# Graphical Interpretation Of Convolution

The graphical interpretation of convolution is very useful in systems analysis as well as communication theory. It permits one to grasp visually the results of many abstract relationships. This is particularly true in communication theory. In linear systems, graphical convolution is very helpful in analysis if $f(t) and\, h(t)$ are known only graphically. To illustrate let us consider $f_1(t) and\, f_2(t)$ as rectangular and triangular pulses as. We shall find the convolution $f_1(t) * f_2(t)$ graphically. By definition,

$$f_1(t) * f_2(t) = \int_{\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau$$

The independent variable in the convolution integral is $\tau$ . The functions $f(\tau) and\, f_2(-\tau)$

Note that $f_2(-\tau)$ is obtained by folding $f_2(\tau)$ about the vertical axis passing through the origin. The term $f_2(t--\tau)$ represents the function $f_2(-\tau)$ shifted by t seconds along the positive $(\tau)$ axis. $f_2(t_1--\tau)$. The value 4 of the convolution integral at $t = t_1$

Is given by the integral in Eq.1.130 evaluated at $t - t_1$ . This area is shown shaded .The value of $f_1(t) * f_2(t)$ at $t = t_1$ is equal to this shaded area and is plotted. We choose different value of t; shift the function $f_2(-\tau)$ accordingly, and find the area under the new products curve. These areas represent the value of the convolution function $f_1(t) * f_2(t)$ at the respective values of t. The plot of the area under the product curve as a function of t represents the desired convolution function $f_1(t) * f_2(t)$.

The graphical mechanism of convolution can be appreciated by visualizing the function $f_2(-\tau)$ as a rigid frame which is being progressed along the $\tau$ axis by $t_1$ seconds. The function represented by this frame is multiplied by $f_1(\tau)$, and the area under the product curve is the value of the convolution function at $t = t_1$. Therefore, to find the value of $f_1(t) * f_2(t)$ at any time, say $t = t_0$, we displace the rigid frame representing $f_2(-\tau)$ by $t_0$ seconds along the $\tau$ axis and multiply this function with $f_1(\tau)$. The area under the product curve is the desired value of $f_1(t) * f_2(t)$ at $t = t_0$. To find the function $f_1(t) * f_2(t)$, we progress the frame successively by different amounts and find the areas of the product curve at various positions. The plot of the area as a function of displacement of the frame represents the required convolution function $f_1(t) * f_2(t)$. To summarize:

1. Fold the function $f_2(\tau)$ about the vertical axis passing through the origin of the $\tau$ axis and obtain the function $f_2(-\tau)$.

2. Consider the folded function as rigid frame and progress it along the $\tau$ axis by an amount, say $t_0$. The rigid frame now represents the function $f_2(t_0 - \tau)$.

3. The product of the function represented by this displaced rigid frame with $f_1(\tau)$ represents the function $f_1(\tau)\, f_2(t_0 - \tau)$, and the area under this curve is given by

$$\int_{-\infty}^{\infty} f_1(\tau) f_2(t_0 - \tau)\, d\tau = \left[f_1(t) * f_2(t)\right]_{t=t_0}.$$

4. Repeat this procedure for different values of t by successively progressing the frame by different amounts and find the values of the convolution function $f_1(t) * f_2(t)$ at those values of t.