Reconstruction of signal

The Ideal Low-Pass Filter:

In this lecture, we examine the Ideal low pass filter and the process of reconstruction of a Band-limited signal.

Let us first see another way of interpreting the action of the Ideal low-pass filter.

Impulse response of Ideal low pass filter:

The Frequency response (the Fourier transform of the impulse response of an LSI system is also called its frequency response) of an ideal low pass filter which allows a bandwidth B, is a rectangle extending from -B to +B, having a constant height as shown in the figure.

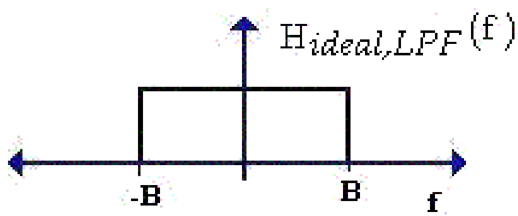

Figure 5.3

Lets look at the Impulse Response of this Ideal low pass filter, taking its height in **[-B, B]** to be **1**. Using the formula for inverse Fourier Transform we have :

$$\text{Impulse Response} = \int_{-B}^{B} e^{j2\pi ft}\,df$$

$$= \frac{2B\ \text{Sin}\pi(2Bt)}{\pi(2Bt)}$$

$$= 2B\ \text{Sinc}(2Bt)$$

(note that $\text{sinc}(t) = \dfrac{\sin(\pi t)}{\pi t}$)

Thus the impulse response of an ideal low pass filter turns out to be a **Sinc function**, which looks like:

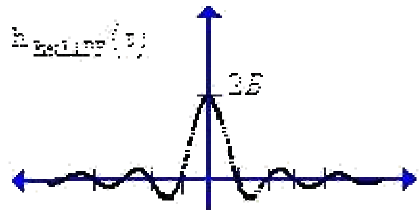

Figure 5.4

Reconstruction of a signal by low pass filter :

Consider a signal **x(t)** having bandwidth less than **B**.

We sample **x(t)** at a rate **2B** and pass $\tilde{x}(t) = \sum_{-\infty}^{\infty} \frac{1}{2B}\, x(\frac{n}{2B})\, \delta(t-\frac{n}{2B})$ into an Ideal low-pass filter of bandwidth **B**. $\text{Assume } T_0 = \frac{1}{2B}$.

The signal **x(t)** and the signal ($\tilde{x}(t)$) obtained by multiplying the signal by a periodic train of impulses, separated by T_0, having strength 1 are shown below.

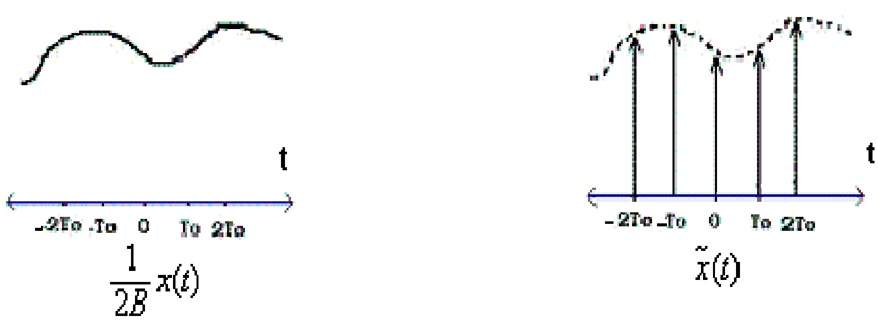

Figure 5.5

What happens when $\tilde{x}(t)$ is fed into the LSI system?

Lets look at the convolution of the impulse response **h(t)** of the Ideal low-pass filter with $\tilde{x}(t)$

$$\tilde{x}(t) * h(t) = \left\{ \sum_{n=-\infty}^{\infty} T_0\, x(nT_0)\, \delta(t - nT_0) \right\} * h(t)$$

$$= \sum_{n=-\infty}^{\infty} T_0\, x(nT_0)\, h(t - nT_0)$$

where we have seen $h(t) = 2B\ \mathrm{sinc}(2Bt) = \frac{1}{T_0}\ \mathrm{sinc}(\frac{t}{T_0})$.

$$\therefore\ \tilde{x}(t) * h(t) = \sum_{-\infty}^{\infty} x(nT_0)\ \mathrm{sinc}(\frac{t}{T_0} - n)$$

When $\tilde{x}(t)$ is passed through a Low pass filter, the output which is the reconstructed signal is nothing but the sum of copies of the impulse response **h(t)** shifted by integral multiples of T_0 and multiplied by the value of **x(t)** at the corresponding integral multiple of T_0. Also observe that the **h(t)** is zero at all sample points (which are integral multiples of T_0) except at zero. Thus, the reconstruction of **x(t)** can be visualized as a sum of the following signals :

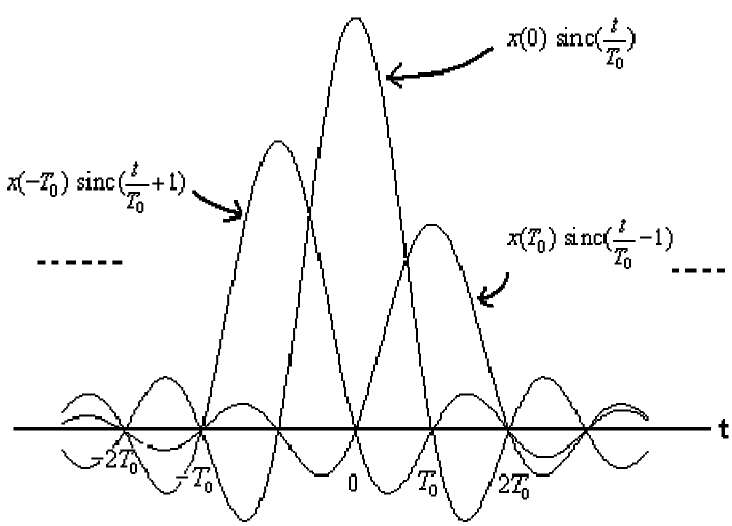

Figure 5.6