Fourier Transform of conjugate signal

Consider a signal **x(t)** with Fourier transform **X(f)**. We'll see what happens to the Fourier

x(t)	X(f)
x(-t)	?
x(t)	?

transform of x(t) on **time-reversal** and **conjugation**. i.e:

Now, we are aware that $$\mathbf{X(f)} = \int_{-\infty}^{\infty} \mathbf{x(t)\, e^{-j2\pi ft}\, dt}$$

Transform **X'(f)** of **x(-t)** is: $$X'(f) = \int_{-\infty}^{\infty} x(-t)\, e^{j2\pi ft}\, dt$$. Substitute $\mathbf{t = -\lambda,}$

Therefore, $$X'(f) = \int_{-\infty}^{\infty} x(\lambda)\, e^{-j2\pi f\lambda}\, d\lambda = X(-f)$$.

Therefore, $$\boxed{\mathbf{x(-t)} \xrightarrow{\mathbf{F.T}} \mathbf{X(-f)}}$$

Applying this result to periodic signals (we have just seen their Fourier transform), you see that if c_k is the k^{th} Fourier Series co-efficient of a periodic signal x(t), c_{-k} is the k^{th} Fourier series co-efficient of x(-t).

Now lets see how the Fourier Transform of $\overline{x(t)}$ is related to that of x(t).

Starting with $$\mathbf{X(f)} = \int_{-\infty}^{\infty} \mathbf{x(t)\, e^{-j2\pi ft}\, dt}$$

taking conjugates, we get :

$$\overline{X(f)} = \int_{-\infty}^{\infty} \overline{x(t)}\ e^{\,j2\pi f t}\ dt$$

Thus,

$$\overline{X(-f)} = \int_{-\infty}^{\infty} \overline{x(t)}\ e^{\,-j2\pi f t}\ dt$$

And, therefore,

$$\overline{x(t)} \xrightarrow{FT} \overline{X(-f)}$$

Applying this in the context of periodic signals, we see that if c_k is the k^{th} Fourier Series co-efficient of a periodic signal **x(t)**, then $\overline{c_{-k}}$ is the k^{th} Fourier series co-efficient of $\overline{x(t)}$.

Let us look at some simple consequences of these properties:

a) What can we say about the Fourier transform of an **even signal x(t)** (with Fourier transform X(f)) ?

x(-t) has Fourier transform **X(-f)**. As x(t) is real, **x(t) = x(-t)**, implying, **X(f) = X(-f)**.

Thus, the Fourier transform of an even signal is even. Similarly, you can show the Fourier transform of an odd signal is odd.

b) What can we say about the Fourier transform of a **real signal x(t)**, with Fourier transform X(f) ?

If x(t) is real,

$$x(t) = \overline{x(t)} \quad \text{for every } t, \qquad \text{implies } X(f) = \overline{X(-f)} \quad \text{for every } f.$$

Thus the Fourier transform of a real signal is **Conjugate Symmetric**.