Problems regarding Fourier Transform

1.

Find the Fourier transform of the rectangular pulse signal $x(t)$ Fig. defined by

$$x(t) = p_a(t) = \begin{cases} 1 & |t| < a \\ 0 & |t| > a \end{cases}$$

By definition

$$X(\omega) = \int_{-\infty}^{\infty} p_a(t)\, e^{-j\omega t}\, dt = \int_{-a}^{a} e^{-j\omega t}\, dt$$

$$= \frac{1}{j\omega}\left(e^{j\omega a} - e^{-j\omega a}\right) = 2\frac{\sin \omega a}{\omega} = 2a\frac{\sin \omega a}{\omega a}$$

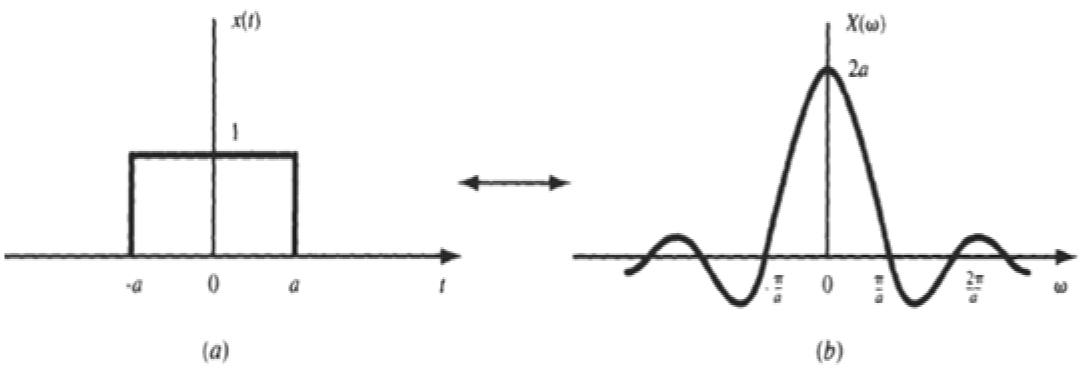


Fig. 3.14 Rectangular pulse and its Fourier transform.

2. Find the Fourier transforms of the following signals:

(a) $x(t) = 1$ (b) $x(t) = e^{j\omega_0 t}$
(c) $x(t) = e^{-j\omega_0 t}$ (d) $x(t) = \cos \omega_0 t$
(e) $x(t) = \sin \omega_0 t$

(a) By Eq. we have

$$\delta(t) \leftrightarrow 1$$

Thus, by the duality property (5.54) we get

$$1 \leftrightarrow 2\pi\delta(-\omega) = 2\pi\delta(\omega)$$

(b) Applying the frequency-shifting property we get

$$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$$

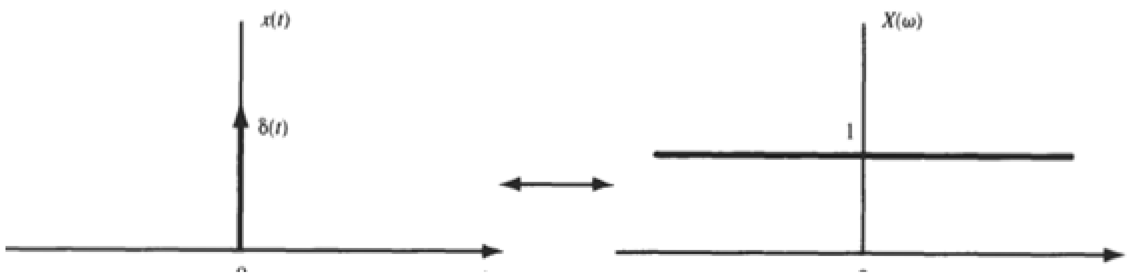


Figure 3.15

(c)

$$e^{-j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega+\omega_0)$$

(d) From Euler's formula we have

$$\cos \omega_0 t = \tfrac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})$$

Thus, using the linearity property we get

$$\cos \omega_0 t \longleftrightarrow \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$$

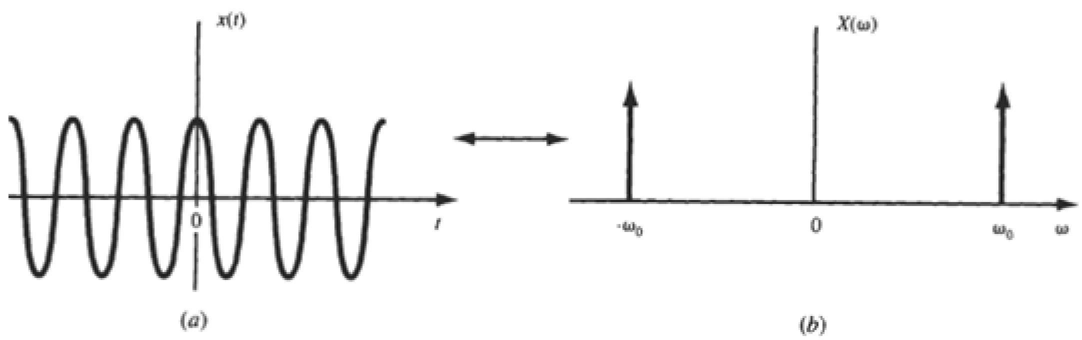
Figure illustrates the relationship in Eq. (*5.144*).

(e) Similarly, we have

$$\sin \omega_0 t = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$$

and again using Eqs. ve get

$$\sin \omega_0 t \longleftrightarrow -j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$$



Cosine signal and its Fourier transform.

Figure 3.15