Sweep error (e_s) : It is defined as the difference between initial slope and final slope expressed as a fraction of a initial slope of the waveform.

Sweep error is also named as slope error or sweep speed error or slope speed error.

$$e_s = \frac{initialslope - finalslope}{initialslope} \text{---------(1)}$$

Expression for sweep error :

Cosider V_S is the waveform generated from

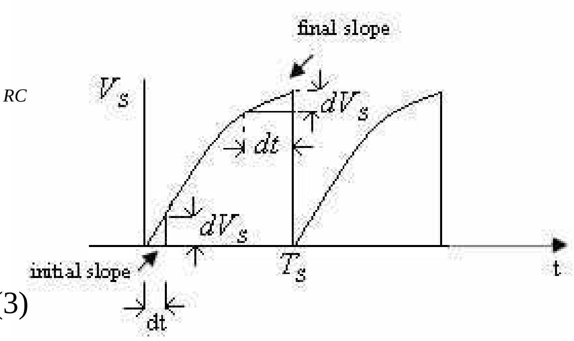
A time base generator as shown in figure.

We know ,

$V_S = V(1 - e^{-\frac{t}{RC}})$ ---(2) because V_s is a raising exponential. Where V is the final value of the waveform

Slope of the waveform is $\frac{dV_S}{dt} = \frac{V}{RC} e^{-t/RC}$

Initial slope of the waveform is $\left(\frac{dV_S}{dt}\right)_{t=0} = \frac{V}{RC}$



Final slope of the waveform is $\left(\frac{dV_S}{dt}\right)_{t=Ts} = \frac{V}{RC} e^{-Ts/RC}$

Therefore from equation (1) ,

$$e_s = 1 - e^{-Ts/RC} \text{-----(3)}$$

If $\frac{Ts}{RC} << 1$ we know $e^{\frac{-Ts}{RC}} = \left(1 - \frac{T_s}{RC} + \left(\frac{T_s}{RC}\right)^2 \frac{1}{2!} - ....\right)$

By neglecting higher order terms,

$$e^{\frac{-Ts}{RC}} = 1 - \frac{T_s}{RC}$$

Then $e_s = \frac{T_s}{RC}$ -----(4)

Sweep error can also be expressed in terms of sweep amplitude.

From equations (2) &(3) ,

$$e_s = \frac{V_s}{V}$$

This can be written as ,

$$e_s = \frac{V_s}{V_o(t = \infty)}$$

So sweep error is the ratio between sweep amplitude and output amplitude at t = ∞

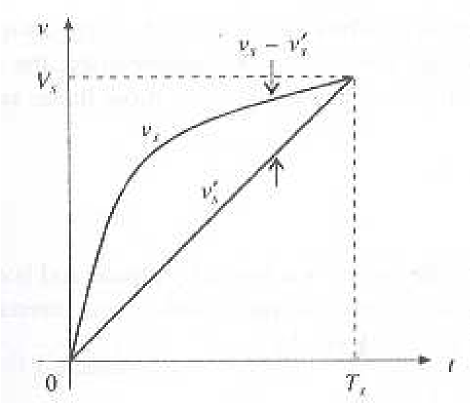
Displacement error (e_d) : It is the ratio of maximum deviation of actual sweep from the linear sweep to the amplitude of sweep voltage.

Expression for displacement error :

Consider v_s is the actual sweep and v_s' is linear sweep

And V_S is the sweep amplitude as shown in figure

.



So ,

$$e_d = \frac{(v_s - v_s')_{max}}{V_S}$$

At t = Ts/2 deviation between actual sweep and linear sweep is maximum.

Therefore $$e_d = \frac{\left(v_s - v_s{}'\right)_{t=\frac{T_S}{2}}}{V_S}$$

We know , $v_S = V(1-e^{-\frac{t}{RC}})$ -----(1)

At t = T_s equation (1) will becomes

$$V_S = V(1-e^{-\frac{T_S}{RC}})$$ since at t = T_s , $v_S = V_S$

If $\frac{T_S}{RC} << 1$ we know $e^{\frac{-Ts}{RC}} = \left(1-\frac{T_s}{RC}+\left(\frac{T_s}{RC}\right)^2\frac{1}{2!}-..............\right)$

By neglecting higher order terms,

$$e^{\frac{-Ts}{RC}} = \left(1-\frac{T_s}{RC}+\left(\frac{T_s}{RC}\right)^2\frac{1}{2!}\right)$$

There fore,

$$V_S = \frac{VT_S}{RC}(1-\frac{T_S}{2RC})$$ ---------(2)

At t =T_s/2 actual sweep amplitude(v_s) is

$$v_S = V(1-e^{-\frac{T_S}{2RC}})$$ (from equation (1))

If $\frac{T_S}{2RC} << 1$ we know $e^{\frac{-Ts}{2RC}} = \left(1-\frac{T_s}{2RC}+\left(\frac{T_s}{2RC}\right)^2\frac{1}{2!}-..............\right)$

By neglecting higher order terms,

$$e^{\frac{-Ts}{2RC}} = \left(1 - \frac{T_s}{2RC} + \left(\frac{T_s}{2RC}\right)^2 \frac{1}{2!}\right)$$

There fore actual sweep amplitude at t = T_S /2 is

$$v_S = \frac{VT_S}{2RC}(1 - \frac{T_S}{4RC}) \text{ ---------(3)}$$

Equation for linear sweep is $v_s{}' = \alpha t$

$$v_s{}' = \frac{V_S}{T_S} t$$ where α is slope of the linear sweep.

At t = T_s / 2 linear sweep amplitude is $v_s{}' = \frac{V_S}{T_S}\frac{T_S}{2} = \frac{V_S}{2}$

By using equation (2),

$$v_S{}' = \frac{VT_S}{RC}(1 - \frac{T_S}{2RC}) \text{ -------(4)}$$

From equations (3) &(4) ,

$$\left(v_s - v_s{}'\right)_{t=\frac{T_S}{2}} = \frac{VT_S}{RC}\left(\frac{T_S}{8RC}\right) \quad \text{-------(5)}$$

Therefore displacement error ,

$$e_d = \frac{\frac{VT_S}{RC}\left(\frac{T_S}{8RC}\right)}{\frac{VT_S}{RC}\left(1 - \frac{T_S}{2RC}\right)}$$ (from equations (2) &(5))

$$e_d = \frac{T_S}{8RC} \quad \text{since } \frac{T_S}{2RC} << 1$$