Problem regarding mean square error

A rectangular function f(t) is defined by figure

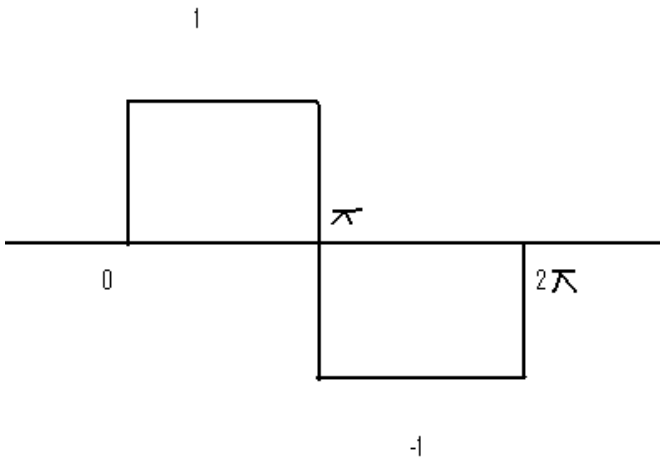

Figure 1.3

$$f(t) = \begin{cases} 1, (0 < t < \pi) \\ -1, (\pi < t < 2\pi) \end{cases}$$

Approximate this function by a waveform sin t over the interval $(0,2\pi)$ such that the mean square error is minimum.

Solution. The function $f(t)$ will be approximated over the interval $(0,2\pi)$, as

$$f(t,) \cong C_{12} \sin t$$

We shall find the optimum value of C_{12} which will minimize the mean square error in this approximation. According to Eq.3.10 to minimize the mean

Square error :

$$C_{12} = \frac{\int_0^{2\pi} f\ (t)\sin t\ dt}{\int_0^{2\pi} \sin^2(t)dt}$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} \sin t\ dt + \int_{\pi}^{2\pi} -\sin t\ dt\right]$$

$$= \frac{4}{\pi}$$

Thus

$$= f(t) \simeq \frac{4}{\pi}\sin t\ .$$