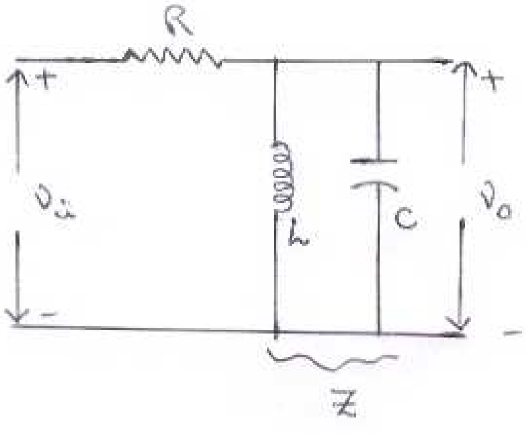
1. RLC circuits:

Response of RLC circuit to step input can be explained by using laplace transform analysis.



Transfer function of RLC circuit:

$V_o(t) = i(t)\ Z$

$V_i(t) = i(t)(R+ Z)$

Where $Z = Ls/(1+s^2LC)$

So $V_o(s) / V_i(s) = Z/R+Z$

$V_o(s) / V_i(s) = Ls/[R+Ls(sCR+1)]$ ----(1)

Step input to RLC circuit:

Here $V_i(t) = V$(for t>0)

So $V_i(s) = V/s$

Equation (1) will becomes,

$V_o(s) / V = L / [RLCs^2+Ls+R]$------(2)

To determine $V_o(t)$ there is a need to determine roots of $RLCs^2+Ls+R$.

Here the roots are , $S_1 = (-2\Pi k / T_o) +J (2\Pi / T_o) \sqrt{1-k^2}$

$S_2 = (-2\Pi k / T_o) - J (2\Pi / T_o)\sqrt{1-k^2}$

Where k = damping constant = $(1/2R) \sqrt{L/C}$

And T_o = time period = $2\Pi\sqrt{LC}$

Case(i): if k = 0, roots are imaginary.

The response is un damped sinusoidal with time period To

(by solving equation (2))

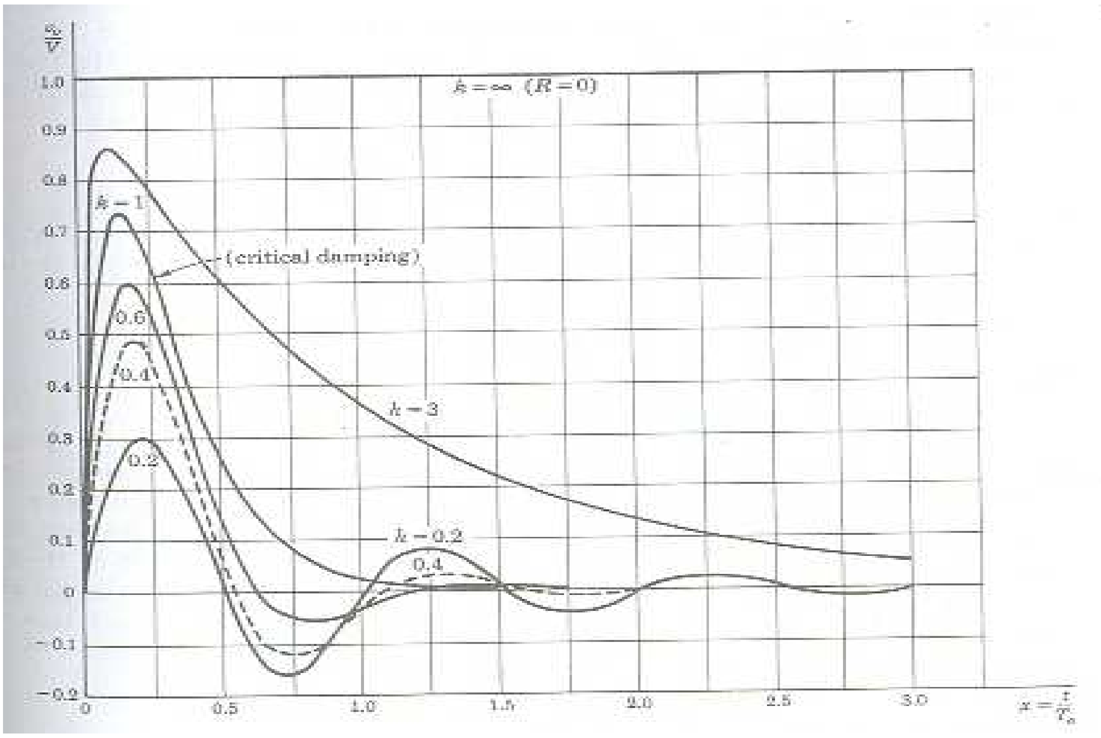
Case(ii) if k=1, roots are real and equal.

This is known as critical damping. here there is no oscillations .

Case(iii) if k>1 ,there are no oscillations in the output and the response is said to be overdamped.

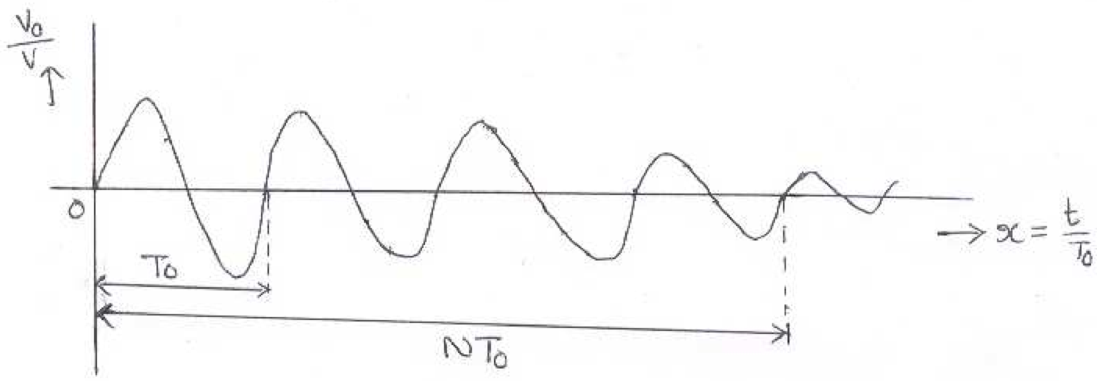
Case(iv) if k<1, the output will be a sinusoid whose amplitude decays with time,and the response is said to be under damped.

Here $V_o(t) /V = (2k/\sqrt{1-k^2})\ e^{-2\pi kx} \sin(2\pi x\sqrt{1-k^2})$



Ringing circuit: The circuit which produces nearly un damped oscillations is known as ringing circuit.
For k<1 ,RLC circuit acts as a ringing circuit.

Number of oscillations produced by RLC circuit:



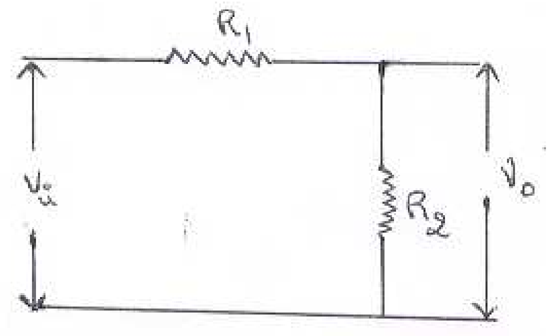
Consider oscillations produced by RLC circuit is significant until the amplitude of oscillations decreases to 1/e of its initial value.
We know for k<1 , $V_o(t) / V = (2k/\sqrt{1-k^2})\, e^{-2\pi kx} \sin(2\pi x\sqrt{1-k^2})$ where x = t/To

If $2\pi kx = 1$ ----(1) its amplitude reduces to 1/e of its initial value.

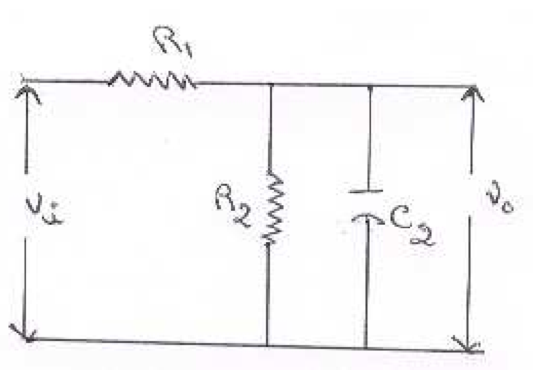
We know $Q = 1/2k$, where Q is the quality factor of circuit.

So $2kQ=1$--------(2)
From eq(1)&(2)
$Q = \pi x$
For N oscillations , x = N
So $N = Q/\pi$

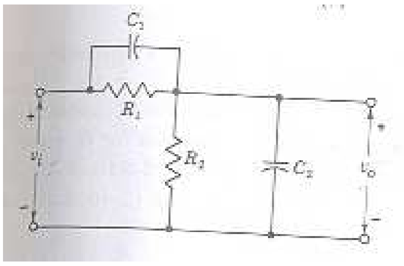
Attenuators: the circuit used to reduce the amplitude of input signal is known as attenuator.
It must be independent of frequency.
Circuit diagram for simple resistive attenuator is shown in figure below,
Here $V_o = V_i R_2 / (R_1 + R_2) = a V_i$
Where a is attenuation constant(<1)

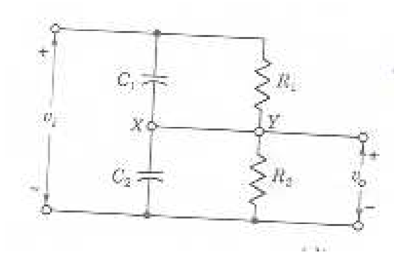


But practically output of attenuator must be connected
to any one electronic device.
We know every electronic device has some input capacitance
, then the circuit diagram is simplified as low pass
RC circuit. This is named as uncompensated attenuator.
Circuit diagram is shown below.



To make the attenuator is independent of frequency there is a need to place a capacitor in parallel with R_1 as shown in figure (it is named as compensated attenuator)
This circuit can be redrawn as bridge network.





Necessary condition for perfect compensation:
If the bridge is perfectly balanced then current through the branch XY is zero. then

$V_o = V_i R_2 / (R_1 + R_2) = a V_i$ it is independent of frequency.

The bridge is said to be perfectly balanced when
$1/j\omega C1 / 1/j\omega C2 = R_1 / R_2$

$\mathbf{R_1C_1 = R_2C_2}$
This is the necessary condition for perfect compensation.