Orthogonality in Vectors

Orthogonality in Vectors:

A vector is specified by magnitude and direction. We shall denote all vectors by boldface type and their magnitudes by lightface type, for example, **A** is a certain vector with magnitude *A*. Consider two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ as shown in Fig. Let the component of $\mathbf{V}_1$ along $\mathbf{V}_2$ be given by $C_{12}V_2$. How do we interpret physically the component of one vector along the other vector? Geometrically the component of a vector $\mathbf{V}_1$ and $\mathbf{V}_2$ is obtained by drawing a perpendicular from the end of $\mathbf{V}_1$ on the vector $\mathbf{V}_2$, as shown in fig. The vector $\mathbf{V}_1$ can now be expressed in terms of vector $\mathbf{V}_2$.

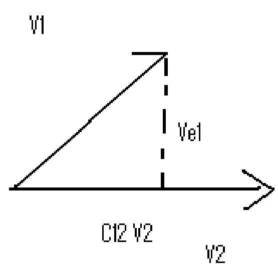

Figure 1.1

$$V_1 = C_{12}V_2 + V_e$$

However, this is not the only way of expressing vector V_1 in terms of vector $\mathbf{V}_2$. Figure illustrates two of the infinite alternate possibilities. Thus, in fig

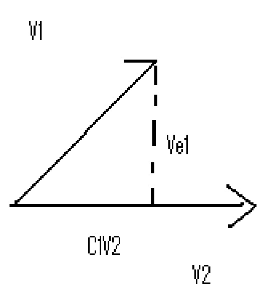

Figure 1.2

$$V_1 = C_1 V_2 + V_{e1}$$

and in fig.

$$V_1 = C_2 V_2 + V_{e2}$$

For convenience, we define the dot product of two vectors **A** and **B** as

$$A \cdot B = AB \cos\theta$$

where θ is the angle between vectors **A** and **B**. It follows from the definition that

$$\mathbf{A} \,.\, \mathbf{B} = \mathbf{B} \,.\, \mathbf{A}$$

According to this notation,

The component of **A** along **B** = $A \cos\theta \dfrac{A \cdot B}{B}$

and

The component of **B** along **A** = $B \cos\theta \dfrac{A \cdot B}{A}$

similarly

The component of $\mathbf{V_1}$ along $\mathbf{V_2}$ $= \dfrac{V_1 \cdot V_2}{V_2}$

Therefore

$$C_{12} = \frac{V_1 \cdot V_2}{{V_2}^2} = \frac{V_1 \cdot V_2}{V_2 \cdot V_2}$$

Note that if $\mathbf{V}_1$ and $\mathbf{V}_2$ are orthogonal then

$$V_1 \cdot V_2 = 0$$

and $C_{12} = 0.$