Vectors and signals

We briefly review the concepts of vector space . A vector space has following key properties, if v, w belongs to V then av+bw belongs to V. For any scalars a and b. That is, linear combinations of vectors give vectors.

Most of your background with vectors has been for vectors in Rn. But: the signals that we deal with are also elements of a vector space, since linear combinations of signals also gives a signal. This is a very important and powerful idea.

Recall that in vector spaces we deal with concepts like the length of a vector, the angle between vectors, and the idea of orthogonal vectors. All of these concepts carry over, by suitable definitions, to vector spaces of signals.

This powerful idea captures most of the significant and interesting notions in signal processing, controls, and communications. This is really the reason why the study of linear algebra is so important.

In this lecture we will learn about geometric representations of signals via signal space (vector) concepts. This straightforward idea is the key to a variety of topics in signals and systems:

1. It provides a distance concept useful in many pattern recognition techniques.
2. It is used in statistical signal processing for the filtering, smoothing, and prediction of noisy signals.
3. It forms the heart and geometric framework for the tremendous advances that have been made in digital communications.
4. It is every waveform-based transform you ever wanted (Fourier series, FFT, DCT, wavelet, etc.)
5. It is also used in the solution of partial differential equations, etc.
6. It relies on our old friend, linearity. One might even say it is the reason that we care so much about linearity in the first place.

We will soon turn our attention to Fourier series, which are a way of analyzing and synthesizing signals.

Vectors will be written in bold font (like the ingredients above. Initially, we can think of a vector v as an ordered set of *n* numbers, written in a column:

Often to conserve writing, this will be written in transposed form,

$$\mathbf{v} = [v_1, v_2, \ldots, v_n]^T.$$

Vector ddition is component-by-component.

While we have written a vector as an n-tuple, that is not what defines a vector. A vector is an element of a vector space, which is to say, it satisfies the linearity property given above.

Scalar multiplication of vectors is in the usual fashion. Matrix multiplication is also taken in the traditional manner.

Let $$\mathbf{g} = [g_1, g_2, \ldots, g_n]^T$$

and

$$\mathbf{f} = [f_1, f_2, \ldots, f_n]^T$$

But the inner product of vectors g and f is written as

$$\langle \mathbf{g}, \mathbf{f} \rangle = \mathbf{g}^T \mathbf{f} = \sum_{i=1}^{n} g_i f_i.$$

The concept and theory of signals and systems are needed in almost all electrical engineering fields and in many other engineering and scientific disciplines as well. In this chapter we introduce the mathematical description and representation of signals and systems and their classifications. We also define several important basic signals essential to our studies.