Adaptation of the Engelund-Hansen formula to Nestos River, Greece

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ABSTRACT

Most well-known sediment transport formulas have been derived using specific field or laboratory data, and this hinders their applicability worldwide. Indeed, the success of some of the top-used sediment transport formulas, in certain applications, is comparable only with their failure in others. The adaptation of established formulas to specific data of rivers and streams is a viable solution to this problem, as shown by the authors with respect to the Meyer- Peter and Müller formula (Sidiropoulos et al., 2021). In this study, we modify the widely used Engelund and Hansen (E-H, 1967) sediment transport formula and calibrate it on the basis of measured data from Nestos River in north-eastern Greece.

1. Study area

The Greek part of Nestos River basin (Fig. 1, north-eastern Greece) drains an area of approximately 2000 km^2 and encompasses the largest cobblestone dam of Greece and the Balkans (Thisavros hydropower dam) with capacity $90 \cdot 10^6$ m^3. The greatest part (82%) of the basin is mountainous and is covered mainly by forested and bushy areas. Agricultural and animal farming activities take place near Paranesti, Stavroupoli and Paschalia, the main villages in the area, but in general no intense human activities take place. The mean slope of Nestos River in the basin is 0.35% (Boskidis et al., 2011; Kaffas et al., 2018).

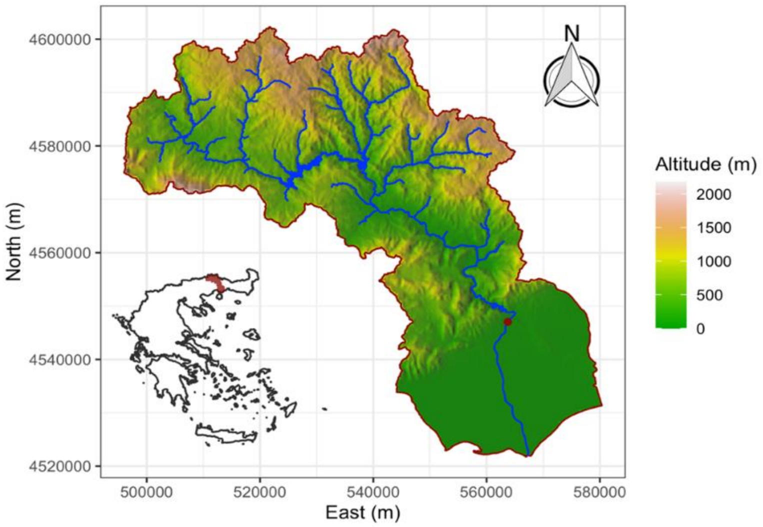

Fig. 1. *Nestos River basin (the red-filled circle marks the location of total load measurements) (Sidiropoulos et al., 2021).*

2. Data–Methods–Results

Measurements were conducted at a location between the outlet of Nestos River basin (Toxotes) and the river delta by the Section of Hydraulic Engineering, Department of Civil Engineering, Democritus University of Thrace. A total of 109 datasets of measured streamflow velocity, bed load and suspended load transport rate, median particle diameter, and cross-sectional geometry were used. The total sediment discharge was determined as a sum of the measured bed load and suspended load transport rates.

2.1. Calculation of total sediment discharge and optimization of E-H formula

The total sediment discharge was calculated, by means of the Engelund and Hansen (1967) formulas by applying two different forms, the original equations and an optimized one. The measured data consist of: (a) the geometric characteristics of the stream (width, slope); (b) the median diameter of sediment particles; (c) the hydraulic characteristics of the stream (depth, mean flow velocity). In the E-H formulas, the total sediment transport is calculated in terms of unitless values using Eq. (1):

$$f_{EH} \cdot \Phi = \alpha \cdot \theta^{\beta} \tag{1}$$

$$\Phi = \frac{m_F}{\rho_F\sqrt{\rho' \cdot g \cdot d^3}} \tag{2}$$

where f_{EH} – dimensionless friction factor, Φ – dimensionless total sediment discharge, θ dimensionless bed shear stress, $\alpha = 0.1$ and $\beta = 5/2$, m_F – total sediment discharge [kg s^{-1} m^{-1}], ρ_F – sediment density [kg m^{-3}], ρ' – relative submerged specific gravity of sediment (1.65), g – acceleration due to gravity [m s^{-2}], d – median particle diameter [m].

Parameters α and β in Eq. (1) were obtained by Engelund and Hansen (1967) using a set of data coming from flume experiments.

The dimensionless bed shear stress, θ, is derived as follows:

$$\theta = \frac{\tau_o}{\rho_W \cdot g \cdot \rho' \cdot d} = \frac{h \cdot I}{\rho' \cdot d}, \text{ (as } \tau_o \approx \rho_W \cdot g \cdot h \cdot I) \tag{3}$$

where τ_o – total bed shear stress [N m^{-2}], h – mean flow depth [m], I – bed slope (for uniform flow). In Eq. (3), the flow depth, h, replaces the hydraulic radius, when the flow width of the considered cross-section is much greater than h.

Finally, the dimensionless friction factor is computed using Eq. (4):

$$f_{EH} = \frac{2 \cdot I}{Fr^2} = \frac{2 \cdot g \cdot h \cdot I}{u_m^2}, \text{ (as } Fr^2 = \frac{u_m^2}{g \cdot h} \text{ for rectangular cross-section)} \tag{4}$$

where Fr – Froude number, u_m – mean flow velocity [m s^{-1}].

In this work, α and β from Eq. (1) were treated as parameters of adjustment through the minimization of the root mean squared error, between computed and measured values of total sediment discharge, by means of the L-BFGS algorithm (Byrd et al., 1995):

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\widehat{m_{F,\iota}} - m_{F,i}\right)^2} \tag{5}$$

where n – number of data points, $\widehat{m_{F,\iota}}$ – estimated total sediment discharge by means of the E-H formulas, $m_{F,i}$ – measured total sediment discharge, for $i = 1,2, \dots, n$.

The results indicate that the calculated values from the optimized equation were more accurate by 32%, in terms of the RMSE error metric [0.58 kg s^{-1} m^{-1} to 0.40 kg s^{-1} m^{-1}]. The corresponding discrepancy ratios, using as margins the quadruple and the one quarter of the corresponding measured total sediment discharge, are 53% and 67%. The optimal parameters were found to be $\alpha = 0.03$ and $\beta = 1.74$. A source of error lies in the fact that the E-H original formulas were based on experimental data from flumes and not on data from natural streams, as are the data used in this study, hence the need for adjusting the formula. The results are deemed satisfactory and highlight the need for further research regarding optimization techniques for the E-H formula.

References

Boskidis I, Gikas GD, Sylaios G, Tsihrintzis VA (2011) Water quantity and quality assessment of lower Nestos river, Greece, Journal of Environmental Science and Health, Part A, 46(10), 1050-1067
Byrd RH, Lu P, Nocedal J, Zhu C (1995) A limited memory algorithm for bound constrained optimization, SIAM Journal on Scientific Computing, 16, 1190-1208
Engelund F, Hansen E (1967) A monograph on sediment transport in alluvial streams, Technical University of Denmark, Copenhagen
Kaffas K, Hrissanthou V, Sevastas S (2018) Modeling hydromorphological processes in a mountainous basin using a composite mathematical model and ArcSWAT, Catena, 162, 108-129
Sidiropoulos E, Vantas K, Hrissanthou V, Papalaskaris T (2021) Extending the Applicability of the Meyer-Peter and Müller Bed Load Transport Formula, Water, 13(20), 2817