The Impulse Response Of A Linear System

The transform of a unit impulse is unity. Hence the response of a system to a unit impulse function will be given by $L^{-1}[H(s)]$ where $H(s)$ is the transfer function of the system. We shall designate the unit impulse response of a system by $H(t)$. Thus

$$h(t) = L^{-1}[H(s)]$$

and

$$H(s) = L\,[h(t)]$$

This is a significant result. *The unit impulse response of a system is the inverse Laplace transform of its transfer function.*

For a lumped-linear system, the transfer function H(s) is a rational algebraic fraction.

$$H(s) = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

Expanding H(s) into partial fractions, we get

$$H(s) = \frac{a_1}{s-p_1} + \frac{a_2}{s-p_2} + \cdots + \frac{a_n}{s-p_n}$$

and

$$h(t) = L^{-1}[H(s)] = a_1 e^{p_1 t} + a_2 e^{p_2 t} + a_n e^{p_n t}$$

The terms p_1, p_2 ,…..p_n are the poles of the transfer function and are known as the *natural frequencies* of the system. We therefore conclude that *the impulse response of a system consists of a linear combination of signals of the natural frequencies of the system.*

In this text the terms source-free response, transient response, and natural response will be used interchangeably. Similarly, the terms response due to source, steady-state response, and forced response will mean one and the same thing. To summarize; the two components of the response are referred to as follows.

(a) Source-free response and the response due to source,

(b) Natural response and the forced response,

(c) Transient response and the steady-state response.

Sometimes these components are also referred to by complimentary function and the particular integral. This choice, however, is not very popular among engineers.

Ex:

A voltage 100 cos 5t is applied to the network with transfer function given below. Find the output voltage $\upsilon_0(s)$.

$$H(s) = \frac{4}{3s^2 + 11s + 4}$$

Solution:

The transfer function H(s) relating the output voltage $\upsilon_0(t)$ to the input voltage across terminals aa' can be found in the usual way. The transfer function H(s) is found to be

$$H(s) = \frac{4}{3s^2 + 11s + 4}$$

$$= \frac{1.33}{(s+0.41)+(s+3.26)}$$

The Laplace transform of the driving function is given by

$$F(s) \neq L[100\cos(5t)u(t)] = \frac{100s}{s^2 + 25}$$

Hence, if $V_0(s)$ is the Laplace transform of the output voltage $\upsilon_0(t)$, we have

$$V_0(s) = \frac{133s}{(s+0.41)(s+3.26)(s^2+25)}$$