Real Estate and Construction Sector Dynamics in the Business Cycle

Online Appendix

Appendix B: Model Equations

The budget constraint for the household is:

$$C_{d,t} + q_{hd,t}H_{d,t} + \frac{S_t}{R_t} \leq q_{hd,t}(1-\delta_{hd})H_{d,t-1} + w_{c,t}N_{c,t} + w_{h,t}N_{hc,t} + w_{h,t}N_{hd,t} + S_{t-1} + q_{l,t}L^{ep}_{hd,t}.$$

The first-order conditions for households are:

$$u_{cd,t}w_{c,t} = z_t\psi_t(N_{c,t}^{1+\xi} + (N_{hc,t}+N_{hd,t})^{1+\xi})^{\frac{\eta-\xi}{1+\xi}})N_{c,t}^{\xi}, \tag{B1}$$

$$u_{cd,t}w_{h,t} = z_t\psi_t(N_{c,t}^{1+\xi} + (N_{hc,t}+N_{hd,t})^{1+\xi})^{\frac{\eta-\xi}{1+\xi}})(N_{hc,t}+N_{hd,t})^{\xi}, \tag{B2}$$

$$u_{cd,t} = \beta_d R_t u_{cd,t+1}, \tag{B3}$$

$$u_{cd,t}q_{hd,t} = \frac{\chi_t z_t}{H_{d,t}} + \beta_d E_t\big(u_{cd,t+1} + (1-\delta_{hd})q_{hd,t+1}\big) \tag{B4}$$

$$u_{cd,t} = z_t\left(\frac{1}{C_{d,t}-\gamma_d C_{d,t-1}} - E_t\frac{\beta_d\gamma_d}{C_{d,t+1}-\gamma_d C_{d,t}}\right). \tag{B5}$$

The budget and borrowing constraint for consumption-good entrepreneur are:

$$\begin{aligned} &C_{c,t} + K_{c,t} + q_{hc,t}H_{c,t} + w_{c,t}N_{c,t} + B_{c,t-1} \\ &\qquad = Y_t + (1-\delta_{kc})K_{c,t-1} + (1-\delta_{hc})q_{hc,t}H_{c,t-1} + \frac{B_{c,t}}{R_t} + q_{l,t}L^{ep}_{hc,t} - \phi_{c,t} \end{aligned} \tag{B6}$$

$$B_{c,t} \leq \rho_b B_{c,t-1} + (1-\rho_b)\theta_c E_t\left[q_{hc,t+1}H_{c,t} + \frac{K_{c,t}}{A_{kc,t}}\right]. \tag{B7}$$

the production technology is:

$$Y_t = K_{c,t-1}^{\alpha_c}H_{c,t-1}^{\mu_c}\left(A_{c,t}N_{c,t}\right)^{1-\alpha_c-\mu_c}. \tag{B8}$$

and the first order conditions are:

$$(1-\alpha_c-\mu_c)Y_t = w_{c,t}N_{c,t}, \tag{B9}$$

$$u_{cc,t}\left(1+\frac{\partial\phi_{c,t}}{\partial K_{c,t}}\right) \tag{B10}$$
$$=\beta_e E_t u_{cc,t+1}\left(\alpha_c\frac{Y_{t+1}}{K_{c,t}}+(1-\delta_{kc})-\frac{\partial\phi_{c,t+1}}{\partial K_{c,t}}\right)+\lambda_{bc,t}(1-\rho_b)\theta_c,$$

$$u_{cc,t}q_{hc,t} \tag{B11}$$
$$=\beta_e E_t u_{cc,t+1}\left(\mu_c\frac{Y_{t+1}}{H_{c,t}}+(1-\delta_{hc})q_{hc,t+1}\right)+\lambda_{bc,t}(1-\rho_b)\theta_c q_{hc,t+1},$$

$$u_{cc,t}=\beta_e R_t u_{cc,t+1}+\lambda_{c,t}-\rho_b\beta_e\lambda_{c,t-1}, \tag{B12}$$

$$u_{cc,t}=\left(\frac{1}{C_{c,t}-\gamma_e C_{c,t-1}}-E_t\frac{\beta_e\gamma_e}{C_{c,t+1}-\gamma_e C_{c,t}}\right), \tag{B13}$$

where $\lambda_{bc,t}$ denotes the multiplied on the borrowing constraint, which is greater than zero in a neighborhood of the equilibrium.
The budget and borrowing constraint for construction-sector entrepreneur are:

$$C_{h,t}+K_{hc,t}+K_{hd,t}+q_{l,t}\left(L_{hc,t}+L_{hd,t}\right)+w_{h,t}\left(N_{hc,t}+N_{hd,t}\right)+B_{h,t-1}$$
$$=q_{hc,t}IH_{c,t}+q_{hd,t}IH_{d,t}+(1-\delta_{kh})K_{hc,t}+(1-\delta_{kh})K_{hd,t}+q_{l,t}\left(L_{hc,t}+L_{hd,t}\right)+\frac{B_{h,t}}{R_t}, \tag{B14}$$

$$B_{h,t}\le\rho_b B_{h,t-1}+(1-\rho_b)\theta_h E_t[q_{l,t+1}\left(L_{hc,t}+L_{hd,t}\right)+K_{hc,t}+K_{hd,t}]. \tag{B15}$$

The production technologies are:

$$IH_{c,t}=K_{hc,t-1}^{\alpha_{hc}}L_{hc,t-1}^{\mu_h}\left(A_{h,t},N_{hc,t}\right)^{1-\alpha_{hc}-\mu_h} \tag{B16}$$

$$IH_{d,t}=A_{hd,t}K_{hd,t-1}^{\alpha_{hd}}L_{hd,t-1}^{\mu_h}\left(A_{hd,t},N_{h,t}\right)^{1-\alpha_{hd}-\mu_h} \tag{B17}$$

and the first order conditions are:

$$(1-\alpha_{hc}-\mu_h)q_{hc,t}IH_{c,t}=w_{h,t}N_{hc,t}, \tag{B18}$$

$$(1-\alpha_{hd}-\mu_h)q_{hd,t}IH_{d,t}=w_{h,t}N_{hd,t}, \tag{B19}$$

$$u_{ch,t}\left(1+\frac{\partial\phi_{hc,t}}{\partial K_{hc,t}}\right) \tag{B20}$$
$$=\beta_e E_t u_{ch,t+1}\left(\alpha_{hc}\frac{q_{hc,t}IH_{c,t+1}}{K_{hc,t}}+(1-\delta_{khc})-\frac{\partial\phi_{hc,t+1}}{\partial K_{hc,t}}\right)+\lambda_{bh,t}(1-\rho_b)\theta_h,$$

$$u_{ch,t}\left(1+\frac{\partial\phi_{hd,t}}{\partial K_{hd,t}}\right) \tag{B21}$$
$$=\beta_e E_t u_{ch,t+1}\left(\alpha_{hd}\frac{q_{hd,t}IH_{c,t+1}}{K_{hd,t}}+(1-\delta_{khd})-\frac{\partial\phi_{hd,t+1}}{\partial K_{hd,t}}\right)+\lambda_{bh,t}(1-\rho_b)\theta_h,$$

$$u_{ch,t}q_{l,t}=\beta_e E_t u_{ch,t+1}\left(\mu_h\frac{q_{hc,t}IH_{c,t+1}}{L_{hc,t}}\right)+\lambda_{bh,t}(1-\rho_b)\theta_h q_{l,t+1}, \tag{B22}$$

$$u_{ch,t}q_{l,t}=\beta_e E_t u_{ch,t+1}\left(\mu_h\frac{q_{hd,t}IH_{d,t+1}}{L_{hd,t}}\right)+\lambda_{bh,t}(1-\rho_b)\theta_h q_{l,t+1}, \tag{B23}$$

$$u_{ch,t}=\beta_e R_t u_{ch,t+1}+\lambda_{bh,t}-\rho_b\beta_e\lambda_{bh,t-1}, \tag{B24}$$

$$u_{ch,t}=\left(\frac{1}{C_{h,t}-\gamma_e C_{h,t-1}}-E_t\frac{\beta_e\gamma_e}{C_{h,t+1}-\gamma_e C_{h,t}}\right). \tag{B25}$$

The market-clearing conditions are:

$$Y_t-\phi_t=C_t+IB_t, \tag{B26}$$

$$GDP_t=Y_t+\bar{q_{hd}}IH_{d,t}. \tag{B27}$$

The evolution of commercial and residential real estate are:

$$IH_{c,t}=H_{c,t}-(1-\delta_{hc})H_{c,t-1}, \tag{B28}$$

$$IH_{d,t}=H_{d,t}-(1-\delta_{hd})H_{d,t-1}. \tag{B29}$$

The land is fixed and equal to:

$$\bar{L_h}=(L_{hc,t}+L_{hd,t}). \tag{B30}$$

The aggregate consumption and business investment are:

$$C_t=C_{d,t}+C_{c,t}+C_{h,t}, \tag{B31}$$

$$IK_{c,t} = K_{c,t} - (1-\delta_{kc})K_{c,t-1}. \tag{B32}$$

Construction capital is equal to

$$IK_{h,t} = K_{hc,t} - (1-\delta_{kh})K_{hc,t-1} + K_{hd,t} - -(1-\delta_{kh})K_{hd,t}. \tag{B33}$$

Non-construction capital is equal to

$$IK_t = IK_{c,t} + IK_{h,t}. \tag{B34}$$

Business Investment

$$IB_t = IK_t + \bar{q}_c IH_{c,t}; \tag{B35}$$

Ex-post land equations are:

$$L^{ep}_{hc} = \frac{H_{c,t}}{(H_{c,t} + H_{c,t})}\bar{L}, \tag{B36}$$

$$L^{ep}_{hd} = \frac{H_{d,t}}{(H_{c,t} + H_{c,t})}\bar{L}. \tag{B37}$$

A competitive equilibrium consists of a sequence of allocation $\{C_{d,t}, H_{d,t}, N_{c,t}, N_{hc,t}, N_{hd,t}, S_t, C_{c,t}, K_{c,t}, H_{c,t}, B_{c,t}, C_{h,t}, IH_{c,t}, K_{hc,t}, L_{hc,t}, IH_{d,t}, K_{hd,t}, L_{hd,t}\}_{t=0}^{\infty}$ and prices $\{w_{c,t}, w_{hc,t}, w_{hd,t}, q_{hd,t}, q_{hd,t}, q_{l,t}, R_t\}_{t=0}^{\infty}$ such that

(i) given the $\{w_{c,t}, w_{hc,t}, w_{hd,t}, q_{hd,t}, R_t\}_{t=0}^{\infty}$ the sequence $\{C_{d,t}, H_{d,t}, N_{c,t}, N_{hc,t}, N_{hd,t}, S_t\}_{t=0}^{\infty}$ solves the household's problem,

(ii) given the $\{w_{c,t}, q_{hc,t}, R_t\}_{t=0}^{\infty}$ the sequence $\{C_{c,t}, K_{c,t}, H_{c,t}, N_{c,t}, B_{c,t}\}_{t=0}^{\infty}$ solves the entrepreneur in the consumption good sector problem,

(iii) given the $\{w_{hc,t}, q_{hc,t}, w_{hd,t}, q_{hd,t}, q_{l,t}, R_t\}_{t=0}^{\infty}$ the sequence $\{C_{h,t}, IH_{c,t}, K_{hc,t}, L_{hc,t}, N_{hc,t}, B_{h,t}, IH_{d,t}K_{hd,t}, L_{hd,t}, N_{hd,t}\}_{t=0}^{\infty}$ solves the entrepreneur in the construction sector problem,

(iv) all markets clear.

7 Appendix C: Steady State

From the Euler equation we can derive that:

$$R = \frac{1}{\beta_d}.$$

The ratio of the Lagrange multipliers to the marginal utility of consumption is equal to:

$$\frac{\lambda_c}{u_{ce}} = \frac{\beta_d - \beta_e}{1 - \rho_b \beta_e} \qquad \& \qquad \frac{\lambda_h}{u_{ce}} = \frac{\beta_d - \beta_e}{1 - \rho_b \beta_e}.$$

The ratio of the construction capital for commercial structures to commercial real estate is:

$$\frac{K_{hc}}{q_{hc} IH_c} = \frac{\alpha_{hc}\beta_e}{1 - \beta_e(1 - \delta_{khc}) - \frac{\lambda_{hc}}{u_{ce}}(1 - \rho_b)\theta_{hc}},$$

and investment for commercial structures is:

$$\frac{I_{hc}}{q_{hc} IH_c} = \delta_{khc} \frac{K_{hc}}{q_{hc} IH_c}.$$

The ratio of the construction capital for residential structures to residential real estate is:

$$\frac{K_{hd}}{q_{hd} IH_d} = \frac{\alpha_{hd}\beta_e}{1 - \beta_e(1 - \delta_{khd}) - \frac{\lambda_{hd}}{u_{ce}}(1 - \rho_b)\theta_{hd}}$$

and investment for residential structures is:

$$\frac{I_{hd}}{q_{hd} IH_d} = \delta_{khd} \frac{K_{hd}}{q_{hd} IH_d}.$$

Land to both type of real estate are:

$$\frac{q_l L_{hc}}{q_{hc} IH_c} = \frac{\mu_h \beta_e}{1 - \beta_e - \frac{\lambda_{hc}}{u_{ce}}(1 - \rho_b)\theta_{hc}},$$

$$\frac{q_l L_{hd}}{q_{hd} IH_d} = \frac{\mu_h \beta_e}{1 - \beta_e - \frac{\lambda_{hd}}{u_{ce}}(1 - \rho_b)\theta_{hd}}.$$

From the FOC in the consumption good we can derive that:

$$\frac{K_c}{Y} = \frac{\alpha_c \beta_e}{1 - \beta_e(1 - \delta_{kc}) - \frac{\lambda_c}{u_{ce}}\theta_c(1 - \rho_b)},$$

$$\frac{I_c}{Y} = \delta_{kc} \frac{K_c}{Y},$$

$$\frac{q_{hc} H_c}{Y} = \frac{\mu_c \beta_e}{1 - \beta_e(1 - \delta_{hc}) - \frac{\lambda_c}{u_{ce}}\theta_c(1 - \rho_b)},$$

$$\frac{q_{hc} IH_c}{Y} = \delta_{hc} \frac{q_{hc} H_c}{Y},$$

$$\frac{B_c}{Y} = \theta_c \left(\frac{q_{hc} H_c}{Y} + \frac{K_c}{Y} \right),$$

and from the budge constraint in the consumption good

$$\frac{C_c}{Y} = \alpha_c + \mu_c - \delta'_{kc}\frac{K_c}{Y} - \delta'_{hc}\frac{q_{hc}H_c}{Y} + R'\frac{B_c}{Y}.$$

From the marginal marginal utility for residential real estate we can derive that:

$$\frac{q_{hd}H_d}{C_d} = \frac{\chi}{(1-\beta_d(1-\delta_{hd}))},$$

and then we use the auxiliary variables x_1 and x_2 to help us with the calculations. We derive that:

$$x_1 = \alpha_{hc} + \mu_h - \frac{I_{hc}}{q_{hc}IH_c} + \theta_h * \left(\frac{q_l L_{hc}}{q_{hc}IH_c} + \frac{K_{hc}}{q_{hc}IH_c}\right),$$
$$x_2 = \alpha_{hc} + \mu_{hha} - \frac{I_{hc}}{q_{hc}IH_c} + \theta_h * \left(\frac{q_l L_{hc}}{q_{hc}IH_c} + \frac{K_{hc}}{q_{hc}IH_c}\right),$$

$$\frac{C_d}{Y} = \frac{1-\left(\frac{C_c}{Y}+\frac{I_c}{Y}\right) - \frac{q_{hc}IH_c}{Y}\left(1-\frac{I_{hc}}{q_{hc}IH_c}+x_1\right)}{1+\delta_{hd}\frac{q_{hd}IH_d}{C_d}\left(\frac{I_{hd}}{q_{hd}IH_d}+x2\right)},$$
$$\frac{q_{hd}H_D}{Y} = \frac{q_{hd}H_d}{C_d}\frac{C_d}{Y},$$

$$\frac{q_{hd}IH_d}{Y} = \delta_{hd}\frac{q_{hd}H_d}{Y}.$$

To calculate the individual hours we need to combine the labour demand for consumption good, commercial and residential real estate:

$$\frac{N_{hd}}{N_{hc}} = \frac{(1-\alpha_{hd}-\mu_h)}{(1-\alpha_{hc}-\mu_h}\frac{q_{hd}IH_d}{q_{hc}IH_c},$$

$$\frac{N_{hc}}{N_c} = \left(\frac{\frac{(1-\alpha_{hc}-\mu_h)}{(1-\alpha_c-\mu_c})\frac{q_{hc}IH_c}{Y}}{\left(1+\frac{N_{hd}}{N_{hc}}\right)^{\xi}}\right)^{\frac{1}{1+\xi}},$$

$$\frac{N_{hd}}{N_c} = \frac{N_{hd}}{N_{hc}}\frac{N_{hc}}{N_c},$$

and then we can find the levels of individual hours with:

$$N_c = \left(\frac{(1-\alpha_c-\mu_c)\frac{Y}{Cd}}{\left(1+\left(\frac{N_{hc}}{N_c}+\frac{N_{hd}}{N_c}\right)^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}}}\right)^{\frac{1}{1+\eta}},$$

$$N_{hc} = \frac{N_{hc}}{N_c} Nc \quad \& \quad N_{hd} = \frac{N_{hd}}{N_c} Nc.$$

From the demand for land for commercial and residential use we can find that:

$$\frac{q_l L_h c}{Y} = \frac{q_l L_{hc}}{q_{hc} IH_c} \frac{q_{hc} IH_c}{Y},$$

$$\frac{q_l L_h d}{Y} = \frac{q_l L_{hd}}{q_{hd} IH_d} \frac{q_{hd} IH_c}{Y}.$$

From land supply we know that $\bar{L}_h = 1$, so:

$$\frac{q_l}{Y} = \frac{q_l L_{hc}}{Y} + \frac{q_l L_{hd}}{Y}$$

And then we can find the individual levels of land with

$$L_{hc} = \frac{q_l L_{hc}}{Y} + \frac{q_l}{Y},$$

$$L_{hd} = \frac{q_l L_{hd}}{Y} + \frac{q_l}{Y}.$$

From the production technologies we find that:

$$q_{hc}^{\frac{\mu_c}{1-\alpha_c-\mu_c}} Y = \left(\frac{K_c}{Y}\right)^{\frac{\alpha_c}{1-\alpha_c-\mu_c}} \left(\frac{q_{hc} H_c}{Y}\right)^{\frac{\mu_c}{1-\alpha_c-\mu_c}} Nc,$$

$$q_{hc}^{-\frac{\alpha_{hc}}{1-\alpha_{hc}}} IH_c = \left(\frac{K_{hc}}{q_{hc} IH_c}\right)^{\frac{\alpha_{hc}}{1-\alpha_{hc}}} L_{hc}^{\frac{\mu_h}{1-\alpha_{hc}}} N_c^{\frac{1-\alpha_{hc}-\mu_h}{1-\alpha_{hc}}}.$$

Combining these two we can find the levels:

$$q_{hc} = \left(\frac{q_{hc}^{\frac{\mu_c}{1-\alpha_c-\mu_c}} Y}{q_{hc}^{-\frac{\alpha_{hc}}{1-\alpha_{hc}}} IH_c} \frac{q_{hc} IH_c}{Y} \right)^{1+\frac{\mu_c}{1-\alpha_c-\mu_c}+\frac{\alpha_{hc}}{1-\alpha_{hc}}}$$

$$Y = \left(\frac{K_c}{Y}\right)^{\frac{\alpha_c}{1-\alpha_c-\mu_c}} \left(\frac{q_{hc} H_c}{Y}\right)^{\frac{\mu_c}{1-\alpha_c-\mu_c}} q_{hc}^{-\frac{\mu_c}{1-\alpha_c-\mu_c}} N_c$$

$$IH_c = \left(\frac{K_{hc}}{q_{hc} IH_c}\right)^{\frac{\alpha_{hc}}{1-\alpha_{hc}}} q_{hc}^{\frac{\alpha_{hc}}{1-\alpha_{hc}}} L_{hc}^{\frac{\mu_h}{1-\alpha_{hc}}} N_{hc}^{\frac{1-\alpha_{hc}-\mu_h}{1-\alpha_{hc}}}$$

$$IH_d = \left(\frac{K_{hd}}{q_{hd} IH_d}\right)^{\alpha_{hc}} \left(\frac{q_{hd} IH_d}{Y} Y\right)^{\alpha_{hd}} L_{hd}^{\alpha_{hd}} N_{hd}^{1-\alpha_{hd}-\mu_h}.$$

Having solved for $Y, IH_c, q_{hc}, IH_d, q_{hd}$ we can substitute and solve for the rest of the variables $q_l, H_d, H_c, C_d, C_c, C_h, K_c, K_{hc}, K_{hd}, B_c, B_h, w_c, w_h \ldots$

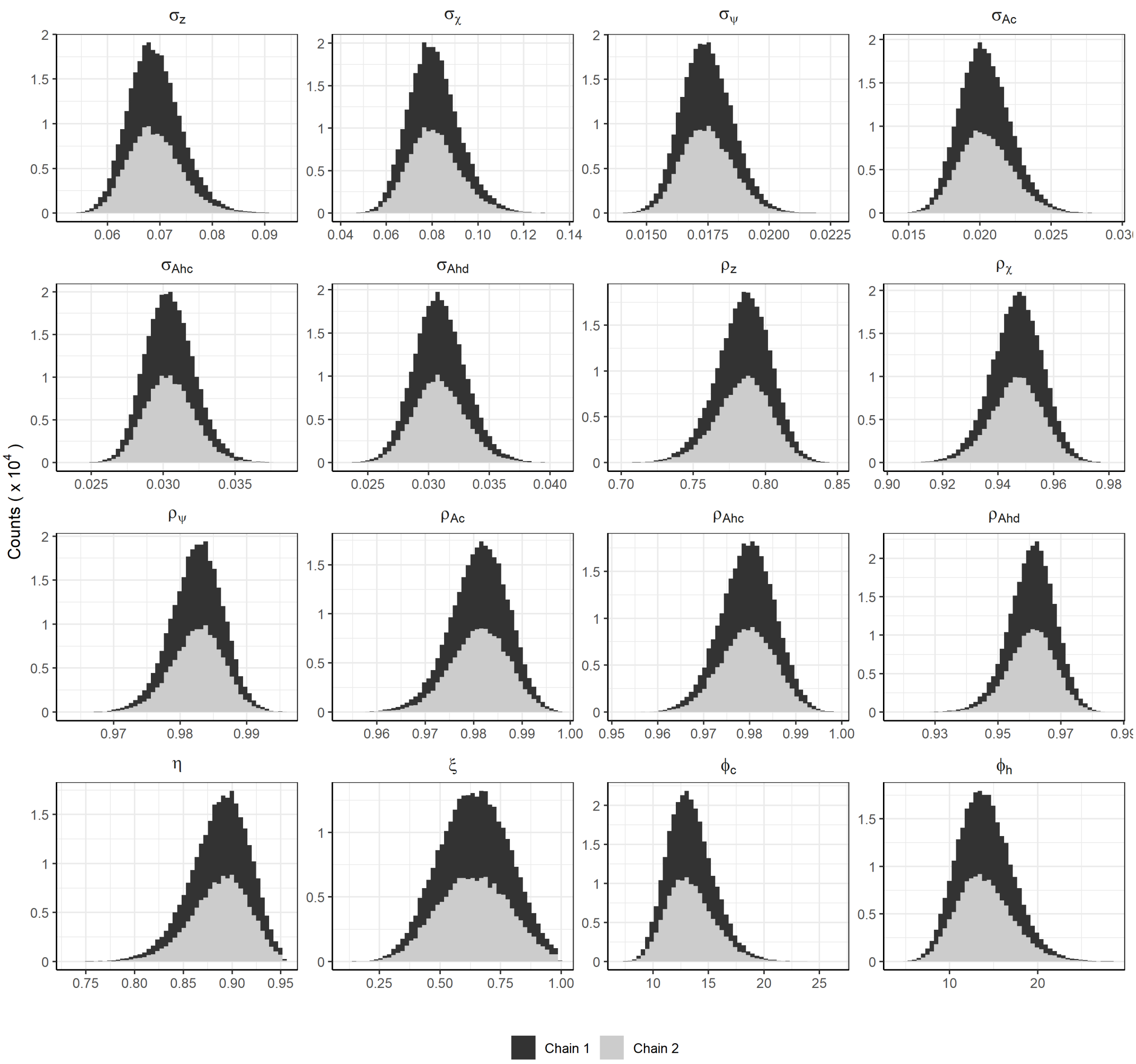
Appendix D: Estimation Details

The parameters of the model are estimated using Bayesian methods. We use Bayesian methods because they allow incorporating a priori information on the parameters of the model. Convergence of the algorithm is assessed by looking at the plots of the draws, the Brooks and Gelman (1998) statistics, and by computing recursively the mean of the marginal posterior distribution of each parameter.

D.1 Posterior Densities

In the following graphs we report the posterior densities of selected parameters for both chains. The posterior ones are based on 200,000 draws from the Metropolis algorithm.

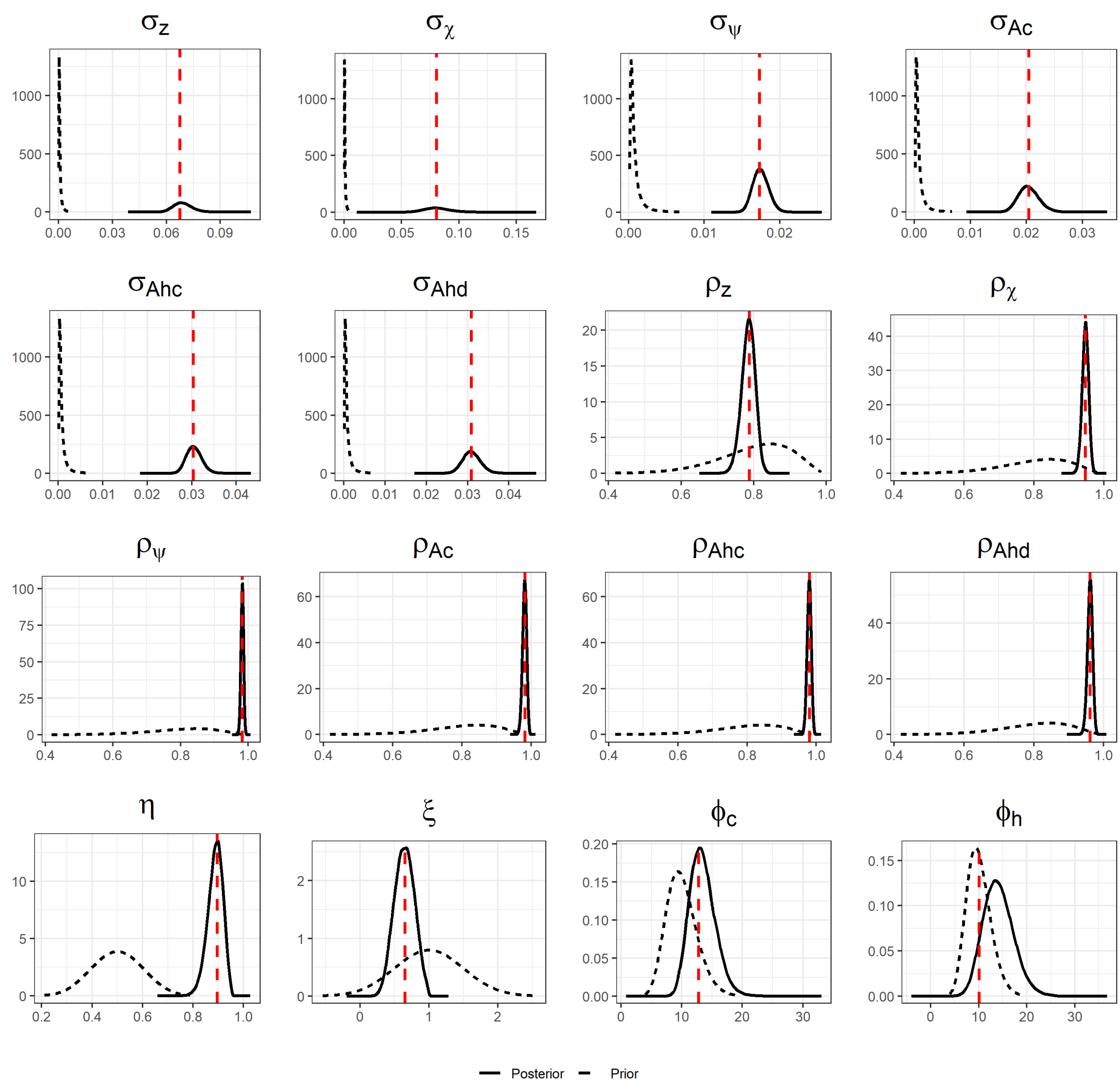
Figure 12 – Posterior Densities



D.2 Prior and posterior densities

In the following graphs we report the prior and posterior densities of selected parameters. The posterior ones are based on 200,000 draws from the Metropolis algorithm and are estimated using a Gaussian kernel.

Figure 13 – Prior & Posterior Densities



Notes: Solid lines denote the posterior density while the dashed lines denote the prior density. Vertical red lines correspond to the posterior mode.

D.3 The output of the Metropolis

The following graphs report the time series of the draws from the posterior distribution generated by the Metropolis algorithm.

Figure 14 – Posterior Density Traceplot

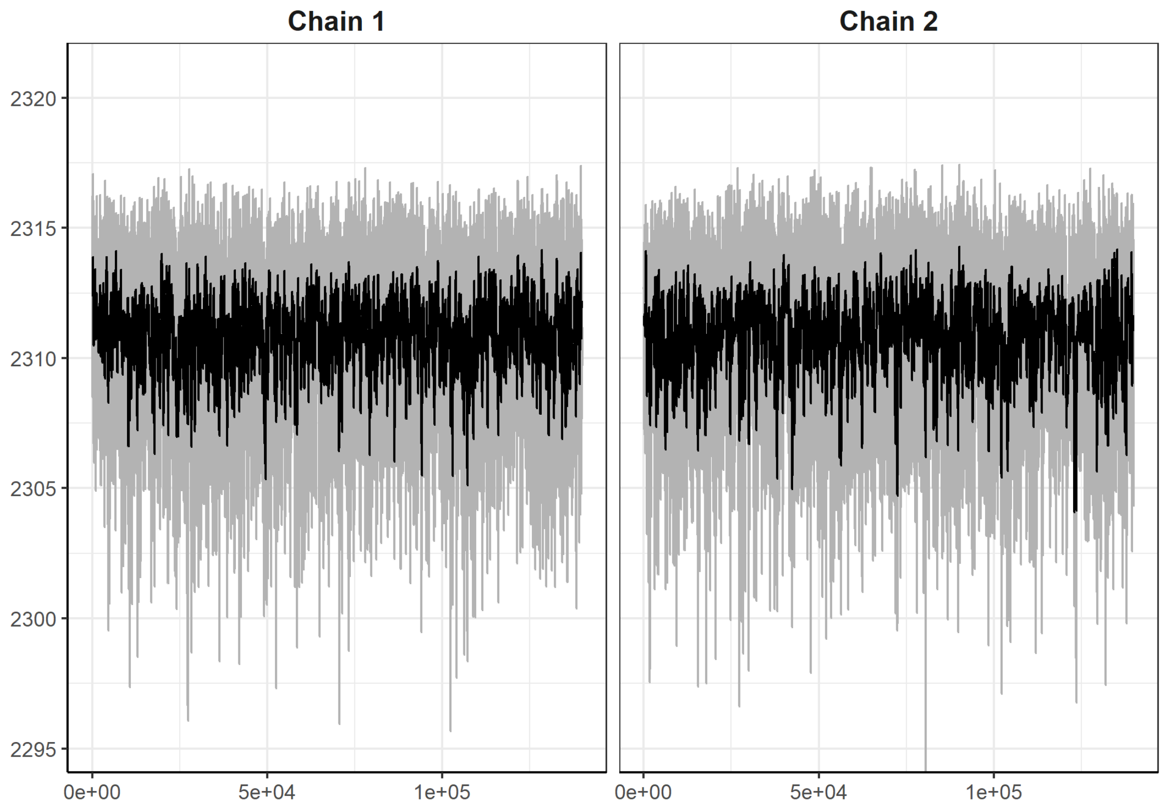


Figure 15 – Structural Shock Traceplot - Chain 1

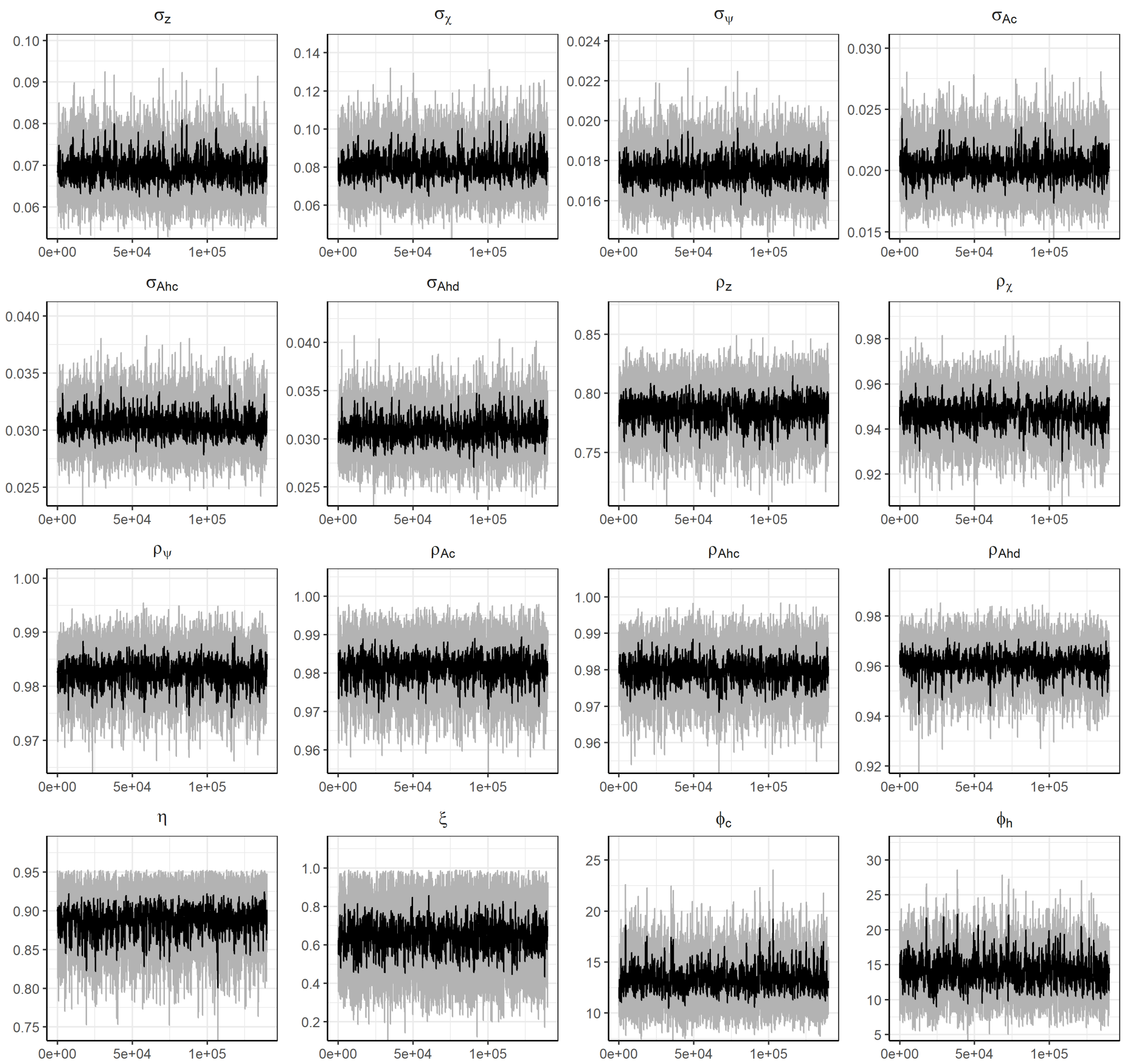
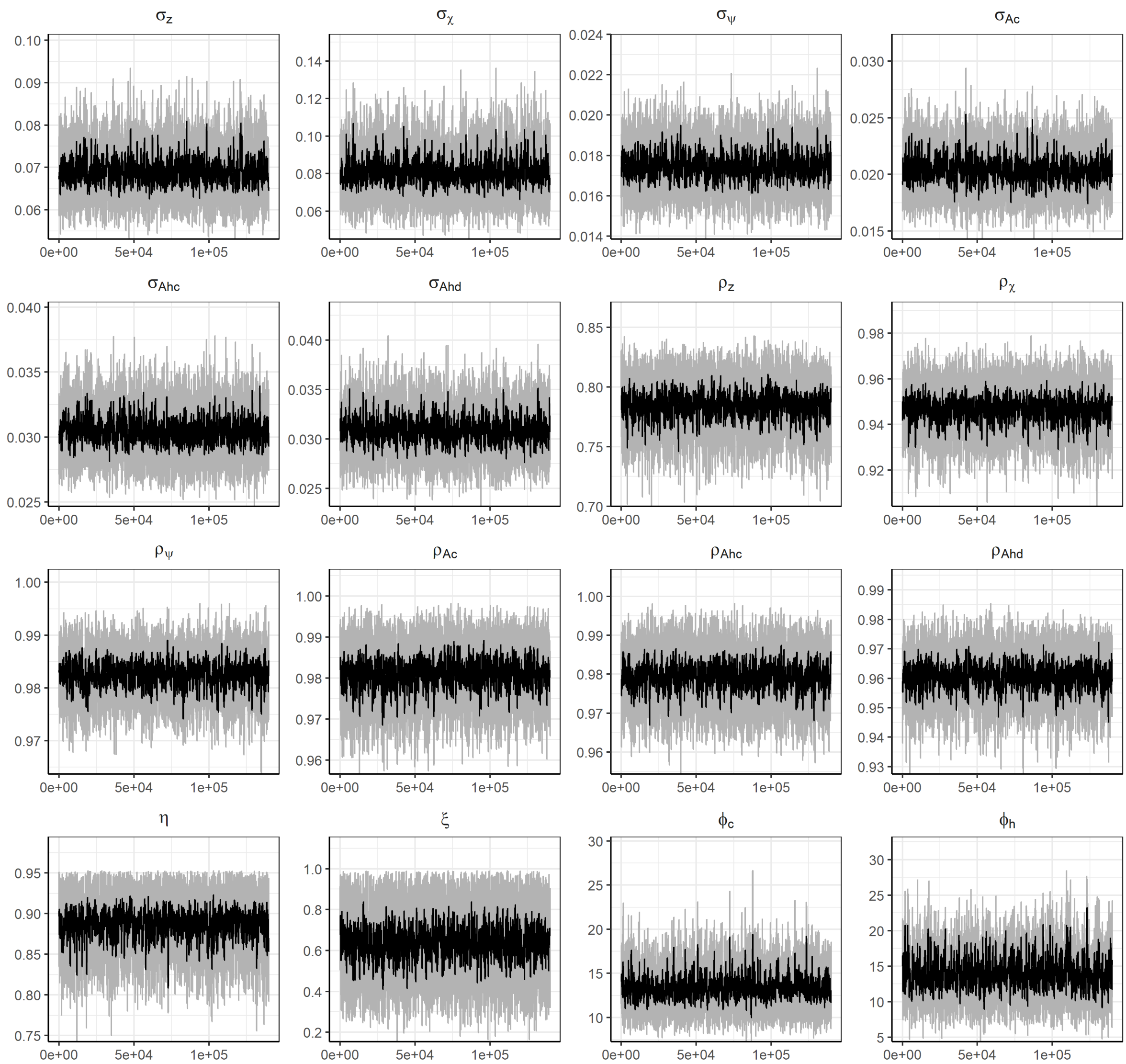


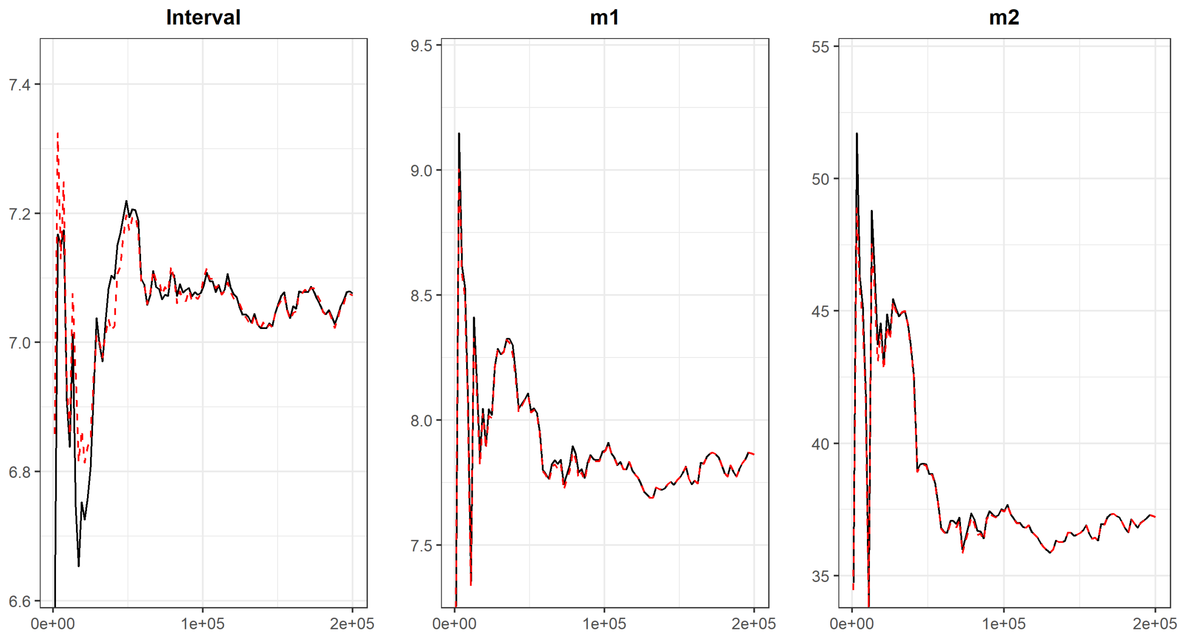
Figure 16 – Structural Shock Traceplot - Chain 2



D.4 Brooks and Gelman (1998) Diagnostics

Brooks and Gelman (1998) convergence diagnostics is based on comparing pooled and within chain variability of MC draws using the 80% interval/quantile range. The convergence diagnostics displays the 80% interval range of draws from the pooled and within chain means as well as the 80% interval range of the second and third central moments (squared and cubed absolute deviations).

Figure 17 – Multivariate Diagnostics



D.4.1 Univariate Diagnostics

Figure 18 – Univariate Diagnostic - Interval

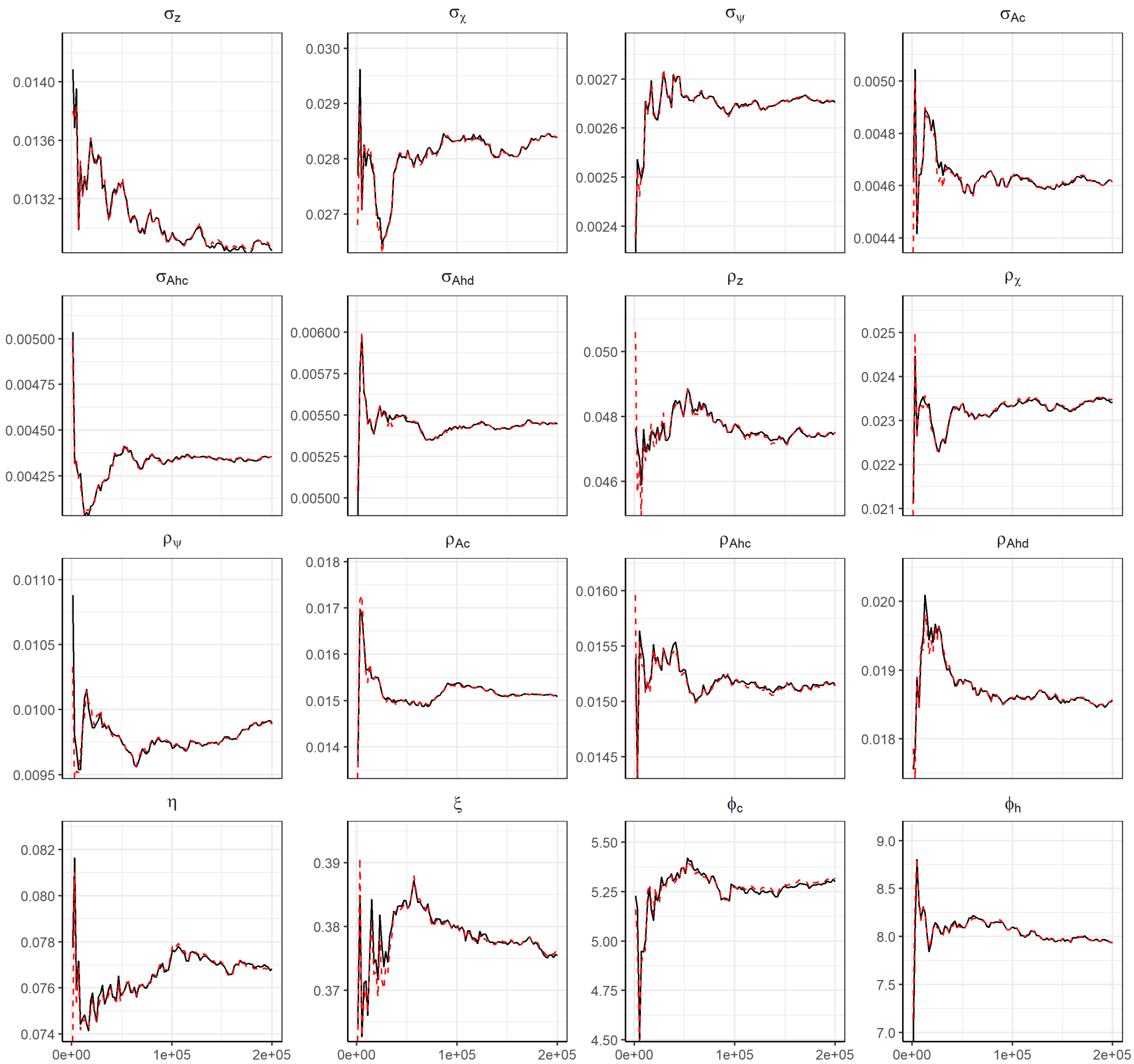


Figure 19 – Univariate Diagnostic - m1

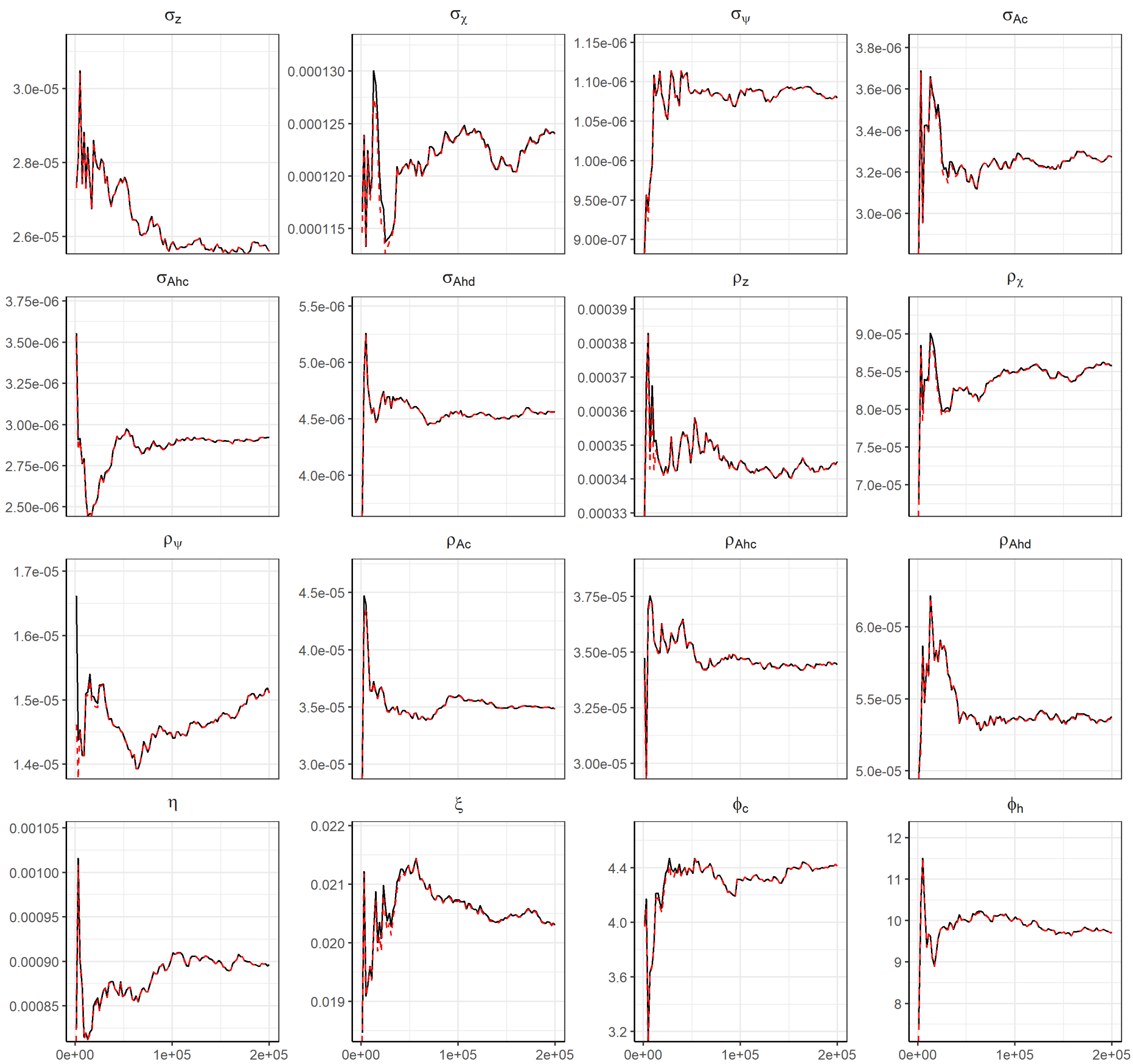
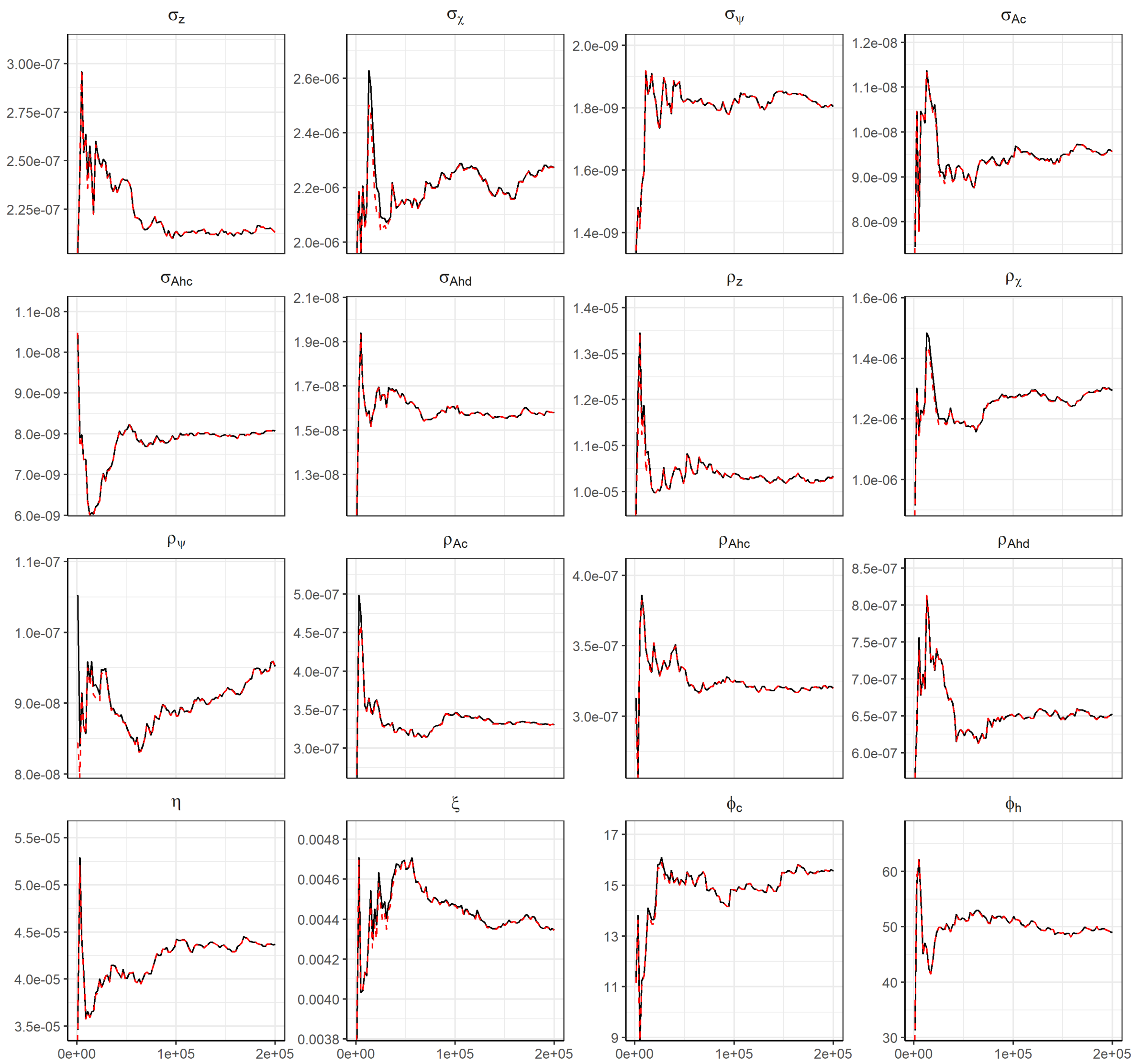


Figure 20 – Univariate Diagnostic - m2



D.5 Recursive Mean

Figure 21 – Recursive Mean - Chain 1

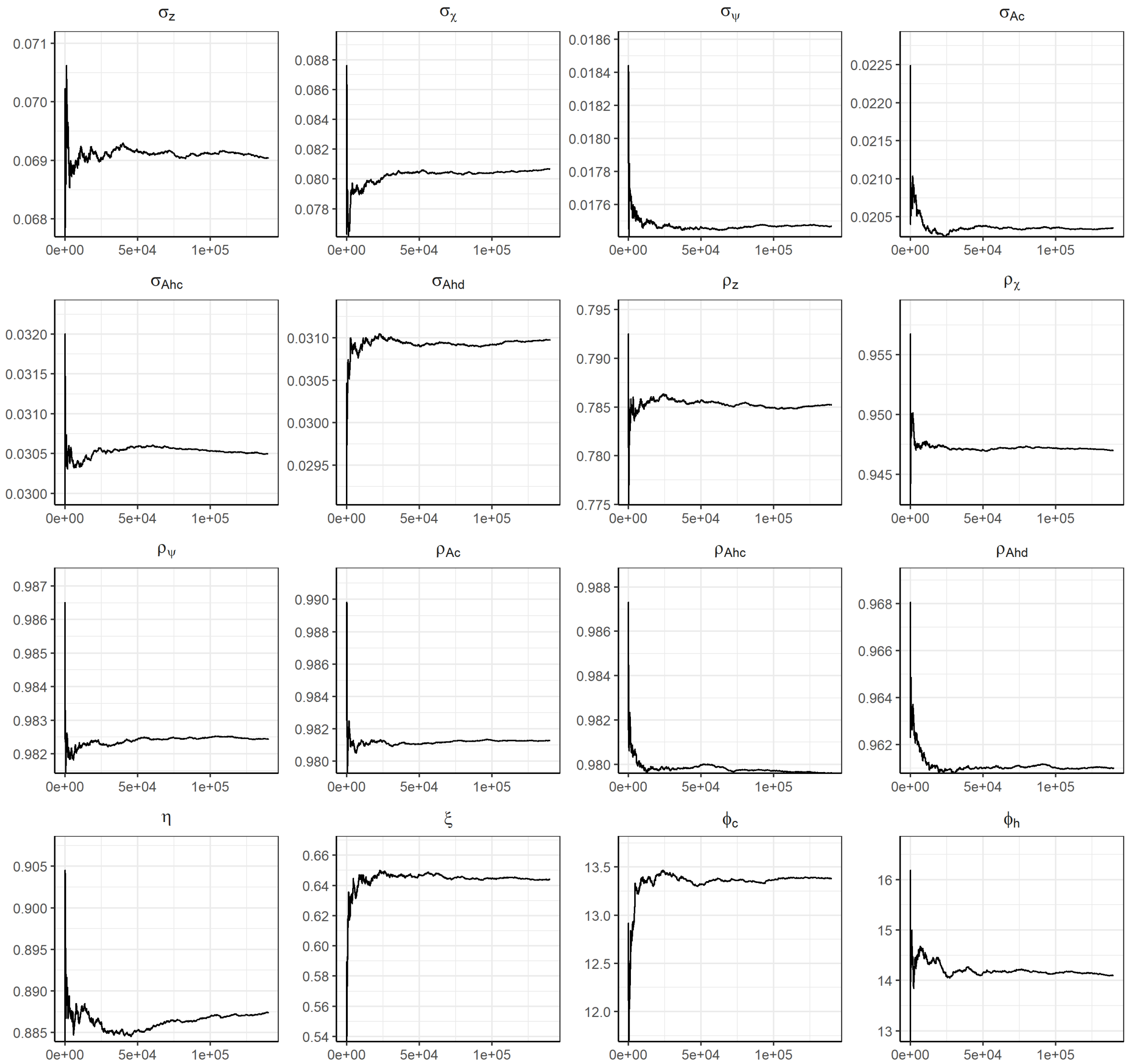


Figure 22 – Recursive Mean - Chain 2