# Convolution theorem for the Fourier Transform

What we have just proved is called the **Convolution theorem** for the Fourier Transform.

It states:

If two signals x(t) and y(t) are Fourier Transformable, and their convolution is also Fourier Transformable, then the Fourier Transform of their convolution is the product of their Fourier Transforms.

If x(t), y(t) and (x*y)(t) are Fourier Transformable and

$$x(t) \xrightarrow{FT} X(F) \quad \& \quad y(t) \xrightarrow{FT} Y(f) \quad \text{then}$$

$$\boxed{(x*y)(t) \xrightarrow{FT} X(F)\,Y(f)}$$

**Dual of the convolution theorem**

We now apply the Duality of the Fourier Transform to the Convolution Theorem to get another important theorem.

Let **x(t)** and **y(t)** be two Fourier transformable signals, with Fourier transforms **X(f)** and **Y(f)** respectively. Assume **X(f)*Y(f)** is Fourier Invertible. We now find its inverse.

What does Duality tell us? If $x(.) \xrightarrow{FT} X(.) \quad \text{then} \quad X(.) \xrightarrow{FT} x(-.)$.

Thus we know: $X(.) \xrightarrow{FT} x(-.) \;\&\; Y(.) \xrightarrow{FT} y(-.) \Rightarrow X(-.) \xrightarrow{FT} x(.) \;\&\; Y(-.) \xrightarrow{FT} y(.)$.

The Convolution theorem says: $X(-.)*Y(-.) \xrightarrow{FT} x(.)\,y(.)$

Applying duality on this result,

$$x(.)\,y(.) \xrightarrow{FT} X(.)*Y(.)$$

Thus we get the Dual version of the Convolution Theorem:

If **x(t)** and **y(t)** are Fourier Transformable, and **x(t) y(t)** is Fourier Transformable , then its Fourier Transform is the convolution of the Fourier Transforms of **x(t)** and **y(t)**. i.e:

If x(t), y(t) and $x(t)\times y(t)$ are Fourier Transformable and

$$x(t) \xrightarrow{FT} X(f) \quad \& \quad y(t) \xrightarrow{FT} Y(f) \quad \text{then}$$

$$\boxed{x(t)\cdot y(t) \xrightarrow{FT} X(f)*Y(f)}$$