Problems on convolution

1. Convolute the two continuous-time signals given below:

$$x_1(t) = e^{-2t}\, u(t)$$
$$x_2(t) = u(t+2)$$

Solution Given :

(1) $x_1(t) = e^{-2t}u(t) \Rightarrow x_1(\tau) = e^{-2\tau}u(\tau)$

(2) $x_2(t) = u(t+2) \Rightarrow x_2(\tau) = u(\tau+2) = \begin{cases} 1, \tau+2>0 \text{ or } \tau>-2 \\ 0\ \tau+2<0 \text{ or } \tau<-2 \end{cases}$

Fig:

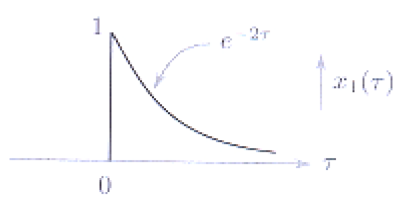


Fig. Ex. 5.2(a): *Signal, $x_1(\tau)$*

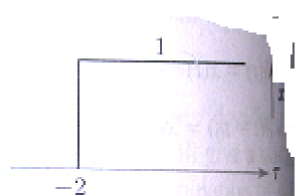


Fig. Ex. 5.2(b): *Signal, x_2(*

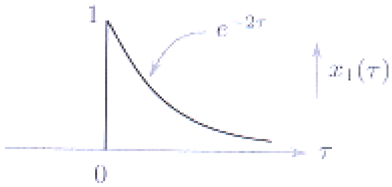


Fig. Ex. 5.2(a): *Signal,* $x_1(\tau)$

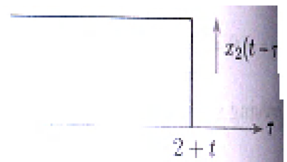


Fig. Ex. 5.2(d): *Folded and shifted signal,* $x_2(t-\tau)$

The sketch of $x_2(t-\tau)$ can also be formed as follows:

We know that

$$x_2(t) = \begin{Bmatrix} 1, t > -2 \\ 0, t < -2 \end{Bmatrix}$$

$$\Rightarrow x_2(t-\tau) = \begin{Bmatrix} 1, & t-\tau > -2 \;\; or\; \tau < t+2 \\ 0, & t-\tau < -2 \;\; or\; \tau > t+2 \end{Bmatrix}$$

Let

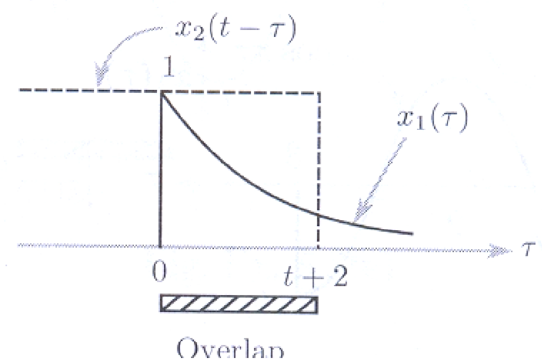
$$y(t) = x_1(t) * x_2(t)$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau) d\tau$$

Let us now proceed to find y(t) for different values of t.

Case (i): For t+2<0 or t<-2, the function $x_1(\tau)$ *and* $x_2(t-\tau)$ do not overlap. Hence, $y(t) = y_1(t) = 0.$

Case: (ii)



For t+2 >0 or t>-2, the functions $x_1(\tau)$ *and* $x_2(t-\tau)$ overlap as shown in

Therefore,

$$y(t) = y_2(t) = \int_0^{t+2} e^{-2\tau} \times 1 d\tau$$
$$= \frac{1}{2} - \frac{1}{2} e^{-2(t+2)}$$

Summarizing the results, we get

$$y(t) = \begin{cases} 0, & t < -2 \\ \frac{1}{2} - \frac{1}{2} e^{-2(t+2)}, & t > -2 \end{cases} \text{Convolution}$$

2.Prove that

(a) $x(t) * \delta(t) = x(t)$

(b) $x(t) * \delta(t - t_0) = x(t_0)$

© $x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\, d\tau$

Solution :

(a) $x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau)\ d\tau$

Applying sifting property, we get

$$x(t) * \delta(t) = x(\tau) \tau = t = x(t)$$

(b) $x(t) * \delta(t - t_0) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau - t_0)\ d\tau$

Applying sifting property, we get

$$x(t) * \delta(t - t_0) = x(\tau) \tau = t - t_0 = x(t - t_0)$$

© Since, $u(t - \tau) = \begin{cases} 1, & t - \tau > 0 \ \ or\ \tau < t \\ & , wecanwrite \\ 0, & t - \tau < 0 \ \ or\ \tau > t \end{cases}$

$$x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau$$

$$= \int_{-\infty}^{t} x(\tau)dt$$