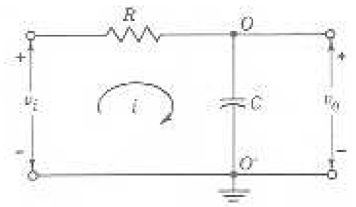
Low pass RC circuit: in an RC circuit if resistor is in series with the input voltage signal and a capacitor is in shunt with the output then that circuit is named as low pass RC circuit.

<u>Operation :</u> we know $X_c = 1/2\pi f C$
If frequency of input signal is low ,capacitive reactance is high ,then
Capacitor is replaced with an open circuit .so output voltage is same as input voltage.

Similarly If frequency of input signal is high ,capacitive reactance is low ,then
Capacitor is replaced with an short circuit .so output voltage is zero.

Hence it behaves like a low pass RC circuit.



Sinusoidal input: the sinusoidal input is defined as $v_i(t)= V_m \sin\omega t$
Applying KVL around the loop

$$v_i(t)- 1/c \int i\,dt - i R =0$$

$$Vi(s) = I(s) [R+ 1/sC]$$

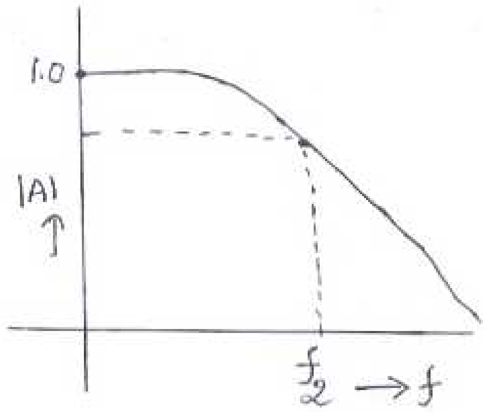
$$Vo(s) = I(s)/ sC$$

$$Vo(s)/Vi(s) = 1/(1+sRC) = 1/(1+j(2\pi fR C))$$

So gain of a circuit is $A= 1/\sqrt{(1+(f/f_2)^2}$
Where $f_2 = 1/2\pi RC$ = upper cutoff frequency
Phase angle = $- \tan^{-1}(f/f2)$

So for an low pass RC circuit if input is a sinusoidal then output is also sinusoidal.

Frequency response:

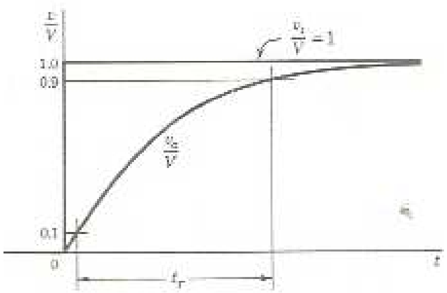


Step voltage input :
For an low pass RC circuit we know Vo = Vc,
Where Vc is the voltage across the capacitor.
Case(i): for step voltage input we know for t<0 , Vi = 0 V , Vc = 0 V so Vo = 0 V

Case(ii): at $t=0^-$ (immediately before t=0)still output voltage is zero
but at $t=0^+$(immediately after t=0) , Vi = V Volts , Vc = 0 V so Vo = 0 Volts

case(iii): for t>0 , voltage across the capacitor increases exponentially(by doing mathematical analysis we can get current is in the form of exponential) so output is a raising exponential.

Time constant of an low pass RC circuit is RC(by neglecting source resistance).
If time constant is high then output takes more time to reach final value similarly
If time constant is low then output takes less time to reach final value.



Rise time : it is the time taken by the output waveform to rise from 10% to 90%of its final value is known as rise time
Expression for rise time:
Here $Vo(t) = V(1-e^{-t/RC})$--------(1)
Consider at t=t1 , Vo(t) = 10% of V
At t=t2 , Vo(t) = 90% of V
So at t=t1, equation 1 willbecomes,
$0.1V = V(1-e^{-t1/RC})$

By simplifying t1 = 0.105 RC
Similarly t2 = 2.3RC

Hence t_r = t1-t2 = 2.2 RC
Also $t_r = 0.35/ f_2$
So time constant of a circuit is directly proportional to the time constant of a circuit and it is inversely proportional to the upper cutoff frequency.

Pulse input to low pass RC circuit :

Case(i): for t<0 and also at $t=0^-$, Vi = 0 V , Vc = 0 V so Vo = 0 V

Case(ii): at $t=0^+$(immediately after t=0) , Vi = V Volts , Vc = 0 V so Vo = 0 Volts

case(iii): during the interval 0 to t_p ,voltage across the capacitor increases exponentially(by doing mathematical analysis we can get current is in the form of exponential) so output is a raising exponential.
Equation for output voltage during 0 to tp is $Vo(t) = V(1-e^{-t/RC})$
At $t = t_p^-$, $Vo(t) = V(1-e^{-tp/RC}) = V_P$

Case(iv): at $t = t_p^+$, input voltage reduces by an amount of V volts but output will also
changes (because Vc can not changes instantaneously).
So at $t = t_p^+$, $Vo(t) = V_P$

Case(v): for $t > t_p$, Vi = 0 V so capacitor will discharges then output will tries to approach zero volts.
We know equation for exponential signal is Vo(t) = Vf +(Vi-Vf) e-(t-tx)/τ
For t>tp , Vf = 0 V , tx = tp , $Vi = V_P$ then $Vo(t) = V_P\ e^{-(t-tp)/RC}$