The Power Density Spectrum

It was mentioned earlier that some signals(for example periodic signals) have infinite energy. Such signals are called power signals.

The meaning full parameter for a power signal f(t) is the average power P. we define the average power(or simply power) or a signal f(t) as the average power dissipated by a current f(t) applied across a 1 ohm resistor (or by current f(t) passing through a 1 ohm resistor). Thus the average power P of a signal f(t) is given by

$$= \lim_{\to\infty} - \int \quad (\ )$$

Note that the average power as defined also the mean square value **of** ( ). h of ( ). If we denote the mean square value of ( ) by ( ) then

$$= \ (\ ) \lim_{\to\infty} \ 1 + \ \int \quad (\ )$$

We can now continue with the same procedure as that used for obtaining energy density Let us form a new function ( )by truncating ( ) outside the interval $| \ | > -.$ h taruncated function can be expressed as

$$(\ ) = \begin{cases} (\ ) \ | \ | < - \\ h \end{cases}$$

As long as T is finite, ( )has finite energy.Let $(\ ) \leftrightarrow \ (\ )$

The)n the energy ( )is given byType equation here.

$$\int_{\infty}^{\infty} (t) \ d = \int_{\infty}^{\infty} | \ \ |(\ | \ \ |$$

ButType equation here.

$$\int_{\infty}^{\infty} (\ ) \quad = \int_{-}^{-} (\ )$$

Hence the average power P is given by

$$= \lim_{\to\infty} - \int (\ ) = \int_{\infty}^{\infty} \lim_{\to\infty} \frac{(\ )}{}$$

As T increases, the energy of $(\ )$ also increases. Thus $|\ (\ |$ increases with T. In the limit as $T \to \infty$, h quatity $|\ (|\ )/$ may approach a limit. Assuming that such a limit exists, we define $(\ )$, the power density spectrum of $(\ ,)$ as

$$(\ ) \lim_{\to\infty} \frac{(\ )}{}$$

$$(\ )$$

$(\ )$ h , (

Power density)of $(\ )$.Hence

$$\text{Average Power P} = (t) = \lim_{\to\infty} - \int_{/}^{/} (t)\, dt$$

$$= \int_{\infty}^{\infty} (\ )$$

$$- \int_{\infty}^{\infty} (\ )$$

Note that

$$(\ ) = (\ ) \quad (-\ )$$

It is obvious from this equation that the power density is an even function of . Hence and may be expressed as

$$\text{Average Power} = (\ ) = 2 \int_{\infty}^{\infty} (\ )$$

$$-\int_{\infty}^{\infty} \quad (\ )$$

It is evident that the power density spectrum of a signal retains only the information of magnitude of the frequency spectrum ( ). The phase information is lost. It follows that all signals with identical frequency spectrum magnitude but different phase functions will have identical power density spectra. Thus for a given signal there is a unique power density spectrum. But the converse is not true; there may be a large number of signals (in fact, infinite) which can have the same power density spectrum.

We have shown that for energy signals, if

$$(\ ) \leftrightarrow \quad (\ )$$

then

$$(\ ) \cos \quad \leftrightarrow [\ (\quad \& \quad + \quad) \ + \ ]$$

and

$$(\ ) \cos \quad \leftrightarrow [\ (\quad \& \quad + \quad) \ + \quad]$$

We can extend these r esults for power signals.

Consider a power signal ( ) with a power density spectrum ( ):

$$(\quad) =) \lim_{\to\infty} \frac{(\ )}{}$$

Consider the signal $\emptyset(\ )$ given by

$$\emptyset(\ ) = \quad (\ ) \cos$$

If ( ) is the power density spectrum+ of $\emptyset(\ )$, h

$$(\quad) =) \lim_{\to\infty} \frac{\emptyset \ (\ )}{}$$

Where

$$(t) \cos \quad = \emptyset\ (t) \leftrightarrow \emptyset\ (\ )$$

From the modulation theorem it follows that

$$\emptyset\ (\ ) - [\ (\ +\ )\ +\ (\ -\ ]$$

and

$$_{\emptyset}(\ ) = \_)\ \lim_{\to\infty} \frac{(\ \ )\ \ (\ -}{}$$

$$= \_)\ \lim_{\to\infty} \frac{(\ \ )\ \ (\ -}{}$$

Note that the cross-product term $(\ +\ )\ (\ -\ )$ vanishes because the two spectra area nonoverlapping

Now yields

$$_{\emptyset}.(\ ) = -\ (\ +\ )\ +\ (\ -$$

$$\emptyset\ (\ )\emptyset = (\ )\sin$$

Then

$$_{\emptyset}(\ ) = \lim_{\to\infty} \frac{(\ \ )\ (\ \ )}{}$$

$$= \lim_{\to\infty} \frac{|[\ (\ \ )\ \ (\ \ )\ ]|}{}$$

$$n = -\ (\ +\ )\ +\ (\ -\ )$$