Fourier Transform Introduction

The Fourier Transform equation

The **Fourier Transform** of a function x(t) can be shown to be:

$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi ft} dt}$$

This equation is called the **Fourier Transform equation.**

(As a convention, we generally use capital letters to denote the Fourier transform)

Obviously not in all cases are we guaranteed that the integral on right hand side will converge.

We'll next discuss the conditions for the Fourier Transform of an aperiodic signal to exist.

Under certain conditions, an aperiodic signal **x(t)** has a Fourier transform **X(f)** and the two are related by:

$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\ e^{-j2\pi ft} dt}$$ (Fourier Transform equation)

$$\boxed{x(t) = \int_{-\infty}^{\infty} X(f)\ e^{j2\pi ft} dt}$$ (Inverse Fourier Transform equation)

Now, lets go on to the conditions for existence of the Fourier Transform. Again notice the similarity of these conditions with the Dirichlet conditions for periodic signals.

Dirichlet Conditions for convergence of Fourier Transform

Consider an aperiodic signal **x(t)**. Its Fourier Transfrom exists (i.e the Transform integral converges) and $\int_{-\infty}^{\infty} X(f)\ e^{j2\pi ft} df$

converges to **x(t)**, except at points of discontinuity provided:

1) **x(t)** is absolutely integrable . i.e: $\int_{-\infty}^{\infty} | x(t) | \, dt$ converges.

2) **x(t)** has only a finite number of extrema in any finite interval.

For example, $f(t) = \sin(\frac{1}{t})$ does not satisfy this condition in, say (0,1).

3) **x(t)** has only a finite number of discontinuities in any finite interval. For example the following function (the so called Dirichlet's function) will not satisfy this condition.

$$x(t) = \begin{matrix} 0 \text{ for } t \text{ being rational} \\ 1 \text{ for } t \text{ being irrational} \end{matrix}$$

These 3 conditions satisfied, $\int_{-\infty}^{\infty} X(f) \, e^{j2\pi ft} df$ will converge to **x(t)** at all points of continuity of **x(t)**. At points of discontinuity of **x(t)**, this integral converges to the average of the left and the right hand limit of **x(t)** at that point.

The Fourier Transform and the Inverse Fourier Transform as systems

The Fourier Transform and The Inverse Fourier transform may be looked at as system transformations. One system for instance takes in a time signal and outputs its Fourier transform, another takes a frequency domain signal (or a spectrum) and produces the corresponding time-domain signal.

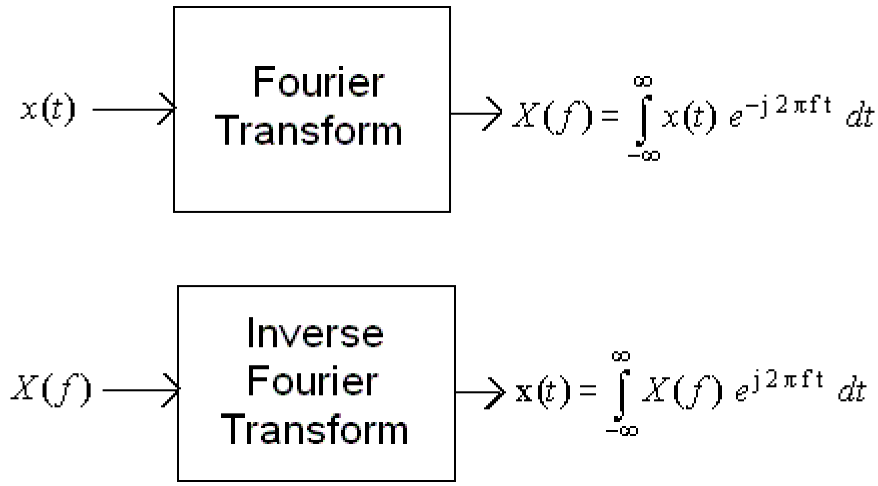

Figure 3.1

Let us now gain some additional insight into the Fourier Transform using this system notion.