# Problems regarding Fourier Series

**1.**

Consider the periodic square wave $x(t)$ shown in Fig.

($a$) Determine the complex exponential Fourier series of $x(t)$.

($b$) Determine the trigonometric Fourier series of $x(t)$.

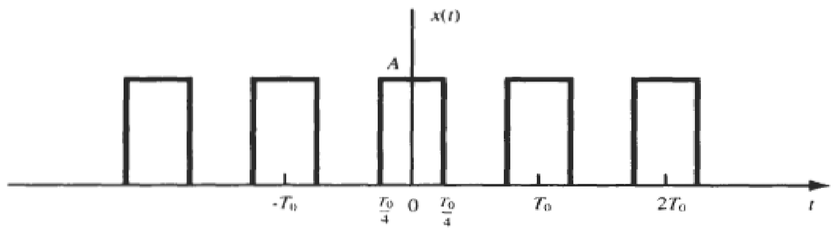



**Fig.**

($a$) Let

$$x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

Using Eq. (*5.102b*), we have

$$c_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} A\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{A}{-jk\omega_0 T_0}\left(e^{-jk\omega_0 T_0/4} - e^{jk\omega_0 T_0/4}\right)$$

$$= \frac{A}{-jk2\pi}\left(e^{-jk\pi/2} - e^{jk\pi/2}\right) = \frac{A}{k\pi}\sin\left(\frac{k\pi}{2}\right)$$

Thus,

$$c_k = 0 \qquad k = 2m \neq 0$$

$$c_k = (-1)^m \frac{A}{k\pi} \qquad k = 2m+1$$

$$c_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt = \frac{1}{T_0}\int_0^{T_0/2} A\, dt = \frac{A}{2}$$

Hence,

$$c_0 = \frac{A}{2} \qquad c_{2m} = 0,\ m \neq 0 \qquad c_{2m+1} = (-1)^m \frac{A}{(2m+1)\pi}$$

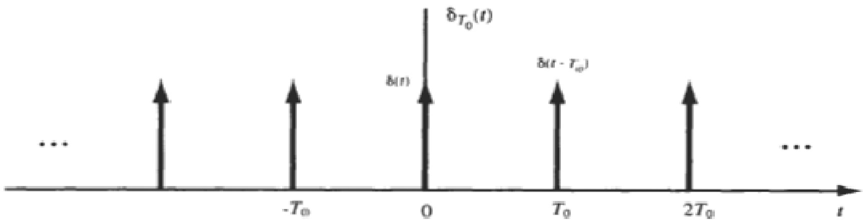
and we obtain

$$x(t) = \frac{A}{2} + \frac{A}{\pi}\sum_{m=-\infty}^{\infty} \frac{(-1)^m}{2m+1} e^{j(2m+1)\omega_0 t}$$

2.

Consider the periodic impulse train $\delta_{T_0}(t)$ shown in Fig. 5-12 and defined by

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$




(*a*) Determine the complex exponential Fourier series of $\delta_{T_0}(t)$.
(*b*) Determine the trigonometric Fourier series of $\delta_{T_0}(t)$.

(*a*) Let

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

Since $\delta(t)$ is involved, we use Eq. to determine the Fourier coefficients and we obtain

$$c_k = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}$$

Hence, we get

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0) = \frac{1}{T_0} \sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} \qquad \omega_0 = \frac{2\pi}{T_0}$$

(*b*) Let

$$\delta_{T_0}(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t) \qquad \omega_0 = \frac{2\pi}{T_0}$$

Since $\delta_{T_0}(t)$ is even, $b_k = 0$, and by Eq. $a_k$ are given by

$$a_k = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cos k\omega_0 t\, dt = \frac{2}{T_0}$$

Thus, we get

$$\delta_{T_0}(t) = \frac{1}{T_0} + \frac{2}{T_0} \sum_{k=1}^{\infty} \cos k\omega_0 t \qquad \omega_0 = \frac{2\pi}{T_0}$$