Casuality And Physical Realizability,The Paley-Wiener Criterion

Physical realizability is defined in the literature in different ways. Here, we shall use the least restrictive definition which will distinguish the systems that are physically possible from those that are not. It is intuitively evident that a physically realizable system cannot have a response before the driving function is applied. This is known as the *causality condition.* This condition may also be expressed alternatively. A unit impulse response h(t) of a physically realizable system must be causal $\left[h(t)=0; t<0\right]$. This is the time –domain criterion of physical realizability. In the frequency domain, this criterion implies that a necessary and sufficient condition for a magnitude function $|H(j\omega)|$ to be physically realizable is that

$$\int_{-\infty}^{\infty} \frac{\left|1n|H(j\omega)|\right|}{1+\omega^2} d\omega < \alpha$$

The magnitude function $|H(j\omega)|$ must, however, be square-integrable before the Paley-Wiener criterionis valid, that is,

$$\int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega < \infty$$

A system whose magnitude function violates the Paley-Weiner criterion has a noncausal impulse response, that is, the response exists forever in the past, prior to the application of the driving function.

We can draw some significant conclusions from the Paley-wiener criterion. It is evident that the magnitude function $|H(j\omega)|$ may be zero at some discrete frequencies, but it cannot be zero over a finite band of frequencies since this will cause the intergral in Eq. to become infinity. It is therefore evident that the ideal filters shown in figures are not physically realizable. We can conclude from eq that the amplitude function cannot fall off to zero faster than a function of exponential order. Thus

$$|H(j\omega)| = ke^{-\alpha|\omega|}$$

is permissible. But the Gaussian error curve

$$\left|H(j\omega)\right| = ke^{-\alpha\omega^2}$$

is not realizable, since it violates eq. In short, a realizable magnitude characterstic cannot have too great a total attenuation. It is interesting to note that although the ideal filter characteristics shown in figure.

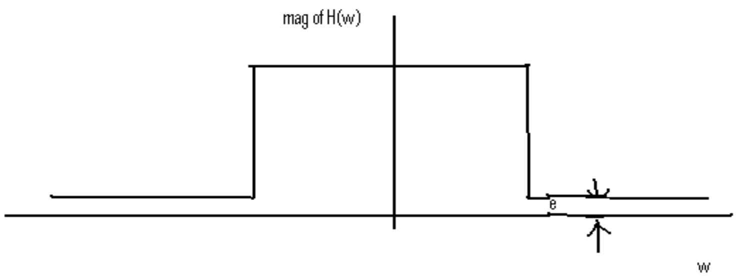

Figure 4.13

and figure are not realizable, it is possible to approach these characteristics as closely as desire. Thus the low-pass filter characteristics shown in figure is physically realizable for arbitrarily small value of ε. The reader can verify that this characteristic does not violate the Paley-Wiener criterion.