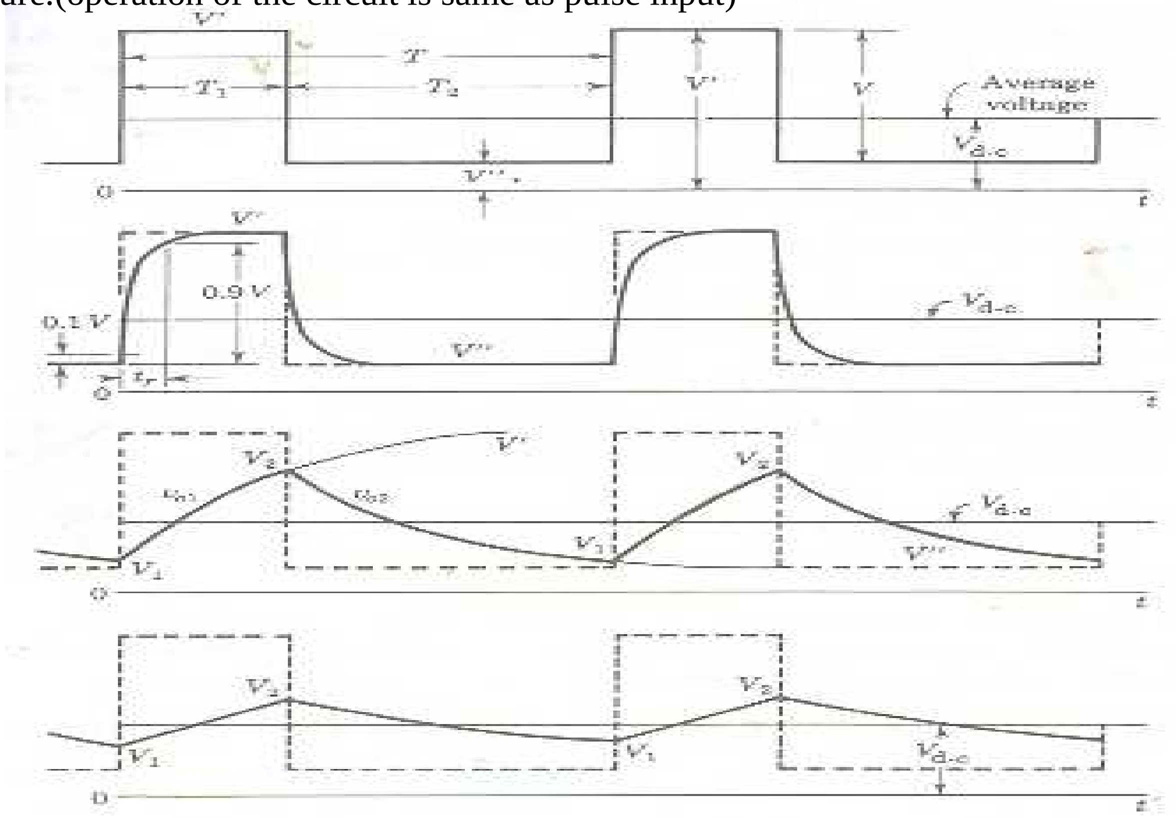
Statement: the area under the pulse input to low pass RC circuit is equal to the area under the output waveform.

Proof:
Area under the input waveform(A),
$A= {}_0\int^{tp} V\ dt = V\ tp$-----(1)

Area under the output waveform =A1+A2
$A1= {}_0\int^{tp} V(1-e^{-t/RC})\ \ dt = V\ tp - V_PRC$
$A2= {}_{tp}\int^{\infty} V_Pe^{-(t-tp)/RC}\ \ dt = V_PRC$
A1+A2 = V tp----(2)
Hence proved.
Square wave input: steady state output waveform for square wave input is shown in figure.(operation of the circuit is same as pulse input)



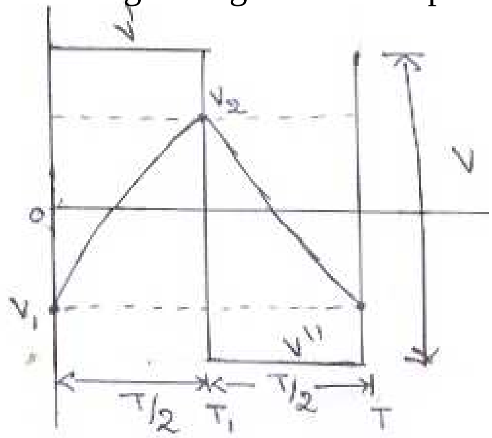
Expressions for v_{o1} & v_{o2} :

We know Vo(t) = Vf +(Vi-Vf) e-(t-tx)/τ
During 0 to T1 ,Vo(t) = $\mathbf{v_{o1}}$

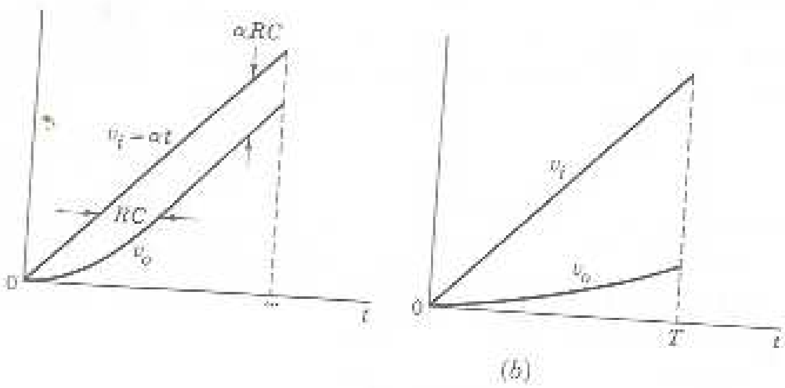
$\mathbf{v_{o1}}$ = V’ +(V_1-V’) e-t/RC

at t=T1 , $\mathbf{v_{o1}} =$ V2
so **V2** = V' +(V_1-V') e-T1/RC
similarly $\mathbf{v_{o2}}$ = V'' +(V_2-V'') e-(t-T1)/RC
at t=T , $\mathbf{v_{o2}} =$ V1
so **V1** = V'' +(V_2-V'') e-T2/RC

Symmetrical square wave input :
For symmetrical square wave input T1=T2= T/2
V_1= - V_2 also V'=V'' = V/2(by considering average value of input is zero)



Expression for V_1 orV_2 :
We know,
V2 = V' +(V_1-V') e-T1/RC
By substituting above values,
V2 = (V/2)tan hx



Ramp input:
We know for an low pass RC circuit,
v_i(t)= Vo(t)+ R I ,(but I= C d Vo(t)/dt)

v_i(t) = Vo(t)+ RC d Vo(t)/dt
but here vi(t) =αt

αt = Vo(t)+ RC d Vo(t)/dt it is a first order differential equation
by solving this equation (or by using laplace transform),
Vo(t)= αt - αRC (1- $e^{-t/RC}$)

If time constant is low, Vo(t)= αt - αRC

If time constant is high, Vo(t)= $\alpha t^2/2RC$

low pass RC circuit as a integrator:

For low pass RC circuit,

$v_i(t)= i R + Vo(t)$

if RC is high then voltage across the capacitor is minimum and output voltage is almost zero . so $v_i(t)= R i$ So $i = v_i(t)/R$

$Vo(t) = 1/RC \int i\, dt$ since $Vo(t) = 1/c \int i\, dt$

Hence low pass RC acts as a integrator when RC >>T

It produces triangular waves by taking square wave input.

Problems on low pass RC circuit:

1. A limited ramp of V volts is applied to an RC integrating circuit .what is the peak value of the output waveform for (a) T = RC (b) T =0.2 RC (c) T =5 RC .

Solution;

Here $Vo(t)= \alpha t- \alpha RC (1- e^{-t/RC})$ and $\alpha = V/T$

Peak value of output is $Vo(T)= V/T -VRC (1- e^{-T/RC})/T$

(a) if T =RC then Vo(T) = 0.368V volts
(b) if T =0.2RC then Vo(T) = 0.1V volts
(c) if T =5RC then Vo(T) = 0.8013V volts