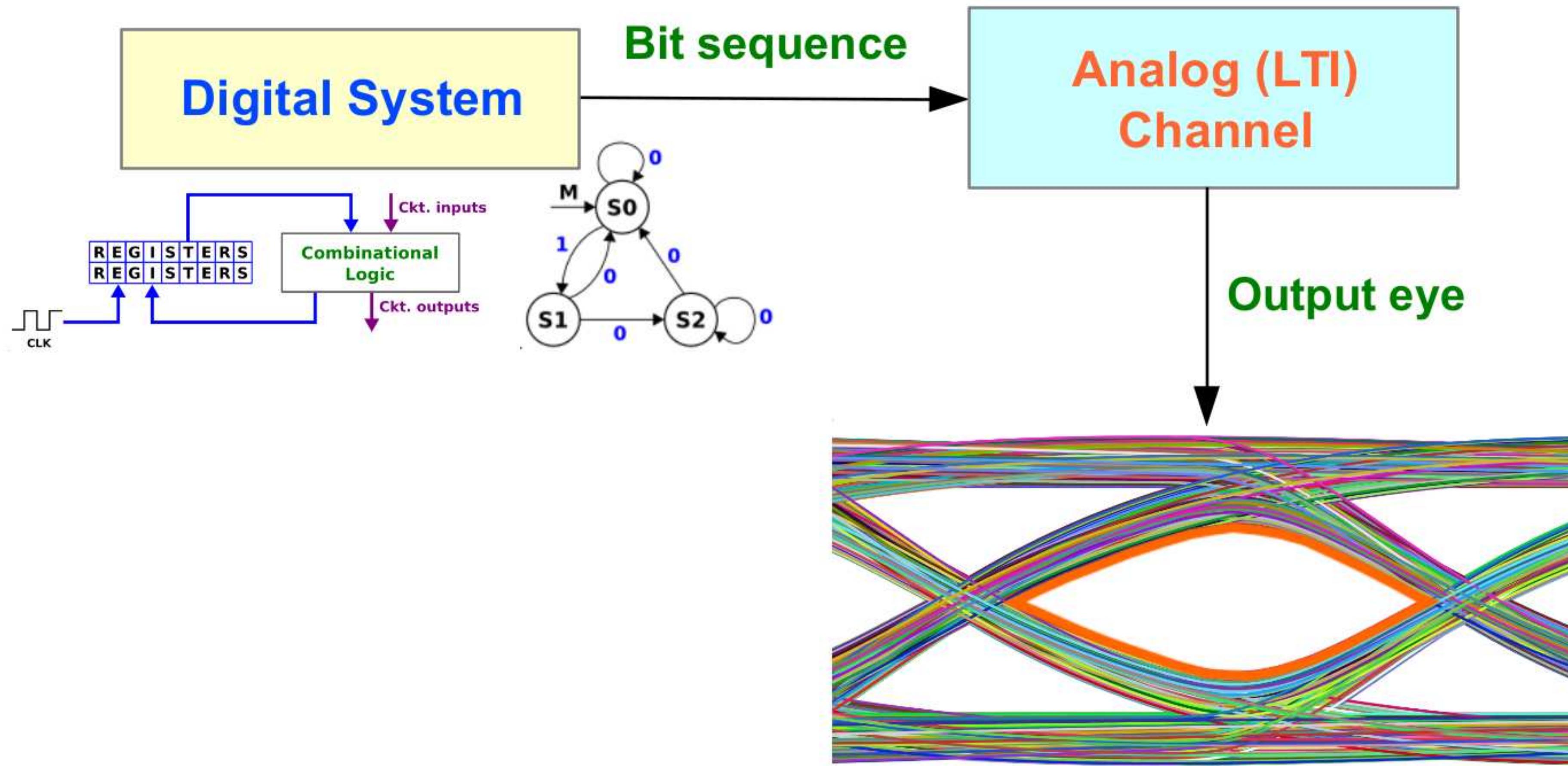


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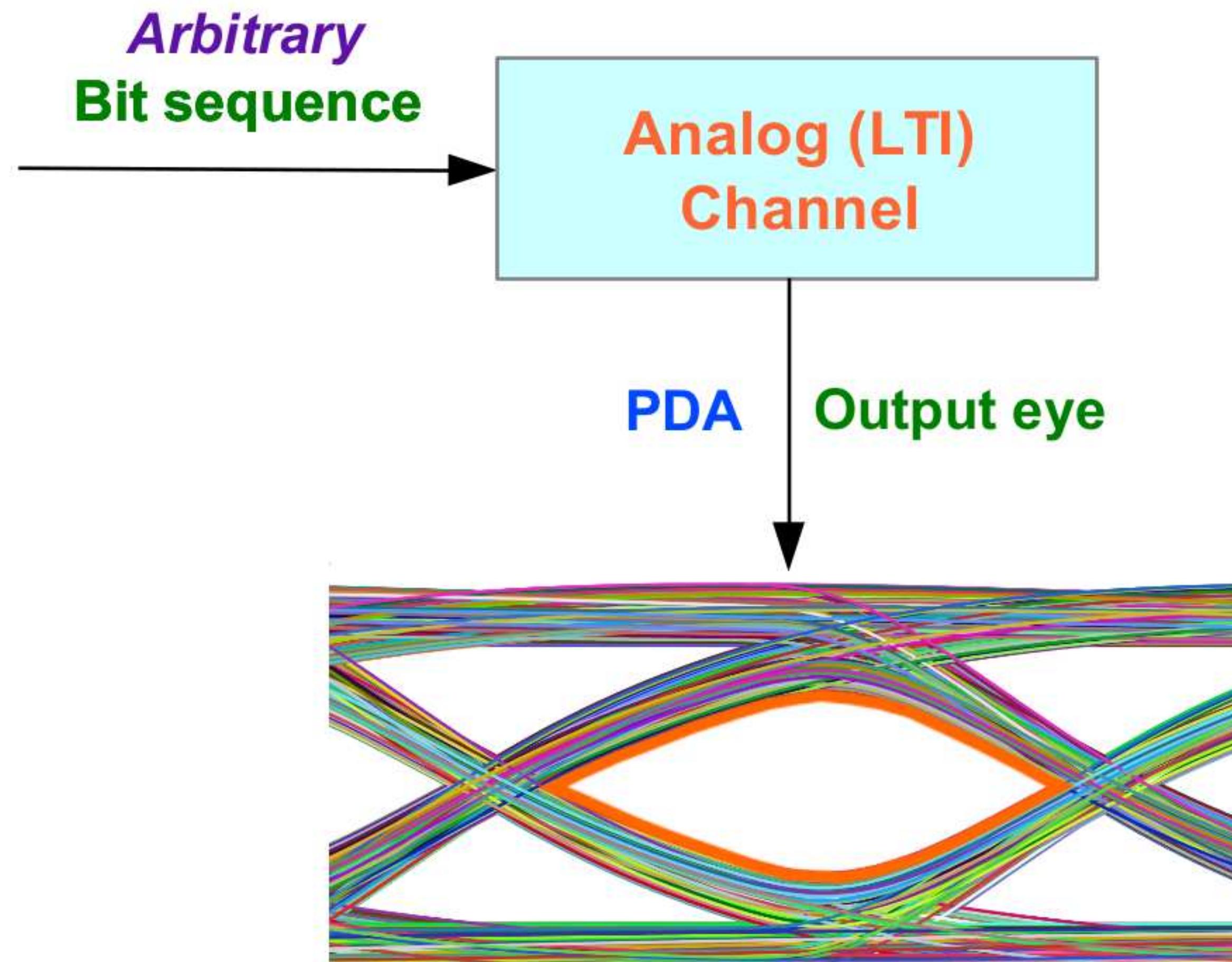
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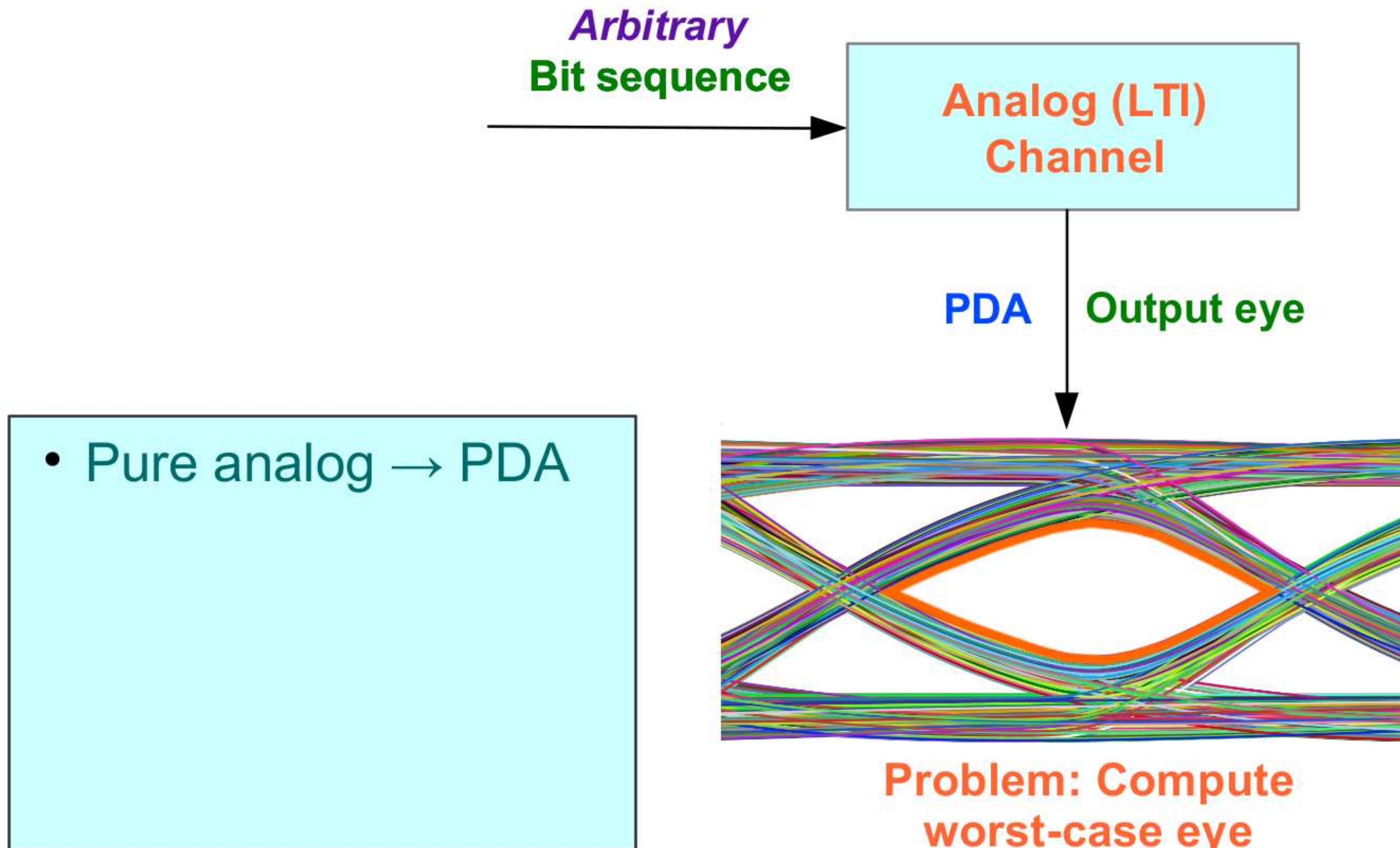
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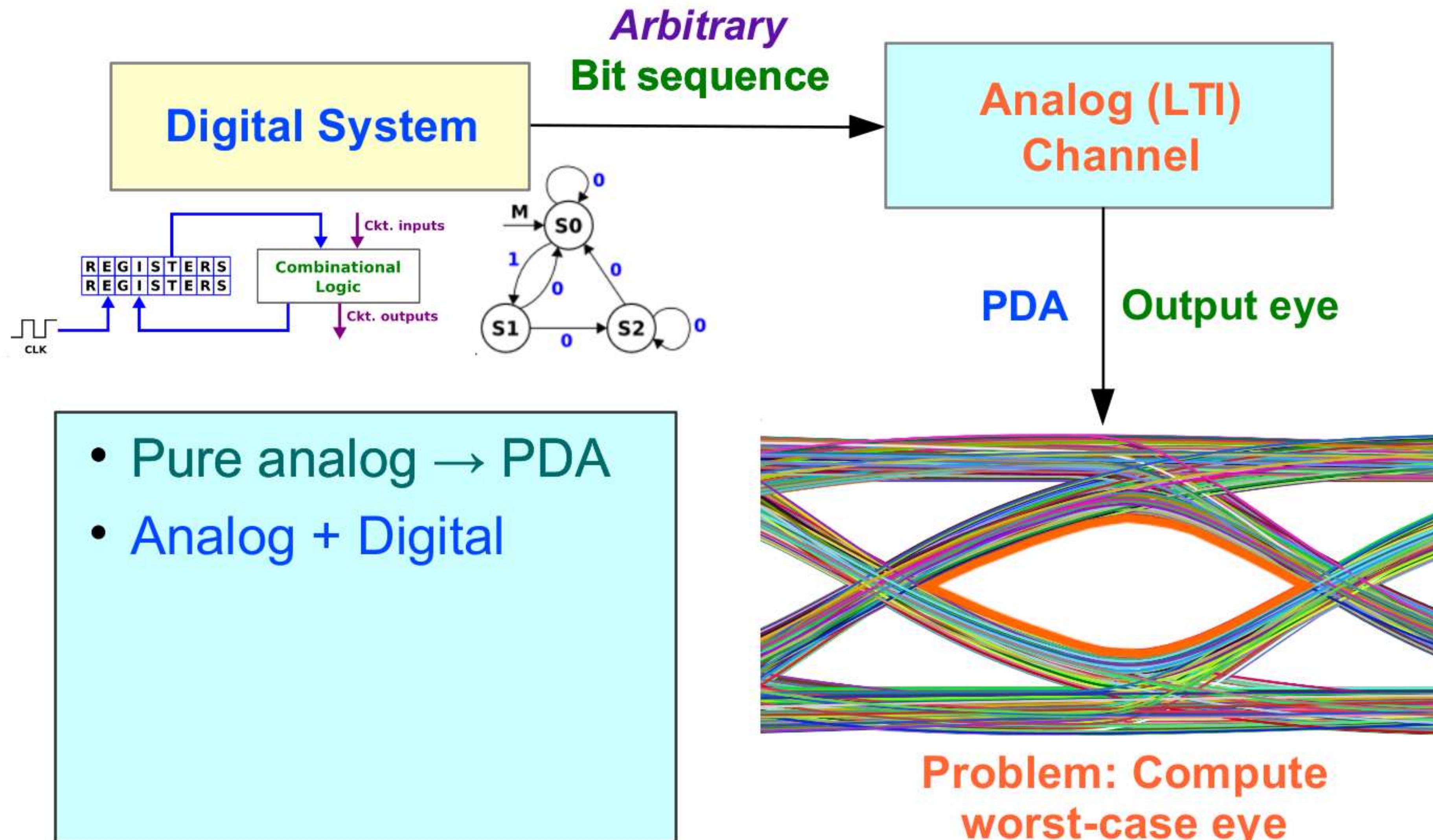


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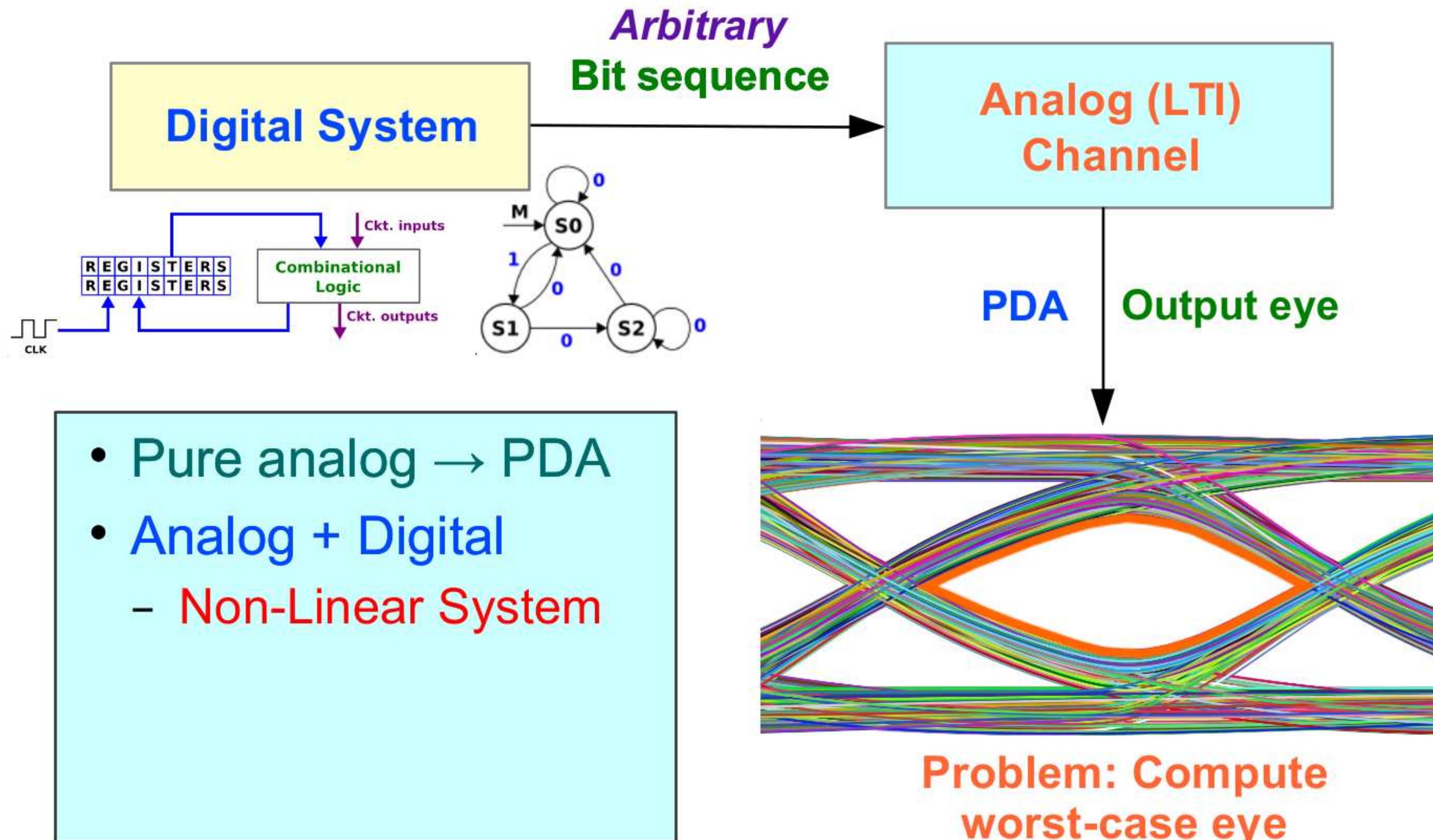
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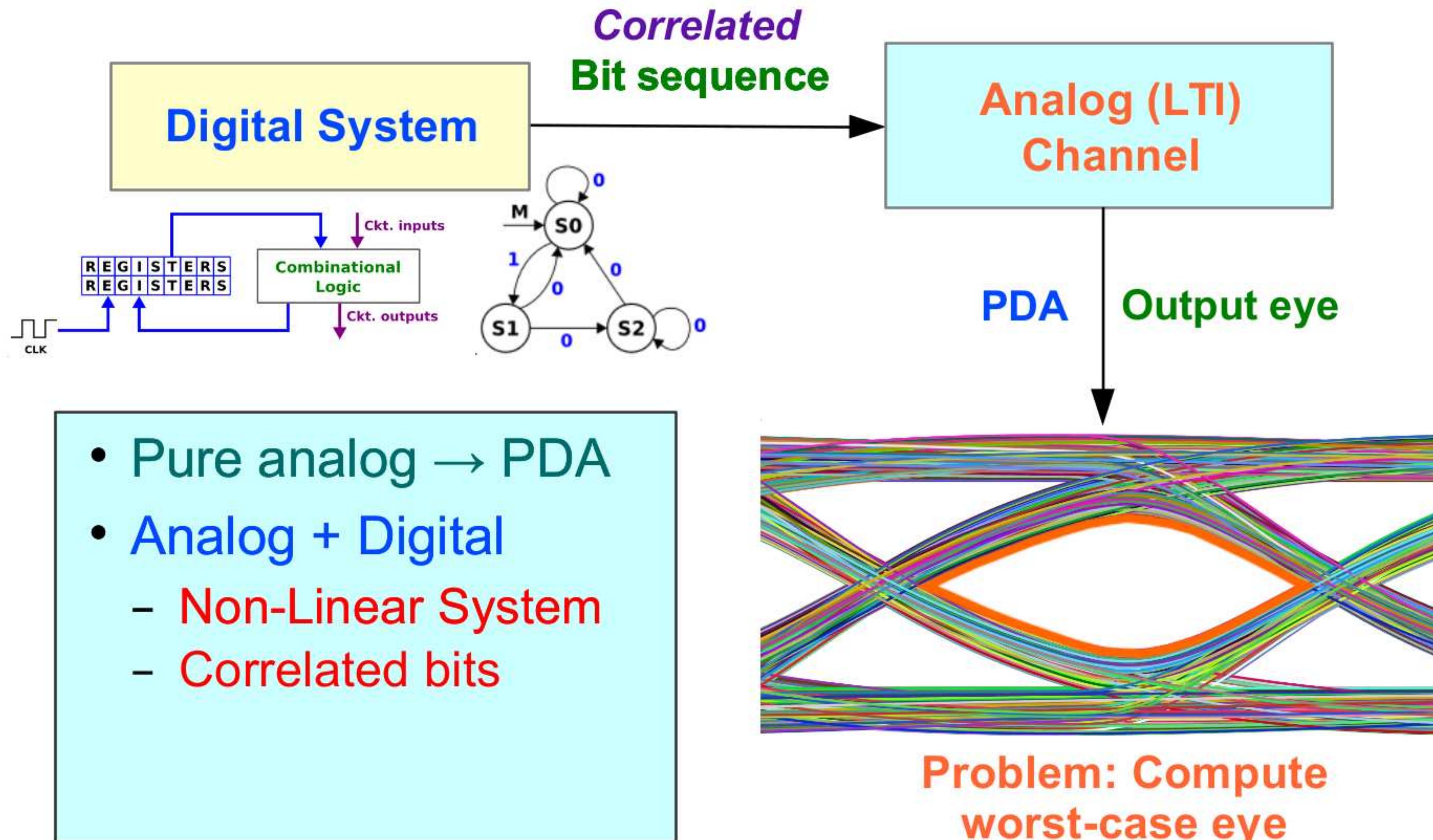
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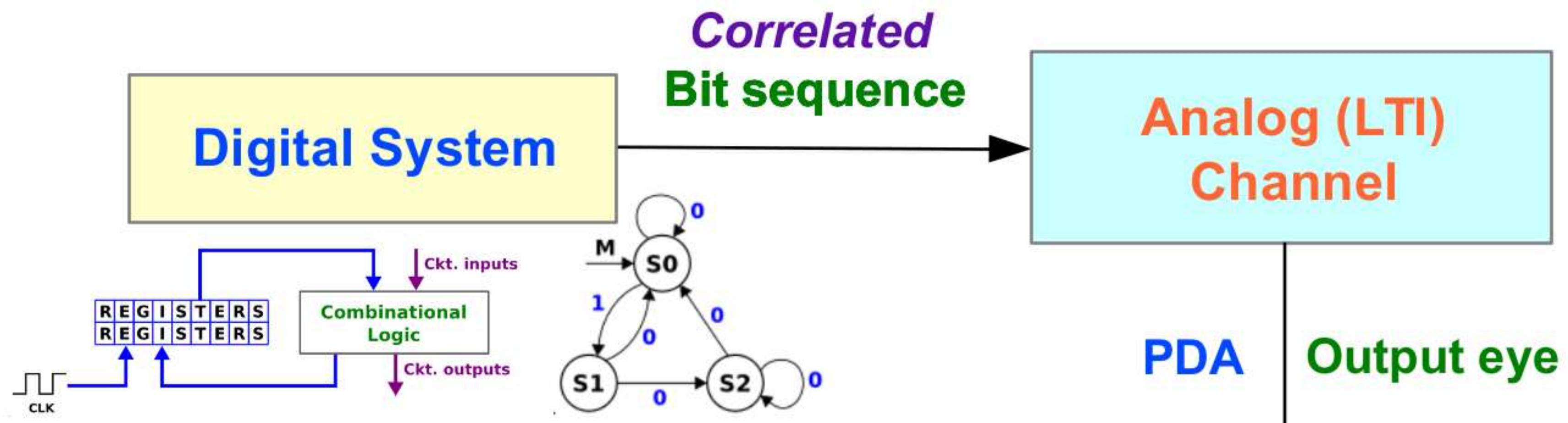
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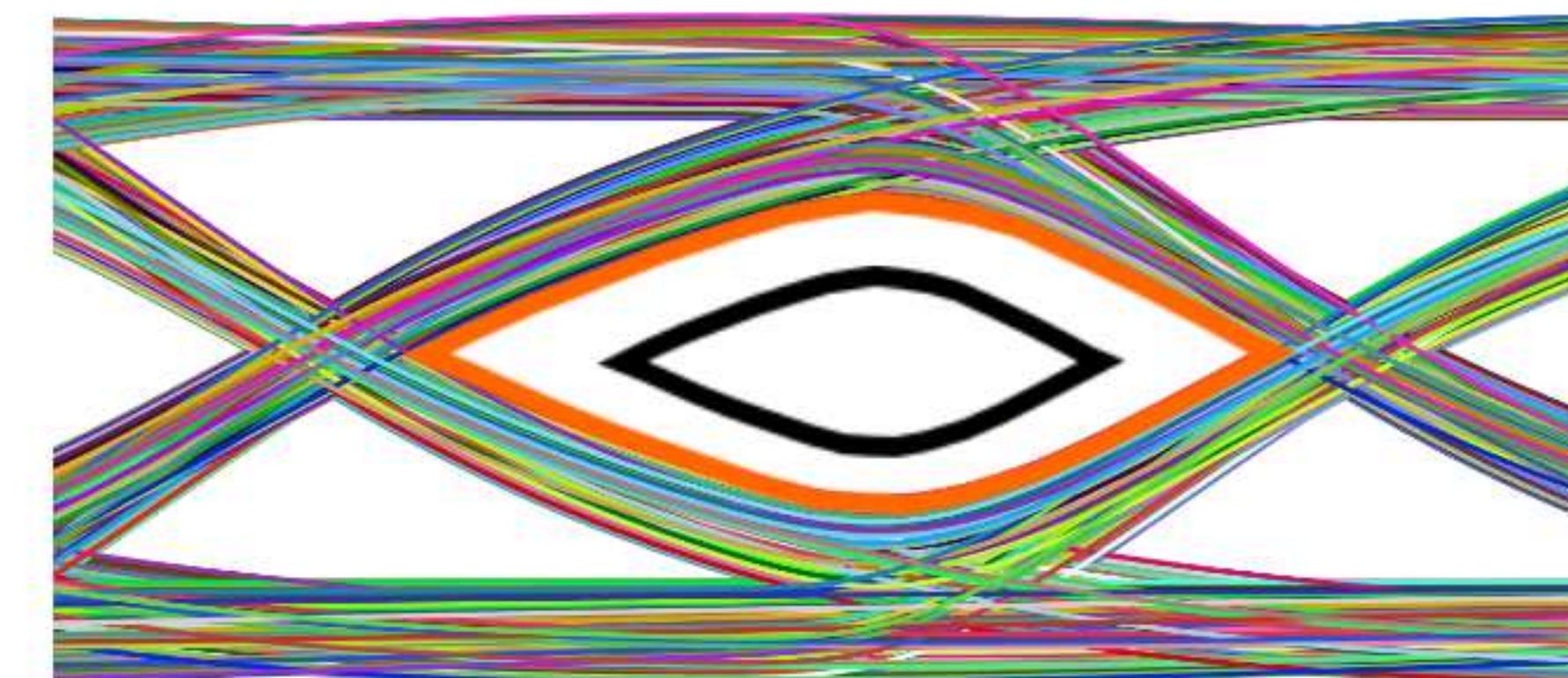
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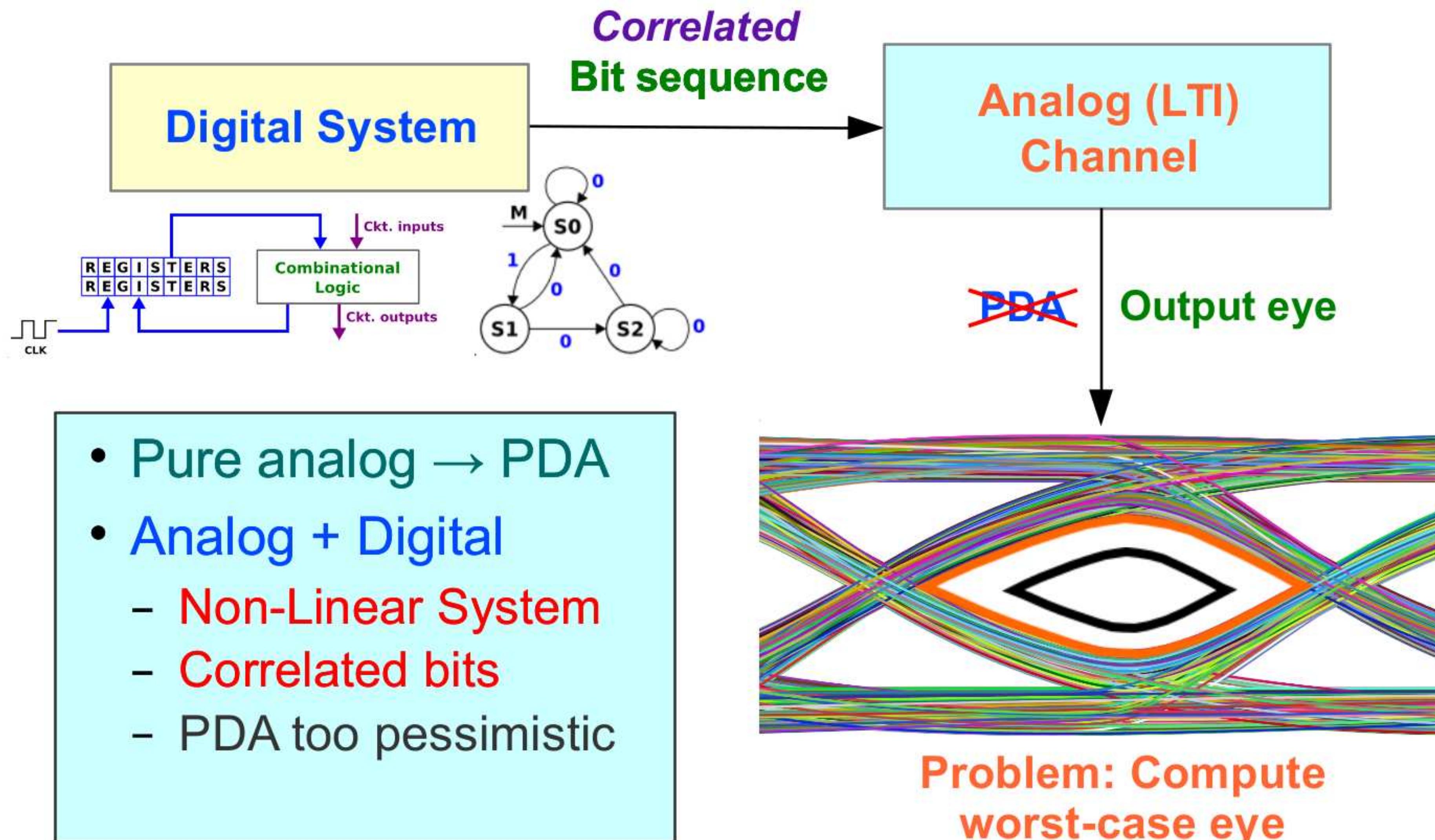


- Pure analog  $\rightarrow$  PDA
- Analog + Digital
  - Non-Linear System
  - Correlated bits
  - PDA too pessimistic

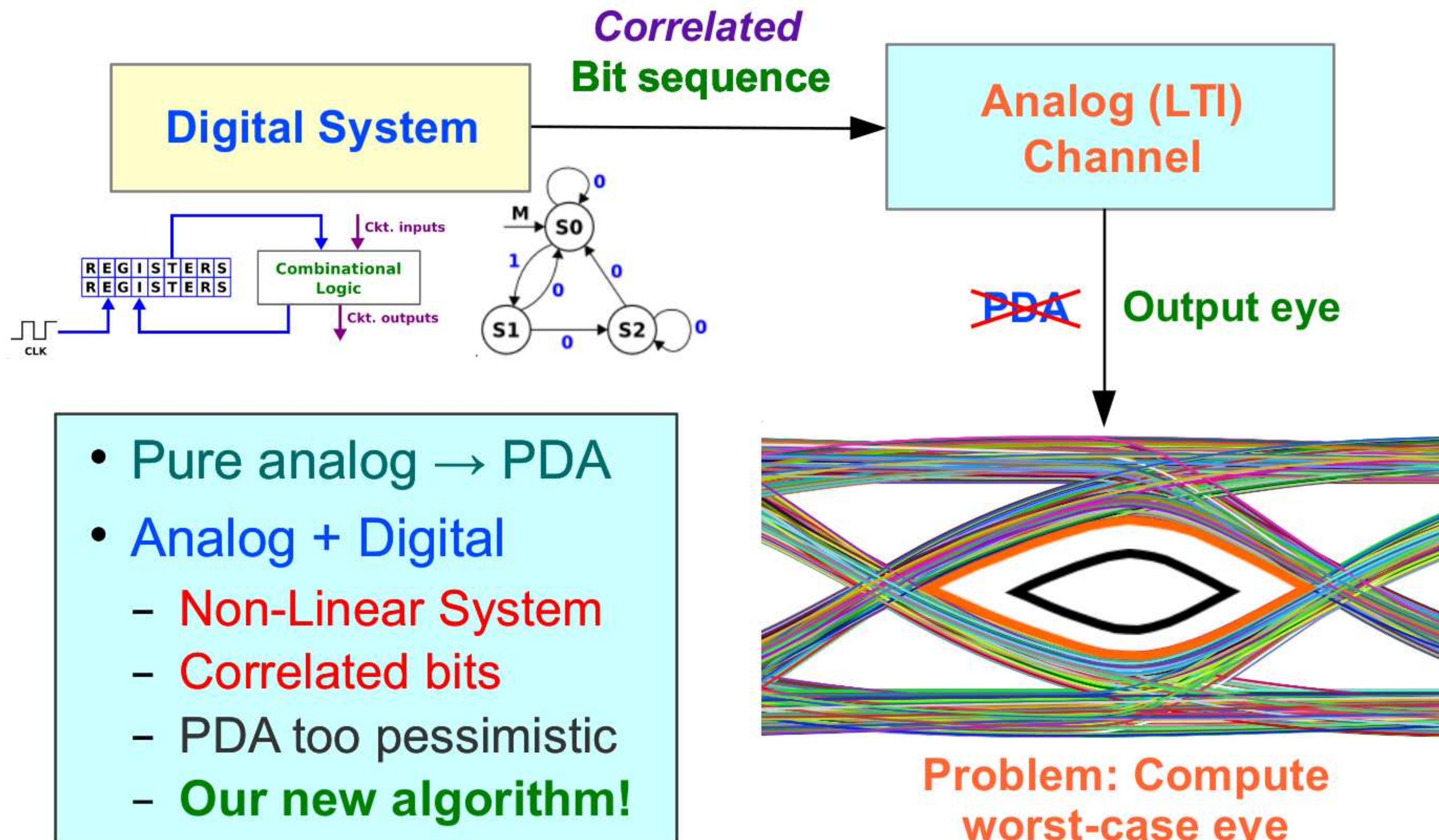


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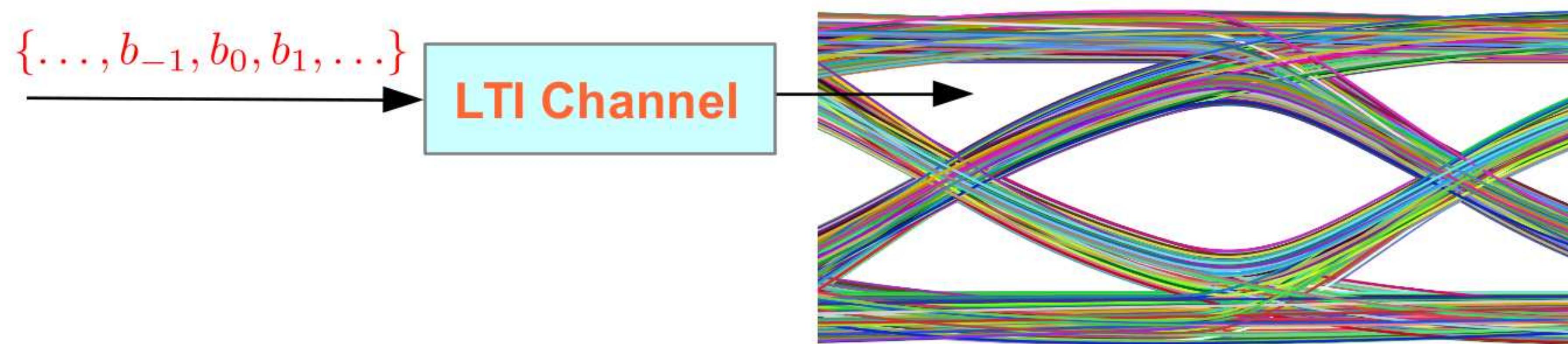


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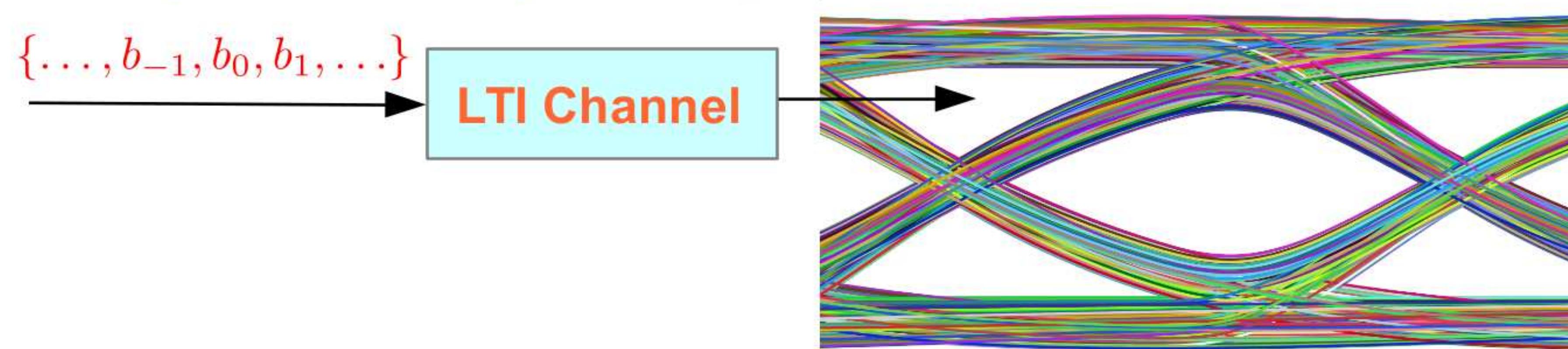
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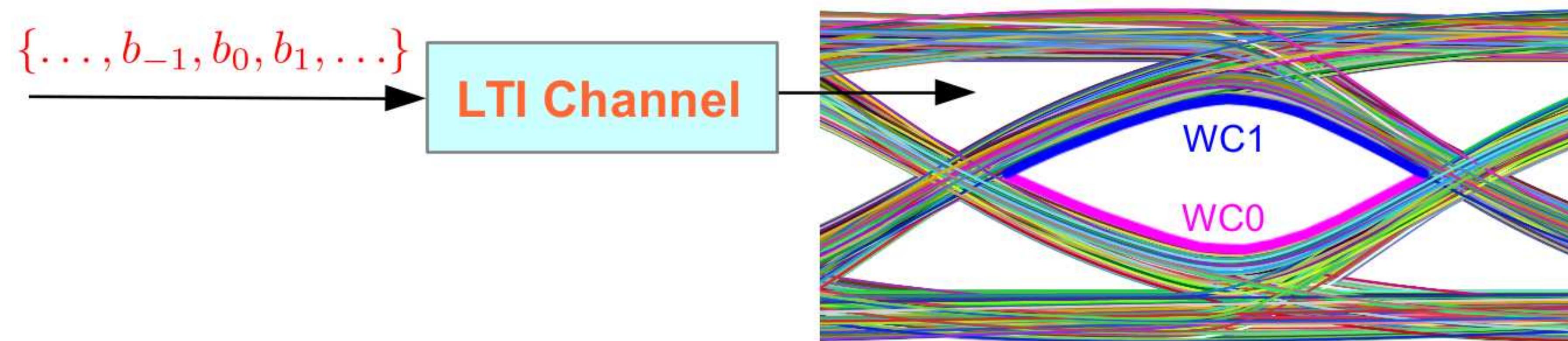
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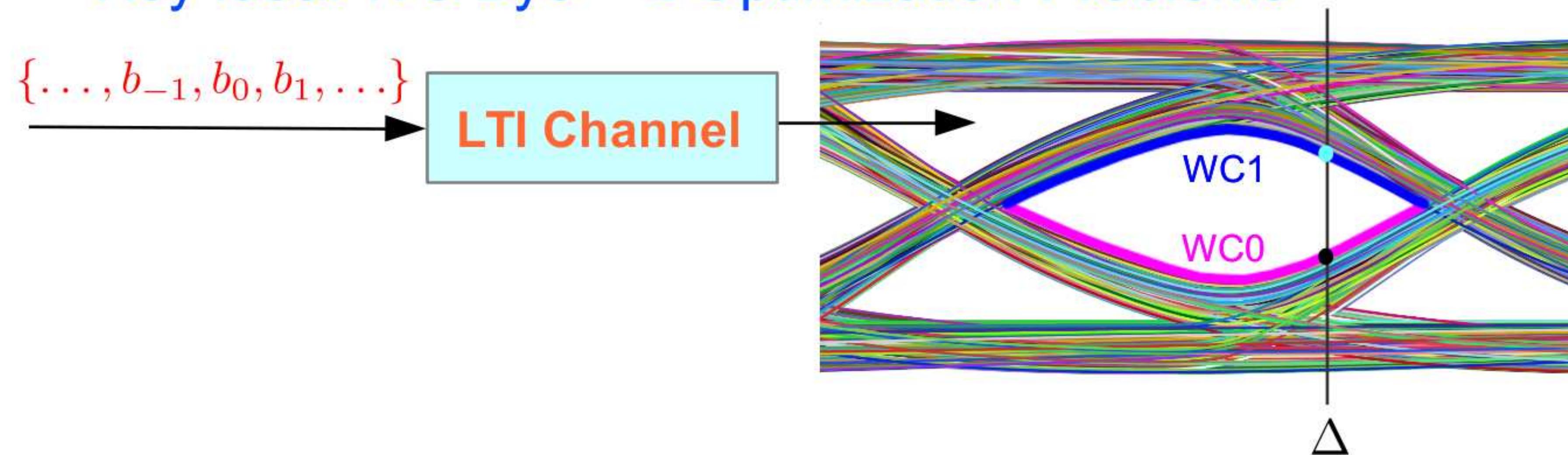
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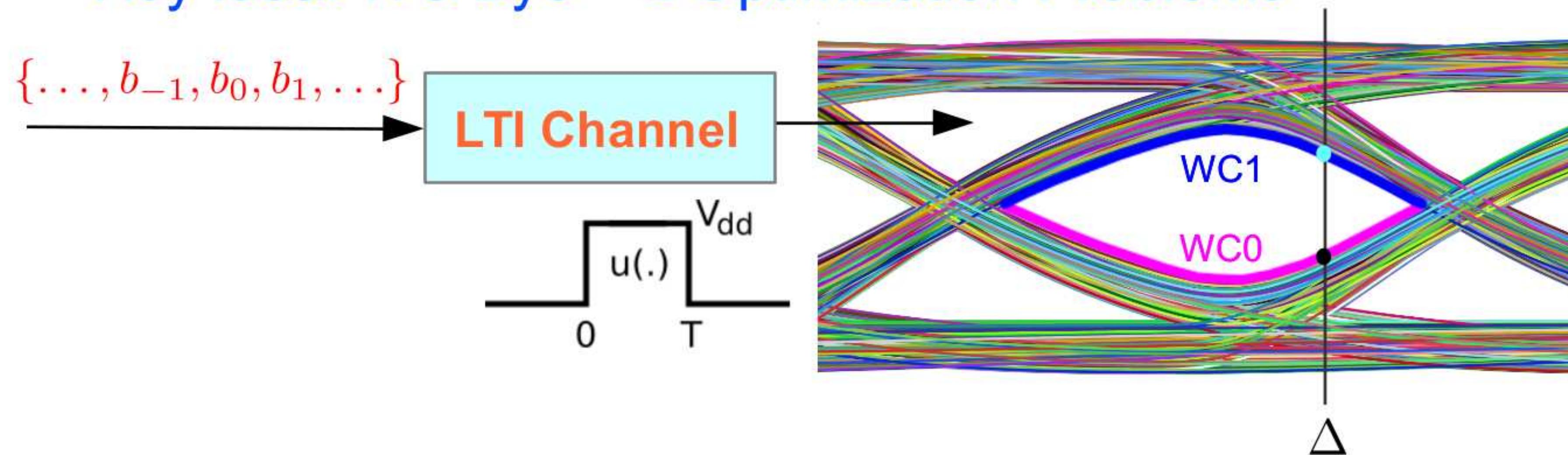
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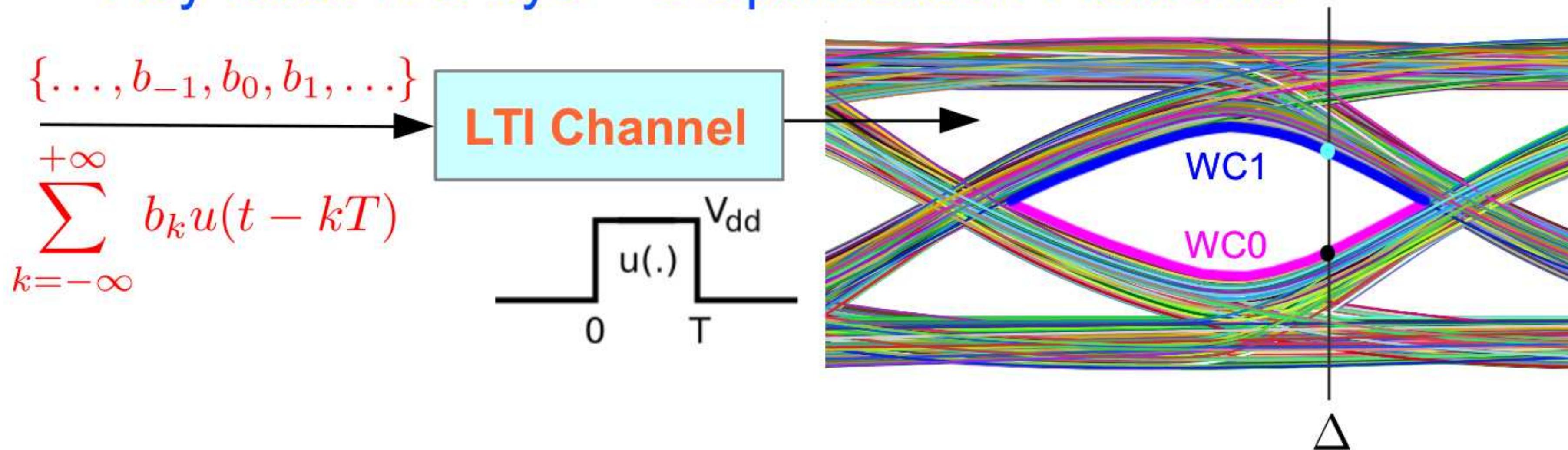
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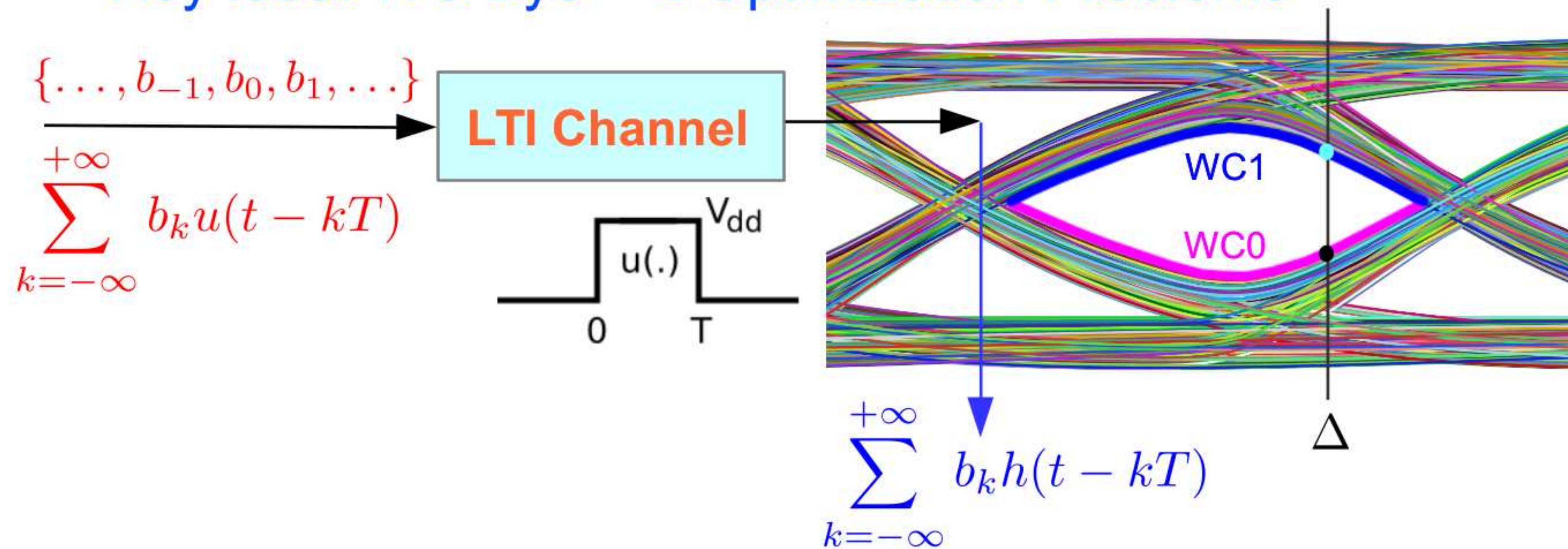
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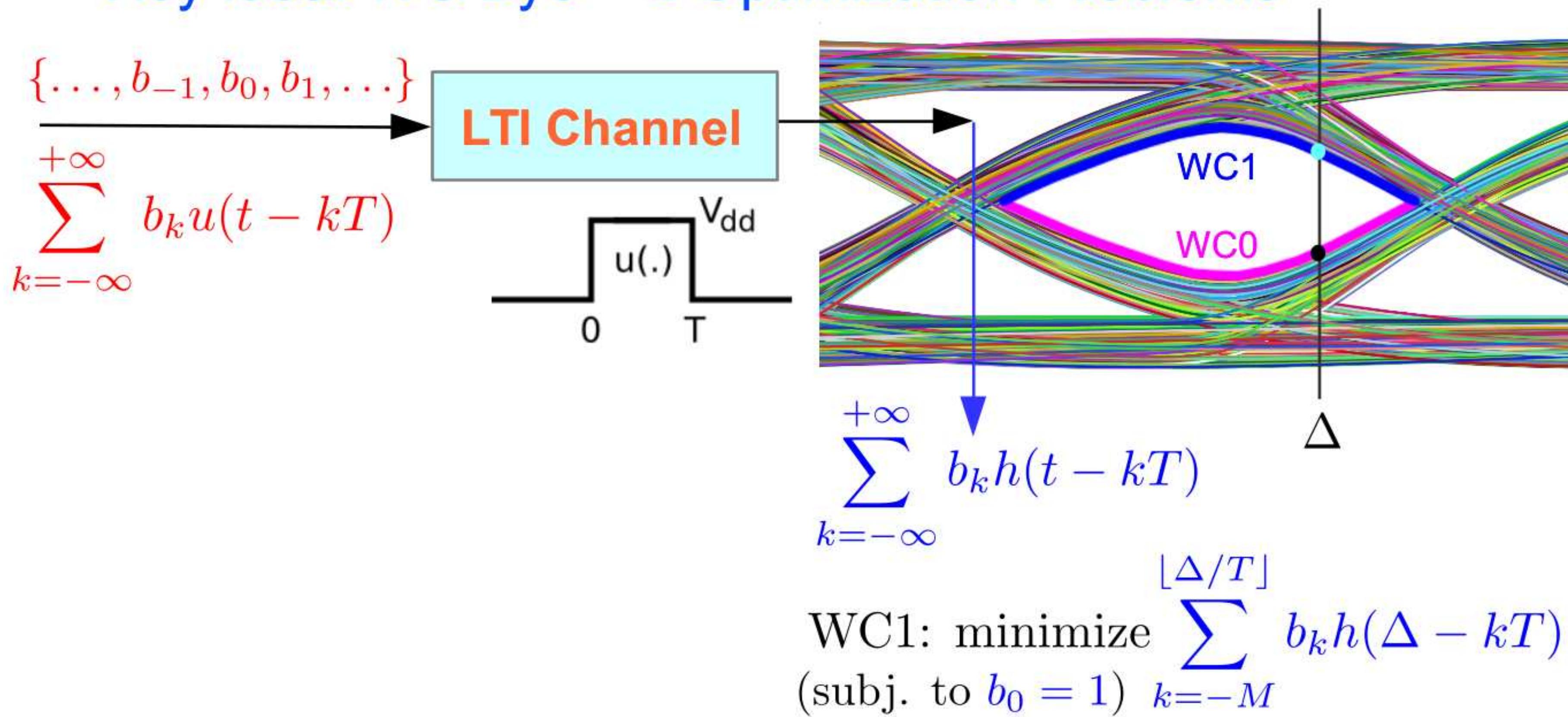
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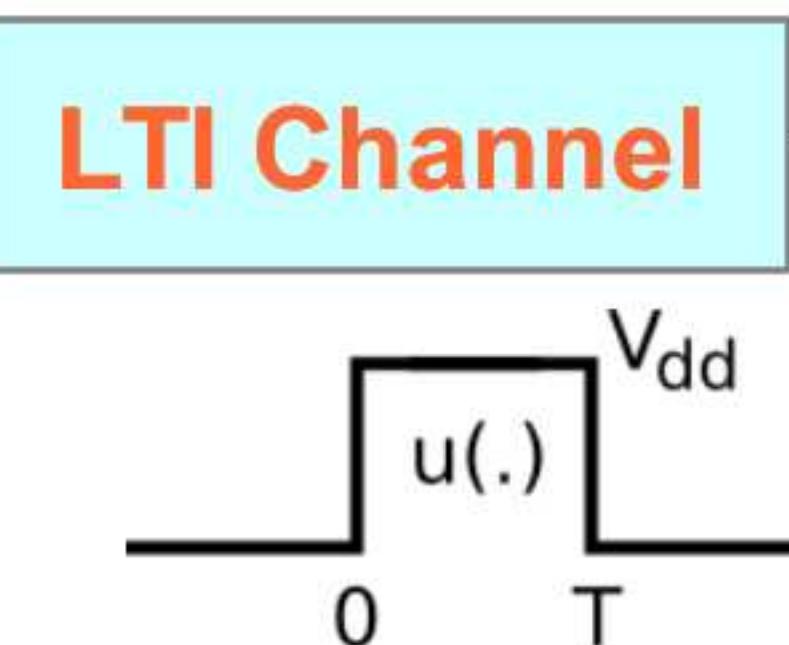
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*Linear combination of the bits!*

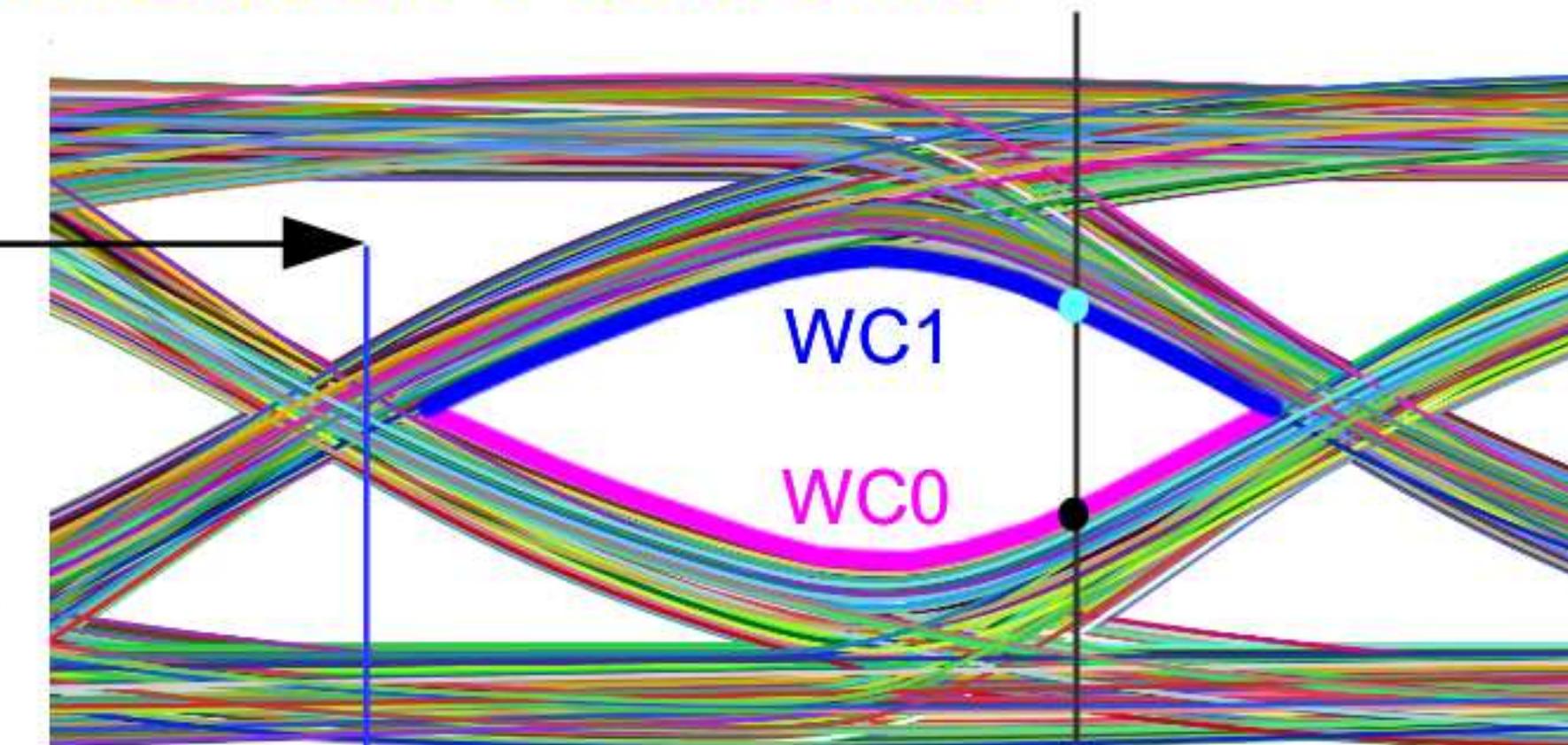
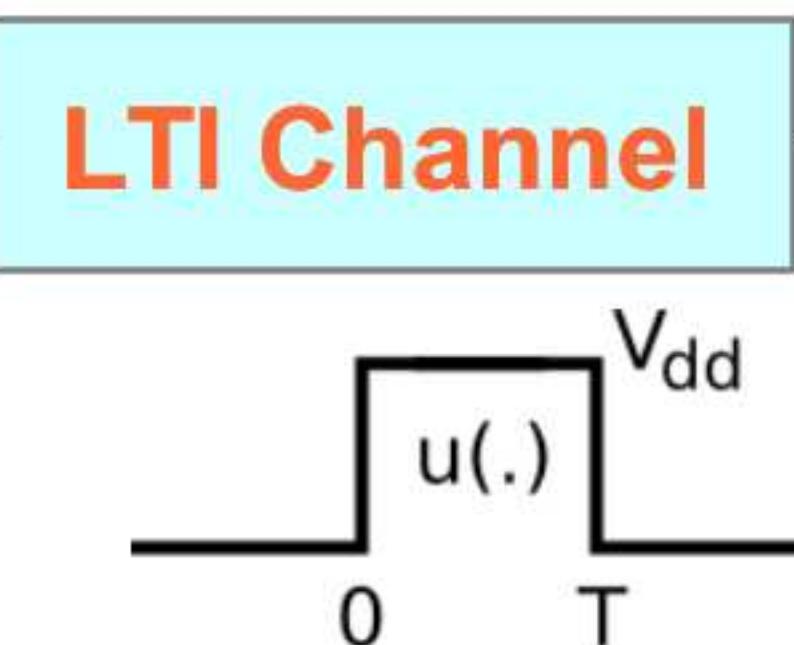
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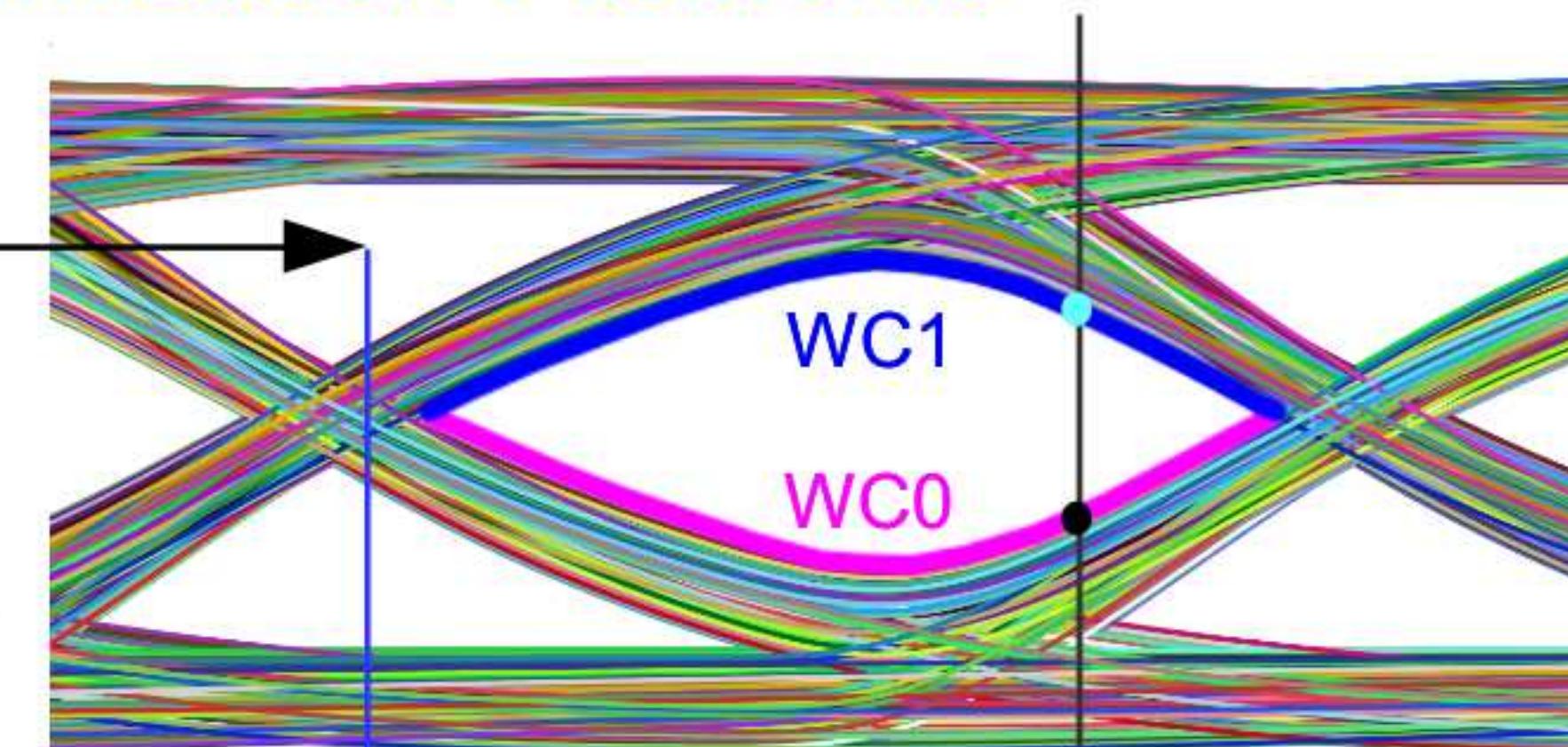
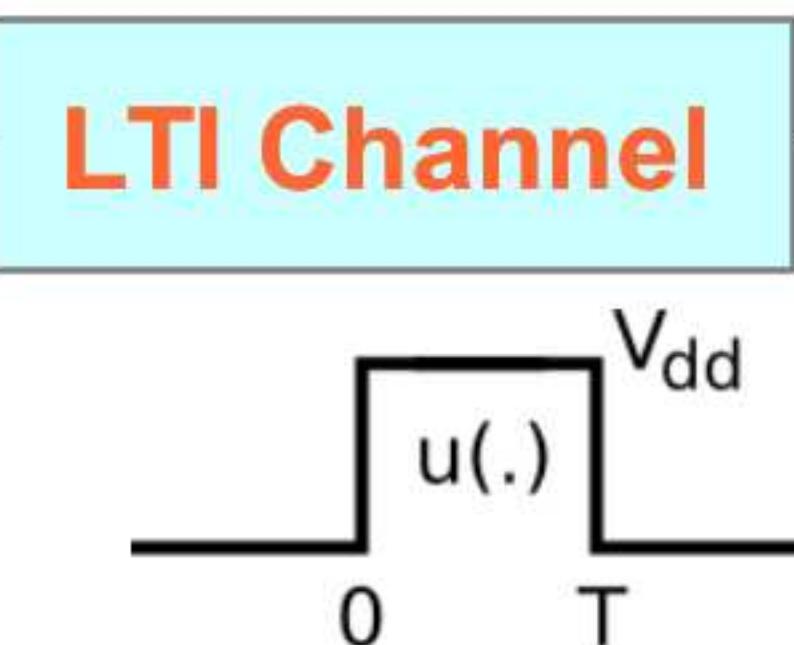
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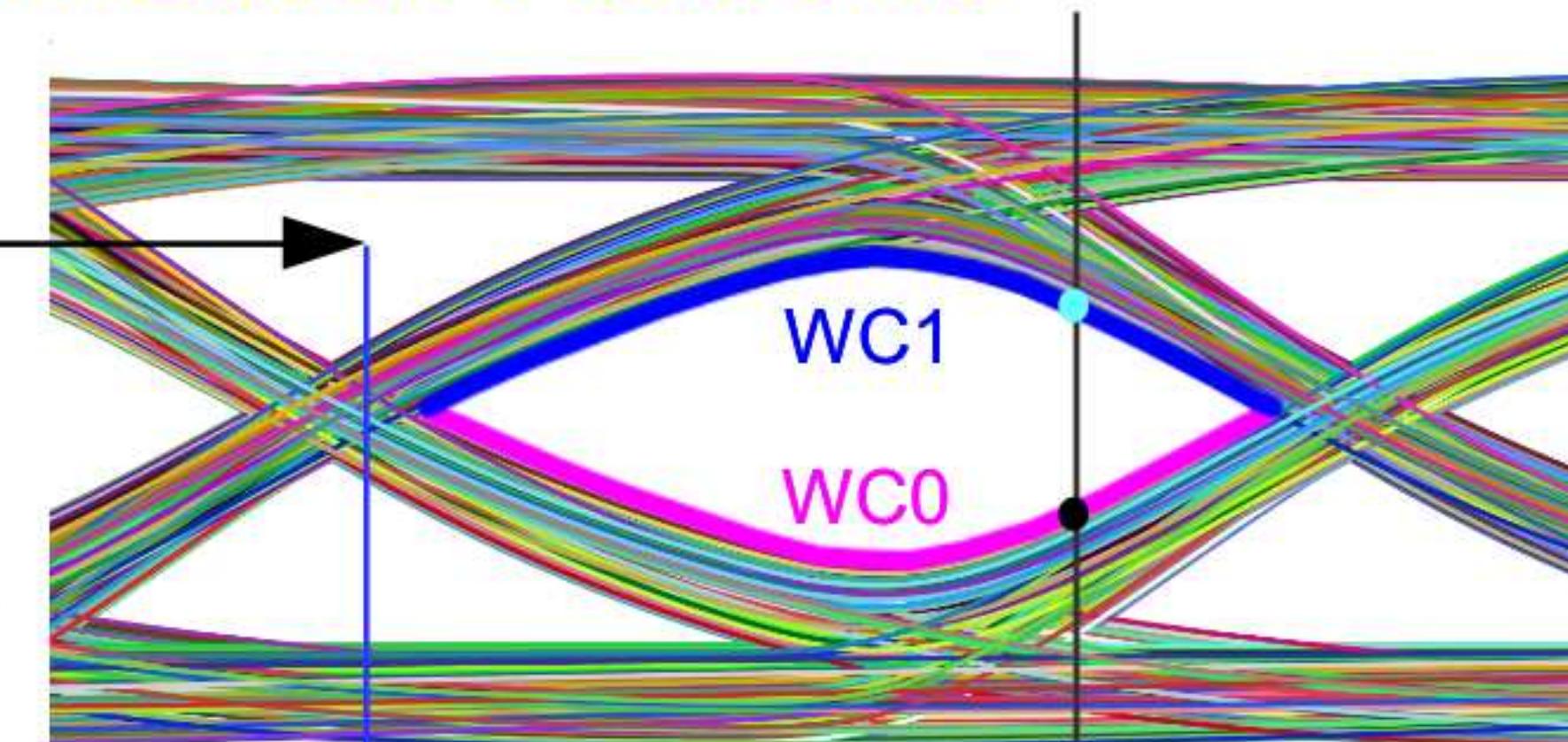
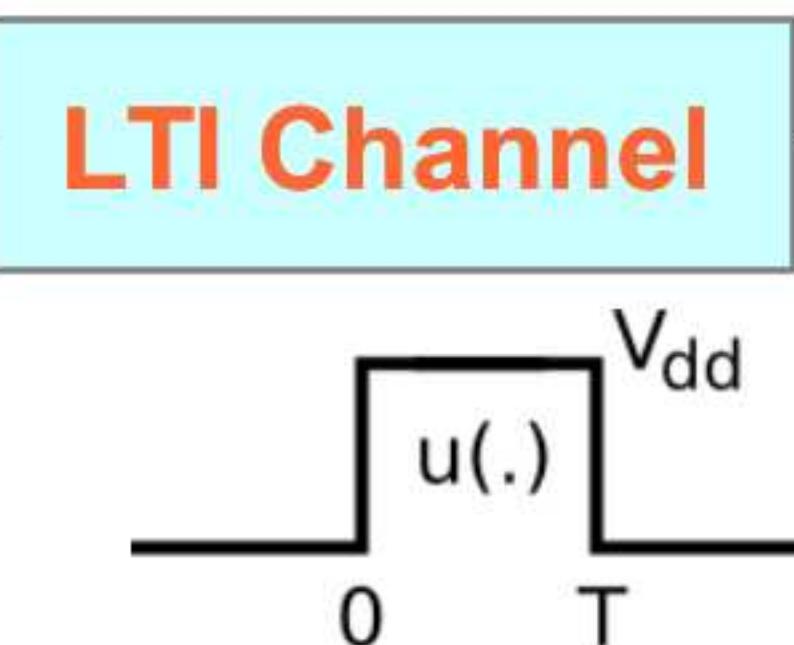
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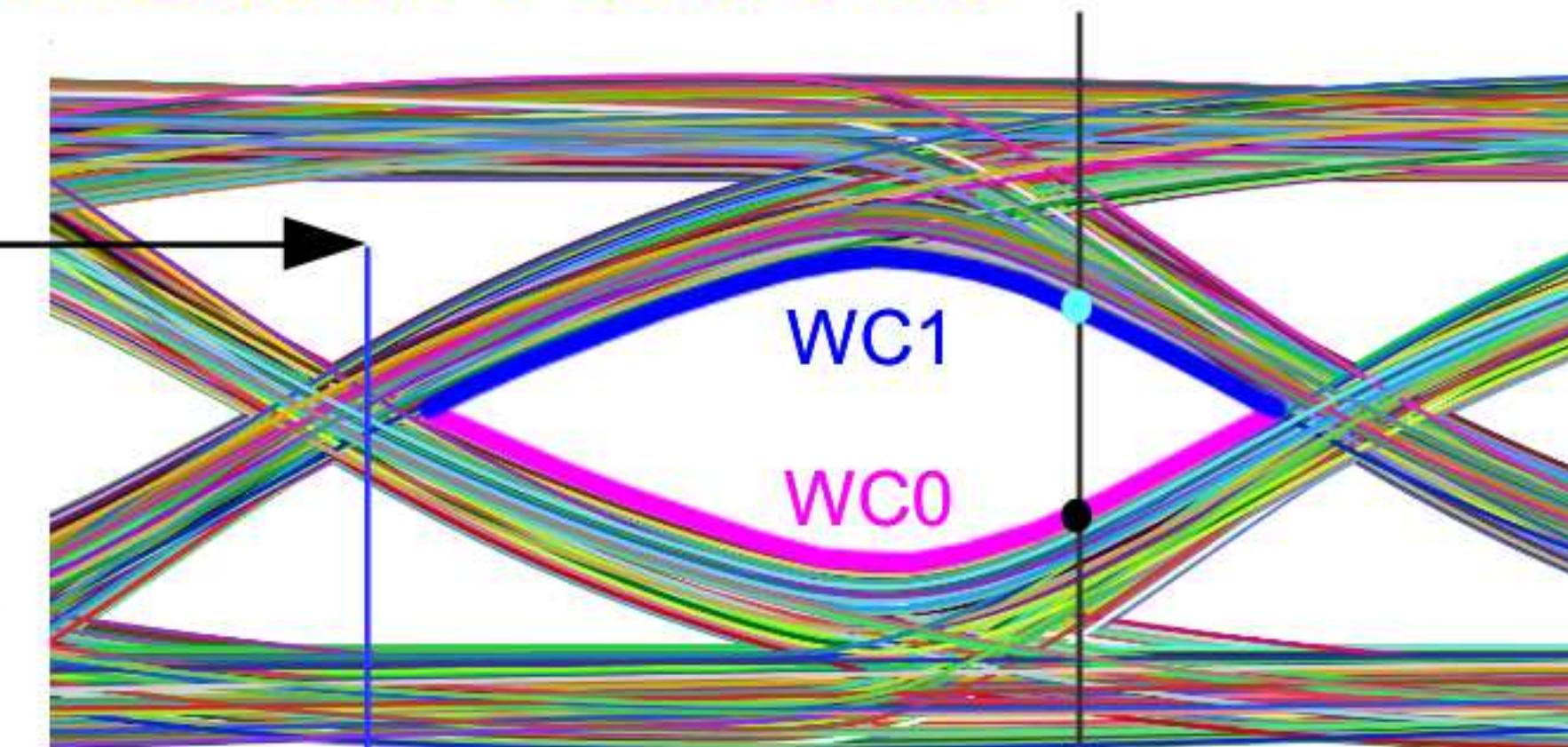
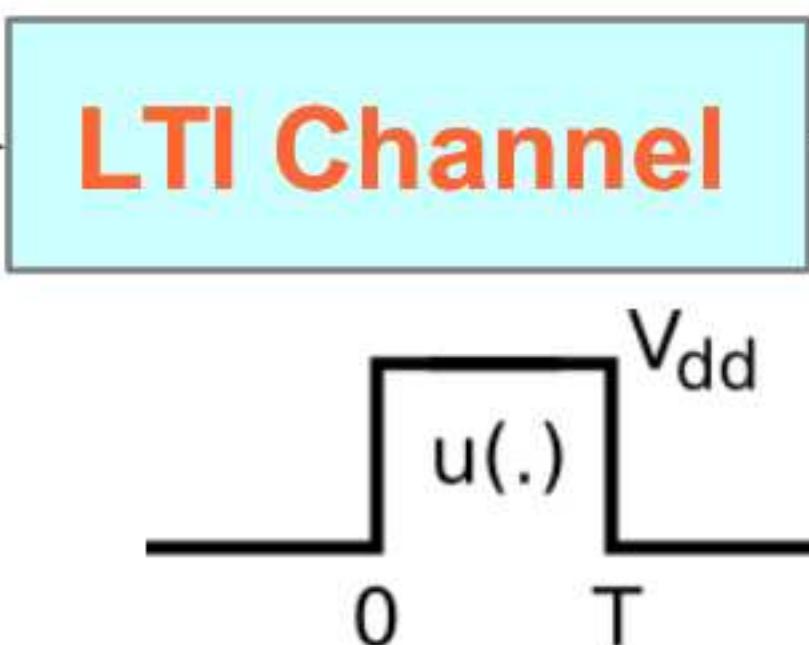
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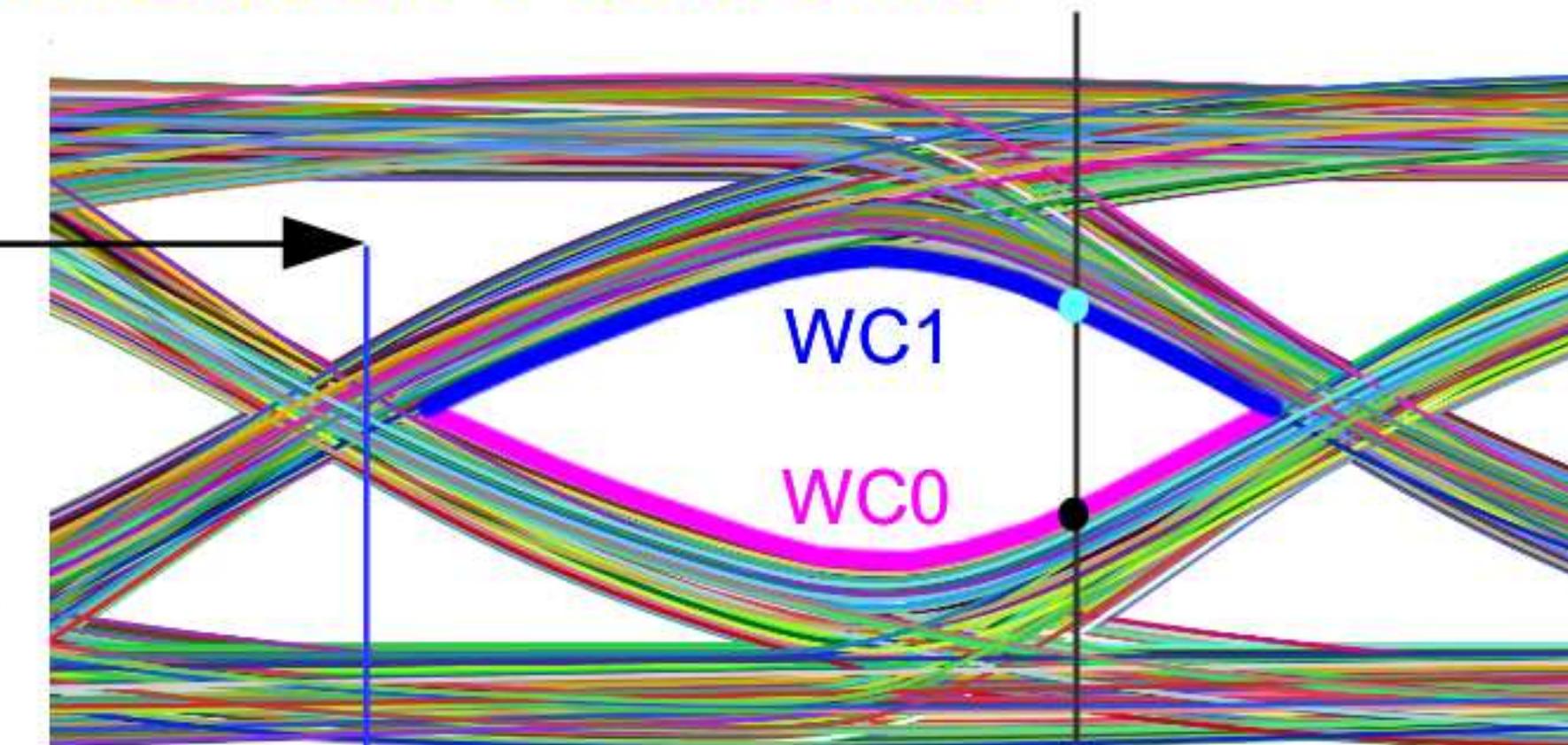
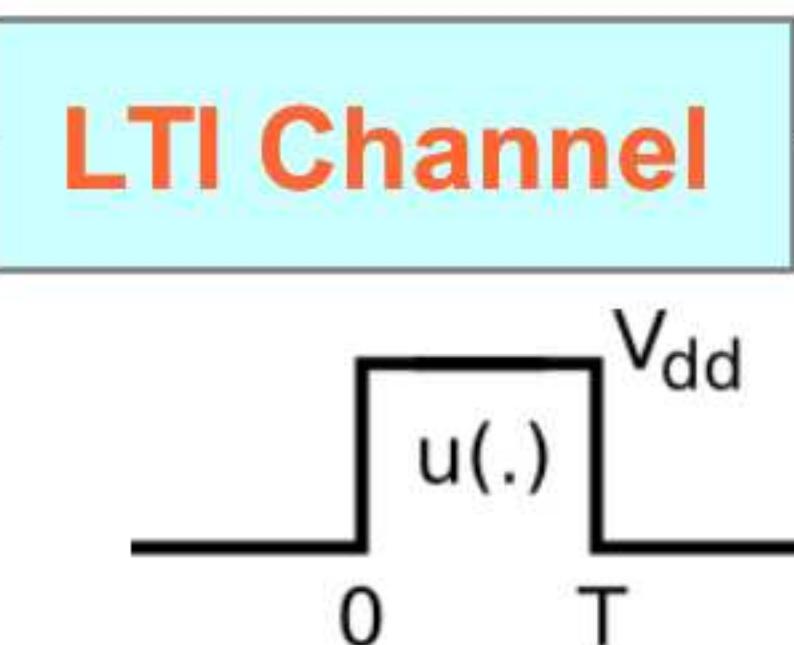
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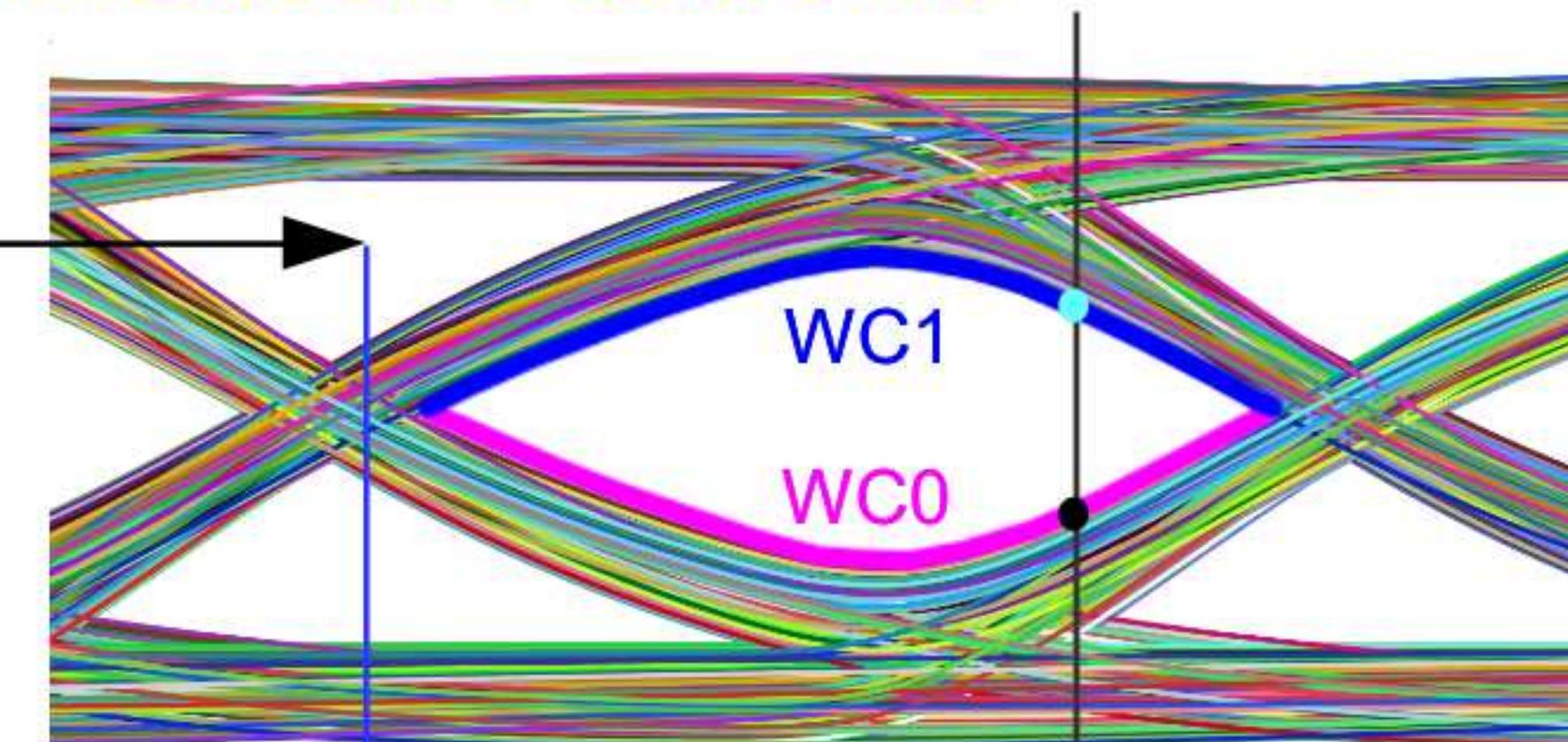
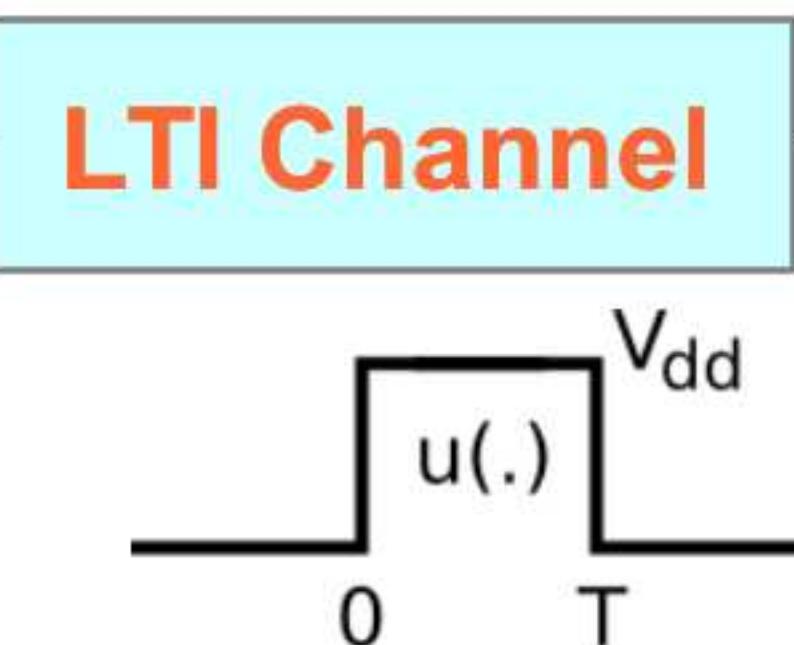
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Correlated bits: PDA fails!

# Questions