Filters

The simplest ideal filters aim at retaining a portion of the spectrum of the input in some pre-defined region of the frequency axis and removing the rest.

A **LOWPASS FILTER** is a filter that passes low frequencies – i.e. around f = 0 and rejects the higher ones, i.e: it multiplies the input spectrum with the following:

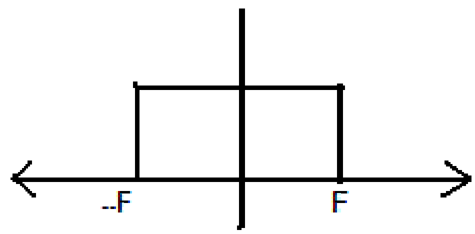

Figure 4.5

A **High pass filter** passes high frequencies and rejects low ones by multiplying the input spectrum by:

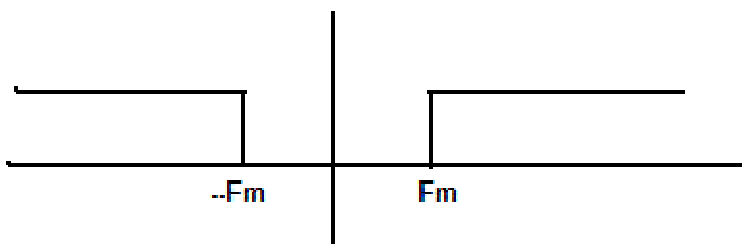

Figure 4.6

A **BANDPASS FILTER** passes a band of frequencies and rejects both higher and lower than those in the band that is passed, thus multiplying the input spectrum by:

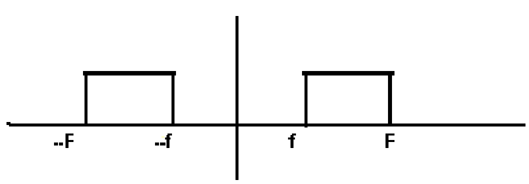

Figure 4.7

A **BANDSTOP FILTER** stops or rejects a band of frequencies and passes the rest of the spectrum, thus multiplying the input spectrum by:

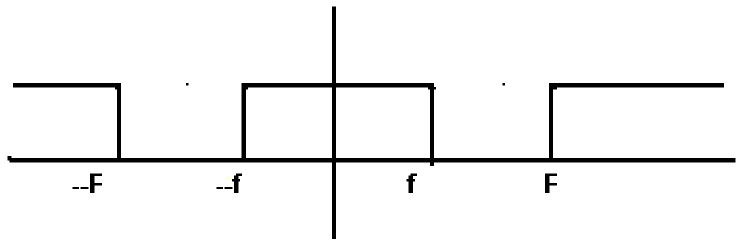

Figure 4.8

How do these filters work? That is, what does multiplication of two signals in the frequency domain imply in the time domain?

If we multiply two Fourier transforms **X(f)** and **H(f)**, let us see what the Inverse Fourier transform of this product is.

Consider the integral $\int_{-\infty}^{\infty} X(f)\, H(f)\, e^{j2\pi f t}\, df$

Let us replace **H(f)** by $\int_{-\infty}^{\infty} h(\lambda)\, e^{-j2\pi f \lambda}\, d\lambda$

This makes the integral, $\int_{-\infty}^{\infty} X(f) \left\{ \int_{-\infty}^{\infty} h(\lambda)\, e^{-j2\pi f \lambda}\, d\lambda \right\} e^{j2\pi f t}\, df$

We can interchange the order of integration, so long as the new double integral converges

$$\int_{-\infty}^{\infty} h(\lambda) \left\{ \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f (t-\lambda)}\, df \right\} d\lambda$$

we note that the term inside the bracket is just the inverse Fourier transform of **X(f)** evaluated at $(t-\lambda)$, Thus the integral simplifies to $\int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$ which is simply the convolution of **h(t)** with **x(t)** !