MOTATION OF CHARGED PARTICAL IN ELECTRICAL FIELD

When the electron is situated b/w the two parallel paltes of the capacitor, the electric filed intensity (ε) is applied towards –ve x-axis. So force acts towards +ve x-axis $(F = -e\varepsilon)$.

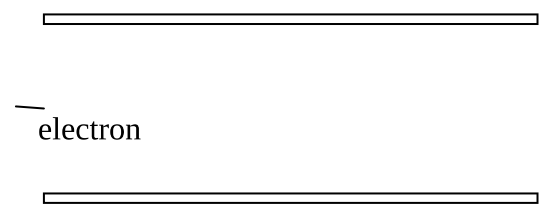
The P.D b/w two plates is given by ‘U’ the distance b/w the two plates is given by ‘d’ and length of two plates is given by l, dimensions of the two plates is equal.

If the distance b/w the two plates (d) is less than the dimensions of plate (l) then that electric filed is said to be uniform.

Mathematical analysis :

Characteristics of motion subjected to initial conditions.

At $t = 0 \quad x = x_0 \;\; and \quad v_x = v_{0x}$



Where $x =$ distance from centre to point under consideration

$x_0 =$ point under consideration or initial distance

$v_{0x} =$ Initial velocity with which it enters.

$v_x =$ velocity At point under consideration.

The initial vel. v_{0x} is chosen along +ve x-axis. So initial distance is also along +ve x-axis since there is no force along y or z-axis the acceleration along y & z is zero.

$\vec{f}_q = m\vec{a} = q\varepsilon = 0 \qquad (\varepsilon = 0)$ along y and z directions.

$\therefore \vec{a}_y = 0, \vec{a}_z = 0$

$$\therefore \vec{a}_y = \frac{d\vec{V}_y}{dt} = 0 \qquad \vec{a}_z = \frac{d\vec{V}_z}{dt} = 0$$

$\vec{V}_y = c =$ constant of integration $\qquad \vec{V}_x = c =$ constant of integration.

Zero acceleration meant constant vel. But initial vel. Is zero along y or z-directions and that zero is constant. Therefore the particle will not move along y or z directions. So the only possible motion is along x-axis (+ve). So this motion is called one-dimensional electron motion.

From newton's second law

$$q\varepsilon = m\vec{a}_x$$

$e\vec{\varepsilon} = m\vec{a}_x$ (considering only magnitude of charge of e^-)

$$\vec{a}_x = \frac{e\vec{\varepsilon}}{m} = \frac{d\vec{V}_x}{dt} = \text{constant}$$

The acceleration is constant along +ve x-direction i.e., rate of change of vel. Is constant [vel. is increasing at regular intervals of time].

The above expression in terms of displacement and vel. is given by

$$\frac{d\vec{V}_x}{dt} = \frac{e\vec{\varepsilon}}{m}$$

By integrating

$$\int \frac{d\vec{V}_x}{dt} = v_x = \int \frac{e\vec{\varepsilon}}{m} dt$$

$$\therefore v_x = \frac{e\varepsilon}{m} t + C_1 \qquad - \qquad \text{constant of integration}$$

At $t = 0$, $v_x = C_1$

From initial conditions $v_x = v_{0x} = c_1$

$$\therefore v_x = \frac{e\varepsilon}{m} t + v_{ox}$$

But $\therefore v_x = \frac{dx}{dt} = \frac{e\varepsilon}{m} t + v_{ox}$

By integrating

$$\int \frac{dx}{dt} dt = x = \frac{e\varepsilon}{m} \int t dt + \int v_{ox} dt$$

$$= \frac{e\varepsilon}{m} \cdot \frac{t^2}{2} + v_{ox} t + C_2$$

Where C_2 is constant of integration

$$\therefore x = \frac{1}{2} a_x t^2 + v_{ox} t + C_2$$

Put $t = 0 \quad \Rightarrow x = C_2$

From initial conditions $x = x_0 = C_2$

$$\therefore x = \frac{1}{2} a_x t^2 + v_{ox} t + x_0$$

Provided a_x is constant and is independent of time since ε is constant.

Potential :

Potential (V) of a point x w.r.t. x_0 is amount of work done aginst the electric field in taking a unit +ve charge from point x_0 at point x.

According to Newton's II law

$$m\vec{a}_x = q\vec{\varepsilon}_x = -e\varepsilon_x$$

$$\vec{a}_x = \frac{-e\varepsilon_x}{m} = \frac{dv_x}{dt} \qquad - \qquad (1)$$

Acceleration along x-direction is defined as rate of change of velocity along x-direction.

Now multiply eq (1) with dx both sides and then integrate

$$\int_{v_{ox}}^{v_x} \frac{dv_x}{dt} dx = \int_{x_0}^{x} \frac{-e\varepsilon_x}{m} dx$$

But $v_x = \frac{dx}{dt}$

$\Rightarrow dx = v_x \cdot dt$

$$\therefore \int_{v_{ox}}^{v_x} v_x dv_x = \frac{-e\varepsilon_x}{m} \int_{x_0}^{x} \varepsilon_x dx \qquad - \qquad (2)$$

$\int_{x_0}^{x} \varepsilon_x dx$ is the expression for work done in carrying a unit +ve charge from point x_0 to x

$$\therefore V = -\int_{x_0}^{x} \varepsilon_x dx$$

This is the amount of W.D. against the electric filed in carrying a unit +ve change from point x_0 to x.

$$\therefore \frac{1}{2}\left[V_x{}^2\right]_{v_{0x}}^{v_x} = \frac{e}{m} \cdot V$$

$$\therefore \frac{eV}{m} = \frac{1}{2}\left[V_x^{\ 2} = V_{0x}^{\ \ 2}\right]$$

$$eV = \frac{1}{2}\left[V_x^{\ 2} = V_{0x}^{\ \ 2}\right] \qquad - \qquad (3)$$

Where eV is nothing but energy in joules (potential energy) e is charge of electron.

Consider any two points A and B with B is at higher potential than at A then

Potential difference = $V_B - V_A = V_{BA}$

Then eqn (3) can be modified as

$$-eV = -\frac{1}{2}m\left[V_x^{\ 2} = V_{0x}^{\ \ 2}\right]$$

$$= \frac{1}{2}m\left[V_x^{\ 2} = V_{0x}^{\ \ 2}\right]$$

$q = -e$

$$qV_{BA} = \frac{1}{2}m\left(V_A^{\ 2} - V_B^{\ 2}\right) \qquad - \qquad (4)$$

Where ‘q’ is charge of particle in coulumb’s

qV_{BA} is in jouled

V_A - initial velocity in m/sec

V_B - final velocity in m/sec

Potential energy b/w the two point = potential x charge

$= qV_{BA}$

L.H.S. is P.E. and the R.H.S. is K.E.

The rise in potential energy = fall in K.E.

i.e., total energy is constant. This is law of conservation of energy this eq(4) is not valid for field varies with time If the particle is an electron, then

$q = -e$

If electron starts from rest position, then

Initial velocity $V_A = 0$

$V_B = V$

$V_{BA} = V$

Eqn(4) is modified as

$$-eV = \frac{1}{2}m\left(0 - v^2\right)$$

$$eV = \frac{1}{2}mv^2$$

$$v^2 = \frac{2eV}{m}$$

$$v = \sqrt{\frac{2eV}{m}}$$

Where 'e' is magnitude of charge of electron

$$e = 1.602 \times 10^{-19}\, c$$

$$m = 9.1 \times 10^{-31}\, kg$$

V is potential in volts

v is final velocity of an electron.

The above expression is valid only when electron starts from rest position.

$$v = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times V}{9.1 \times 10^{-31}}}$$

$$= 5.93\sqrt{V} \times 10^{5}$$

$$\therefore v = 5.93 \times 10^{5}\sqrt{V}$$

If an electron falls through apotential difference of 1volt its final speed $= 5.93 \times 10^{5}\sqrt{V}$