Problems regarding Fourier Transform

1.

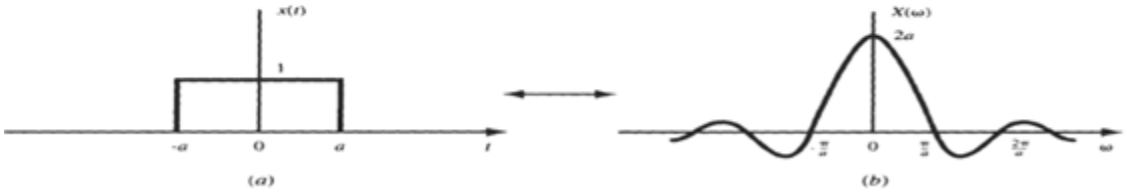
Find the Fourier transform of the rectangular pulse signal $x(t)$ Fig. defined by

$$x(t) = p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$$

By definition

$$X(\omega) = \int_{-\infty}^{\infty} p_a(t)\, e^{-j\omega t}\, dt = \int_{-a}^{a} e^{-j\omega t}\, dt$$

$$= \frac{1}{j\omega}(e^{j\omega a} - e^{-j\omega a}) = 2\frac{\sin \omega a}{\omega} = 2a\frac{\sin \omega a}{\omega a}$$



(a) (b)

Fig. 3.12 Rectangular pulse and its Fourier transform.

2. Find the Fourier transform of the signal Fig.

$$x(t) = \frac{\sin at}{\pi t}$$

From Eq. we have

$$p_a(t) \longleftrightarrow 2\frac{\sin \omega a}{\omega}$$

Now by the duality property we have

$$2\frac{\sin at}{t} \longleftrightarrow 2\pi p_a(-\omega)$$

Dividing both sides by 2π (and by the linearity property), we obtain

$$\frac{\sin at}{\pi t} \longleftrightarrow p_a(-\omega) = p_a(\omega)$$

where $p_a(\omega)$ is defined by [see Eq.

$$p_a(\omega) = \begin{cases} 1 & |\omega| < a \\ 0 & |\omega| > a \end{cases}$$

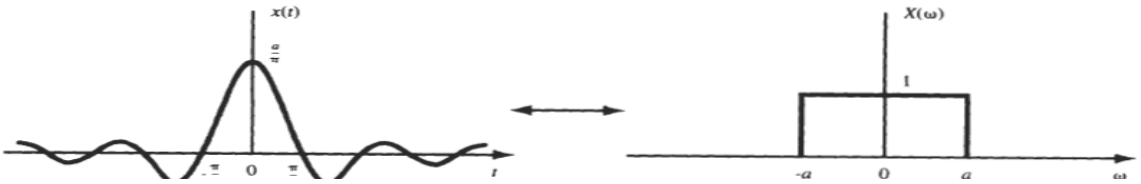


Figure 3.13