Properties of Autocorrelation

We have, $\phi_{xy}(t) = \phi_{yx}(-t)$.*If x=y, we get* , $\phi_{xx}(t) = \phi_{xx}(-t)$ **.** This means that autocorrelation function is an even function of time. For an even signal x(-t)=x(-t). This means that autocorrelation of an even function x(t) also equals the convolution of x(t) with x(t) itself.

Autocorrelation function is maximum at the origin. That is,

$$\left|\phi_{xx}(t)\right| \leq \phi_{xx}(0)$$

The result stated it can be proved as follows:

We know that,

$$\left[x(\tau) \pm x(\tau - t)\right]^2 \geq 0$$

Integrating with respect to τ , we get

$$\int_{-\infty}^{\infty} \left[x(\tau) \pm x(\tau - t)\right]^2 d\tau \geq 0$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} x^2(\tau)\, d\tau + \int_{-\infty}^{\infty} x^2(\tau - t)\, dt \pm 2 \int_{-\infty}^{\infty} x(\tau)x(\tau - t)\, dt \geq 0$$

$$\Rightarrow \quad \phi_{xx}(0) + \phi_{xx}(0) \pm 2\phi_{xx}(t) \geq 0$$

$$\Rightarrow \quad \phi_{xx}(0) \geq \pm \phi_{xx}(t)$$

$$\Rightarrow \quad \left|\phi_{xx}(t)\right| \leq \phi_{xx}(0)$$