# PRIME IMPLICANTS

Any single 1 or group of 1's that can be combined together on Karnaugh map of the function F represents a product term which is called an IMPLICANT. A PRIME IMPLICANT is a product term that cannot be combined with another term to eliminate a variable. A single 1 is a prime implicant if it is not adjacent to any other 1's. Two adjacent 1's form a prime implicant if they are not contained in a group of four adjacent 1's. Four adjacent 1's form a prime implicant if they are not contained in a group of eight adjacent 1's. The minimum sum-of-products expression for a function consists of some (BUT NOT NECESSARILY ALL) of the prime implicants of a function.

```
  \ AB
CD\  00  01  11  10
   \________________
    |   |   |   |   |
 00 |   | 1 | 1 |   |
    |___|___|___|___|
    |   |   |   |   |
 01 | 1 | 1 | 1 |   |
    |___|___|___|___|
    |   |   |   |   |
 11 | 1 |   | 1 | 1 |
    |___|___|___|___|
    |   |   |   |   |
 10 |   |   | 1 | 1 |
    |___|___|___|___|
```

Prime implicants: A'B'D, BC', AC, A'C'D
AB, B'CD

Minimum solution: F = A'B'D + BC' + AC

To ensure that a minimum solution is found, select essential prime implicants first. Then find a minimum set of prime implicants that cover the remaining 1's on the map.

```
  \ AB
CD\  00  01  11  10
   \________________
    |   |   |   |   |
 00 | X | 1 |   | X |
    |___|___|___|___|
    |   |   |   |   |
 01 | 1 | 1 |   |   |
    |___|___|___|___|
    |   |   |   |   |
 11 |   | 1 | 1 | 1 |
    |___|___|___|___|
    |   |   |   |   |
 10 | 1 |   |   |   |
    |___|___|___|___|
```

Essential prime implicants are:
A'C', ACD, and A'B'D'

Either A'BD or BCD can be chosen for final minimum solution.

## PROBLEMS

For reference, this section introduces the terminology used in some texts to describe the minterms and maxterms assigned to a Karnaugh map. Otherwise, there is no new material here.

Σ (sigma) indicates sum and lower case "m" indicates minterms. Σm indicates sum of minterms. The following example is revisited to illustrate our point. Instead of a Boolean equation description of unsimplified logic, we list the minterms.

$f(A,B,C,D) = \Sigma\, m(1, 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15)$ or

$f(A,B,C,D) = \Sigma(m_1,m_2,m_3,m_4,m_5,m_7,m_8,m_9,m_{11},m_{12},m_{13},m_{15})$

The numbers indicate cell location, or address, within a Karnaugh map as shown below right. This is certainly a compact means of describing a list of minterms or cells in a K-map.

$$\begin{aligned} Out = {} & \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,C\,D \\ & + \overline{A}\,B\,\overline{C}\,\overline{D} + \overline{A}\,B\,\overline{C}\,D + \overline{A}\,B\,C\,D \\ & + A\,B\,\overline{C}\,\overline{D} + A\,B\,\overline{C}\,D + A\,B\,C\,D \end{aligned}$$

$$f(A,B,C,D) = \Sigma\, m(0,1,3,4,5,7,12,13,15)$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 8 | 9 | 11 | 10 |

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

$$f(A,B,C,D) = \overline{A}\,\overline{C} + \overline{A}D + B\overline{C} + BD$$

The Sum-Of-Products solution is not affected by the new terminology. The minterms, **1**s, in the map have been grouped as usual and a Sum-OF-Products solution written.

Below, we show the terminology for describing a list of maxterms. Product is indicated by the Greek Π (pi), and upper case "M" indicates maxterms. ΠM indicates product of maxterms. The same example illustrates our point. The Boolean equation description of unsimplified logic, is replaced by a list of maxterms.

f(A,B,C,D) = Π M(2, 6, 8, 9, 10, 11, 14) or

f(A,B,C,D) = Π($M_2$, $M_6$, $M_8$, $M_9$, $M_{10}$, $M_{11}$, $M_{14}$)

Once again, the numbers indicate K-map cell address locations. For maxterms this is the location of **0**s, as shown below. A Product-OF-Sums solution is completed in the usual manner.

$$Out = (\overline{A}+\overline{B}+\overline{C}+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+\overline{B}+\overline{C}+D)(\overline{A}+B+C+D)$$
$$(\overline{A}+B+\overline{C}+\overline{D})(\overline{A}+B+\overline{C}+\overline{D})(\overline{A}+B+\overline{C}+D)$$

$$f(A,B,C,D) = \Pi M(2,6,8,9,10,11,14)$$

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 1 | 3 | 2 |
| 01 | 4 | 5 | 7 | 6 |
| 11 | 12 | 13 | 15 | 14 |
| 10 | 8 | 9 | 11 | 10 |

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

| AB \ CD | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

$$f(A,B,C,D) = (\overline{A}+B)(\overline{C}+D)$$