The Auto-correlation and the Cross-correlation

The autocorrelation involves correlation of identical signals. Autocorrelationtion is commutative in nature, that is, it can be performed any order.

Autocorrelation is a measure of similarity of a given function with itself when sghifted by some amount. Therefore, under no shift, the two functions match and under this condition, autocorrelation is maximum. Physically if an individual compares his two photographs taken at the same instant, there is maximum similarity. If one compares his two photographs taken at two different instants of time, the similarly decreases as the dates on which the similarity and hence the correlation between a function and its shifted version decreses. As the shift approaches infinity, the similarity almost vanishes and hence, the autocorrelation function tends to zero. Proceeding with our work on the Fourier transform, let us define two important functions, the Auto-correlation and the Cross-correlation.

Auto Correlation

You have seen that for a Periodic signal y(t), $y(t) \otimes \overline{y(-t)}$ has Fourier series coefficients that the modulus square of the Fourier series coefficients of y(t).

Lets look at an equivalent situation with aperiodic signals, i.e:

Assume that $x(t) \xrightarrow{FT} X(f)$

then $? = \xrightarrow{FT} |X(f)|^2$

Notice that $|X(f)|^2 = X(f)\overline{X(f)}$

Since $\overline{x(t)} \xrightarrow{FT} \overline{X(-f)}$

We have, $\overline{x(-t)} \xrightarrow{FT} \overline{X(f)}$

Using the dual of the convolution theorem,

$$|X(f)|^2 \xrightarrow{FT^{-1}} x(t) * \overline{x(-t)}$$
$$= \int_{-\infty}^{\infty} x(t-\lambda)\,\overline{x(-\lambda)}\,d\lambda$$
$$= \int_{-\infty}^{\infty} x(t+\gamma)\,\overline{x(\gamma)}\,d\gamma$$

The **auto-correlation** of x(t), denoted by R_{xx} is defined as:

$$R_{XX}(t) = \int_{-\infty}^{+\infty} x(t+\gamma)\,\overline{x(\gamma)}\,d\gamma$$

Its Spectrum is the modulus square of the spectrum of x(t).

It can also be interpreted as the projection of **x(t)** on its own shifted version, shifted back by an interval 't'.

It can be shown that $R_{xx}(t) <= R_{xx}(0)$ (note that $R_{xx}(0)$ is nothing but the energy in the signal **x(t)**

Cross Correlation

The cross correlation between two signals x(t) and y(t) is defined as :

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t+\tau)\,\overline{y(t)}\,dt$$

Note that the cross-correlation R_{xy} is the convolution of $x(t)$ and $\overline{y(-t)}$.

If $y(t) = x(t-\tau_0)$ then using the fact that the auto-correlation integral peaks at 0 , the cross correlation peaks at $\tau = -\tau_0$.

It may be said that cross-correlation function gives a measure of resemblance between the shifted versions of signal x(t) and y(t). Hence it is used to in Radar and Sonar applications to measure distances . In these systems, a transmitter transmits signals which on reflection from a target are received by a receiver. Thus the received signal is a time shifted version of the transmitted signal . By seeing where the cross-correlation of these two signals peaks, one can determine the time shift and hence the distance of the target.

The Fourier transform of $R_{xy}(t)$ is of-course $\widehat{R}_{xy}(f) = X(f)\,\overline{Y(F)}$