Adaptive Neuro Fuzzy Inference System (ANFIS)

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Outline

- Fuzzy logic
- Fuzzy membership
- Fuzzy inference system
- Neural fuzzy inference system
  - Architecture
  - Learning model
  - Implementation

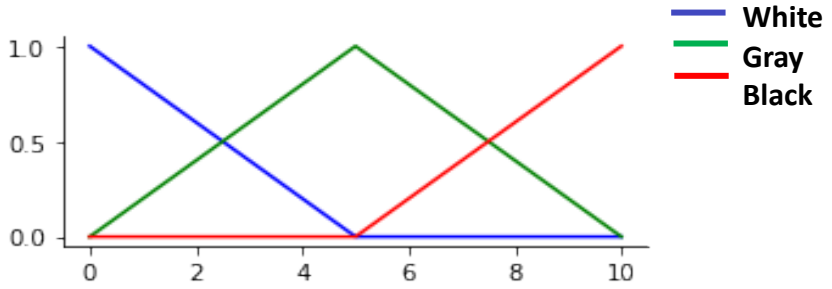
- Activities in this session will be: Building a FIS and ANFIS

Fuzzy Logic

- Fuzzy Logic teaches computers to make human-like decisions in an uncertain world.

- “The closer one looks at a real-problem, the fuzzier becomes the solution” - Zadeh, 1973

- Can deal with the decision-making problems involving uncertainty and vagueness of real world applications.



Terminology

- **Fuzzy Logic** is a methodology predicated on the idea that the "truthiness" of something can be expressed over a continuum.
- This is to say that something isn't *true* or *false* but instead *partially true* or *partially false*.

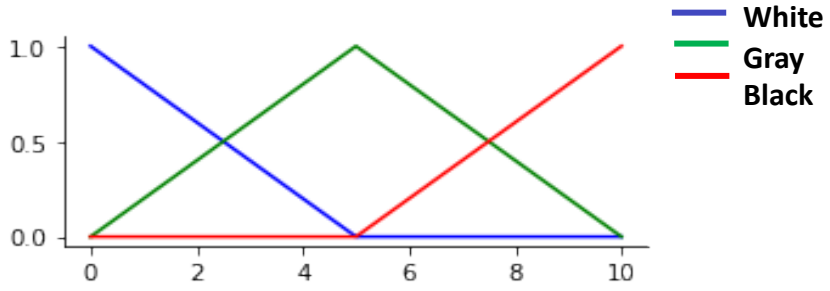
Terminology

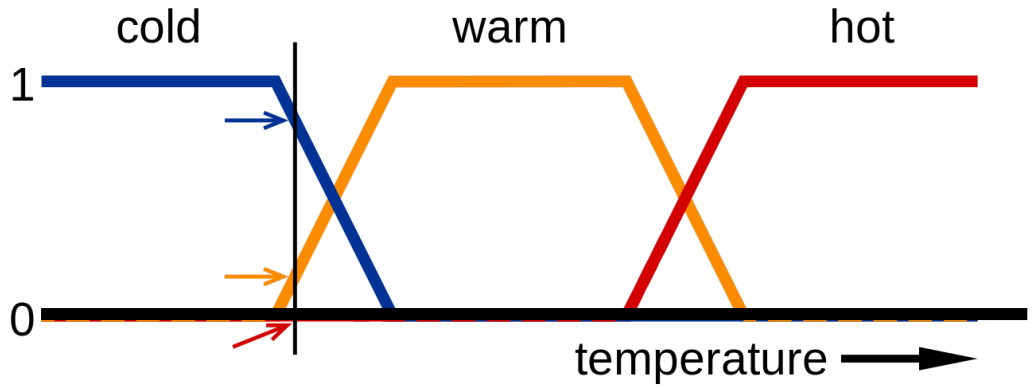
- A **fuzzy variable** has a **crisp value** which takes on some number over a pre-defined domain (in fuzzy logic terms, called a **universe**). The crisp value is how we think of the variable using normal mathematics.

- A fuzzy variable also has several **terms** that are used to describe the variable. These terms are usually adjectives like “poor,” “mediocre,” and “good.”

Terminology

- Each term has a **membership function** that defines how a crisp value maps to the term on a scale of 0 to 1. In essence, it describes “how good” something is.





Terminology

- A fuzzy system consists of a set of fuzzy **IF-THEN rules** that describe the input-output mapping relationship of the networks.

- These rules are simply mappings that describe how one or more fuzzy variables relates to another. These are expressed in terms of an IF-THEN statement; the IF part is called the **antecedent** and the THEN part is the **consequent**.

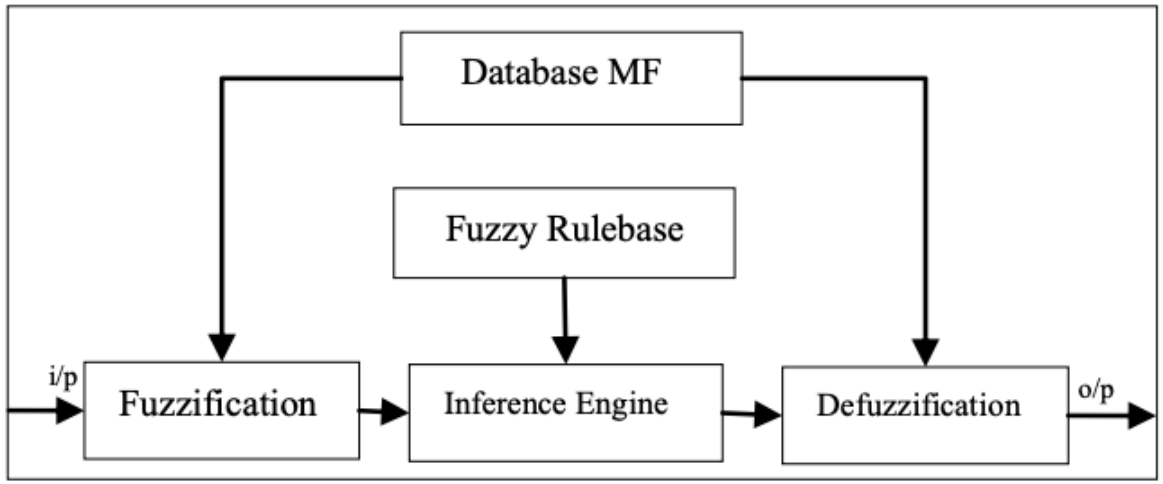
Fuzzy Inference

- **Fuzzy inference** is the process of formulating input/output mappings using fuzzy logic.

- Knowledge is encoded as using a set of explicit **linguistic rules**, which can be easily understood by people without technical expertise.

- Fuzzy systems implement nonlinear systems using linguistic variables when adequate knowledge about the system is available.

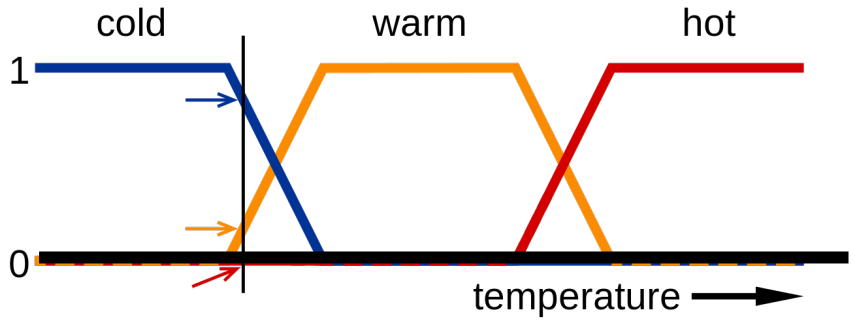
Framework of Fuzzy Logic System



- **Fuzzification**, which translates crisp (real-valued) inputs into fuzzy values;
- **Rule base reasoning**, an inference engine that applies a fuzzy reasoning mechanism to obtain a fuzzy output using rules;
- **Defuzzification**, translates this latter output into a crisp value, as shown in the figure.

Fuzzification

- The purpose of fuzzification is to map system input values from 0 to 1 via defined input membership functions.



- **Input**: A crisp value restricted to to the universe.
- **Output**: a fuzzy degree of membership in the qualifying linguistic set

μ_A: membership function
χ: universe set

$$\mu: \chi \rightarrow [0, 1]$$

Rule-based Reasoning

- In rule-based reasoning, the fuzzy input values membership values are mapped to classify fuzzy output through a table containing if-then rules.

- Rules are expressed as a logic implication **p → q** where p is called the **antecedent** of the rule and q is called the **consequence** of the rule.

Defuzzification

- Defuzzification is a process which produces single system output (crisp) values by using a defuzzification formula and fuzzy output membership outputs.

Tipping Example

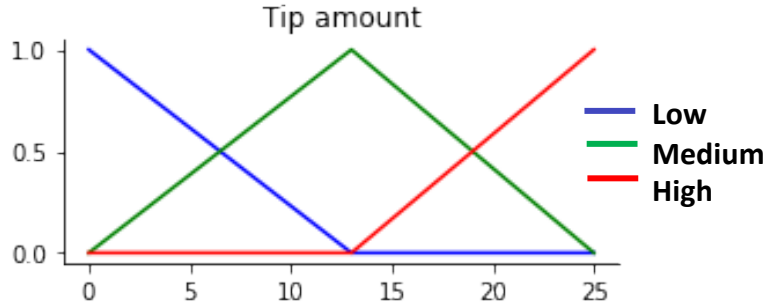
- **Antecednets (Inputs)**
  - **Service**
    - Universe (ie, crisp value range): How good was the service of the wait staff, on a scale of 0 to 10?
    - Fuzzy set (ie, fuzzy value range): poor, acceptable, amazing.
  - **Food quality**
    - Universe: How tasty was the food, on a scale of 0 to 10?
    - Fuzzy set: bad, decent, great
- **Consequents (Outputs)**
  - **Tip**
    - Universe: How much should we tip, on a scale of 0% to 25%
    - Fuzzy set: low, medium, high
- **Rules**
  - IF the service was poor and the food quality was poor THEN the tip will be low.
  - IF the service was average, THEN the tip will be medium.
  - IF the service was good and the food quality was good, THEN the tip will be high.

Source: https://pythonhosted.org/scikit-fuzzy/auto_examples/plot_tipping_problem_newapi.html

Usage

6.5

9.8



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Let's see the code example

Rules

IF the service was poor and the food quality was poor THEN the tip will be low.
IF the service was average, THEN the tip will be medium.
IF the service was good and the food quality was good, THEN the tip will be high.

Adaptive Neuro-Fuzzy Inference Systems (ANFIS)

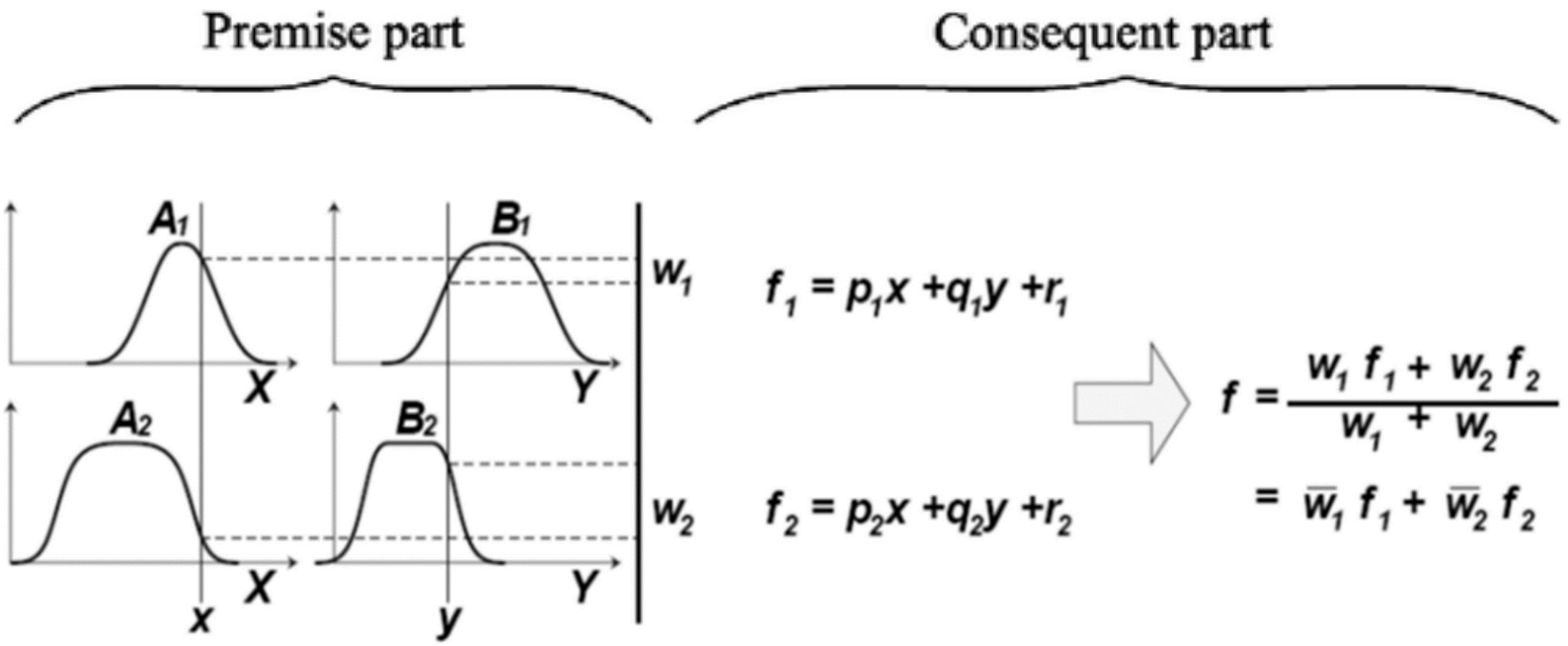
Adaptive Neuro-Fuzzy Inference Systems (ANFIS)

- ANFIS is a kind of **artificial neural network** that is based on Takagi–Sugeno **fuzzy inference system**.

- Network nodes in different layers have different structures.

ANFIS

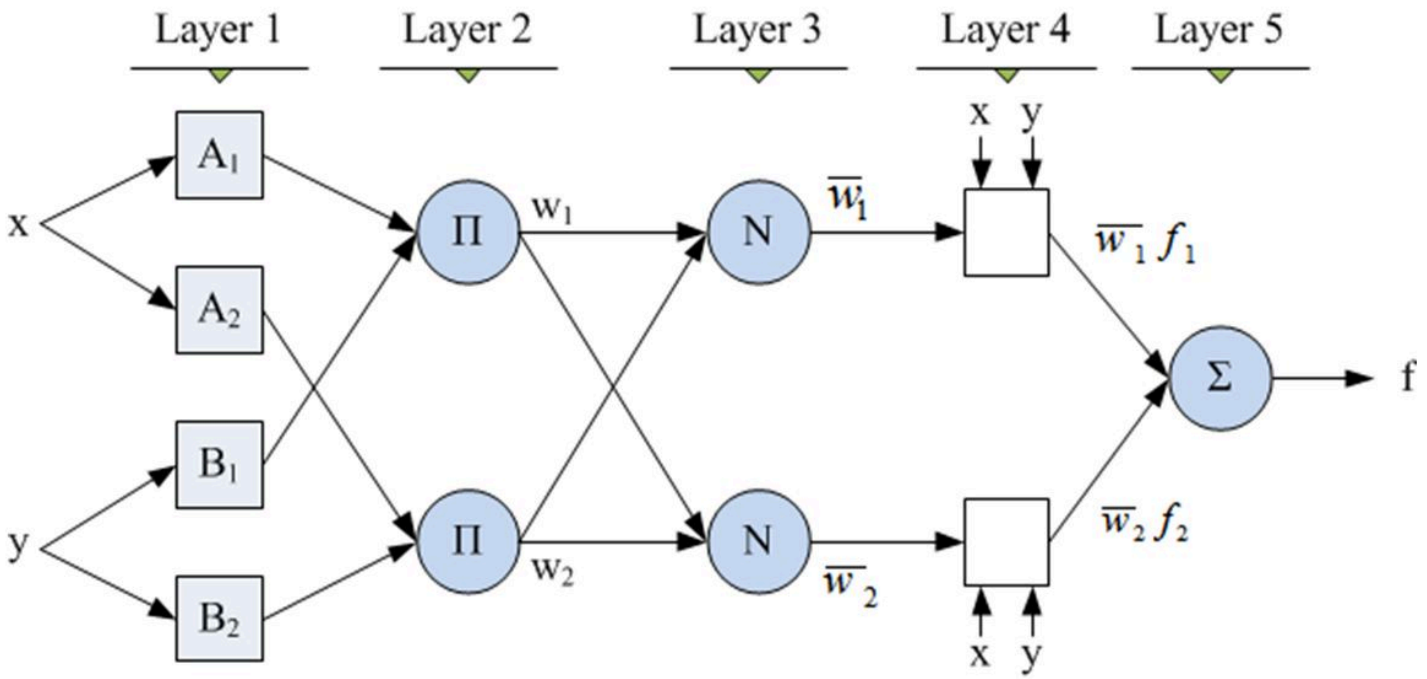
- Consider a first-order Sugeno fuzzy model, with two inputs, x and y, and one output, z.
- Rule set
  - Rule 1: If x is A_1 and y is B_1 , then $f_1 = p_1x + q_1y + r_1$
  - Rule 2: If x is A_2 and y is B_2 , then $f_2 = p_2x + q_2y + r_2$



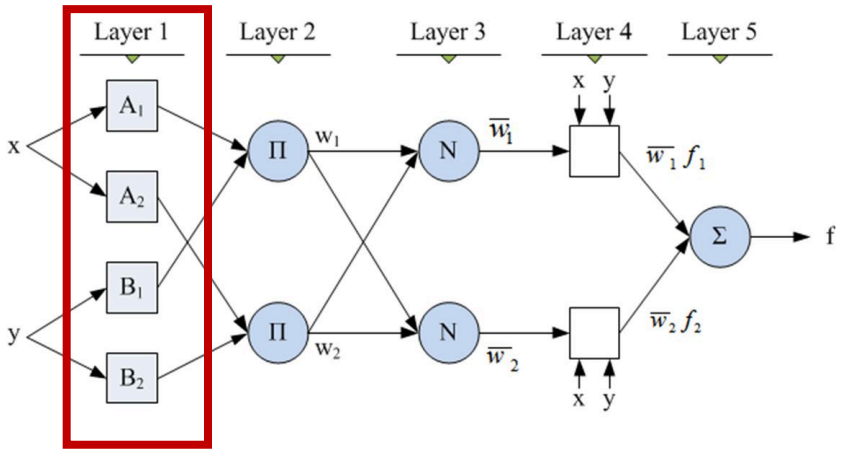
ANFIS Architecture

- 5-Layer ANFIS architecture:
  - Fuzzy layer
  - Product layer
  - Normalized layer
  - De-fuzzy layer
  - Total output layer

- Square: Fixed node
- Circle: Adaptive node



ANFIS Layers – L1

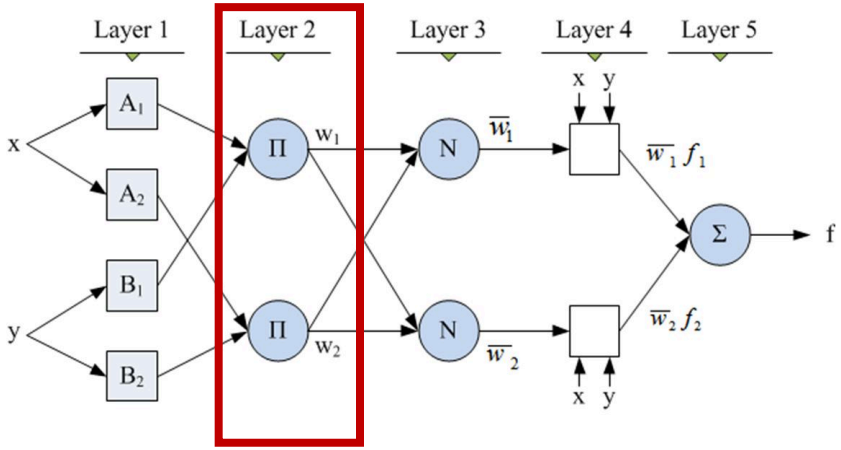


- **Layer 1 – Fuzzy Layer**
- **Layer 1**: every node is an adaptive node with node function:

$$O_{1,i} = \mu_i\,(x_i)$$

- Parameters in this layer are called premise parameters.

ANFIS Layers – L2

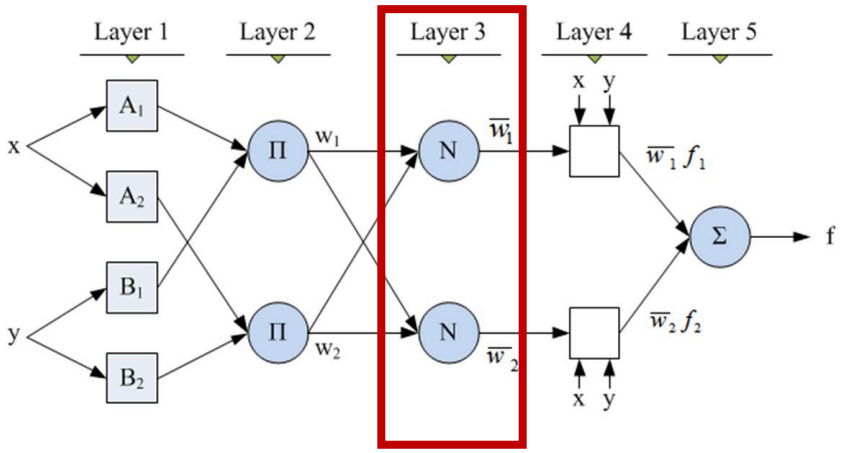


- **Layer 2: Product Layer**
- **Layer 2**: every node is **fixed** whose output (representing **firing strength**) is the product of the inputs:

$$O_{2,i} = w_i = \pi_j \mu_i$$

- Any operator that performs fuzzy AND.

ANFIS Layers – L3

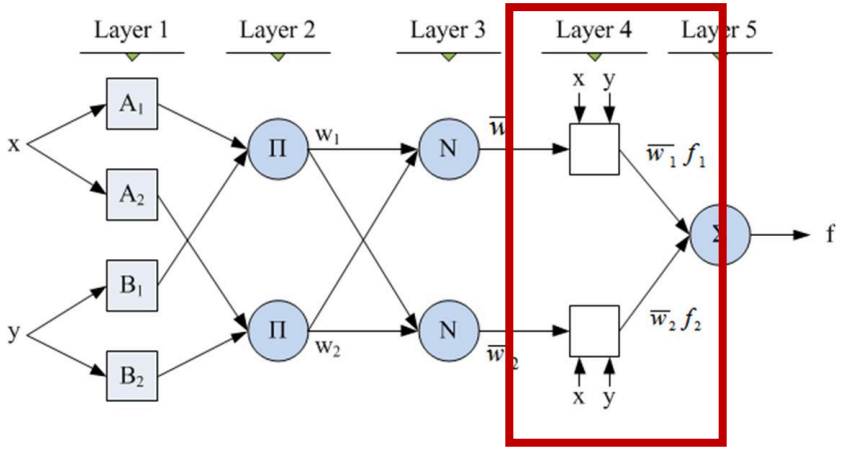


- **Layer 3: Normalized Layer**
- **Layer 3**: every node is **fixed** (normalization):

$$O_{3,i} = \overline{w_i} = \frac{w_i}{\sum_j w_j}$$

- Indicated normalization to the firing strength from previous layer.
- Output is called **normalized firing strengths**.

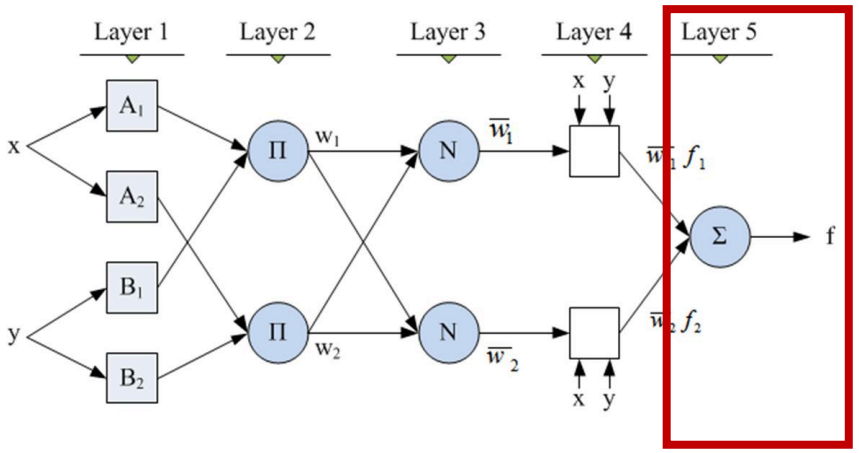
ANFIS Layers – L4



- **Layer 4: De-fuzzy Layer**
- **Layer 4**: every node is adaptive (**consequent parameters**) :

$$O_{4,i} = O_{3,i} f_i = \overline{w_i}(p_0 + p_1 x_1 + \cdots + p_n x_n)$$

ANFIS Layers – L5



- **Layer 5: Total Output Layer**
- **Layer 5**: **single fixed node**, sums up inputs:

$$O_{5,i} = \sum_i \overline{w_i} f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}$$

- Adaptive network is functionally equivalent to a Sugeno fuzzy model!

Learning Algorithm of ANFIS

the membership function parameters that best allow the associated fuzzy inference system to track the given input/output data.

- The parameters associated with the membership function change through the learning process.
- **Goal**: tune all the modifiable parameters, to formulate the ANFIS output match the training data.

	Forward Pass	Backward Pass
Premise Parameters	Fixed	Gradient descent
Consequent Parameters	Least square	Fixed
Signals	Node output	Error signal

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