Fourier Series Introduction

Introduction

A very basic concept in Signal and System analysis is Transformation of signals. It involves a whole new paradigm of viewing signals in a context different from the natural domain of their occurence. For example, the transformation of a signal from the time domain into a representation of the frequency components and phases is known as Fourier analysis.

Why do we need transformations?

We can't analyse all the signals that we want to, in their existing domain. Transforming a signal means looking at a signal from a different angle so as to gain new insight into many properties of the signal that may not be very evident in their natural domain. Transformation is usually implemented on an independent variable.

Examples:

1. A doctor examines a patient's heart beat, which is a function of time in real world but is represented as a function of space for easier diagnosis of the problem. The job is done by ECG (Electro-Cardio-Gram) which shows the variations in the pulse rate in spatial coordinates.

2. For a musician, all the ragas played are actually in time domain but frequency is more important for him than time. Why frequency has more value must be somewhat intuitive as the variations in sound are due to change in frequency.

3. For a circuit, the input and output signals are functions of time. If we need to study or monitor these signals, we use an Oscilloscope to display these signals using spatial coordinates.

Every periodic signal can be written as a summation of sinusoidal functions of frequencies which are multiples of a constant frequency (known as fundamental frequency). This representation of a periodic signal is called the Fourier Series. An aperiodic signal can always be treated as a periodic signal with an infinite period. The frequencies of two consecutive terms are infinitesimally close and summation gets converted to integration. The resulting pattern of this representation of an aperiodic signal is called the Fourier Transform.

Signals Treated as Vectors

Any vector in N-dimensional space can be fully specified by a set of N numbers (i.e. its components in various directions). Similarly we can also treat signals in continuous and discrete times as special cases of vectors with infinite dimensions.

Why do we need signals to be treated as vectors?

The mathematical analysis of vectors is highly advanced compared to signals. Treating signals as vectors helps us to attribute many additional properties to them. Moreover we do feel comfortable taking signals as vectors in a problem involving number of signals.

Countable Infinity:

A set is called countably infinite if and only if its all elements have 1-1 correspondence with set of natural numbers or any other countable infinite set. We can easily see set of integers satisfying this property. Now we can call a set countably infinite if its elements have 1-1 correspondence with integers (it will ensure automatically that the condition be satisfied for natural numbers). Every rational number can be taken as a tuple of two integers (numerator and denominator) making the set of rational numbers also countably infinite.

Exercise: Prove that the set of real numbers is not countably infinite.

Note:

A Discrete Signal x[n] can be thought of as a " Vector " with countably infinite dimensions.

A Continuous Signal x(t) can be thought of as a vector with uncountably infinite dimensions.