Corporate Finance

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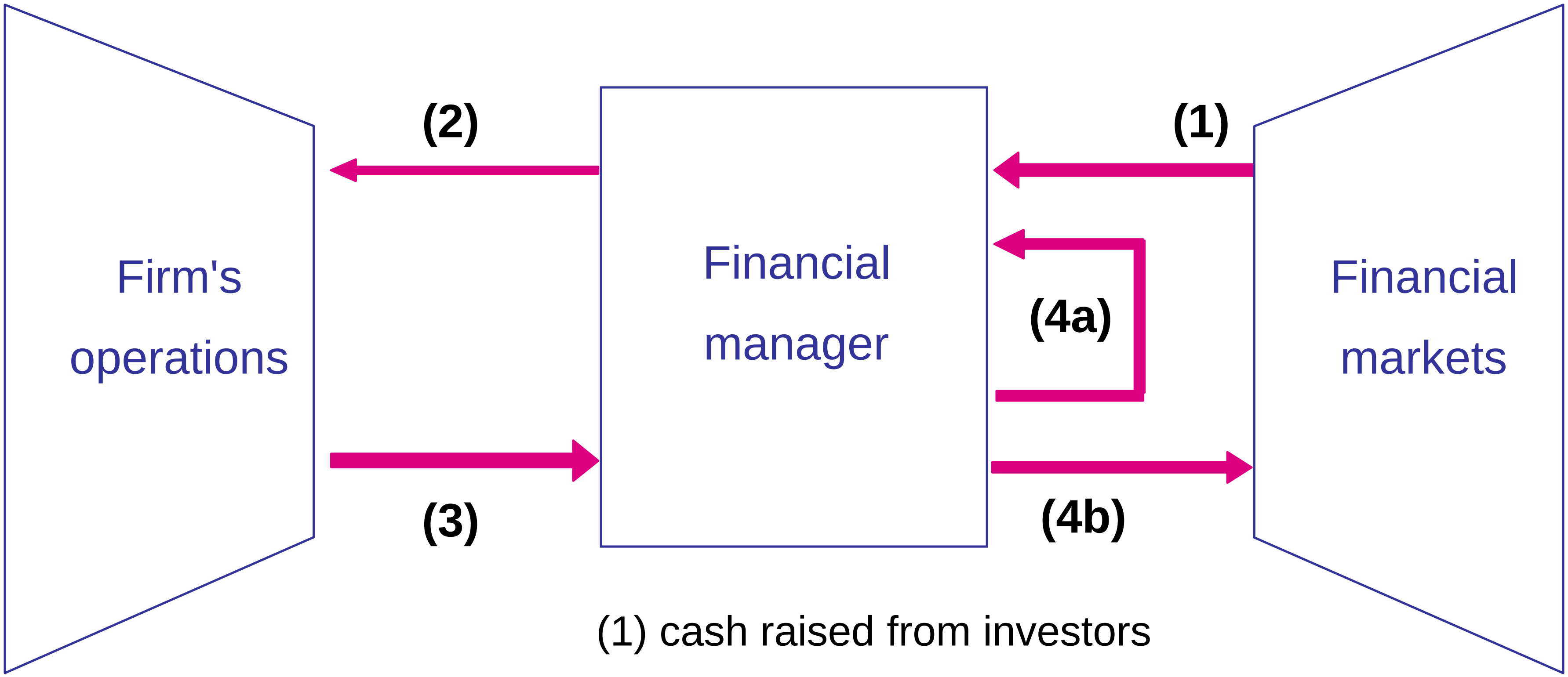

(1) cash raised from investors

(2) cash invested in firm

(3) cash generated by operations

(4a) cash reinvested

(4b) cash returned to investors

Main concept:
wealth creation

How to create value?

How to create value?

DEBIT	Balance sheet EUC CF at t=0		CREDIT
Cash	3.500	Equity	3.500
			N = 350
			P = 10

N = number of shares
P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on 'market values'?

How to create value?

DEBIT	Balance sheet EUC CF at t=1		CREDIT
Cash	0	Equity	3.500
Project	?		

N = 350

P = 10

N = number of shares

P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on 'market values'?

How to create value?

DEBIT		Balance sheet EUC CF at t=1	CREDIT
Cash	0	Equity	?
Project	?		
			N = 350
			P = ?

N = number of shares
P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on ‘market values’?

How to create value?

DEBIT	Balance sheet EUC CF at t=1		CREDIT
Cash	0	Equity	?
Project	4.375		

N = 350

P = ?

N = number of shares

P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on 'market values'?

How to create value?

DEBIT	Balance sheet EUC CF at t=1		CREDIT
Cash	0	Equity	4.375
Project	4.375		

N = 350

P = ?

N = number of shares

P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on ‘market values’?

How to create value?

DEBIT	Balance sheet EUC CF at t=1		CREDIT
Cash	0	Equity	4.375
Project	4.375		

N = 350

$\boxed{P = 12{,}50}$

N = number of shares
P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on 'market values'?

The wealth creation is equal to 875 (4.375 - 3.500).
The wealth creation per share is 875 / 350 = 2,50 P increases by 25%.

DEBIT	Balance sheet EUC CF at t=1		CREDIT
Cash	0	Equity	4.375
Project	4.375		
			N = 350
			$\boxed{P = 12,50}$

N = number of shares
P = price per share

Assume we invest the 3.500 in a very promising project.
What happens with the balance sheet based on ‘market values’?

The wealth creation is equal to 875 (4.375 - 3.500).
The wealth creation per share is 875 / 350 = 2,50 P increases by 25%.

DEBIT — Balance sheet EUC CF at t=1 — CREDIT

Debit		Credit	
Cash	0	Equity	4.375
Project	4.375		

N = 350

$\boxed{P = 12,50}$

WE CREATE VALUE IF WE INVEST IN PROJECTS THAT ARE WORTH MORE THAN THEY COST.

N = number of shares
P = price per share

DEBIT	Balance sheet EUC CF at t=0		CREDIT
Cash	3.500	Equity	3.500
Project	0		

N = 350

P = 10

N = number of shares

P = price per share

Assume you invest the 3.500 in a project that is worth 2.800 instead of 4.375?
What happens with the balance sheet based on 'market values'?

The destruction of wealth is equal to 700 (2.800 - 3.500) or 700 / 350 = 2 per share.

P decreases from 10 to 8 (-20%)!

DEBIT	Balance sheet EUC CF at t=1		CREDIT
Cash	0	Equity	2.800
Project	2.800		

N = 350

$\boxed{P = 8}$

WE DESTROY VALUE IF WE INVEST IN PROJECTS THAT ARE WORTH LESS THAN THEY COST.

N = number of shares

P = price per share

Assume you invest the 3.500 in a project that is worth 2.800 instead of 4.375?
What happens with the balance sheet based on 'market values'?

Fundamental topics in FINANCE:

a. **The capital budgeting decision**
(het investeringsvraagstuk)

b. **The financing decision / the capital structure decision**
(financieringsvraagstuk / vermogensstructuurvraagstuk)

c. **The relation between the capital budgeting and the financing decision**

d. **The financial investment decision**
(het beleggingsvraagstuk)

e. **Asset pricing**
(prijsvormingsvraagstuk)

Fundamental topics in FINANCE:

a. **The capital budgeting decision**
(het investeringsvraagstuk)

Another example of an investment decision …

The acquisition of firm Target by firm Bidder.

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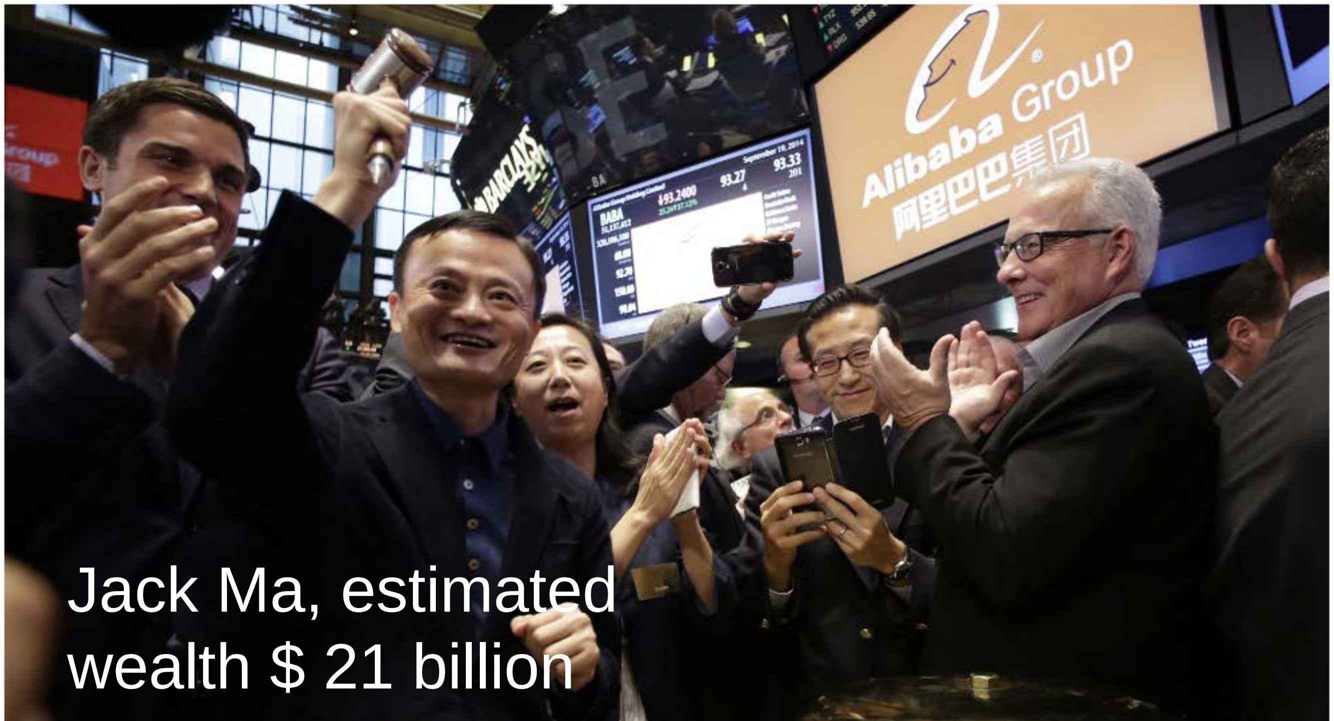
Asset pricing

Forbes / Tech

SEP 22, 2014 @ 11:51 AM 73,387 VIEWS

Alibaba Claims Title For Largest Global IPO Ever With Extra Share Sales



Alibaba Chairman Jack Ma celebrates his company's IPO at the New York Stock Exchange on Friday. (Photo: Mark Lennihan/AP)

September 22, 2014

http://www.forbes.com/sites/ryanmac/2014/09/22/alibaba-claims-title-for-largest-global-ipo-ever-with-extra-share-sales/

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Structure

Week	Topic	Chapters	Homework (click on the homework assignment to view the structure of the homework review session)	Assessment
Week 1	Hirshleifer Model and concepts in Finance	Berk & DeMarzo: 1 + 2 + 3 + 4 + 5 Finance, Text- and Workbook: 1	*Chapter 1: 1-5.* *Chapter 2: 1-9, 11-14.*	
Week 2	Capital budgeting	Berk & DeMarzo: 7 + 8	*Chapter 3: 1-5.* *Chapter 4: 1-2.*	
Week 3	Bonds and Shares	Berk & DeMarzo: 6 + 9	*Chapter 5: 1-6.* *Chapter 6: 1-2.*	
Week 4	MV-analysis and portfolio theory	Berk & DeMarzo: 10 + 11 Finance, Text- and Workbook: 7 + 8	*Chapter 7: 1-6.* *Chapter 8: 1-8.*	
Week 5	Capital Asset Pricing Model (CAPM) and cost of capital	Berk & DeMarzo: 11 + 12	*Chapter 9: 1-4.* *Chapter 13: 1-5.*	
Week 6	Efficient market hypothesis (EMH) and Mergers & Acquisitions (M&A)	Berk & DeMarzo: 9.5 + 28	*Chapter 12: 1-7.* *Chapter 14: 1-4.*	
Week 7	Discount rates for international projects and review of all topics	Berk & DeMarzo: 1-12 + 28 Finance, Text- and Workbook: 1-9 + 12-15	Review questions	Assignment
Week 8		Exam		

Programme today

Hirshleifer model

- Hirshleifer model without financial market and without real market
- Hirshleifer model with financial market but without real market
- Hirshleifer model with financial market and real market
- Fisher separation theorem

Concepts

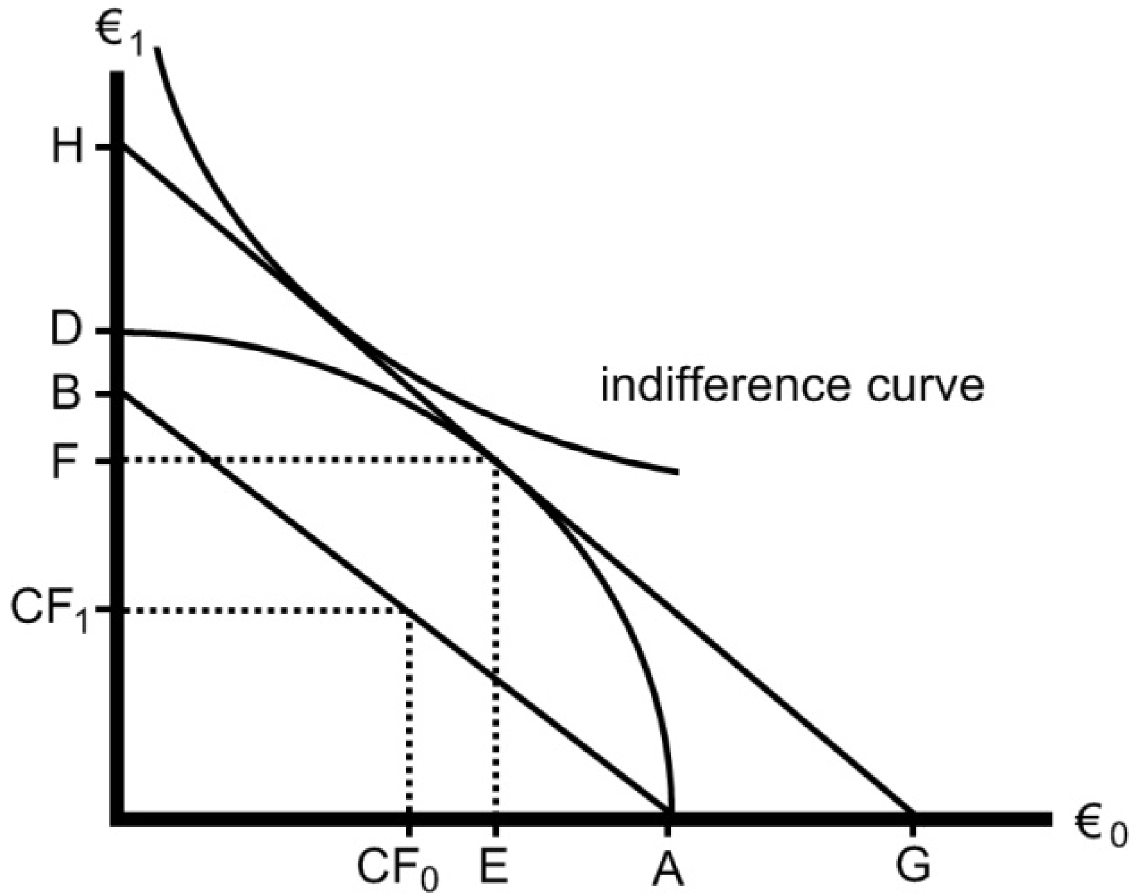
- Time
- Risk
- Law of one price

Allocation decision

The financial economic decision for each individual is how much to:

- consume;
- invest in the financial markets;
- invest in the real markets.

	Financial market	**Real market**
Objects:	Financial assets	Real assets / Projects
t = 0	Fin. investments	Real investments
t > 0	Cash flows	Cash flows



€1
H
D
B
F
CF1
indifference curve
CF0
E
A
G
€0

Hirshleifer model

Assumptions (17 in total):

A1. A certain world is assumed: the individual knows all the decision alternatives and the corresponding outcomes

A2. There is a one-period model where only two moments are important: the start of the period (now, t=0) and the end of the period (later, t=1).

A3. The individual has a current income of CF_0 and a future income of CF_1.

Hirshleifer model without financial and without real market

At t=0 you receive CF_0 and at t=1 CF_1
(income at t=0 and t=1 respectively)

- What do you do?
  - At t=0 you can consume CF_0 completely, partly or nothing.
  - *If* there is money left at t=0, you put this amount under your pillow and you consume it including the CF_1 at t=1.

Question 1

Given:

- CF_0 € 1000
- CF_1 € 1100
- There is no financial market.

The maximum consumption at t=0 is …

A. € 500

B. € 1000

C. € 2000

D. € 2100

Question 2

Given:

- CF_0 € 1000
- CF_1 € 1100
- There is no financial market.

The maximum consumption at t=1 is …

A. € 500

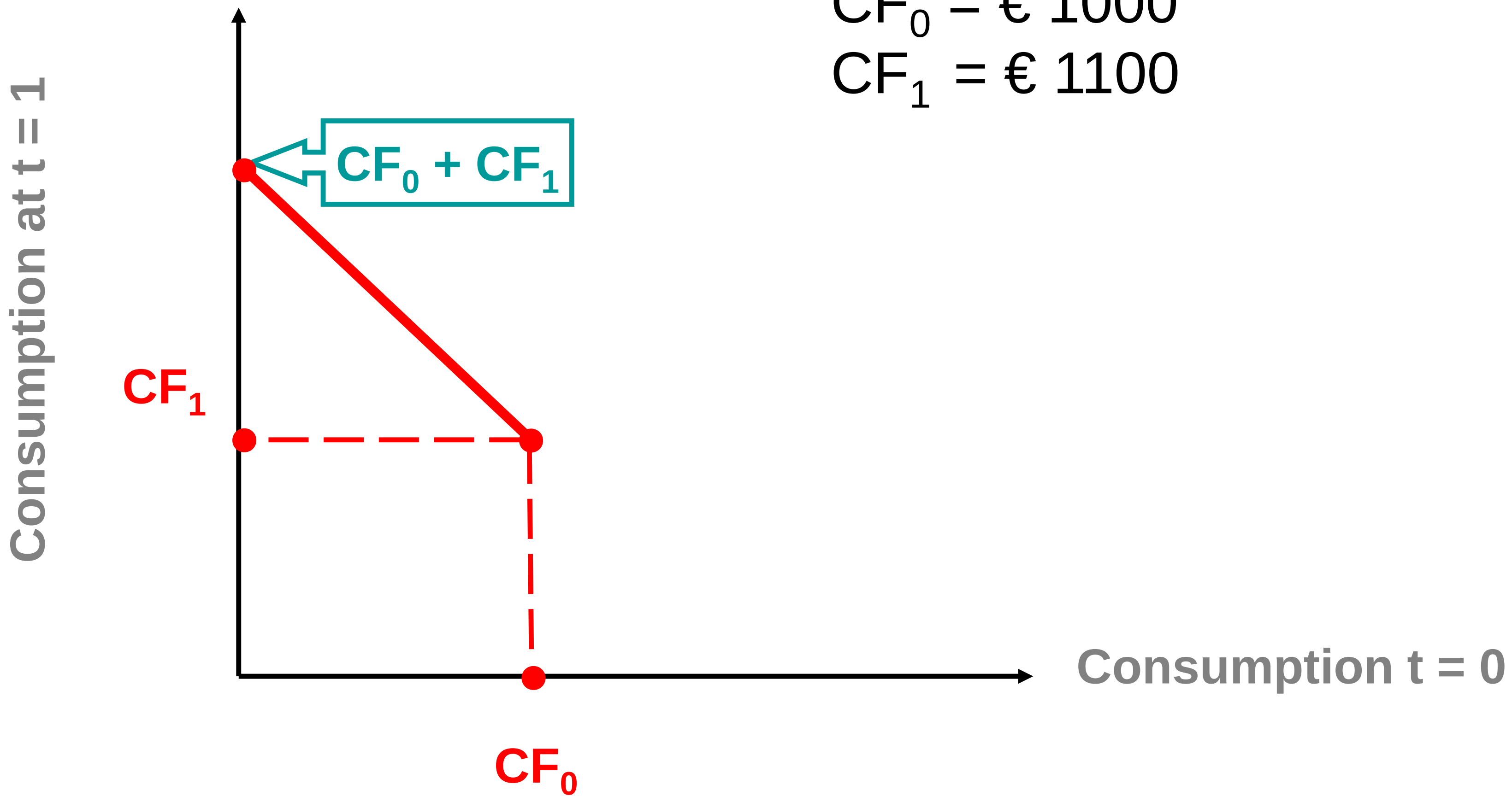
B. € 1000

C. € 2000

D. € 2100

Hirshleifer model without financial and without real market

CF_0 = € 1000
CF_1 = € 1100



Hirshleifer model with financial but without real market

Assumtion 14 of the 17 assumptions:

A14 Each participant can borrow or lend unlimitedly against the risk-free market interest rate r_f

Question 3

Given:

- CF_0 € 1000
- CF_1 € 1100
- r_f = 10%

The maximum consumption at t=0 is …

A. € 500

B. € 1000

C. € 2000

D. € 2200

Question 4

Given:

- CF_0 € 1000
- CF_1 € 1100
- $r_f = 10\%$

The maximum consumption at t=1 is …

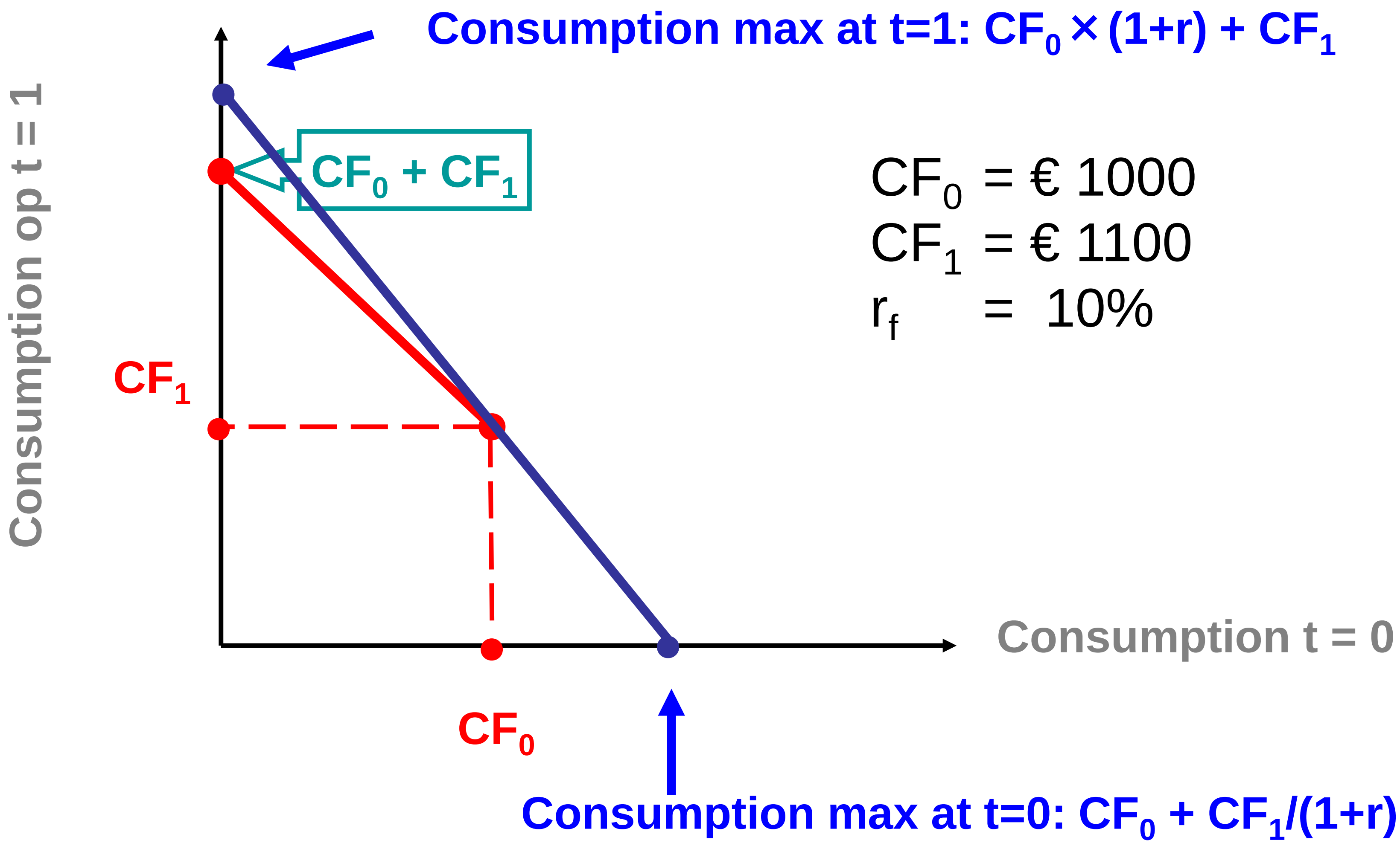
A. € 500

B. € 1000

C. € 2000

D. € 2200

Hirshleifer model with <u>financial</u> but without real market



CF_0 = € 1000
CF_1 = € 1100
r_f = 10%

Question 5

Given:

- CF_0 € 1000
- CF_1 € 2100
- $r_f = 5\%$

The maximum consumption at t=0 is …

A. € 1000

B. € 3000

C. € 3100

D. € 3150

Again the same questions but now with other amounts and another r_f

Question 6

Given:

- CF_0 € 1000
- CF_1 € 2100
- $r_f = 5\%$

The maximum consumption at t=1 is …

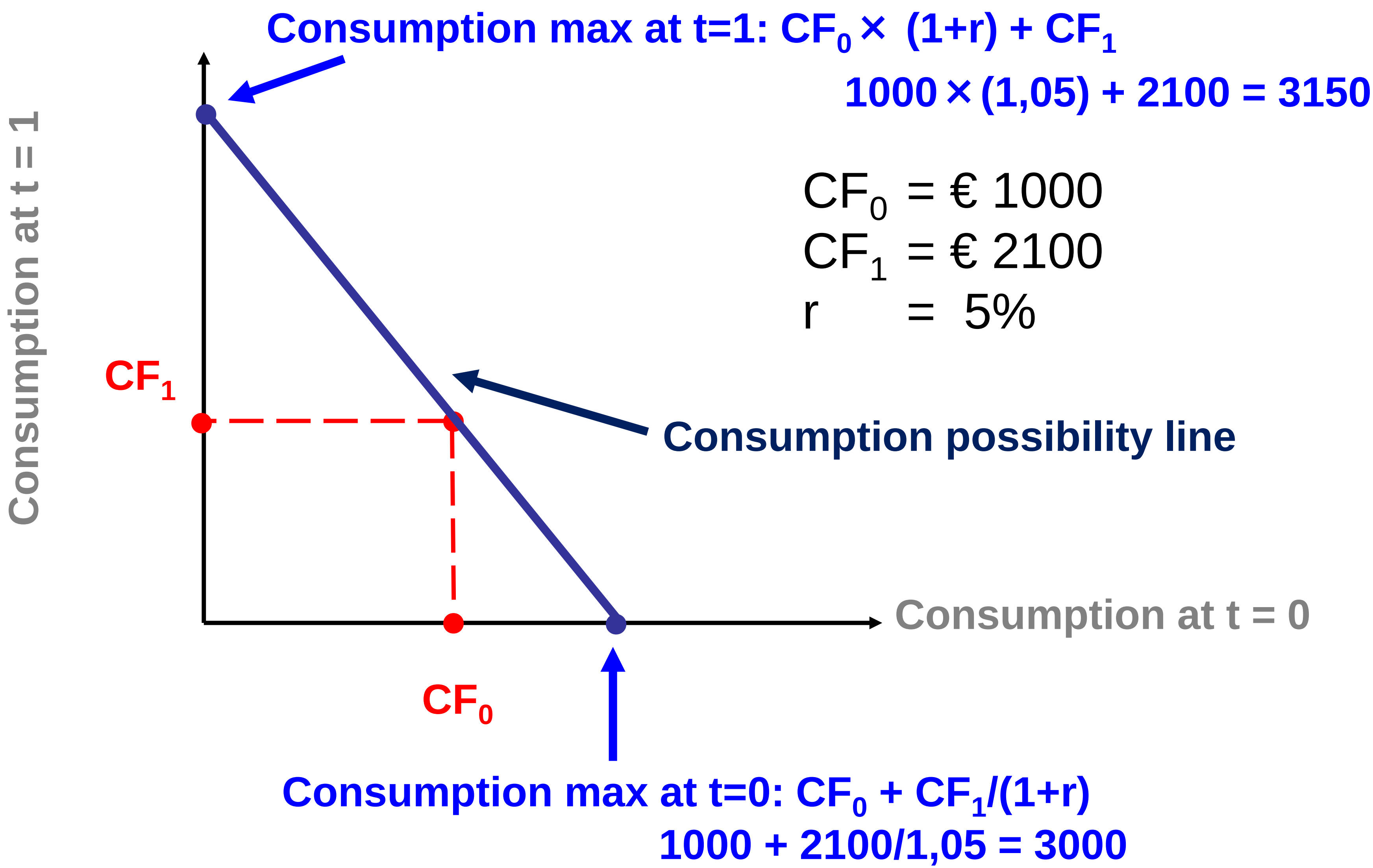
A. € 1000

B. € 3000

C. € 3100

D. € 3150

Hirshleifer model with financial but without real market



CF_0 = € 1000
CF_1 = € 2100
r = 5%

Question 7

Given:

- CF_0 € 1000
- CF_1 € 2100
- $r = 5\%$
- **C_0 € 1500**

C_0 max = 3.000
C_1 max = 3.150
(see questions 5 and 6)

The consumption at t=1 (C_1) is equal to

A. € 1000

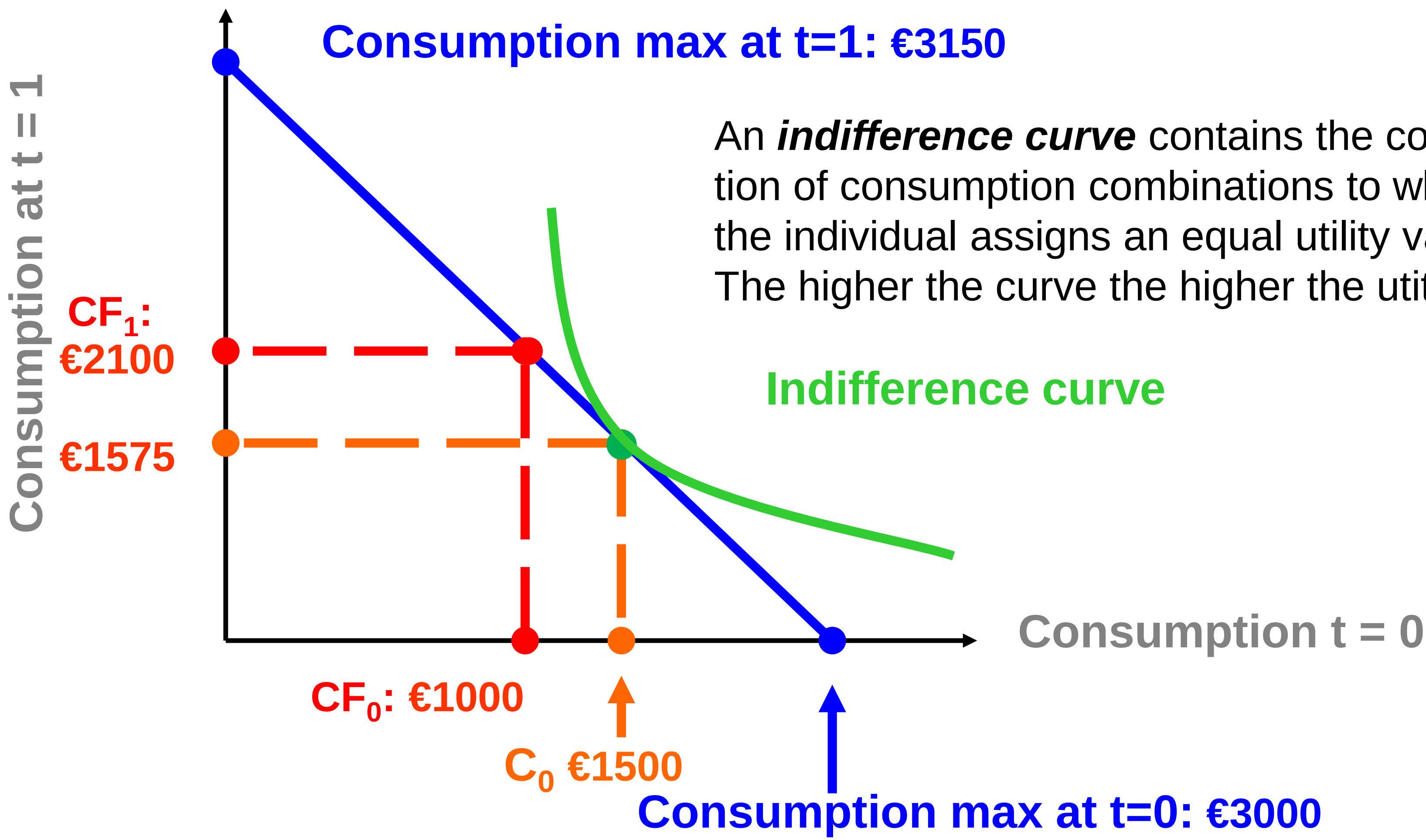
B. € 1500

C. € 1550

D. € 1575

Hirshleifer model with financial but without real markt

An ***indifference curve*** contains the collection of consumption combinations to which the individual assigns an equal utility value. The higher the curve the higher the utitliy.



Summary

With **financial markets** it is possible to *reallocate cash flows in time* >> consumption possibity line.

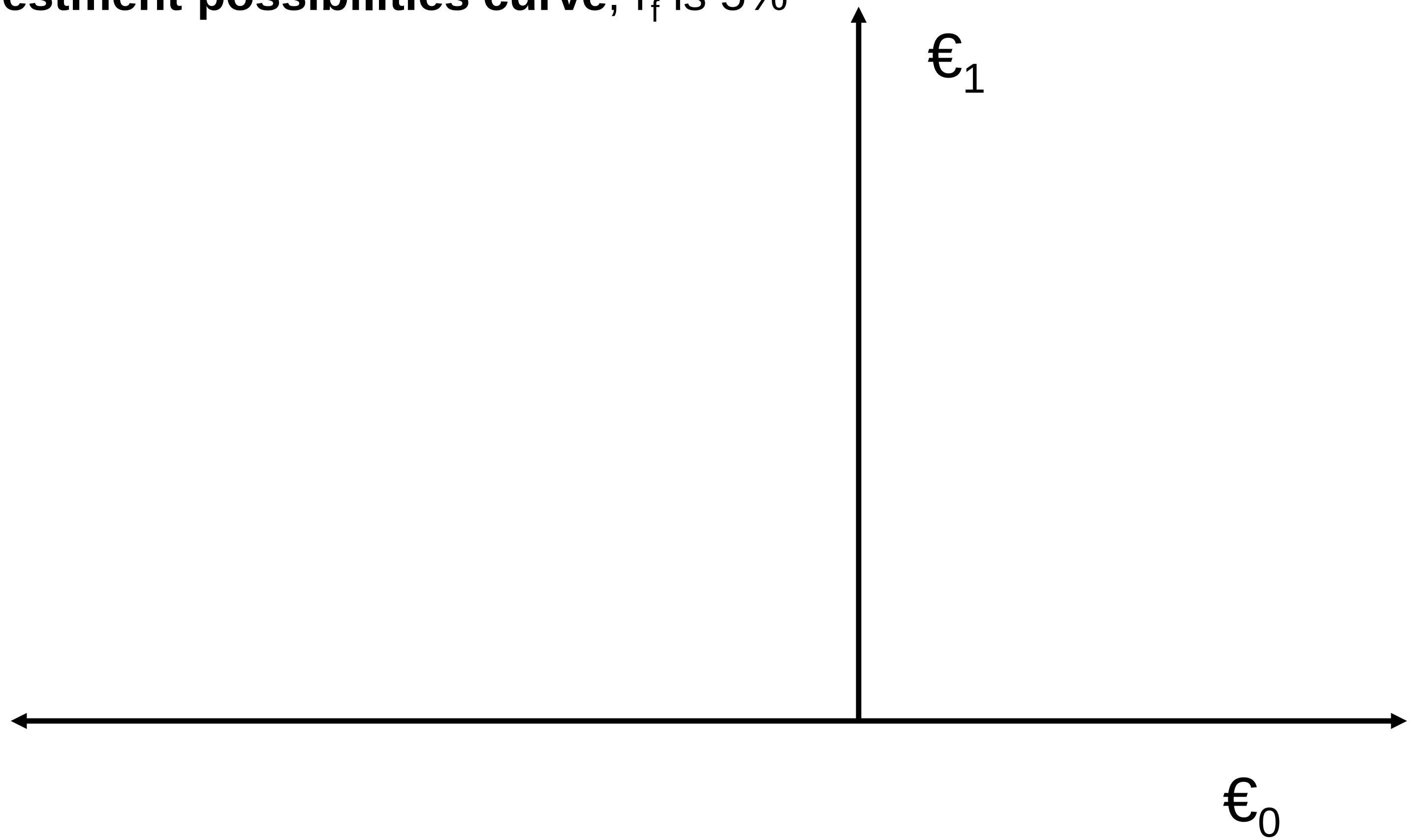
The optimal consumption combination (C_0,C_1) of the individual can be determined as the point where the *indifference curve is tangent to the consumption possibilities line*.

Hirshleifer model with financial and real market

Hirshleifer model with financial and real market

Assume: CF_0 = 0 and CF_1 = 0

Real investment possibilities curve, r_f is 5%

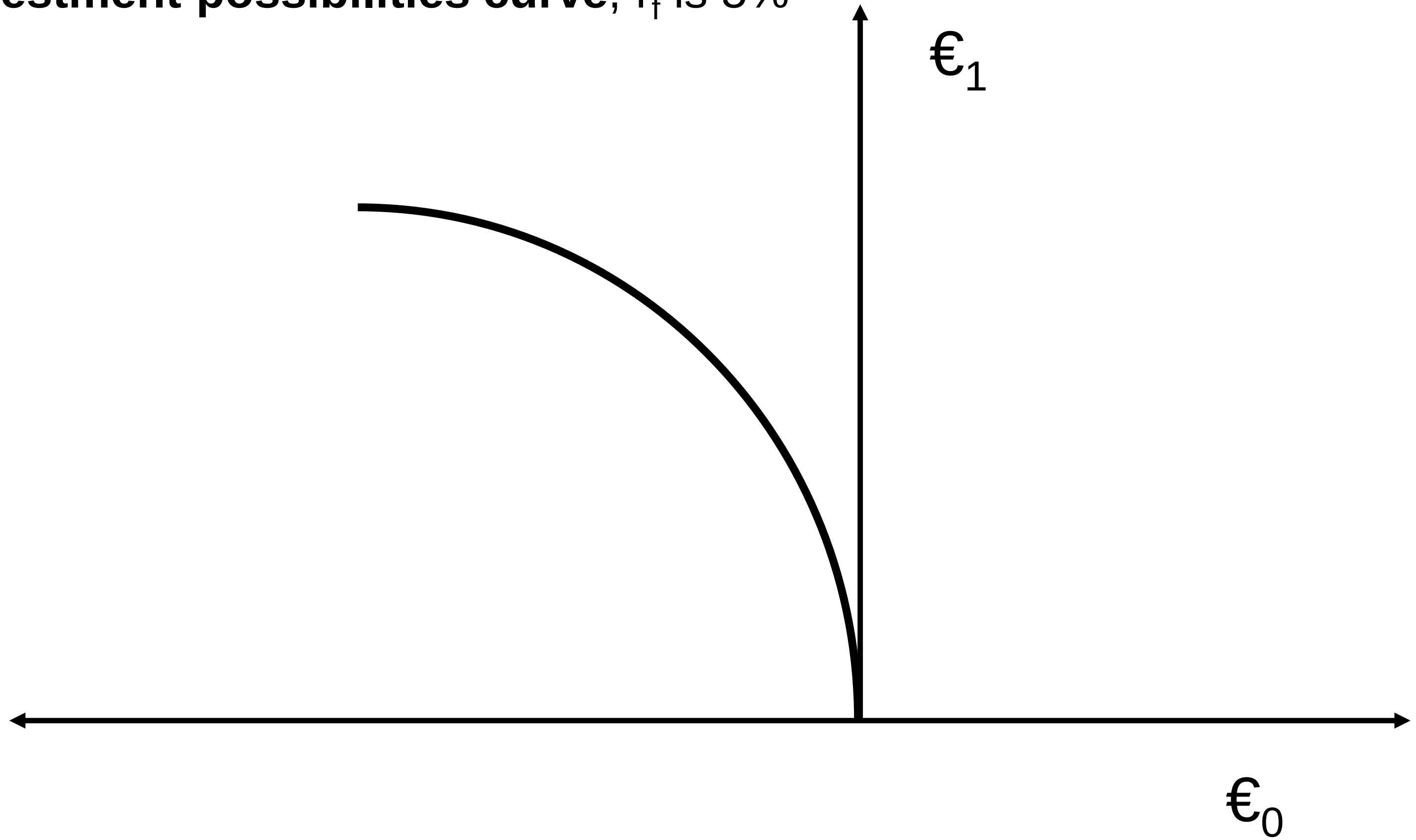


- Investment at t=0 € 500
- Revenue at t=1 € 630
- NPV = -500 + 630/1,05 = -500 + 600 = 100
- $C_{0\ max}$ = 100 (at t=0 you borrow 600, you invest 500 and consume 100; at t=1 you redeem 600 and pay interest of 30)
- $C_{1\ max}$ = 105 (at t=0 you borrow 500, you invest 500 and consume nothing; at t=1 you redeem the loan (500) and pay interest of 25; the residual is 630 -525 = 105

Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 5%



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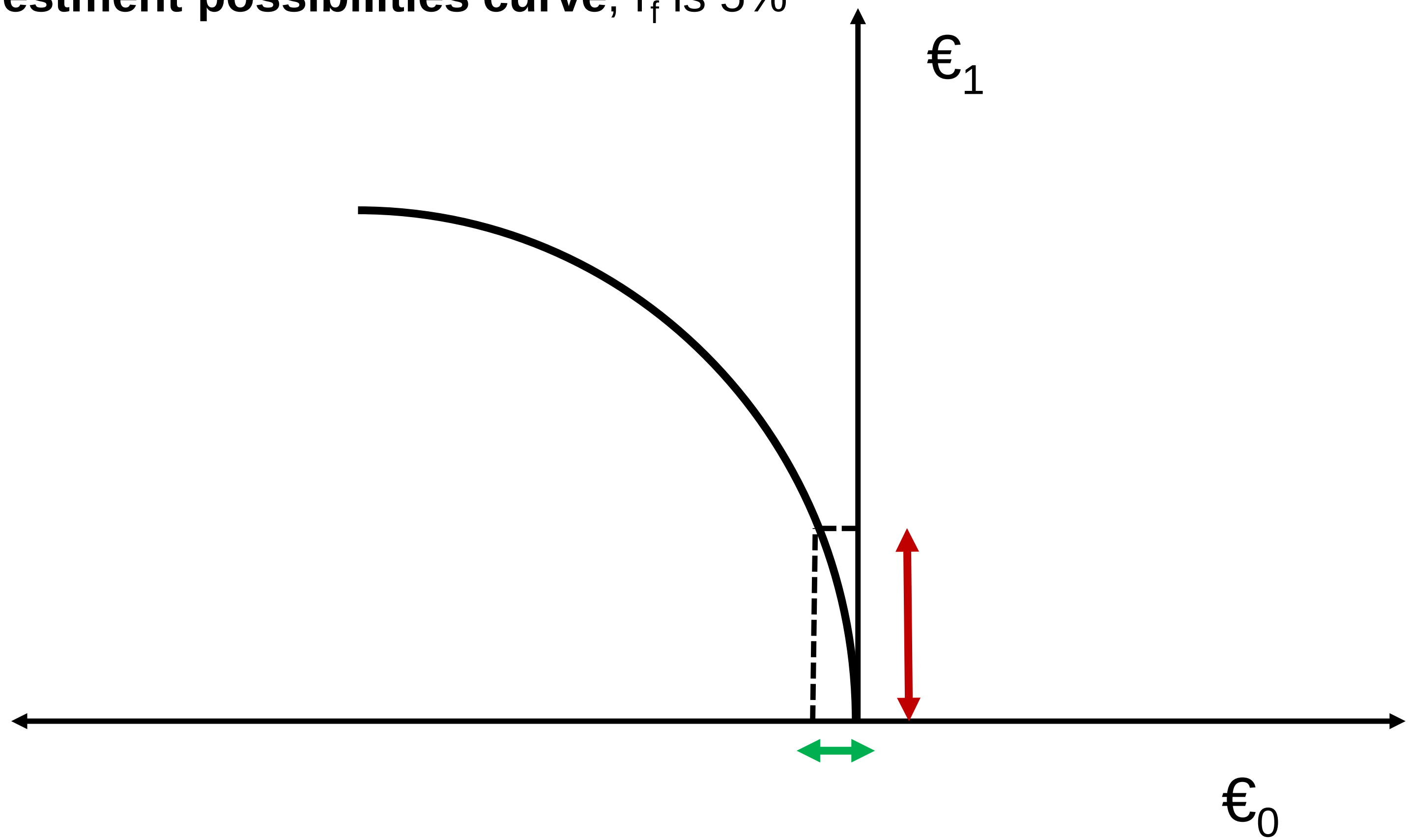
Real investment possibilities curve, r_f is 5%



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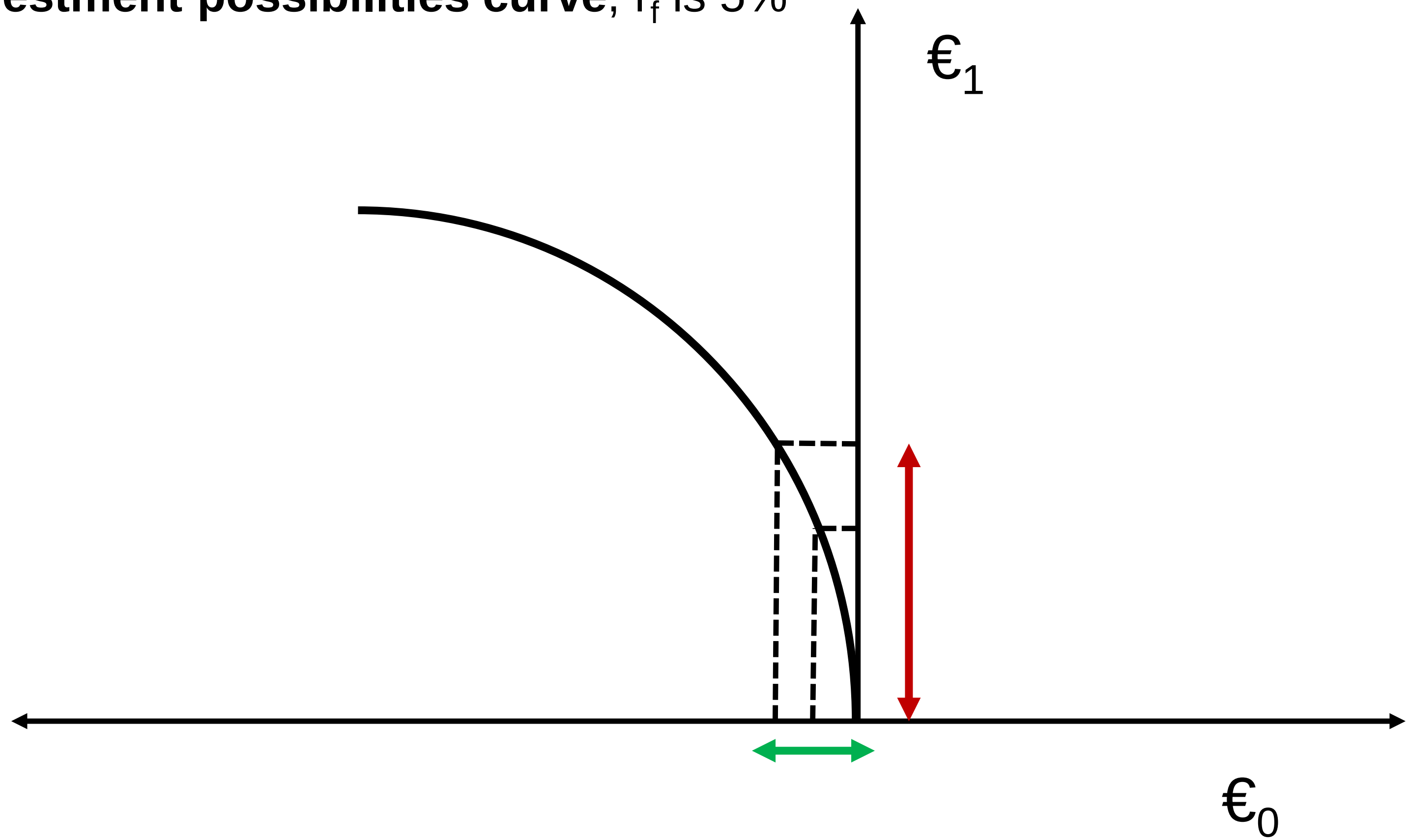
Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 5%



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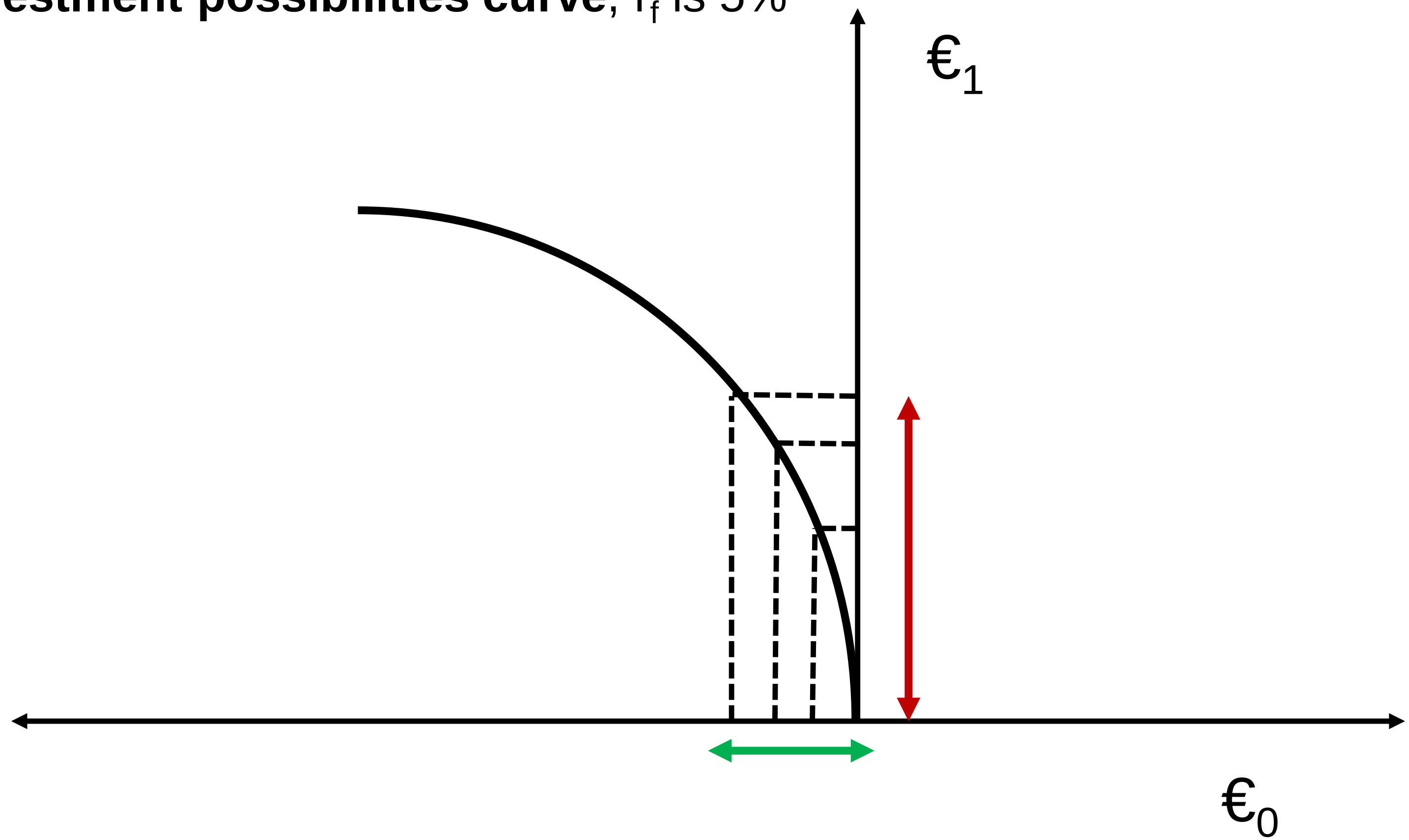
Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 5%



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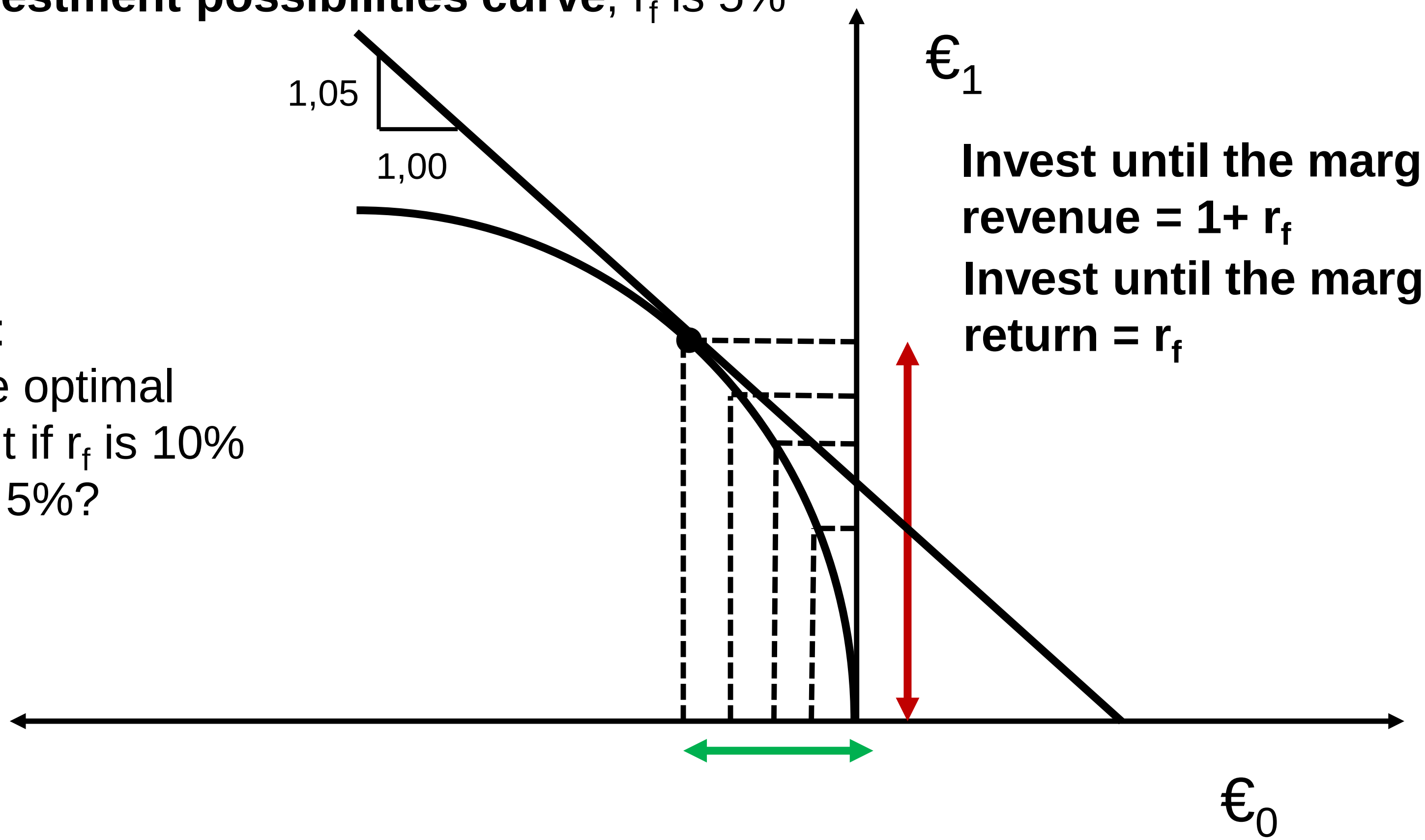
Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 5%



Invest until the marginal revenue = 1+ r_f

Invest until the marginal return = r_f

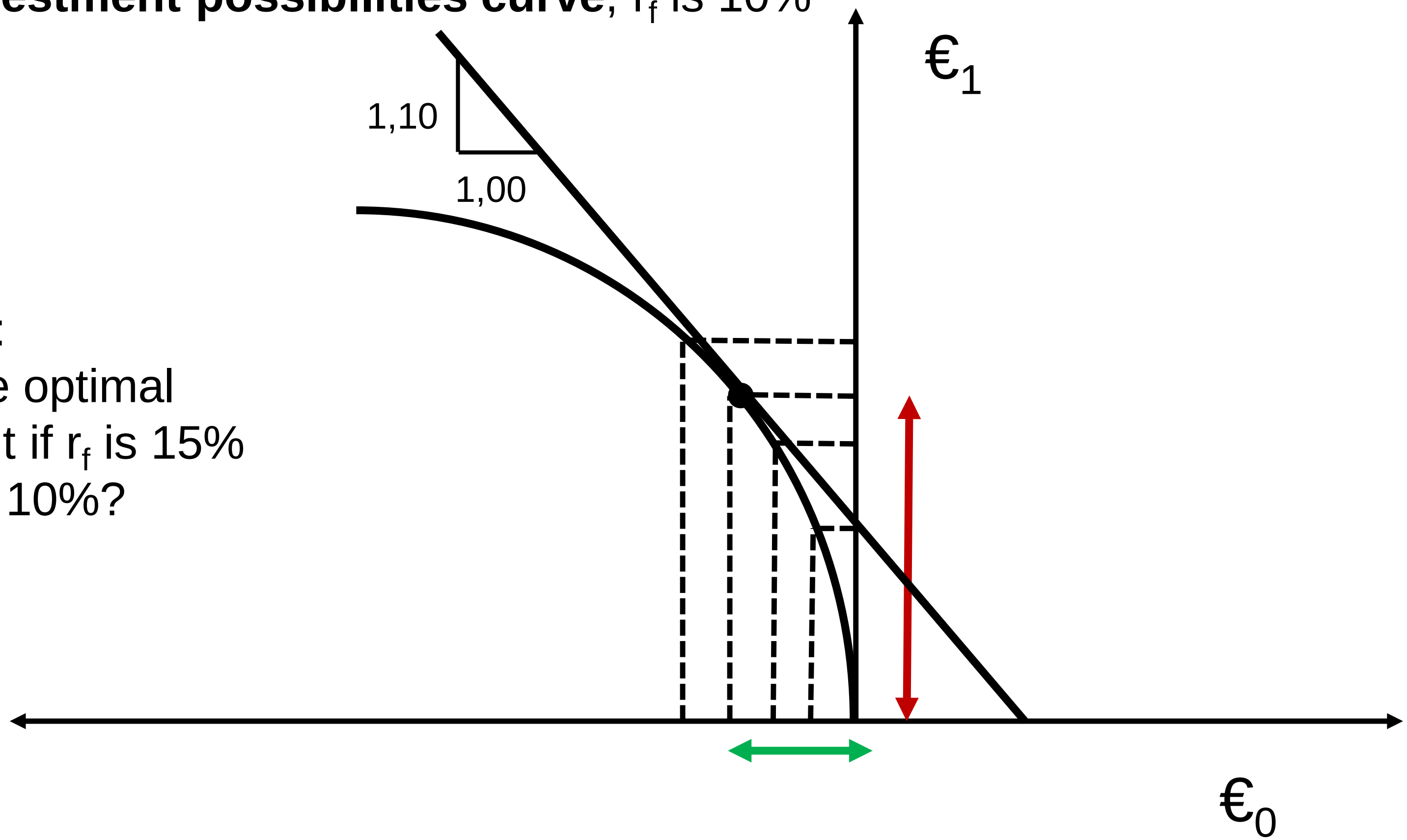
Question:
What's the optimal investment if r_f is 10% instead of 5%?

- Investment at t=0 € 500
- Revenu at t=1 € 630
- NPV = -500 + 630/1,05 = -500 + 600 = 100
- $C_{0\ max}$ = 100 (at t=0 you borrow 600, you invest 500 and consume 100; at t=1 you redeem 600 and pay interest of 30)
- $C_{1\ max}$ = 105 (at t=0 you borrow 500, you invest 500 and consume nothing; at t=1 you redeem the loan (500) and pay interest of 25; the residual is 630 -525 = 105

Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 10%



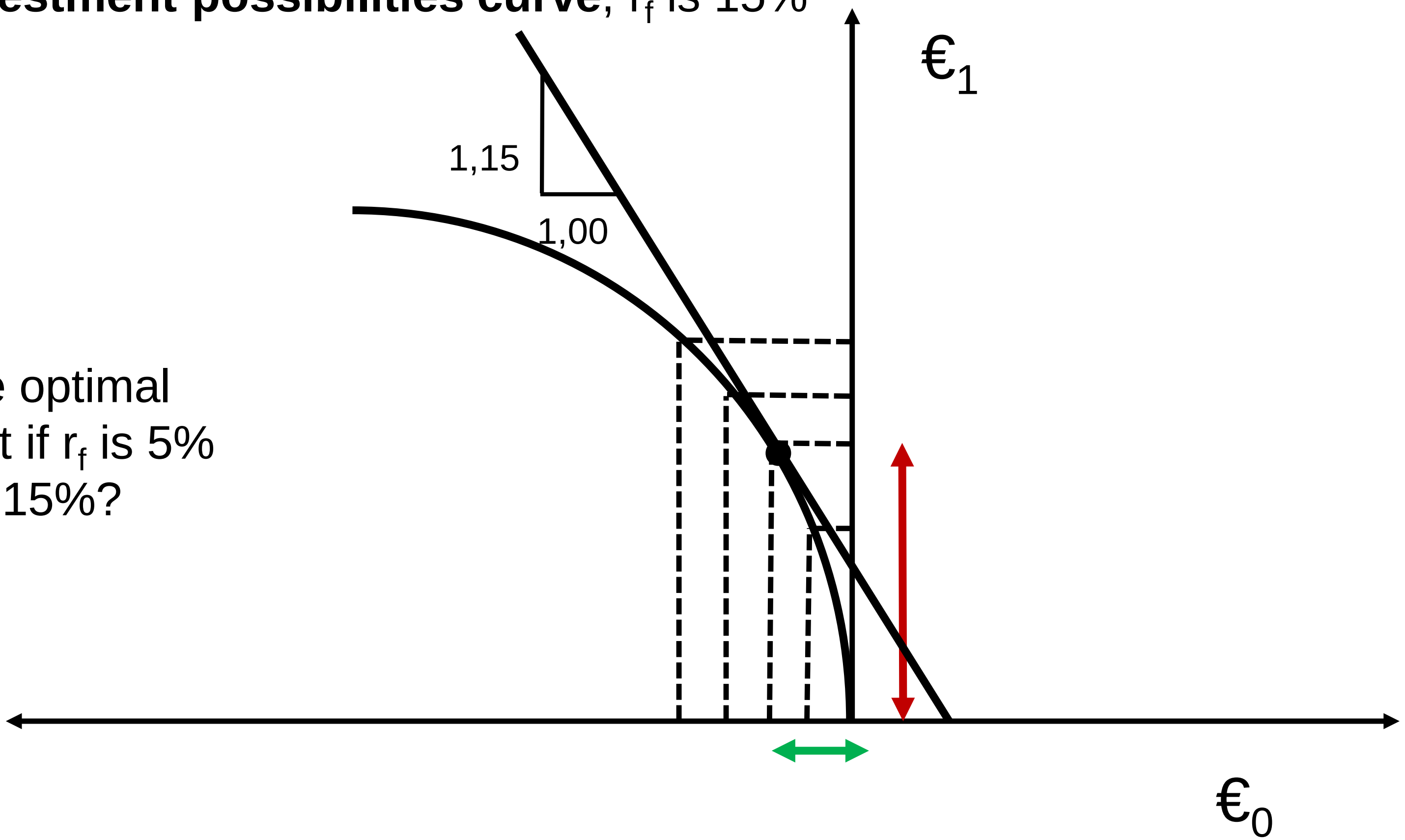
Question:
What's the optimal investment if r_f is 15% instead of 10%?

- Investment at t=0 € 500
- Revenu at t=1 € 630
- NPV = -500 + 630/1,05 = -500 + 600 = 100
- $C_{0\ max}$ = 100 (at t=0 you borrow 600, you invest 500 and consume 100; at t=1 you redeem 600 and pay interest of 30)
- $C_{1\ max}$ = 105 (at t=0 you borrow 500, you invest 500 and consume nothing; at t=1 you redeem the loan (500) and pay interest of 25; the residual is 630 -525 = 105

Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 15%



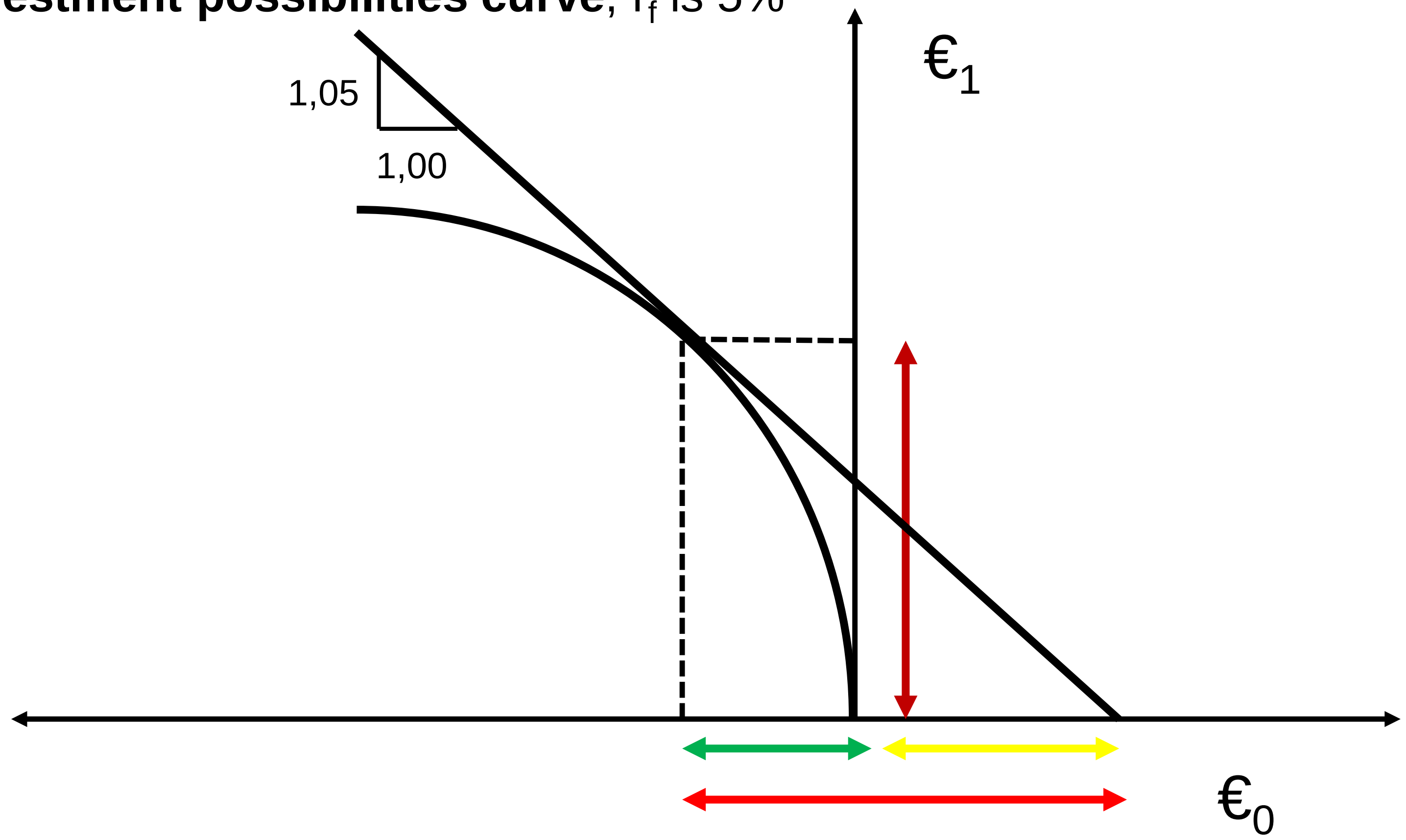
Question:
What's the optimal investment if r_f is 5% instead of 15%?

- Investment at t=0 € 500
- Revenue at t=1 € 630
- NPV = -500 + 630/1,05 = -500 + 600 = 100
- $C_{0\ max}$ = 100 (at t=0 you borrow 600, you invest 500 and consume 100; at t=1 you redeem 600 and pay interest of 30)
- $C_{1\ max}$ = 105 (at t=0 you borrow 500, you invest 500 and consume nothing; at t=1 you redeem the loan (500) and pay interest of 25; the residual is 630 -525 = 105

Hirshleifer model with financial and real market

Assume: $CF_0 = 0$ and $CF_1 = 0$

Real investment possibilities curve, r_f is 5%



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Question 8

Without real market:
C_0 max = 3000
C_1 max = 3150

Given:

- CF_0	€ 1000
- CF_1	€ 2100
- r	5%
- Investment at t=0	€ 500
- Revenue at t=1	€ 630

The maximum consumption at t=0 is …

A. € 1000
B. € 3100
C. € 3255
D. € 3350

Question 8

Given:

-	CF_0	€ 1000
-	CF_1	€ 2100
-	r	5%
-	Investment op t=0 (EA)	€ 500
-	Revenue at t=1 (OF)	€ 630

Answer 8

At t=0 you invest €500.
The consumption at op t=0 is at max if you consume nothing at t=1.

At t=0 you borrow the maximum amount:

- the revenue of the investment is €630. Borrow € 630/1,05 =	€ 600
- CF_1 is € 2.100. Borrow in addition € 2.100/1,05 =	€ 2.000
	€ 2.600

At t=0 you have in total:

- CF_0 plus the amount borrowed: € 1.000 + € 2.600 =	€ 3.600
- minus the investment outlay of	€ 500
The consumption max at t=0 is:	**€ 3.100**

Or: $\mathbf{C_0}$ **max** = $CF_0 + CF_1/1{,}05$ + NPV = 1.000 + 2.100/1,05 + 100 = 3.100

Question 9

Without real market we had:
C_0 max = 3.000
C_1 max = 3.150

Given:

- CF_0	€ 1000
- CF_1	€ 2100
- r	5%
- Investment at t=0	€ 500
- Revenue at t=1	€ 630

The maximun consumption at t=1 is …

A. € 1000
B. € 3100
C. € 3255
D. € 3350

Question 9

Answer 9

At t=0 you invest € 500. The maximum consumption at t=1 is reached if you consume nothing at t=0. Since CF_0 is €1.000 at t=0 you lend € 500.

De maximum consumption at t=1 then is:

- amount lent plus interest: € 500 × 1,05 =	€ 525
- CF_1	€ 2.100
- Revenue of the investment	€ 630
Maximum consumption at t=1	**€ 3.255**

Or: C_1 max = C_0 max × (1+r) = 3.100 × 1,05 = **3.255**

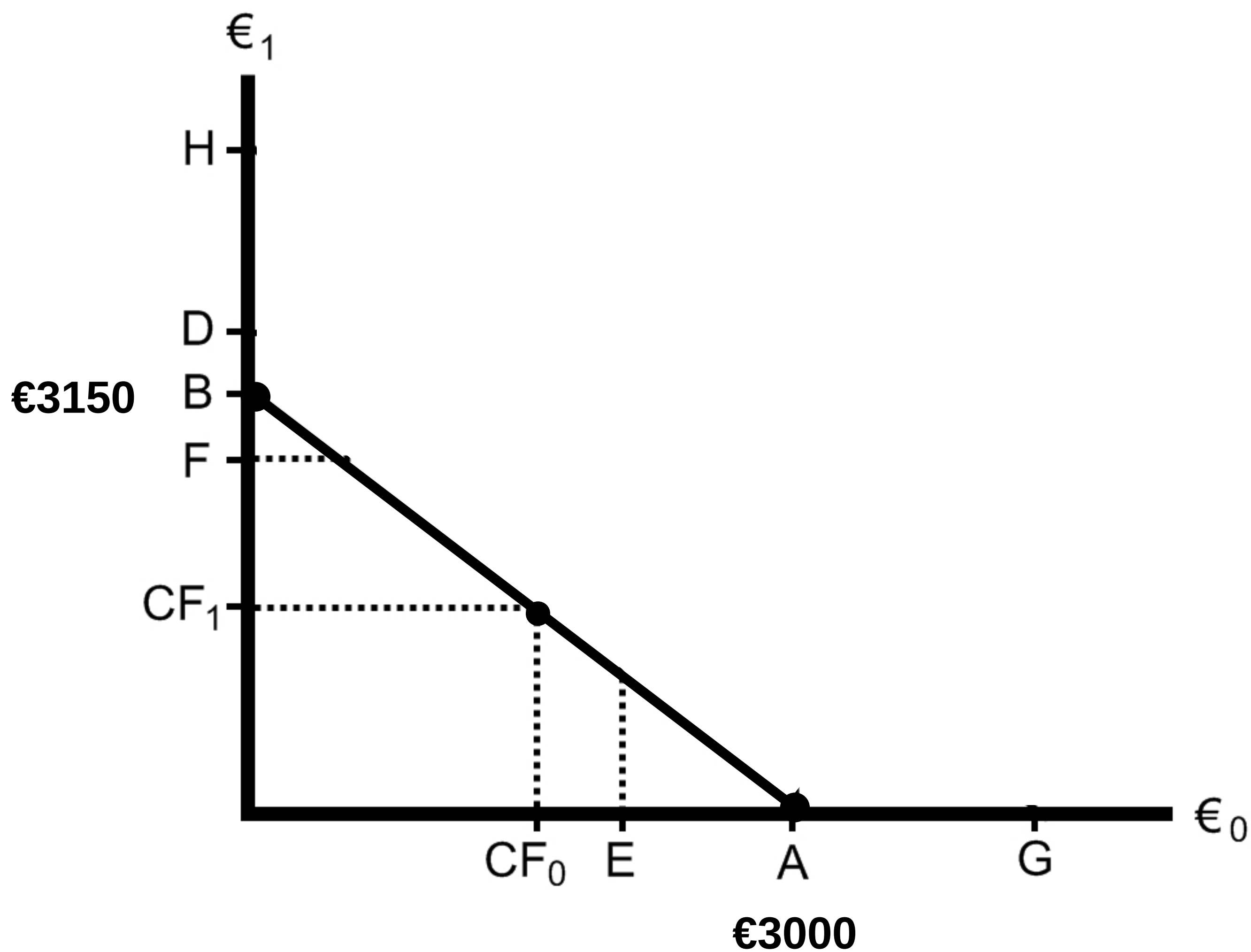
Hirshleifer model with financial and real market

Assume: CF_0 = €1000; CF_1 = € 2100; r_f=5%; investment = 500, revenue = 630

Graphical presentation of questions 8 and 9

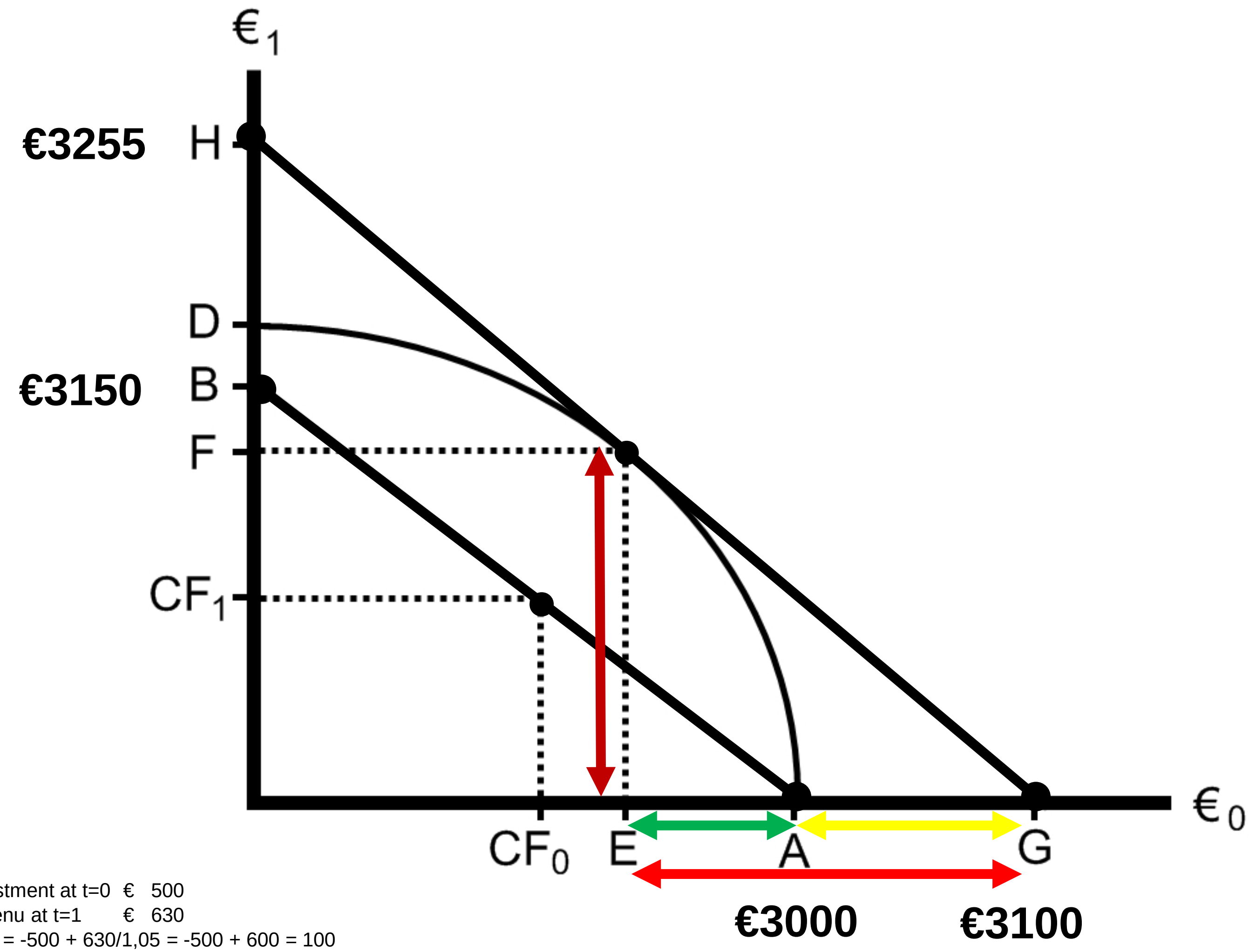
Hirshleifer model with financial and real market

Assume: CF_0 = €1000; CF_1 = € 2100; r_f=5%; investment = 500, revenue = 630



Hirshleifer model with financial and real market

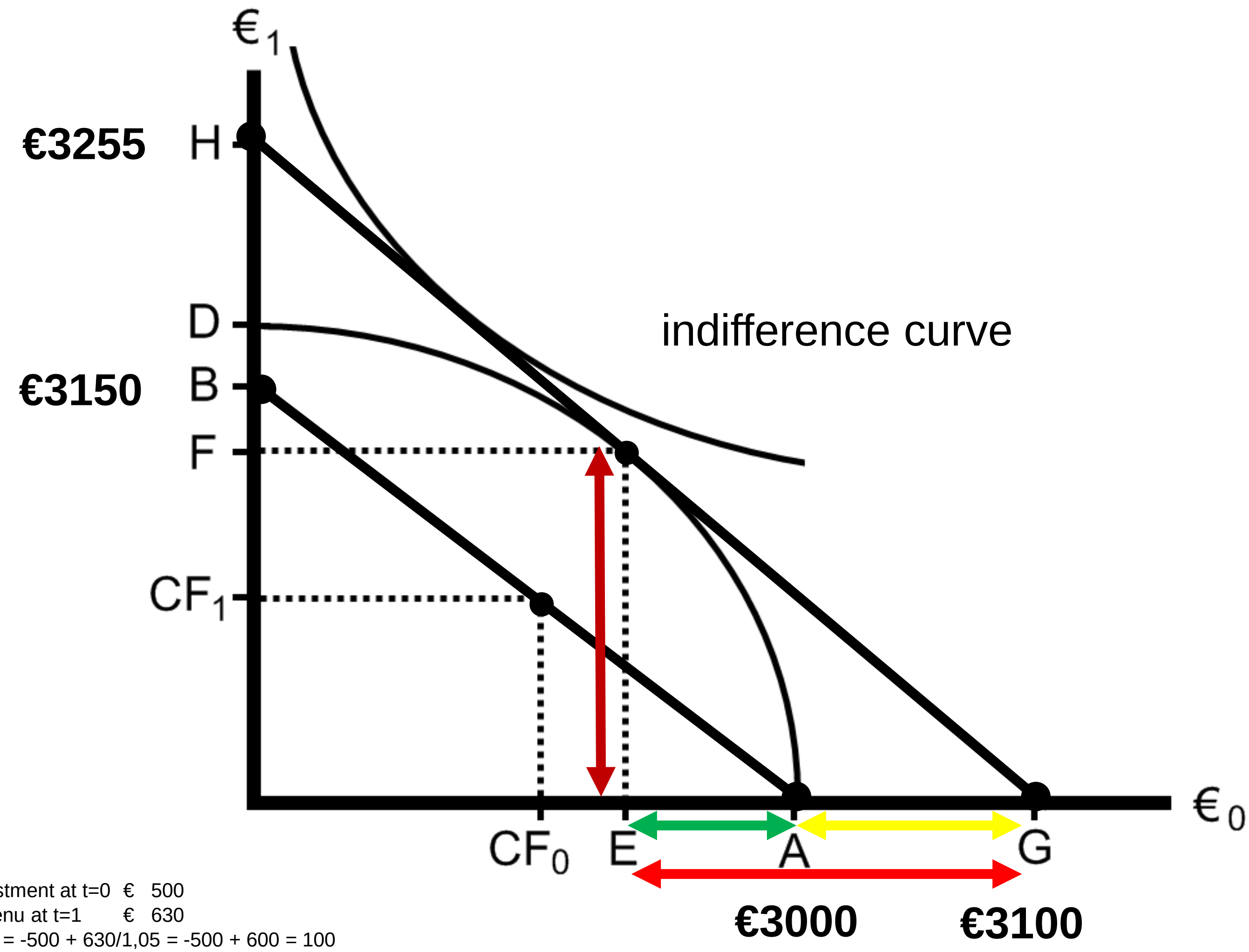
Assume: CF_0 = €1000; CF_1 = € 2100; r_f=5%; investment = 500, revenue = 630



- Investment at t=0 € 500
- Revenu at t=1 € 630
- NPV = -500 + 630/1,05 = -500 + 600 = 100

Hirshleifer model with financial and real market

Assume: CF_0 = €1000; CF_1 = € 2100; r_f=5%; investment = 500, revenue = 630



- Investment at t=0 € 500
- Revenu at t=1 € 630
- NPV = -500 + 630/1,05 = -500 + 600 = 100

Summary

- The existence of financial and real markets makes people “happier”, i.e. gives them higher utilitiy.

  - Without financial markets people have to consume in the same period as they have income.
  - Without real markets profitable projects will be unused.
  - The existence of both markets makes it possible for people to achieve at higher indifference curves.

Fisher separation theorem

Theorem:
The real investment decision is taken independently from the consumption decision.

Step 1, optimal investments:
Marginal return = interest rate (opportunity cost of capital*)

Step 2, optimal consumption
Marginal utility of consumption now and later = interest rate

* “**The opportunity cost of capital** (or more simply, the cost of capital), which is the best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted.” BDM, p.159

Fisher separation theorem

Meaning of the separation theorem:

- Everybody wants the same level of capital investments (until the marginal return = interest rate).

- Managers can focus on real investment projects that add value.

- The financial market takes care of the individual choice with respect to consumption now and later.

Programme today

Hirshleifer model

- Hirshleifer model without financial market and without real market
- Hirshleifer model with financial market but without real market
- Hirshleifer model with financial market and real market
- Fisher separation theorem

Concepts

- Time
- Risk
- Law of one price

Concepts in Finance

1. Time
2. Risk
3. Law of one price

Time

Assume that:

- I offer to pay you € 55 one year from now.
- There is no risk (can can be sure I'll pay)
- The interest rate is 10%.

Would you accept my offer?

Yes?

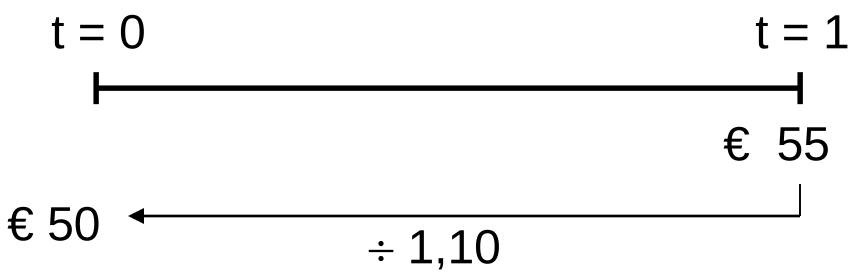
Assume:
for this generous offer, I ask you to pay me a certain amount of money.
How much would you pay for my offer?

Maximum € 55 / 1,1 = € 50

In the financial market you need € 50 to replicate a pay-off of € 55 at t=1

Time

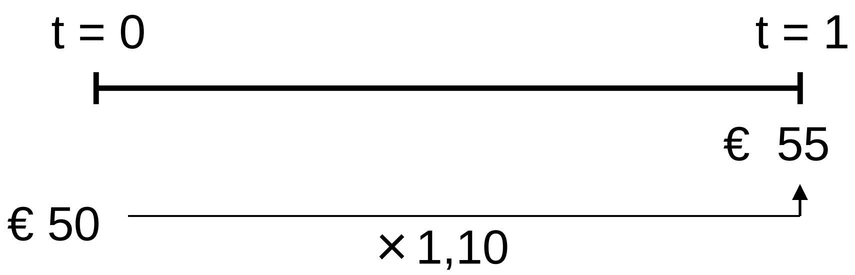
Hint: always draw a time line for clarity



“I’d rather have € 1 today than € 1 tomorrow”

Time

Hint: always draw a time line for clarity



“I’d rather have € 1 today than € 1 tomorrow”

Time

- **Calculating the present value (PV) of a cash flow**
  $PV_0 = CF_T \times (1 + R_T)^{-T} = CF_T / (1 + R_T)^T$
  $PV_0 = 55 / (1 + 10\%) = 50$

- **Calculating the future value**
  $FV_T = CF_0 \times (1 + R_T)^T$
  $FV_1 = 50 \times (1 + 10\%) = 55$

- **Calculating the present value from a series of cashflows**
  $PV_0 = CF_1 \times (1 + R_1)^{-1} + CF_2 \times (1 + R_2)^{-2} + \ldots$

$$= \sum_{t=1}^{T} CF_t \cdot (1+R)^{-t}$$

RailWay X

- Two types of rails of the same quality
- X has to choose between type A and B
- Costs for purchase and maintenance

Rail type	t=0	t=1	t=2	t=3
A	-€ 15	-€ 5	-€ 5	-€ 5
B	-€ 7	-€ 8	-€ 8	-€ 8

- The interest rate is 10%, which rails to choose?

A. Type A
B. Type B

RailWay X

Rail type	t=0	t=1	t=2	t=3
A	-€ 15	-€ 5	-€ 5	-€ 5
B	-€ 7	-€ 8	-€ 8	-€ 8

Determine the present value of the expected CFs

$-15 \times (1{,}1)^{0} = -15{,}00$

$-5 \times (1{,}1)^{-1} = -\ 4{,}55$

$-5 \times (1{,}1)^{-2} = -\ 4{,}13$

$-5 \times (1{,}1)^{-3} = -\ 3{,}76 \quad +$

PV switch A $\quad -27{,}43$

RailWay X

Rail type	t=0	t=1	t=2	t=3
A	-€ 15	-€ 5	-€ 5	-€ 5
B	-€ 7	-€ 8	-€ 8	-€ 8

Determine the present value of the expected CFs

$-7 / (1{,}1)^0 = -\ 7{,}00$

$-8 / (1{,}1)^1 = -\ 7{,}27$

$-8 / (1{,}1)^2 = -\ 6{,}61$

$-8 / (1{,}1)^3 = -\ 6{,}01 \quad +$

PV switch B $\quad -26{,}89$

Time: annuities

Calculate the present value

$PVA_T = A\cdot(1+R)^{-1} + A\cdot(1+R)^{-2} + \ldots + A\cdot(1+R)^{-T}$ (1)

where,

PVA_T = the present value at t=0 of a T-year annuity

A = periodic cashflows (starts at t=1)

$$PVA_T = \frac{A}{R}[1-(1+R)^{-T}]$$

See BDM (p. 112-113)

Time: annuities

- **Example**: you get four years € 2.000 at the end of the year. The interest rate is 5%.

2.000 / $1{,}05^1$	= 1.905
2.000 / $1{,}05^2$	= 1.814
2.000 / $1{,}05^3$	= 1.728
2.000 / $1{,}05^4$	= 1.645 +
Present value	7.092

- Value according to the formula: $PVA_T = \frac{A}{R}[1-(1+R)^{-T}]$:

(2000/0,05) x (1- $(1{,}05)^{-4}$) = € 7.092

Time: annuities

From the previous slide, the value of the 4 cash flows is:

$$(2.000/0.05) \times (1- (1{,}05)^{-4}) = € \ 7.092$$

Suppose the interest rate is 6% instead of 5%, how much would the value be?

A. €6.930

B. €7.092

C. €7.293

$$(2.000/0{,}06) \times (1- (1{,}06)^{-4}) = € \ 6.930$$

Impact of increased longevity on the price of pensions

- Value of € 1 per year of pension until you die
  - Expected pension entitlement: 66-82 years (17)
  - Interest rate = 4%

  $= (1/0{,}04) \times (1- (1{,}04)^{-17}) =$ € 12,17

- Value of € 1 per year of pension until you die
  - Expected pension entitlement: 66-84 years (19)
  - Interest rate = 4%

  $= (1/0{,}04) \times (1- (1{,}04)^{-19}) =$ € 13,13

Pensions become (13,13/12,17-1 =) 7,8% more expensive!

Note: see BDM example 4.8 about the lottery prize (p.114).

Time: the perpetuity

Special case: the perpetuity

Als T $\rightarrow \infty$, then $PVA_T = \frac{A}{R}[1-(1+R)^{-T}] \rightarrow PVA_T = \frac{A}{R}$

Example:

Rent of a house is € 2.000 per month $\rightarrow$ € 24.000 per year;

Expenses per year 2 × monthly rent; R = 6%.

***Value* house?** $PVA = \frac{20.000}{0,06} = 333.333$

***Price* house?**

See BDM (p. 112-113)

Time: the growing perpetuity

Special case: the growing perpetuity

$$PV(\text{growing perpetuity}) = \frac{A}{R - G}$$

Example:

Rent of a house is € 2.000 per month $\rightarrow$ € 24.000 per year;

Expenses per year 2 × monthly rent; R = 6%; G = 2%

Value **house?** $PV = \dfrac{20.000}{0{,}06 - 0{,}02} = 500.000$

Price **house?**

Risk

certainty	A	Pr{A}	B	Pr{B}
	€ 1.000	100%	€ 0	0%

uncertainty	A	Pr{A}	B	Pr{B}
	€ 1.000	?	€ 0	?

risk	A	Pr{A}	B	Pr{B}
	€ 1.000	25%	€ 0	75%

Risk

risk	A	Pr{A}	B	Pr{B}
	€ 1,000	25%	€ 0	75%

- Calculate the expected payoff

  € 1.000 x 25% + € 0 x 75% = € 250

- How much are you prepared to pay to participate?

A. Slightly less than € 250 then you are risk averse

B. Exactly € 250 then you are risk neutral

C. Slightly more than € 250 then you are risk loving

Time and risk

Previously I offered to pay you:

- €1.040 in one year time (no risk)
- Interest rate is 4%
- Value of this promised payment is:
  $E(CF_T)$ / 1,04 = €1.040 / 1,04 = €1.000

Same offer, but instead:

- There is a 20% probability that I won't pay anything
- $E(CF_T)$ = 80% x €1.040 + 20% x €0 = €832
- Risk neutral value = $E(CF_T)$ / 1,04 = €832 / 1,04 = €800

Question:

How much would the value be if the agent is risk averse?

Time and risk

Question:

$E(CF_T)$ = 80% x €1.040 + 20% x €0 = €832

How much would the value be when the agent is risk averse?

A. Slightly less than €800
B. Exactly €800
C. Slightly more than €800

- If the agent is risk averse the discount rate is:

  the interest rate (4%) + a risk premium

- For example, if the risk premium is 1%:

  the value is = € 832 / 1.05 = € 792

Time and risk

Application to financial markets later in this course and further in the bachelor & MSc Finance programme.

- How risky are bonds?
- How risky are stocks?
- How do we measure risk in financial markets?

Law of one price

If equivalent investment opportunities trade simulaneously in different competitive markets, then they must trade for the same price in both markets