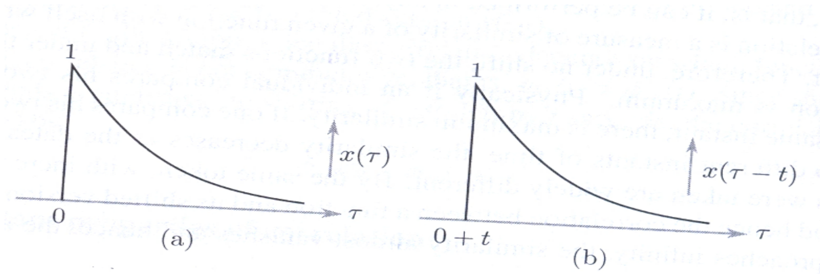
Problem on correlation

Find the autocorrelation of the signal, x(t) $= e^{-at}u(t)$

Solution:



The autocorrelation of x(t) is defined as follows:

$$\phi_{xx}(t) = \int_{-\infty}^{\infty} x(\tau)\, x(\tau - t)dt$$

The function $x(\tau)$ *and* $x(\tau - t)$ are as depicted in respectively.

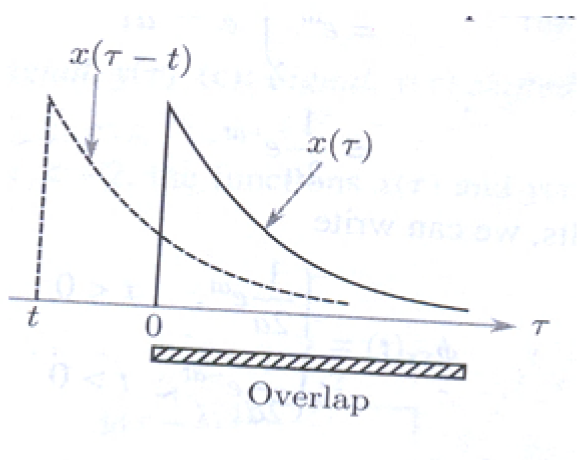
Also,

$$x(\tau) = e^{-a\tau}\ u(\tau)$$

and

$$x(\tau - t) = e^{-a\tau}\ u(\tau - t)$$

Case(i): For t<0, the functions $x(\tau)$ *and* $x(\tau - t)$ Overlap as

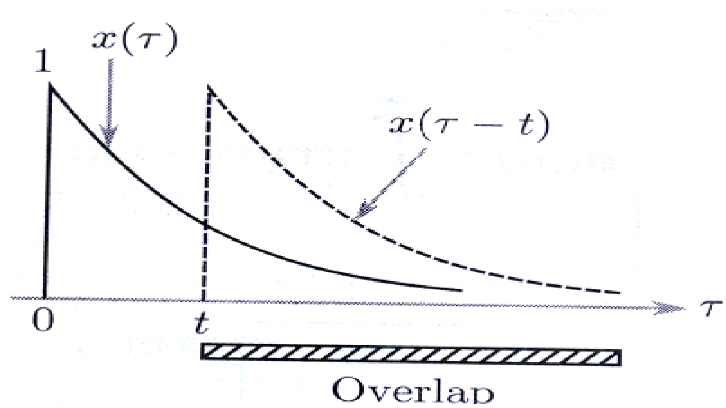


We have,

$$
\begin{aligned}
\phi_{xx}(t) &= \int_{-\infty}^{\infty} e^{-a\tau} e^{-a\tau} \, d\tau \\
&= e^{at} \int_{-\infty}^{\infty} e^{-2a\tau} \, d\tau \\
&= e^{at\left[\frac{e^{-2a\tau}}{-2a}\right]_0^{\infty}} \\
&= \frac{1}{2a} e^{at}
\end{aligned}
$$

Case (ii) : For t>0, the functions, $x(\tau - t)$ overlap as depicted

Fig:



Referring Fig. Ex, we can write

$$\phi_{xx}(t) = \int_{-\infty}^{\infty} e^{-a\tau} e^{-a(\tau-t)} d\tau$$

$$= e^{at} \int_{-\infty}^{\infty} e^{-2a\tau} d\tau$$

$$= \frac{1}{2a} e^{-at}$$

Summarizing the results, we can write

$$\phi_{xx}(t) = \begin{cases} \frac{1}{2a} e^{at}, & t < 0 \\ \frac{1}{2a} e^{-at}, & t > o \end{cases}$$

$$= \frac{1}{2a} e^{-a|t|}$$