III. Deadline Nov 23, 12:30pm. Only via email to nata.ivanova@ualberta.ca.

5. Advection. [30pt]

Write a code to solve the 1-d linear advection equation using

a) the upwind discretization

$$\frac{a_i^{n+1} - a_i^n}{\Delta t} = -u \frac{a_i^n - a_{i-1}^n}{\Delta x} \tag{2}$$

b) FTSC method

$$\frac{a_i^{n+1} - a_i^n}{\Delta t} = -u \frac{a_{i+1}^n - a_{i-1}^n}{2\Delta x} \tag{3}$$

c) implicit-in-time discretization

$$\frac{a_i^{n+1} - a_i^n}{\Delta t} = -u \frac{a_i^{n+1} - a_{i-1}^{n+1}}{\Delta x} \tag{4}$$

For all the cases, consider the domain $[0, 1]$ with $u = 1$, and periodic boundary conditions. For initial conditions, use a top-hat:

$$a = \begin{cases} 0 & if \quad x < 0.4 \\ 1 & if \quad 0.4 \leq x \leq 0.6 \\ 0 & if \quad 0.6 < x \end{cases} \tag{5}$$

Note that for periodic BCs, the foremost left and the foremost right points are identical, so you could do the update without the need for ghost points.

Define one period of time as $T = 1/u$. Run you program for T=1 with CFL numbers=0.1, 0.33(3), 0.7, and using 3 different resolutions (so that $\Delta x = 0.05, 0.033(3), 0.01$). Discuss the outcomes while considering the solutions at T=0.1 and 1.

Note that for all homeworks that request you write a code, the languages include C, C++, Fortran or python. Your code must be self-contained. No use of packages like Mathematics, Mathlab etc is permitted. No use of Numerical recepies or similar packages that can be called from C, etc is permitted. Your code, plots and analyzis are to be submitted via email.