

## ONCE'S & TWO'S COMPLEMENT

### r's Complement and (r-1)'s Complement Subtraction of Unsigned Numbers

In digital computers, complements are used for simplifying subtraction operation and logical manipulation. There are two types of complements for binary as well as decimal number system.

Radix Complement (r's complement)

Diminished Radix Complement (r-1 complement)

For Binary Number System, the radix complement is known as 2's complement and Diminished Radix Complement is 1's complement. For Decimal Number System, radix complement is known as 10's complement and Diminished Radix Complement (r-1 complement) is 9's Complement.

### Radix Complement (r's complement) :

- **The r's complement**

$$\Rightarrow r^n - N \quad \text{for} \quad N \neq 0$$

$$\Rightarrow r^n - N = 0 \quad \text{for} \quad N = 0$$

- **Examples**

$$10\text{'s Complement of } (52520)_{10} = 10^5 - 52520 = 47480$$

$$10\text{'s Complement of } (0.3267)_{10} = 10^0 - 0.3267 = 0.6733$$

$$10\text{'s Complement of } (25.639)_{10} = 10^2 - 25.639 = 74.361$$

$$2\text{'s Complement of } (101100)_2 = (2^6)_{10} - (101100)_2 = 1000000 - 101100 = 010100$$

$$2\text{'s Complement of } (0.0110)_2 = 2^0 - 0.0110 = 1 - 0.0110 = 0.1010$$

**Simple Method**

$$10\text{'s Complement of } (52520)_{10} = \begin{array}{r} 99910 \\ 52520 \\ \hline 47480 \end{array}$$

$$2\text{'s Complement of } (101100)_2 = \begin{array}{r} 1112 \\ 101100 \\ \hline 010100 \end{array}$$

The 10's complement of 52520 is obtained by subtracting the first non zero least significant digit in the given number from 10 and subtracting the rest from 9. If the least significant digit is 0 in the given number, they are left unchanged as shown in the example above. While calculating the 2's complement, the first non zero least significant bit is subtracted from 2 and the remaining are subtracted from 1. If the least significant digit is 0 in the given number, they are left unchanged as shown in the example above. 2's complement can also be calculated by first taking the 1's complement (explained below) of the given number and then adding 1 to the least significant bit of the 1's complement.

Remember that:

1. 10's complement is calculated for  $N=1,2,3,4,5,6,7,8,9,0$  and 2's complement for binary numbers i.e 0 & 1 only.
2. Taking Complement of the complement, we get the original number restored .

### Diminished Radix Complement( $r-1$ 's Complement):

Assume that we are given any number  $N$  having digits  $n$  in base  $r$  system, then  $r-1$ 's complement can be defined as  $(r^n) - r^m - N$ .

### Subtraction with $r$ 's complement:

Assume that you want to subtract two unsigned numbers i.e., a number  $N$  (Subtrahend) from  $M$  (Minuend) then  $M-N$  is

1. Add  $M$  to the  $r$ 's complement of  $N$ .
2. Check the result for an end carry
  - a. If an end carry occurs then discard it.
  - b. If an end carry doesn't occur, take the  $r$ 's complement of the result and place a negative sign in front.

**Examples**

$$\begin{aligned} & \bullet (72532 - 3250)_{10} \\ & \quad M = 72532 \quad N = 03250 \\ & \quad M = 72532 \\ & \quad 10\text{'s comp of } N = 96750 \\ & \quad \text{End Carry} \quad 1 \sqrt{69282} \quad \therefore \text{Answer} = 69282 \end{aligned}$$

$$\begin{aligned} & \bullet (3250 - 72532)_{10} \\ & \quad M = 03250 \quad N = 72532 \\ & \quad M = 03250 \\ & \quad 10\text{'s comp of } N = 27468 \\ & \quad \text{No end Carry} \rightarrow \sqrt{30718} \\ & \quad 10\text{'s comp of } 30718 = 69282 \\ & \quad \text{Answer} = -69282 \end{aligned}$$

**Two's complement of signed numbers**

For example, beginning with the signed 8-bit binary representation of the decimal value 5, using subscripts to indicate the base of a representation needed to interpret its value:

$$00000111_2 = 7_{10}$$

The most significant bit is 0, so the pattern represents a non-negative (positive) value. To convert to  $-5$  in two's-complement notation, the bits are inverted; 0 becomes 1, and 1 becomes 0:

$$1111100$$

At this point, the numeral is the ones' complement of the decimal value 5. To obtain the two's complemented, 1 is added to the result, giving:

$$11111001_2 = -5_{10}$$

The result is a signed binary number representing the decimal value  $-5$  in two's-complement form. The most significant bit is 1, so the value represented is negative.

The two's complement of a negative number is the corresponding positive value. For example, inverting the bits of  $-5$  (above) gives:

00000100

And adding one gives the final value:

$$00000101_2 = 5_{10}$$

The value of a two's-complement binary number can be calculated by adding up the power-of-two weights of the "one" bits, but with a negative weight for the most significant (sign) bit; for example:

$$11111011_2 = -128 + 64 + 32 + 16 + 8 + 0 + 2 + 1 = (-2^7 + 2^6 + \dots) = -5$$

Note that the two's complement of zero is zero: inverting gives all ones, and adding one changes the ones back to zeros (the overflow is ignored). Also the two's complement of the most negative number represent able (e.g. a one as the most-significant bit and all other bits zero) is itself. Hence, there appears to be an 'extra' negative number.

A more formal definition of a two's-complement negative number (denoted by  $N^*$  in this example) is derived from the equation  $N^* = 2^n - N$ , where  $N$  is the corresponding positive number and  $n$  is the number of bits in the representation.

For example, to find the 4 bit representation of  $-5$ :

$$N = 5_{10} \text{ therefore } N = 0101_2$$

$$n = 4$$

Hence:

$$N^* = 2^n - N = 2^4 - 5_{10} = 10000_2 - 0101_2 = 1011_2$$

The calculation can be done entirely in base 10, converting to base 2 at the end:

$$N^* = 2^n - N = 2^4 - 5 = 11_{10} = 1011_2$$

Find the once complement of following numbers

a)10001                      b)111000                      c)00110

answers

a)01110                      b)000111                      c)11001

Find the two's complement of following numbers

a)10001                      b)111000                      c)00110

answers

a) 01110

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      1
     --

```

01111

b) 111000

```

      1
     --

```

111001

c) 001100

```

      1
     --

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001101