

Contents

1	Introduction	1
1.1	Features and naming convention	1
1.2	Estimation	1
1.3	Prior tools for BVARs	1
1.4	Identification	2
1.5	Applications/Methods	2
1.6	Extra	2
1.7	Tests	2
1.8	Counterfactuals	2
1.9	Forecast Scenarios	2
1.10	Diagnostics	2
1.11	See also other packages	3
2	Estimation	3
2.1	Reduce form VAR(p)	3
2.2	Structural Form VAR(p)	3
2.3	Compact Form	3
2.4	Minnesota prior	4
3	Methods	5
4	Applications	5
4.1	Example one	5
4.2	Example two	5
5	Final Words	5
6	Literature	5
7	FAVAR	6
8	Algorithms	6
8.1	IRF	6
8.2	Error Bands for Impulse Response Function	7

1 Introduction

1.1 Features and naming convention

- **fa**: Factor Augementing
- **sv**: Stochastic Volatility
- **tv**: Time Varying Parametrs

1.2 Estimation

1.2.1 Classical Models

- **var_ls** : Classical
- **var_ml** : Maximum Likelihood (for illustration purposes)
- **var_gls**: Generalized Linear Squares
- **var_bc** : Bias-Correction
- **var_tvp**: Time Varing Parameter

- `var_sv`: Stochastic Volatility
- `var_tvp_sv`: Time Varing Parameter & Stochastic Volatility
- `favar`: Factor Augmented
- `favar_tvp`: Factor Augement Time Varying Parameter
- `bvar_lrg`: Large Bayesian VAR (see pkg `lbvar`)

1.2.2 Markov Switching

- `msvar`: regime-switching models with fixed transition probabilities

1.2.3 Bayesian VAR with analytical Solution

- `bvar_diff`: Diffuse prior
- `bvar_conj`: Conjugate prior
- `bvar_minn`: Minnesota prior
- `bvar_ss` : Steady State prior

1.2.4 Bayesian VAR with Gibbs Sampler

- `bvar_svss`: Stochastic Search Variables Selection
- `bvar_fvs` : Flexible Variable Selection
- `bvar_cnw` : Conditional Normal-inverse-Wishart prior
- `bvar_inw` : Independent Normal-inverse-Wishart prior

1.3 Prior tools for BVARs

- Hyperparameter optimasation by grid search
- Block exogeneity
- Dummy observation ...

1.4 Identification

- `id_chol`: Choleski factorisation (short-run restrictions)
- `id_triangular`:Triangular factorisation (short-run restrictions)
- `id_lr`: Long run restrictions
- `id_sign`: Sign, magnitude and zero restrictions (Arias et al., 2014)
- `id_iv`: Instrumental variables estimation Stock and Watson (2012) & Mertens and Ravn (2013)

1.5 Applications/Methods

- `irf`: Impulse response functions
- `forecast`: Unconditional forecasts
- `fevd`:Forecast error variance decomposition
- `hd`: Historical Decomposition
- `forecast_shock`: Conditional forecasts: shock approach (Waggoner and Zha, 1999)
- `forecast_tilt`: Conditional forecasts: tilting approach (Robertson et al., 2005)
- `forecast_eval`: Forecast Evaluation

1.6 Extra

- Non-fundamentanless
- Lag Order Selection Procedures
 - Top-down Sequential Testing
 - Bottom-up Sequential Testing
- Granger Causality
- vecm
- Information Criteria (aic, hqc, sic)

1.7 Tests

- Residual Autocorrelation
 - Portmanteau Test for Residual Autocorrelation
 - LM Test for Residual Autocorrelation
- Stability
- Time Invariance
- Heteroskedasticity
- Normality

1.8 Counterfactuals

- Simulation
- Policy

1.9 Forecast Scenarios

1.10 Diagnostics

- Convergence test
- Histograms of the posterior
- Gaussian kernel estimation
- Recursive Moments
- Trace plots

1.10.1 Accompanying

- Filtering (forthcoming transx)
- Simulation (forthcoming simdgp)
- Data (forthcoming econdata)
- Unit Root test (see urca)

1.11 See also other packages

- vars
- svars
- bvars: <https://github.com/joergrieger/bvars>
- bvarsv: <https://cran.r-project.org/web/packages/bvarsv/index.html>
- lbvar: <https://github.com/gabrielrvsc/lbvar>
- bvartools: <https://github.com/franzmohr/bvartools>
- bvar: <https://github.com/nk027/bvar>
- bvarrKK: <https://github.com/bdemeshev/bvarrKK>
- mfbvar: <https://github.com/ankargren/mfbvar>
- BMR
- GediminasB/bayesVAR_TVP
- ragt2ridges

2 Estimation

2.1 Reduce form VAR(p)

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + u_t$$

2.2 Structural Form VAR(p)

$$B_0 y_t = B_1 y_{t-1} + \dots + B_p y_{t-p} + e_t$$

Let try some text here:

- $E(e_t) = 0$ — every error term has mean zero α
- $E(e_t e_t') = \Omega$ — the contemporaneous covariance matrix of error terms is Ω (a $k \times k$ positive-semidefinite matrix)
- $E(e_t e_{t-k}') = 0$ — for any non-zero k — there is no correlation across time; in particular, no serial correlation in individual error terms.[1]

$$y_t = \underbrace{B_0^{-1} B_1 y_{t-1}}_{A_1} + \dots + \underbrace{B_0^{-1} B_p y_{t-p}}_{A_p} + \underbrace{B_0^{-1} e_t}_{w_t}$$

2.3 Compact Form

Consider the VAR(p) model (??) written in more compact form

\$\$\$y_t =

$$Y = AZ + U$$

where $Z = [1, y'_{t-1}, \dots, y'_{t-p}]$

$$\hat{A} = [\hat{\nu}_t, \hat{A}_1, \dots, \hat{A}_p] = (Z'Z)^{-1} Z'Y$$

$$\hat{U} = Y - Z\hat{A}$$

$$\hat{\Sigma}_u = \frac{(\hat{U}'\hat{U})^{-1}}{(N - Kp - p - 1)}$$

2.3.1 Impulse Response Function

Given B_j and u_t , we immediately obtain $w_t = B_0 u_t$. Having identified the structural shocks w_t , our interest usually is not in the shocks themselves, however, but in the responses of each element of $y_t = (y_{1t}, \dots, y_{Kt})$ to a one-time impulse in $w_t = (w_{1t}, \dots, w_{Kt})$

$$\frac{\partial y_{t+i}}{\partial w_t} = \Theta_i, \quad i = 0, 1, 2, \dots, H,$$

where Θ_i is a $K \times K$ matrix. The elements of this matrix for given i are denoted as

$$\theta_{jk,i} = \frac{\partial y_{j,t+i}}{\partial w_{kt}}$$

such that $\Theta_i = [\theta_{jk,i}]$.

By successive substitution for Y_{t-i} , equation can be written as

$$Y_{t+i} = A_{i+1}Y_{t-1} + \sum_{j=0}^i A_j U_{t+i-j}$$

Left-multiplying this equation by $J \equiv [I_K, 0_{K \times K(p-1)}]$ yields

$$y_{t+i} = JA_{i+1}Y_{t-1} + \sum_{j=0}^i JA^j U_{t+i-j} \quad (1)$$

$$= JA^{i+1}Y_{t-1} + \sum_{j=0}^i JA^j J' J U_{t+i-j} \quad (2)$$

$$= JA^{i+1}Y_{t-1} + \sum_{j=0}^i JA^j J u_{t+i-j} \quad (3)$$

$$(4)$$

Thus, the response of the variable $j = 1, \dots, K$ in the VAR(p) system to a unit shock u_{kt} , $k = 1, \dots, K$, i periods ago, is given by:

$$\Phi_{i, K \times K} = [\phi_{jk,i}] \equiv JA^i J'$$

The ϕ_i are also sometimes referred to as responses to VAR forecast errors, as dynamic multipliers, or simply as reduced-form impulse responses.

2.4 Minnesota prior

The simplest form of prior distributions for VAR models is known as the Minnesota (or Litterman) prior. In this framework, it is assumed that the VAR residual variance-covariance matrix Σ is known. Hence, the only object left to estimate is the vector of parameters β . To obtain the posterior distribution for β from 1.2.3, one needs two elements: the likelihood function $f(y|\beta)$ for the data, and a prior distribution $\pi(\beta)$ for β .

$y \sim \mathcal{N}(\tilde{X}\beta, \tilde{\Sigma})$ Therefore, one may write the likelihood for y as:

$$f(y|\beta, \tilde{\Sigma}) = (2\pi)^{-nT/2} |\tilde{\Sigma}|^{-1/2} \exp\left[-\frac{1}{2}(y - \tilde{X}\beta)\tilde{\Sigma}^{-1}(y - \tilde{X}\beta)\right]$$

Ignoring terms independent from β relegated to proportionality simplifies to:

$$f(y|\beta, \tilde{\Sigma}) \propto \exp\left[-\frac{1}{2}(y - \tilde{X}\beta)\tilde{\Sigma}^{-1}(y - \tilde{X}\beta)\right]$$

Now turn to the prior distribution for β . It is assumed that β follows a multivariate normal distribution, with mean β_0 and covariance matrix Ω_0 :

$$\pi(\beta) \sim \mathcal{N}(\beta_0, \Omega_0)$$

To identify β_0 and Ω_0 , Litterman (1986) proposed the following strategy. As most observed macroeconomic variables seem to be characterized by a unit root (in the sense that their changes are impossible to forecast), our prior belief should be that each endogenous variable included in the model presents a unit root in its first own lags, and coefficients equal to zero for further lags and cross-variable lag coefficients. In the absence

of prior belief about exogenous variables, the most reasonable strategy is to assume that they are neutral with respect to the endogenous variables, and hence that their coefficients are equal to zero as well. These elements translate into β_0 being a vector of zeros, save for the entries concerning the first own lag of each endogenous variable which are attributed values of 1. Note though that in the case of variables known to be stationary, this unit root hypothesis may not be suitable, so that a value around 0.8 may be preferred to a value of 1.

3 Methods

We describe our methods in this chapter.

4 Applications

Some *significant* applications are demonstrated in this chapter.

4.1 Example one

4.2 Example two

5 Final Words

We have finished a nice book.

6 Literature

Here is a review of existing methods.

You can label chapter and section titles using `{#label}` after them, e.g., we can reference Chapter 1. If you do not manually label them, there will be automatic labels anyway, e.g., Chapter 3.

Figures and tables with captions will be placed in `figure` and `table` environments, respectively.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

Reference a figure by its code chunk label with the `fig:` prefix, e.g., see Figure 1. Similarly, you can reference tables generated from `knitr::kable()`, e.g., see Table 1.

```
knitr::kable(
  head(iris, 20), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

You can write citations, too. For example, we are using the **bookdown** package [?] in this sample book, which was built on top of R Markdown and **knitr** [?].

7 FAVAR

Factor augmented VAR model introduced in Bernanke et al (2005). The FAVAR model can be written compactly as

$$y_{i,t} = \lambda_i f_t = \gamma_i r_t + \epsilon_{it}$$

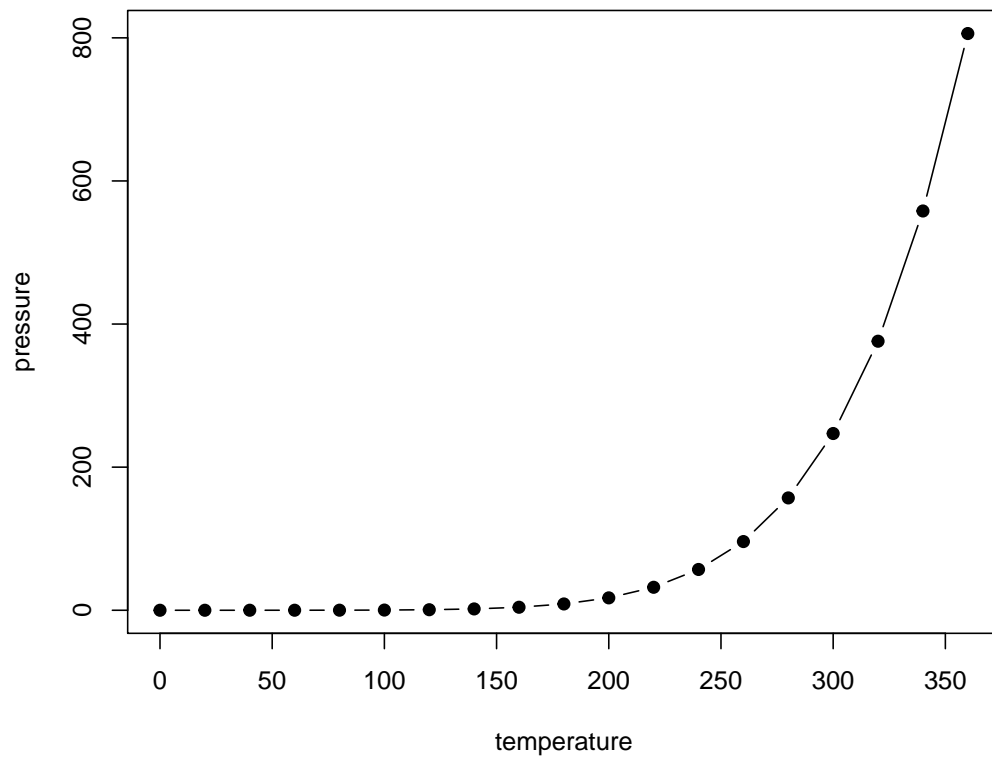


Figure 1: Here is a nice figure!

Table 1: Here is a nice table!

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width	Species
5.1	3.5	1.4	0.2	setosa
4.9	3.0	1.4	0.2	setosa
4.7	3.2	1.3	0.2	setosa
4.6	3.1	1.5	0.2	setosa
5.0	3.6	1.4	0.2	setosa
5.4	3.9	1.7	0.4	setosa
4.6	3.4	1.4	0.3	setosa
5.0	3.4	1.5	0.2	setosa
4.4	2.9	1.4	0.2	setosa
4.9	3.1	1.5	0.1	setosa
5.4	3.7	1.5	0.2	setosa
4.8	3.4	1.6	0.2	setosa
4.8	3.0	1.4	0.1	setosa
4.3	3.0	1.1	0.1	setosa
5.8	4.0	1.2	0.2	setosa
5.7	4.4	1.5	0.4	setosa
5.4	3.9	1.3	0.4	setosa
5.1	3.5	1.4	0.3	setosa
5.7	3.8	1.7	0.3	setosa
5.1	3.8	1.5	0.3	setosa

8 Algorithms

8.1 IRF

8.1.1 Companion

8.1.2 Algorithm 1

1. Create a 2D-array of $Kp \times (h + 1) \times Kp$
2. Assign the shock to the first observation (unit, se, etc)
3. Iterate over the horizons 1, 2, ..., h where

$$irf_{i+1} = irf_i A'$$

8.1.3 Algorithm 2

1. Create a array $J = [I_k, 0_{K \times K(p-1)}]$
2. Create an 3D-array of $Kp \times (h + 1) \times Kp$ to store the irfs
3. Assign the shock to the first observation (unit, se, etc)
4. Iterate over the horizons for $i = 1, 2, \dots, h$ where

$$irf_{i+1} = J A^i J'$$

8.2 Error Bands for Impulse Response Function

8.2.1 Asymptotic

8.2.2 Monte Carlo

8.2.3 Bootstrap

8.2.4 Bootstrap after Bootstrap

Montecarlo method proceeds as follows. 1. Draw $\hat{\Omega}$ from $N(\hat{\Omega}, \hat{Q})$. 2. compute $C(L)$. 3. Repeat 1-2M (with Mbig, i.e. 1000) times. 4. For all the elements $C_{i,j,h}$, $i, j = 1, \dots, n$, $h = 1, 2, \dots$ of the impulse response functions collect the th and $1 - th$ percentile across the draws as a confidence interval for $C_{i,j,h}$.

8.2.5 Algorithm 2.2.1 (impulse response functions, all priors):

1. Define the number of iterations (It – Bu) of the algorithm, and the time horizon h.
2. Fix $i = 1$. Then set $y_i, T = 1$.
3. At iteration n, draw β_n from its posterior distributions. Simply recycle draw n from the Gibbs sampler.
4. Generate recursively the simulated values $\tilde{y}_{T+1}^{(n)}, \tilde{y}_{T+2}^{(n)}, \dots, \tilde{y}_{T+h}^{(n)}$ from 1.1.2: $\tilde{y}_{T+1}^{(n)} = A_1 y_t + A_2 y_{t-1} + \dots + A_p y_{t+1-p}$. Once \tilde{y}_{T+1} is computed, use: $\tilde{y}_{T+2} = A_1 \tilde{y}_{T+1} + A_2 y_T + \dots + A_p y_{T+2-p}$ And continue this way until \tilde{y}_{T+h} is obtained. Once again, both the exogenous terms and the shocks are ignored since they are assumed to take a value of 0 at all periods. The values of A_1, A_2, \dots, A_p come from β_n .
5. Discard β_n to obtain draws $\tilde{y}_{T+1}, \tilde{y}_{T+2}, \dots, \tilde{y}_{T+h}$ from the predictive distribution $f(y_{T+1:T+h} | y_T)$.
6. Repeat until (It – Bu) iterations have been performed. This produces: $\tilde{y}_{T+1}, \tilde{y}_{T+2}, \dots, \tilde{y}_{T+h}$

- (p) $T+h | y^T oIt-j$ = a sample of independent draws from the joint predictive distribution in the case $y_i, T = 1$.
7. Go back to step 2, and fix $i = 2$. Then go through steps 3-6 all over again. Then repeat the process for $i = 3, \dots, n$. This generates the impulse response functions for all the shocks in the model.