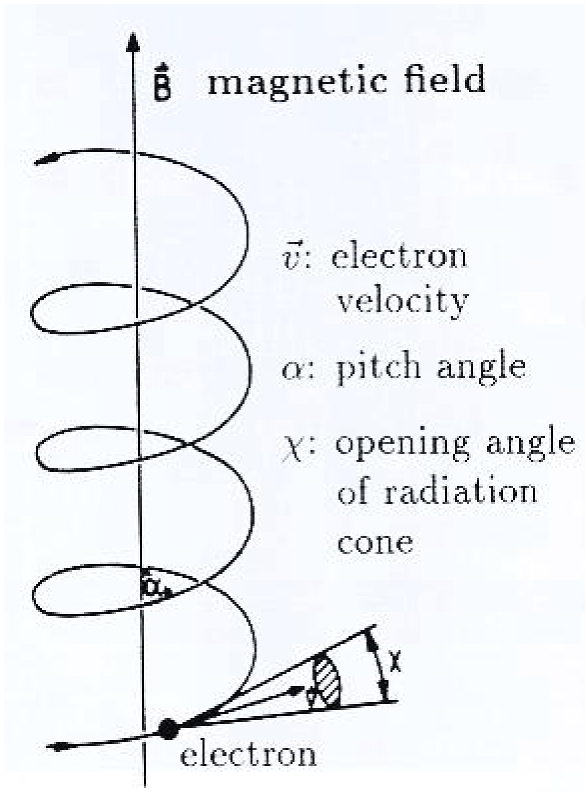
PARALLEL AND PERPENDICULAR E&B FIELDS

Consider the case where both electric and magnetic fields acts simultaneously on the electron (ε & B are parallel to each other).



Case (i):

If initial velocity of electron is either zero or along magnetic field B then there is no force on electron due to magnetic field intensity ε. Therefore the electron will move parallel to these fields with constant acceleration. The electron is moving in a direction of –ve y-axis.

Velocity of electron $V_y = V_{0y} - a_y t$

Where –ve sign indicates that acceleration is towards –ve y-axis

$$y = V_{0y} - \frac{1}{2} a_y t^2$$

If component of velocity which is $\perp' r$ to 'B' then it becomes a circular motion.

The radius of circular path is independent of $'\varepsilon'$ and it is only dependent on magnetic field. But velocity component along electric field changes w.r.t. time (constant acceleration towards –y-axis). The resultant path is helical path with pitch changes w.r.t time. Here pitch of helix is defined as distance travelled along –ve y-axis per revolution increases with each revolution.

Perpendicular electric and magnetic filed :

Let the magnetic field is directed towards –ve y-axis electric filed directed towards –ve x-axis. Then force on e- due to electric field is directed towards +ve x-axis. The force due to magnetic field is perpendicular to 'B' and which is assumed along +ve z-direction. Therefore resultant plane of motion of electron is xz plane.

Since there is no component of force along y-direction i.e., $f_y = 0$ then $a_y = 0$

$\Rightarrow v_y = v_{0y}$ which is negligible

$v_y = v_{0y} t$ which is also negligible.

Assume that electron starts from rest position. Since the initial velocity is zero then initial magnetic force is zero. But due to electric field the electron is directed towards +ve x-axis. But now onwards magnitude force is not zero and it directed toward +ve z-axis. The path with bend away from +ve x-direction to +ve z-direction (xz-plane).

Mathematical Analysis :

The force due to electric field is $= e\varepsilon$ (in +ve x-direction)

Force due to magnetic field will be analysed in 3 axis.

Velocity component along x-direction $= v_x$

Velocity component along y-direction $= v_y$

Velocity component along z-direction $= v_z$

Since 'B' is in –ve y-direction $f_y = 0$ (because $v_y = 0$)

Force due to v_x i.e., $f_x = ev_x B$ (m +ve z-direction)

Force due to v_z i.e., $f_z = eBv_z$ (m –ve x-direction)

Net force, $fx = e\varepsilon - eBv_z = m\frac{dv_x}{dt}$ (from newtons law)

Let $\omega = \frac{v}{R} = \frac{eB}{m}$ and $\mu = \frac{\varepsilon}{B}$

Eq(1) can be written

$$\frac{dvx}{dt} = \frac{e\varepsilon}{m} - \frac{eBv_z}{m}$$
$$= \omega\mu - \omega v_z \qquad - \qquad (2)$$

Force due to $v_x = eBv_x$

Force in +ve z-direction $= eBv_x = ma_3 = m\frac{dv_x}{dt} \qquad - \qquad (3)$

$$\frac{dv_3}{dt} = \frac{eBv_x}{m}$$

$$\frac{dv_z}{dt} = \omega v_x \qquad - \qquad (4)$$

Differentiate eq(2) w.r.t. 't'

$$\frac{d^2 v_x}{dt^2} = \frac{d}{dt}(\omega\mu) - \frac{d}{dt}(\omega v_z)$$
$$= 0 - \omega\frac{dv_z}{dt}$$

But from eq(4) $\frac{dv_z}{dt} = \omega v_x$

$$\Rightarrow \frac{d^2 v_x}{dt^2} = -\omega(\omega v_x)$$
$$= -\omega^2 v_x$$
$$\Rightarrow \frac{d^2 v_x}{dt^2} + \omega^2 v_x = 0 \qquad - \qquad (5)$$

This is a second order linear D.E. with constant coefficient and the initial conditions are

$v_x = v_z = 0$ at $t = 0$

This eq(5) can be solved by laplace transform

(for general solution refer back side of notes)

From eq(2)

$$\frac{dv_x}{dt} = \omega\mu - \omega v_z$$

Apply laplace transforms on both sides of eq(2)

$$L\left[\frac{dv_x}{dt}\right] = L[\omega\mu] - L[\omega v_z]$$
$$SL[v_x] = \omega\mu L[1] - \omega L[v_z]$$
$$= \omega\mu L\left[\frac{1}{S}\right] - \omega L[v_z] \qquad - \qquad (6)$$

From eq (4)

$$\frac{dv_z}{dt} \omega v_x$$

Apply laplace transformations.

$$L\left[\frac{dv_z}{dt}\right] = \omega L[v_x]$$
$$SL[v_z] = \omega L[v_x] \qquad - \qquad (7)$$

Substitute eq(7) in eq(6)

$$L[v_x] = \omega\mu\left[\frac{1}{S}\right] - \omega L[v_z]$$
$$\frac{S^2}{\omega} L[v_z] = \omega\mu\left[\frac{1}{S}\right] - \omega L[v_z]$$
$$L[v_z]\left[S^2 + \omega^2\right] = \frac{\omega^2}{S}\mu$$
$$L[v_z] = \frac{\omega^2 \mu}{S\left(S^2 + \omega^2\right)}$$
$$= \frac{\mu\left[\left(S^2 + \omega^2\right) - S^2\right]}{S\left(S^2 + \omega^2\right)}$$
$$= \mu\left[\frac{1}{s} - \frac{S}{S^2 + \omega^2}\right]$$

By taking inverse laplace transformation

$$v_z = \mu\left[L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{S}{S^2 + \omega^2}\right)\right]$$
$$v_z = \mu[1 - \cos\omega t] \qquad - \qquad (8) \qquad \left[L\cos\omega t = \frac{S}{S^2 + \omega^2}\right]$$

From eq(4)

$$\omega v_x = \frac{dv_z}{dt}$$

Differentiate eq(8)

$$\Rightarrow \frac{dv_z}{dt} = \mu(\sin\omega t)\omega$$
$$(4) \Rightarrow \quad \omega v_x = \mu\omega(\sin\omega t)$$
$$v_x = \mu\sin\omega t \qquad - \qquad (9)$$

Coordinates of x and z are obtained by integrating eq(8) and eq(9) w.r.t. initial conditions $v_x = v_z = 0 \quad at \quad t = 0$

$$\int v_x dt = x = \int \mu \sin \omega t dt$$
$$\Rightarrow x = \frac{-\mu \cos \omega t}{\omega} + c_1$$

But as $x = 0 \ at \ t = 0$

$$C_1 = \frac{\mu}{\omega}$$

$$\therefore x = \frac{-\mu \cos \omega t}{\omega} + \frac{\mu}{\omega}$$
$$x = \frac{\mu}{\omega}[1 - \cos \omega t] \qquad - \qquad (10)$$

$$\int v_z dt = z = \int \mu (1 - \cos \omega t)$$
$$\Rightarrow z = \mu \left[t - \frac{\sin \omega t}{\omega} \right] + C_2$$

Put t = 0, z = 0

$$\Rightarrow 0 = \mu(0) + C_2$$
$$\Rightarrow z = \mu \left(t - \frac{\sin \omega t}{\omega} \right) \qquad - \qquad (11)$$

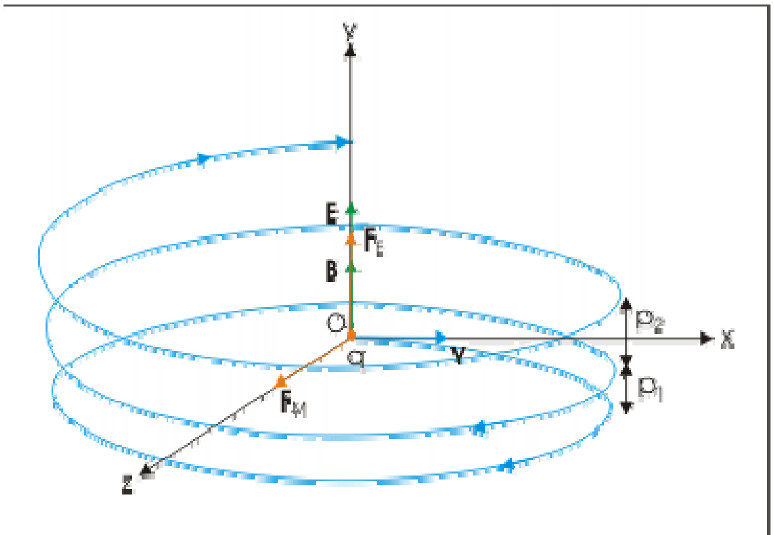
$$\therefore x = \frac{\mu}{\omega}[1 - \cos \omega t]$$
$$z = \mu t - \frac{\mu}{\omega} \sin \omega t$$

Let $\theta = \omega t \ \ and \qquad Q = \frac{\mu}{\omega}$

$$\Rightarrow x = Q[1 - \cos \theta]$$
$$z = Q\theta - Q \sin \theta$$
$$= Q[\theta - \sin \theta]$$

Parametric equations of cycloid

$$\Rightarrow x = Q[1 - \cos\theta]$$
$$z = Q[\theta - \sin\theta]$$



Cycloid is the path generated by a point on the circumference of circle of radius 'Q' which rolls along the straight line (z-axis).

Where Q is raidus of rolling circle

θ is no. of rotations through which the circle is rotated.

ω is ang. Velocity of rotation of rolling circle

$2\pi Q$ is distance covered along z-axis which is equal to circumference of rolling Circle

$2Q$ is maximum distance covered along x-axisa