Parseval's Theorem in Fourier series

We now obtain the result equivalent to the Parseval's theorem we have already seen in the context of periodic signals.

Let x(t) and y(t) be periodic with a common period T.

$$x(t) = \sum_{-\infty}^{\infty} c_k e^{j 2\pi \frac{k}{T} t} \quad \& \quad h(t) = \sum_{-\infty}^{\infty} d_k e^{j 2\pi \frac{k}{T} t}$$

Applying the Convolution theorem equivalent we have just proved on $x(t)$ and $\overline{y(-t)}$ we get:

$$\frac{1}{T}\int_{(T)} x(\lambda)\,\overline{y(\lambda - t)}\, d\lambda = \sum_{-\infty}^{\infty} c_k\, \overline{d_k}\, e^{j 2\pi \frac{k}{T} t}$$

Put t = 0, to get: $\boxed{\frac{1}{T}\int_{(T)} x(\lambda)\,\overline{y(\lambda)}\, d\lambda = \sum_{-\infty}^{\infty} c_k\, \overline{d_k}}$

Compare this equation with the Parseval's theorem we had proved earlier.

If we take x = y, then T becomes the fundamental period of x and:

$$\boxed{\frac{1}{T}\int_{(T)} |x(t)|^2\, dt = \sum_{-\infty}^{+\infty} |c_k|^2}$$

Note the left-hand side of the above equation is the power of x(t).

Note also that the periodic convolution of $x(t)$ and $\overline{x(-t)}$ yields a periodic signal with Fourier coefficients that are the modulus square of the coefficients of x(t).

Another important result

If, $y(t) = x(t) \otimes h(t)$

Then $\frac{1}{T}\int_{(T)} |y(t)|^2\, dt$ represents the power of y(t), where T is a period common to x(t) and h(t).

If , $x(t) = \sum_{-\infty}^{\infty} c_k e^{j2\pi\frac{k}{T}t}$ & $h(t) = \sum_{-\infty}^{\infty} d_k e^{j2\pi\frac{k}{T}t}$

$$y = x \otimes h = \sum_{k=-\infty}^{\infty} c_k d_k e^{j2\pi\frac{k}{T}t}$$

Applying the Parseval's theorem to y,

$$\frac{1}{T}\int_{(T)} |y(t)|^2 dt = \sum_{-\infty}^{\infty} |c_k|^2 |d_k|^2$$