Trigonometric Forier series

Trigonometric Fourier Series:

The trigonometric Fourier series representation of a periodic signal $x(t)$ with fundamental period T_0 is given by

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t) \qquad \omega_0 = \frac{2\pi}{T_0}$$

where a_k and b_k are the Fourier coefficients given by

$$a_k = \frac{2}{T_0} \int_{T_0} x(t) \cos k\omega_0 t \, dt$$

$$b_k = \frac{2}{T_0} \int_{T_0} x(t) \sin k\omega_0 t \, dt$$

The coefficients a_k and b_k and the complex Fourier coefficients c_k are related by

$$\frac{a_0}{2} = c_0 \qquad a_k = c_k + c_{-k} \qquad b_k = j(c_k - c_{-k})$$

From Eq. we obtain

$$c_k = \tfrac{1}{2}(a_k - jb_k) \qquad c_{-k} = \tfrac{1}{2}(a_k + jb_k)$$

When $x(t)$ is real, then a_k and b_k are real and by Eq. we have

$$a_k = 2\,\mathrm{Re}[c_k] \qquad b_k = -2\,\mathrm{Im}[c_k]$$

Even and Odd Signals:

If a periodic signal $x(t)$ is even, then $b_k = 0$ and its Fourier series (5.8) contains only cosine terms:

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t \qquad \omega_0 = \frac{2\pi}{T_0}$$