Fourier Transform of Periodic signals

We know the Fourier transform of the signal that assumes the value 1 identically is the dirac-delta function.

$$1 \xrightarrow{F.T.} \delta(f)$$

By the property of translation in the frequency domain, we get:

$$e^{j2\pi f_0 t} \xrightarrow{F.T.} \delta(f - f_0)$$

This is the result we will make use of in this section.

Suppose **x(t)** is a periodic signal with the period **T**, which admits a Fourier Series representation. Then,

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t}$$

$$\text{where } c_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) e^{-j2\pi k f_0 t}\, dt$$

Now since the Fourier transformation is **linear**, the above result can be used to obtain the Fourier Transform of the periodic signal x(t):

$$X(f) = \Sigma\ c_k \text{ (Fourier transform of } e^{j(2\pi/T)kt} \text{)}$$

Therefore,

$$\boxed{X(f) = \sum_{-\infty}^{\infty} c_k\ \delta(f - \frac{k}{T})}$$

By putting this transform in inverse Fourier transform equation, one can indeed confirm that one obtains back the Fourier series representation of **x(t)**.

$$\int_{-\infty}^{\infty} X(f)e^{j2\pi ft}df = \int_{-\infty}^{\infty}\sum_{-\infty}^{\infty} c_k\delta(f-\frac{k}{T})\ e^{j2\pi ft}\ df$$
$$= \sum_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_k\delta(f-\frac{k}{T})\ e^{j2\pi ft}\ df$$
$$= \sum_{-\infty}^{\infty} c_k e^{j2\pi\frac{k}{T}t}$$

Thus, the Fourier transform of a periodic signal having the Fourier series coefficients c_k is a train of impulses, occurring at multiples of the fundamental frequency, the strength of the impulse at $\frac{k}{T}$ being c_k.

This looks like:

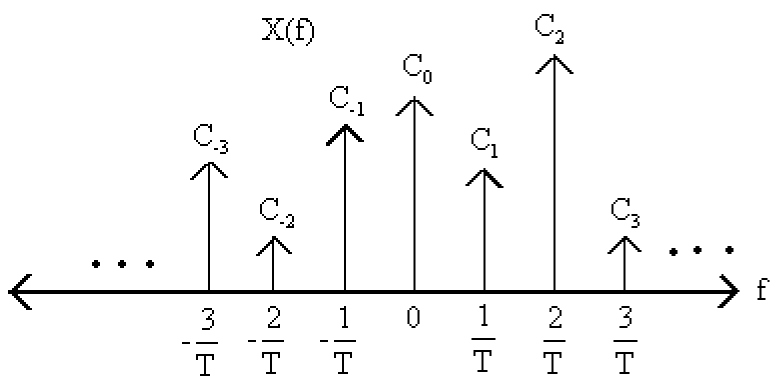

Figure 3.11