Commercial Real Estate, Housing and the Business Cycle

Online Appendix

Appendix B: Model Equations

The budget constraint for the household is:

$$C_{d,t} + q_{hd,t}H_{d,t} + \frac{S_t}{R_t} = q_{hd,t}(1-\delta_{hd})H_{d,t-1} + w_{c,t}N_{c,t} + w_{h,t}N_{hc,t} + w_{h,t}N_{hc,t} + S_{t-1}$$

The first-order conditions for households are:

$$u_{cd,t}w_{c,t} = z_t\psi_t(N_{c,t}^{1+\xi} + (N_{hc,t}+N_{hd,t})^{1+\xi})^{\frac{\eta-\xi}{1+\xi}})N_{c,t}^{\xi} \tag{B1}$$

$$u_{cd,t}w_{h,t} = z_t\psi_t(N_{c,t}^{1+\xi} + (N_{hc,t}+N_{hd,t})^{1+\xi})^{\frac{\eta-\xi}{1+\xi}})(N_{hc,t}+N_{hd,t})^{\xi} \tag{B2}$$

$$u_{cd,t} = \beta_d R_t u_{cd,t+1} \tag{B3}$$

$$q_{hd,t}u_{cd,t} = \frac{\chi_t z_t}{H_{d,t}} + \beta_d E_t\big(u_{cd,t+1}(1-\delta_{hd})q_{hd,t+1}\big) \tag{B4}$$

$$u_{cd,t} = z_t\left(\frac{1}{C_{d,t}-\gamma_d C_{d,t-1}} - E_t\frac{\beta_d\gamma_d}{C_{d,t+1}-\gamma_d C_{d,t}}\right) \tag{B5}$$

The budget and borrowing constraint for consumption-good entrepreneur are:

$$\begin{aligned} C_{c,t} + \frac{K_{c,t}}{A_{kc,t}} + q_{hc,t}H_{c,t} + w_{c,t}N_{c,t} + B_{c,t-1} \\ = Y_t + (1-\delta_{kc})K_{c,t-1} + (1-\delta_{hc})q_{hc,t}H_{c,t-1} + \frac{B_{c,t}}{R_t} - \phi_{c,t} \end{aligned} \tag{B6}$$

$$B_{c,t} \le \rho_{bc}B_{c,t-1} + (1-\rho_{bc})\theta_c E_t\left[q_{hc,t+1}H_{c,t} + \frac{K_{c,t}}{A_{kc,t}}\right] \tag{B7}$$

the production technology is:

$$Y_t = K_{c,t-1}^{\alpha_c}H_{c,t-1}^{\mu_c}\left(A_{c,t}N_{c,t}\right)^{1-\alpha_c-\mu_c} \tag{B8}$$

and the first order conditions are:

$$(1-\alpha_c-\mu_c)Y_t = w_{c,t}N_{c,t} \tag{B9}$$

$$u_{cc,t}\left(1+\frac{\partial\phi_{c,t}}{\partial K_{c,t}}\right) \tag{B10}$$
$$=\beta_e E_t u_{cc,t+1}\left(\alpha_c\frac{Y_{t+1}}{K_{c,t}}+(1-\delta_{kc})-\frac{\partial\phi_{c,t+1}}{\partial K_{c,t}}\right)+\lambda_{bc,t}(1-\rho_{bc})\theta_c$$

$$u_{cc,t}q_{hc,t} \tag{B11}$$
$$=\beta_e E_t u_{cc,t+1}\left(\mu_c\frac{Y_{t+1}}{H_{c,t}}+(1-\delta_{hc})q_{hc,t+1}\right)+\lambda_{bc,t}(1-\rho_{bc})\theta_c q_{hc,t+1}$$

$$u_{cc,t}=\beta_e R_t u_{cc,t+1}+\lambda_{c,t}-\rho_{bc}\beta_e\lambda_{c,t-1} \tag{B12}$$

$$u_{cc,t}=\left(\frac{1}{C_{c,t}-\gamma_e C_{c,t-1}}-E_t\frac{\beta_e\gamma_e}{C_{c,t+1}-\gamma_e C_{c,t}}\right) \tag{B13}$$

where $\lambda_{bc,t}$ denotes the multiplied on the borrowing constraint, which is greater than zero in a neighborhood of the equilibrium.

The budget and borrowing constraint for construction-sector entrepreneur are:

$$C_{h,t}+K_{hc,t}+K_{hd,t}+q_{l,t}\left(L_{hc,t}+L_{hd,t}\right)+w_{h,t}\left(N_{hc,t}+N_{hd,t}\right)+B_{h,t-1}$$
$$=q_{hc,t}IH_{c,t}+q_{hd,t}IH_{d,t}+(1-\delta_{kh})K_{hc,t}+(1-\delta_{kh})K_{hd,t}+q_{l,t}\left(L_{hc,t}+L_{hd,t}\right)+\frac{B_{h,t}}{R_t} \tag{B14}$$

$$B_{h,t}\le\rho_{bh}B_{h,t-1}+(1-\rho_{bh})\theta_h E_t[q_{l,t+1}\left(L_{hc,t}+L_{hd,t}\right)+K_{hc,t}+K_{hd,t}] \tag{B15}$$

The production technologies are:

$$IH_{c,t}=K_{hc,t-1}^{\alpha_{hc}}L_{hc,t-1}^{\mu_{hc}}\left(A_{h,t}N_{hc,t}\right)^{1-\alpha_{hc}-\mu_{hc}} \tag{B16}$$

$$IH_{d,t}=A_{hd,t}K_{hd,t-1}^{\alpha_{hd}}L_{hd,t-1}^{\mu_{hd}}\left(A_{hd,t}N_{h,t}\right)^{1-\alpha_{hd}-\mu_{hd}} \tag{B17}$$

and the first order conditions are:

$$(1-\alpha_{hc}-\mu_{hc})q_{hc,t}IH_{c,t}=w_{h,t}N_{hc,t} \tag{B18}$$

$$(1-\alpha_{hd}-\mu_{hd})q_{hd,t}IH_{d,t}=w_{h,t}N_{hd,t} \tag{B19}$$

$$u_{ch,t}\left(1+\frac{\partial\phi_{hc,t}}{\partial K_{hc,t}}\right) \tag{B20}$$
$$=\beta_e E_t u_{ch,t+1}\left(\alpha_{hc}\frac{q_{hc,t}IH_{c,t+1}}{K_{hc,t}}+(1-\delta_{khc})-\frac{\partial\phi_{hc,t+1}}{\partial K_{hc,t}}\right)+\lambda_{bh,t}(1-\rho_{bh})\theta_h$$

$$u_{ch,t}\left(1+\frac{\partial\phi_{hd,t}}{\partial K_{hd,t}}\right) \tag{B21}$$
$$=\beta_e E_t u_{ch,t+1}\left(\alpha_{hd}\frac{q_{hd,t}IH_{c,t+1}}{K_{hd,t}}+(1-\delta_{khd})-\frac{\partial\phi_{hd,t+1}}{\partial K_{hd,t}}\right)+\lambda_{bh,t}(1-\rho_{bh})\theta_h$$

$$u_{ch,t}q_{l,t}=\beta_e E_t u_{ch,t+1}\left(\mu_{hc}\frac{q_{hc,t}IH_{c,t+1}}{L_{hc,t}}\right)+\lambda_{bh,t}(1-\rho_{bh})\theta_h q_{l,t+1} \tag{B22}$$

$$u_{ch,t}q_{l,t}=\beta_e E_t u_{ch,t+1}\left(\mu_{hd}\frac{q_{hd,t}IH_{d,t+1}}{L_{hd,t}}\right)+\lambda_{bh,t}(1-\rho_{bh})\theta_h q_{l,t+1} \tag{B23}$$

$$u_{ch,t}=\beta_e R_t u_{ch,t+1}+\lambda_{bh,t}-\rho_{bh}\beta_e\lambda_{bh,t-1} \tag{B24}$$

$$u_{ch,t}=\left(\frac{1}{C_{h,t}-\gamma_e C_{h,t-1}}-E_t\frac{\beta_e\gamma_e}{C_{h,t+1}-\gamma_e C_{h,t}}\right) \tag{B25}$$

The market-clearing conditions are:

$$Y_t-\phi_t=C_t+IB_t \tag{B26}$$

$$GDP_t=Y_t+\bar{q_{hd}}IH_{d,t} \tag{B27}$$

where aggregate consumption and the business investment are:

$$C_t=C_{d,t}+C_{c,t}+C_{h,t}$$
$$IB_t=\frac{IK_{c,t}}{A_{kc,t}}+IK_{h,t}+\bar{q_{hc}}IH_{c,t}$$

and the two components of business investment are:

$$IK_{c,t}=K_{c,t}-(1-\delta_{kc})K_{c,t-1}$$
$$IK_{h,t}=K_{hd,t}-(1-\delta_{kh})K_{hd,t-1}+K_{hd,t}-(1-\delta_{kh})K_{hd,t-1}$$

The evolution of commercial and residential real estate are:

$$IH_{c,t}=H_{c,t}-(1-\delta_{hc})H_{c,t-1} \tag{B28}$$

$$IH_{d,t} = H_{d,t} - (1 - \delta_{hd})H_{d,t-1} \tag{B29}$$

Finally, the land is fixed and equal to:

$$\bar{L_h} = (l_h L_{hc,t} + (1 - l_h)L_{hd,t}) \tag{B30}$$

A competitive equilibrium consists of a sequence of allocation $\{C_{d,t}, H_{d,t}, N_{c,t}, N_{hc,t}, N_{hd,t}, S_t, C_{c,t}, K_{c,t}, H_{c,t}, B_{c,t}, C_{h,t}, IH_{c,t}, K_{hc,t}, L_{hc,t}, IH_{d,t}, K_{hd,t}, L_{hd,t}\}_{t=0}^{\infty}$ and prices $\{w_{c,t}, w_{hc,t}, w_{hd,t}, q_{hd,t}, q_{hd,t}, q_{l,t}, R_t\}_{t=0}^{\infty}$ such that

(i) given the $\{w_{c,t}, w_{hc,t}, w_{hd,t}, q_{hd,t}, R_t\}_{t=0}^{\infty}$ the sequence $\{C_{d,t}, H_{d,t}, N_{c,t}, N_{hc,t}, N_{hd,t}, S_t\}_{t=0}^{\infty}$ solves the household's problem,

(ii) given the $\{w_{c,t}, q_{hc,t}, R_t\}_{t=0}^{\infty}$ the sequence $\{C_{c,t}, K_{c,t}, H_{c,t}, N_{c,t}, B_{c,t}\}_{t=0}^{\infty}$ solves the entrepreneur in the consumption good sector problem,

(iii) given the $\{w_{hc,t}, q_{hc,t}, w_{hd,t}, q_{hd,t}, q_{l,t}, R_t\}_{t=0}^{\infty}$ the sequence $\{C_{h,t}, IH_{c,t}, K_{hc,t}, L_{hc,t}, N_{hc,t}, B_{h,t}, IH_{d,t}K_{hd,t}, L_{hd,t}, N_{hd,t}\}_{t=0}^{\infty}$ solves the entrepreneur in the construction sector problem,

(iv) all markets clear.

7 Appendix C: Steady State

From the Euler Equation we can derive that:

$$\beta = \frac{1}{R} \tag{C1}$$

$$\frac{\lambda_c}{u_{ce}} = \frac{\beta_d - \beta_e}{1 - \rho_{bc}\beta_e} \qquad \& \qquad \frac{\lambda_h}{u_{ce}} = \frac{\beta_d - \beta_e}{1 - \rho_{bh}\beta_e} \tag{C2}$$

$$\frac{K_c}{Y} = \frac{\alpha_c \beta_e}{1 - \beta_e(1 - \delta_{kc}) - \frac{\lambda_c}{u_{ce}}\theta_c(1 - \rho_{bc})} \tag{C3}$$

$$\frac{q_{hc}H_c}{Y} = \frac{\mu_c \beta_e}{1 - \beta_e(1 - \delta_{hc}) - \frac{\lambda_c}{u_{ce}}\theta_c(1 - \rho_{bc})} \tag{C4}$$

$$\frac{B_c}{Y} = \theta_c \left(\frac{q_{hc}H_c}{Y} + \frac{K_c}{Y} \right) \tag{C5}$$

$$\frac{C_c}{Y} = \alpha_c + \mu_c - \delta'_{kc}\frac{K_c}{Y} - \delta'_{hc}\frac{q_{hc}H_c}{Y} + R'\frac{B_c}{Y} \tag{C6}$$

$$\frac{K_{hc}}{q_{hc}IH_c} = \frac{\alpha_{hc}\beta_e}{1 - \beta_e(1 - \delta_{khc}) - \frac{\lambda_{hc}}{u_{ce}}(1 - \rho_{bh})\theta_{hc}} \tag{C7}$$

$$\frac{K_{hd}}{q_{hd}IH_d} = \frac{\alpha_{hd}\beta_e}{1 - \beta_e(1 - \delta_{khd}) - \frac{\lambda_{hd}}{u_{ce}}(1 - \rho_{bh})\theta_{hd}} \tag{C8}$$

$$\frac{q_{hd}H_d}{C_d} = \frac{\chi}{(1 - \beta_d(1 - \delta_{hd}))} \tag{C9}$$

$$x_1 = \alpha_{hc} + \mu_{hc} - \frac{I_{hc}}{q_{hc}IH_c} + \theta_h * \left(\frac{q_l L_{hc}}{q_{hc}IH_c} + \frac{K_{hc}}{q_{hc}IH_c} \right)$$

$$x_1 = \alpha_{hc} + \mu_{hc} - \frac{I_{hc}}{q_{hc}IH_c} + \theta_h * \left(\frac{q_l L_{hc}}{q_{hc}IH_c} + \frac{K_{hc}}{q_{hc}IH_c} \right)$$

$$\frac{C_d}{Y} = \frac{1 - \left(\frac{C_c}{Y} + \frac{I_c}{Y} \right) - \frac{q_{hc}IH_c}{Y}\left(1 - \frac{I_{hc}}{q_{hc}IH_c} + x_1 \right)}{1 + \delta_{hd}\frac{q_{hd}IH_d}{C_d}\left(\frac{I_{hd}}{q_{hd}IH_d} + x2 \right)} \tag{C10}$$

$$\frac{N_{hd}}{N_{hc}} = \frac{(1 - \alpha_{hd} - \mu_{hd})}{(1 - \alpha_{hc} - \mu_{hc}}\frac{q_{hd}IH_d}{q_{hc}IH_c} \tag{C11}$$

$$\frac{N_{hc}}{N_c} = \left(\frac{\frac{(1-\alpha_{hc}-\mu_{hc})}{(1-\alpha_c-\mu_c}) \frac{q_{hc} IH_c}{Y}}{\left(1+\frac{N_{hd}}{N_{hc}}\right)^{\xi}} \right)^{\frac{1}{1+\xi}} \tag{C12}$$

$$\frac{N_{hd}}{N_c} = \frac{N_{hd}}{N_{hc}} \frac{N_{hc}}{N_c} \tag{C13}$$

$$N_c = \left(\frac{(1-\alpha_c-\mu_c)\frac{Y}{Cd}}{\left(1+\left(\frac{N_{hc}}{N_c}+\frac{N_{hd}}{N_c}\right)^{1+\xi}\right)^{\frac{\eta-\xi}{1+\xi}}} \right)^{\frac{1}{1+\eta}} \tag{C14}$$

$$N_{hc} = \frac{N_{hc}}{N_c} Nc \tag{C15}$$

$$N_{hd} = \frac{N_{hd}}{N_c} Nc \tag{C16}$$

$$\frac{q_l L_{hc}}{q_{hc} IH_c} = \frac{\mu_{hc}\beta_e}{1-\beta_e-\frac{\lambda_{hc}}{u_{ce}}(1-\rho_{bh})\theta_{hc}} \tag{C17}$$

$$\frac{q_l L_{hd}}{q_{hd} IH_d} = \frac{\mu_{hd}\beta_e}{1-\beta_e-\frac{\lambda_{hd}}{u_{ce}}(1-\rho_{bh})\theta_{hd}} \tag{C18}$$

$$\frac{q_l \bar{L_h}}{Y} = \frac{q_l L_{hc}}{Y} + \frac{q_l L_{hd}}{Y} \tag{C19}$$

$$\frac{q_l \bar{L_h}}{Y} = \left(\left(\frac{q_l L_{hc}}{Y}\right)^{\omega} + \left(\frac{q_l L_{hd}}{Y}\right)^{\omega} \right)^{\frac{1}{\omega}} \tag{C20}$$

$$q_{hc}^{\frac{\mu_c}{1-\alpha_c-\mu_c}} Y = \left(\frac{K_c}{Y}\right)^{\frac{\alpha_c}{1-\alpha_c-\mu_c}} \left(\frac{q_{hc}H_c}{Y}\right)^{\frac{\mu_c}{1-\alpha_c-\mu_c}} Nc \tag{C21}$$

$$q_{hc}^{-\frac{\alpha_{hc}}{1-\alpha_{hc}}} IH_c = \left(\frac{K_{hc}}{q_{hc} IH_c}\right)^{\frac{\alpha_{hc}}{1-\alpha_{hc}}} L_{hc}^{\frac{\mu_{hc}}{1-\alpha_{hc}}} N_c^{\frac{1-\alpha_{hc}-\mu_{hc}}{1-\alpha_{hc}}} \tag{C22}$$

Combining these two:

$$q_{hc} = \left(\frac{q_{hc}^{\frac{\mu_c}{1-\alpha_c-\mu_c}} Y}{q_{hc}^{-\frac{\alpha_{hc}}{1-\alpha_{hc}}} IH_c} \frac{q_{hc} IH_c}{Y} \right)^{1+\frac{\mu_c}{1-\alpha_c-\mu_c}+\frac{\alpha_{hc}}{1-\alpha_{hc}}} \tag{C23}$$

$$Y = \left(\frac{K_c}{Y}\right)^{\frac{\alpha_c}{1-\alpha_c-\mu_c}} \left(\frac{q_{hc}H_c}{Y}\right)^{\frac{\mu_c}{1-\alpha_c-\mu_c}} q_{hc}^{-\frac{\mu_c}{1-\alpha_c-\mu_c}} N_c \tag{C24}$$

$$IH_c = \left(\frac{K_{hc}}{q_{hc} IH_c}\right)^{\frac{\alpha_{hc}}{1-\alpha_{hc}}} q_{hc}^{\frac{\alpha_{hc}}{1-\alpha_{hc}}} L_{hc}^{\frac{\mu_{hc}}{1-\alpha_{hc}}} N_{hc}^{\frac{1-\alpha_{hc}-\mu_{hc}}{1-\alpha_{hc}}} \tag{C25}$$

$$IH_d = \left(\frac{K_{hd}}{q_{hd} IH_d}\right)^{\alpha_{hc}} \left(\frac{q_{hd} IH_d}{Y} Y\right)^{\alpha_{hd}} L_{hd}^{\alpha_{hd}} N_{hd}^{1-\alpha_{hd}-\mu_{hd}} \tag{C26}$$

Having solved for $Y, IH_c, q_{hc}, IH_d, q_{hd}$ we can substitute and solve for the rest of the variables $q_l, H_d, H_c, C_d, C_c, C_h, K_c, K_{hc}, K_{hd}, B_c, B_h, w_c, w_h \ldots$

Appendix D: Estimation Details

The parameters of the model are estimated using Bayesian methods. We use Bayesian methods because they allow incorporating a priori information on the parameters of the model and also because pure maximum likelihood tends to produce fragile results, particularly in situations in which some parameters are weakly identified.

D.1 The output of the Metropolis

The following graphs report the time series of the draws from the posterior distribution generated by the Metropolis algorithm.

Figure 12 – Posterior Density Traceplot

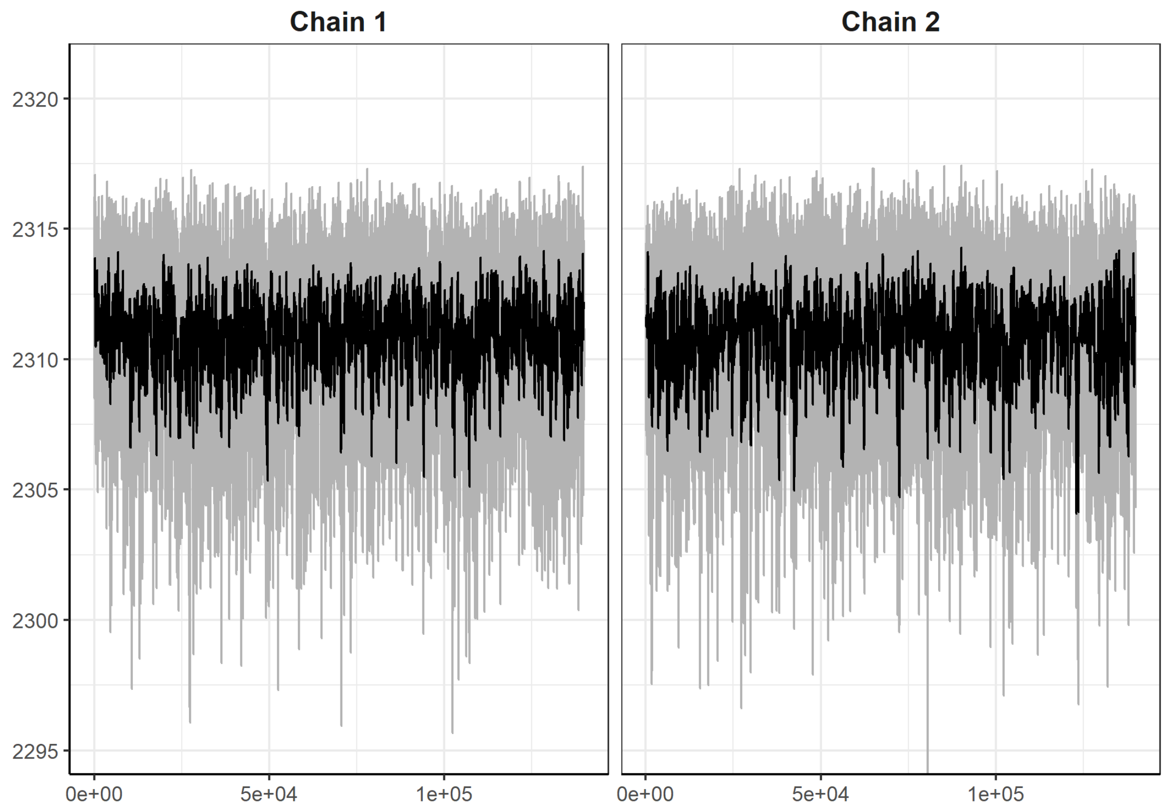


Figure 13 – Structural Shock Traceplot - Chain 1

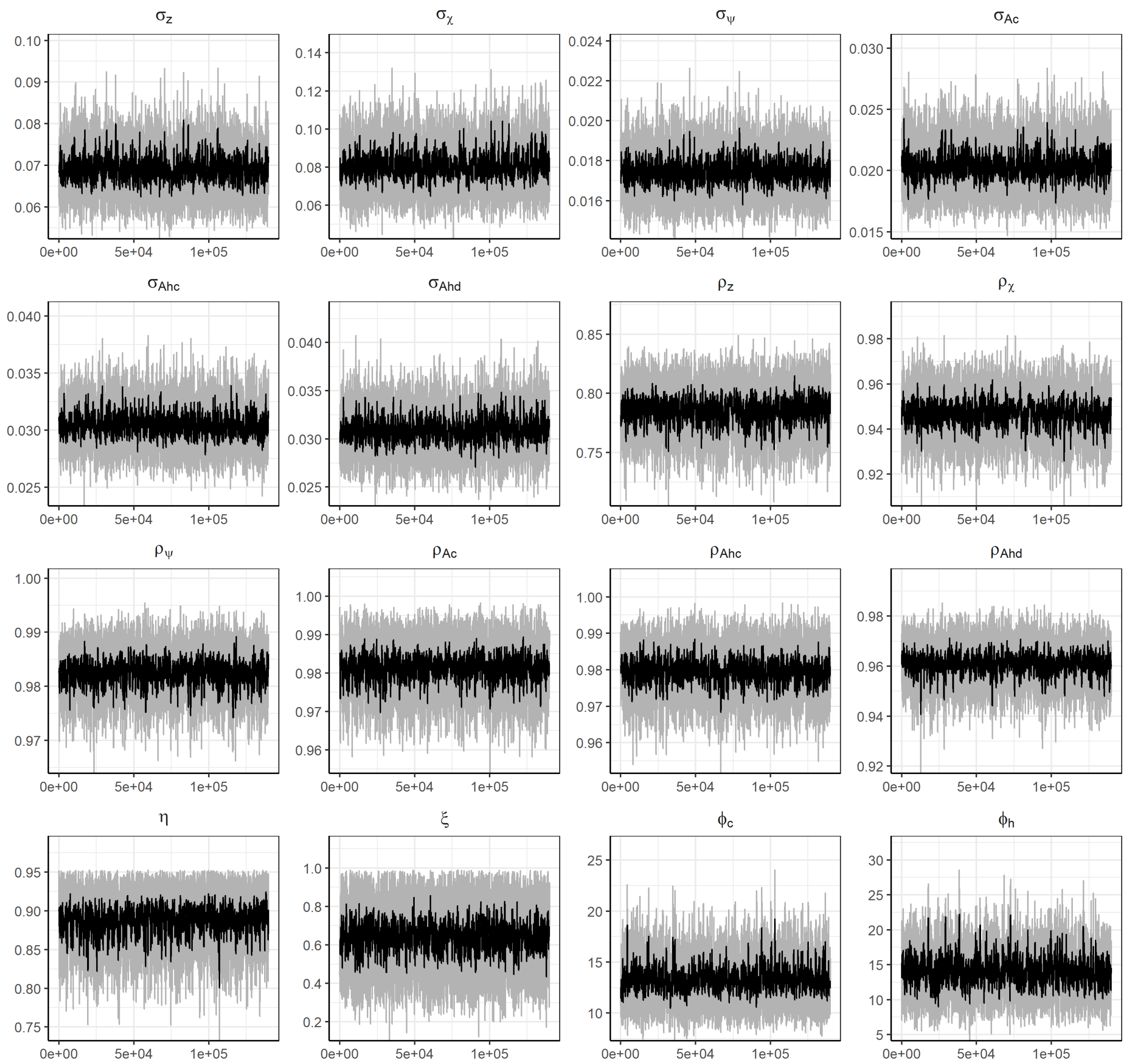
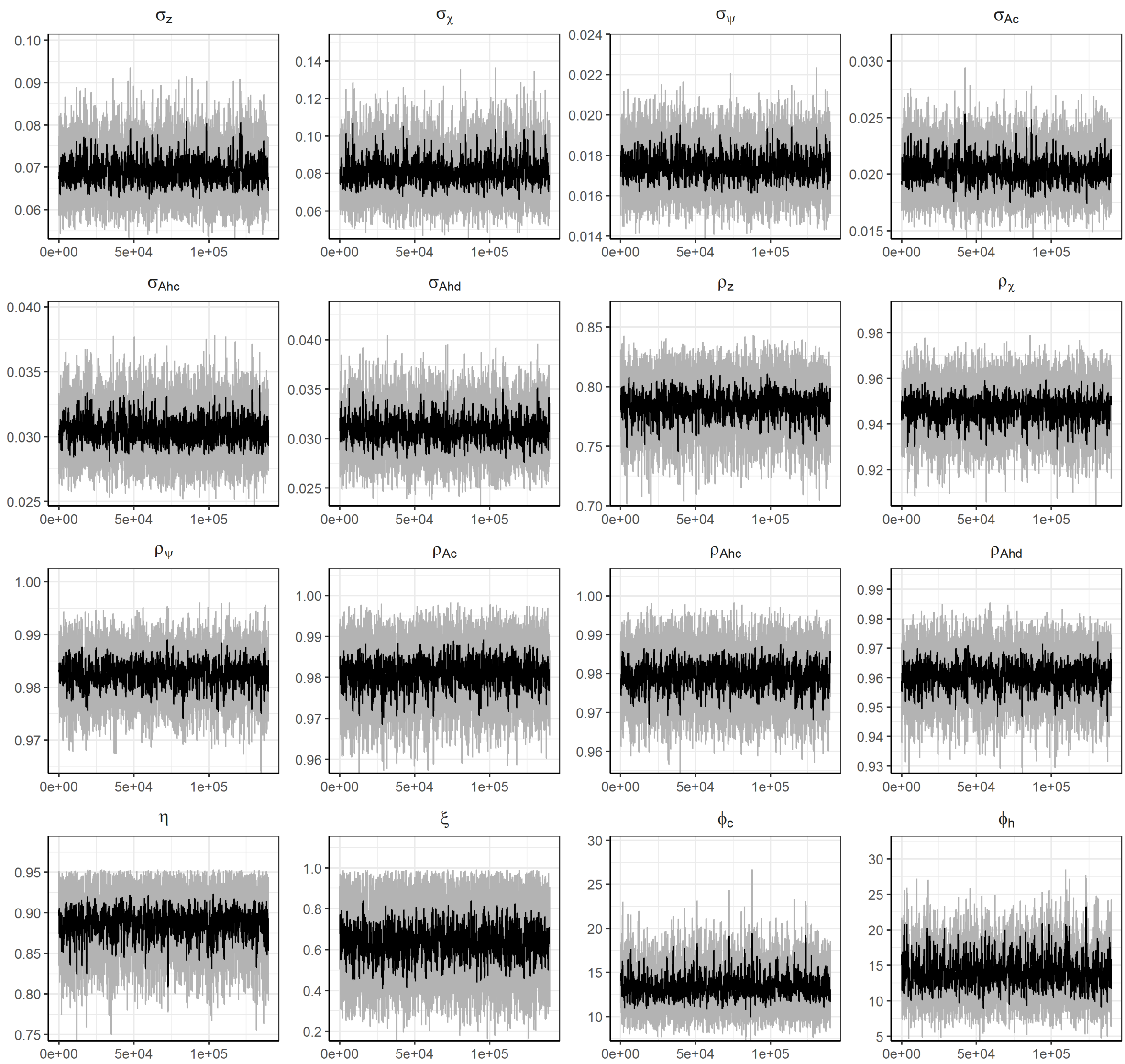


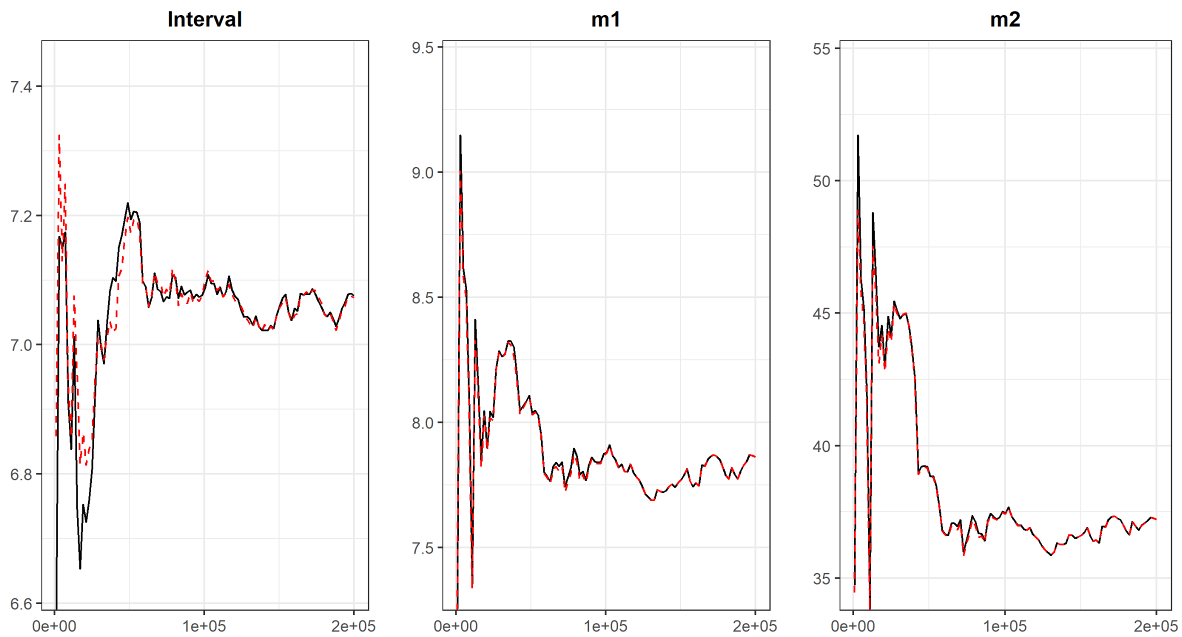
FIGURE 14 – STRUCTURAL SHOCK TRACEPLOT - CHAIN 2



D.2 Brooks and Gelman (1998) Diagnostics

Brooks and Gelman (1998) convergence diagnostics is based on comparing pooled and within chain variability of MC draws using the 80% interval/quantile range. The convergence diagnostics displays the 80% interval range of draws from the pooled and within chain means as well as the 80% interval range of the second and third central moments (squared and cubed absolute deviations). Due to computational reasons, the multivariate convergence diagnostics does not follow of Brooks and Gelman (1998) directly, but rather applies their idea for univariate convergence diagnostics to the range of the posterior likelihood function instead of the individual parameters. Thus, the posterior kernel is used to aggregate the parameters into a scalar statistics whose convergence is then checked using the Brooks and Gelman (1998) univariate convergence diagnostic.

D.2.1 Multivariate Diagnostics



D.2.2 Univariate Diagnostics

Figure 15 – Univariate Diagnostic - Interval

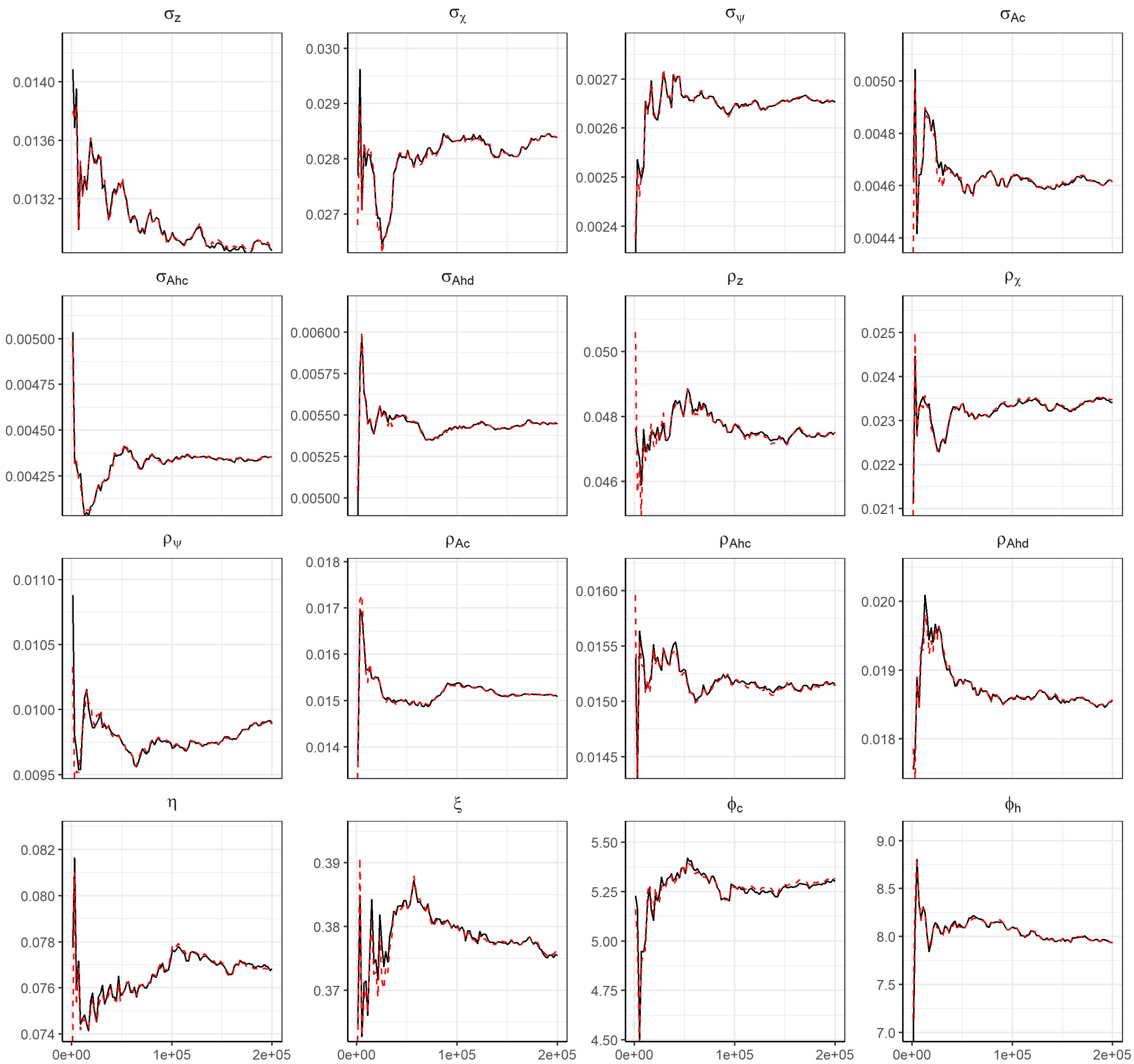


FIGURE 16 – UNIVARIATE DIAGNOSTIC - M1

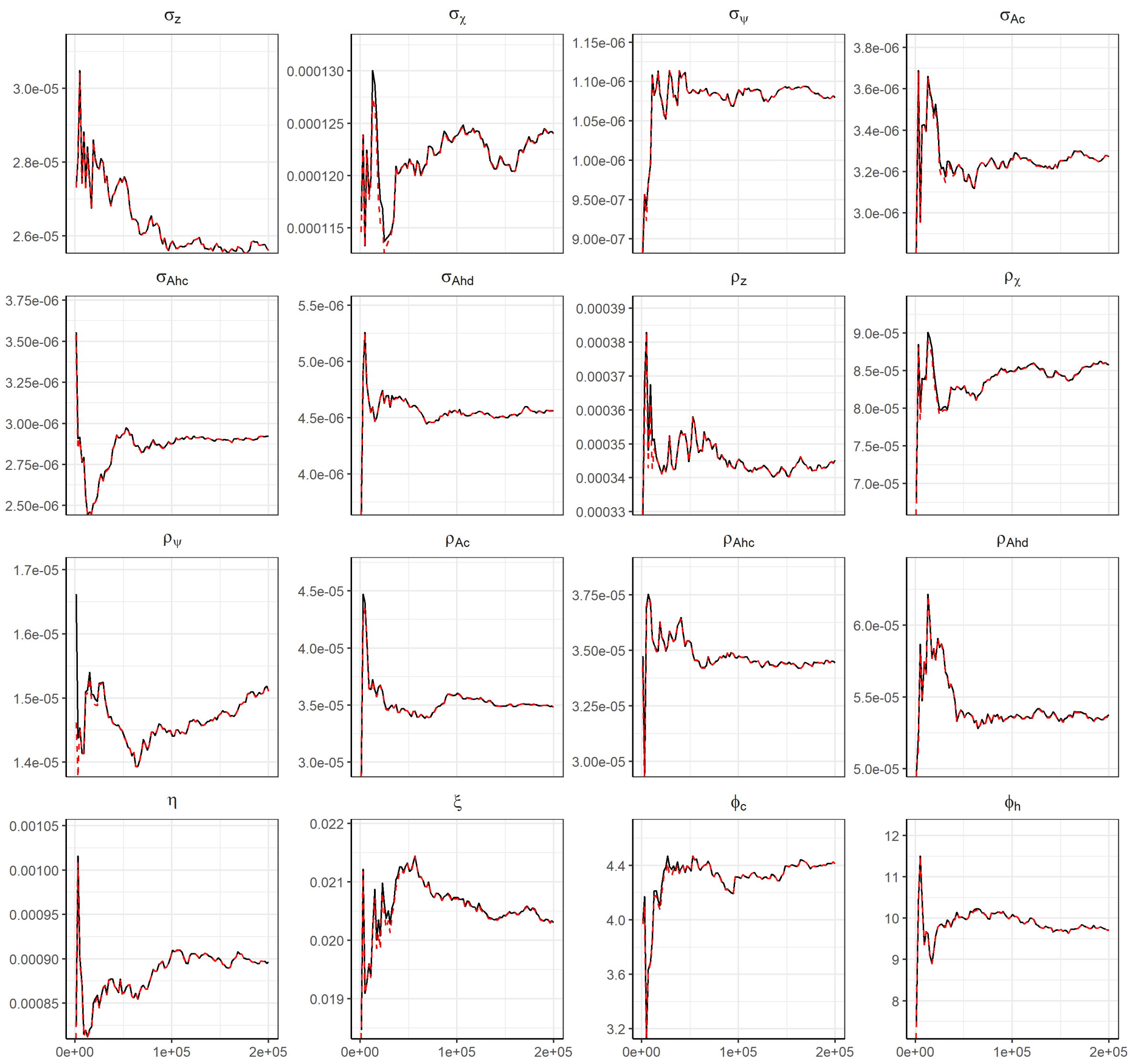
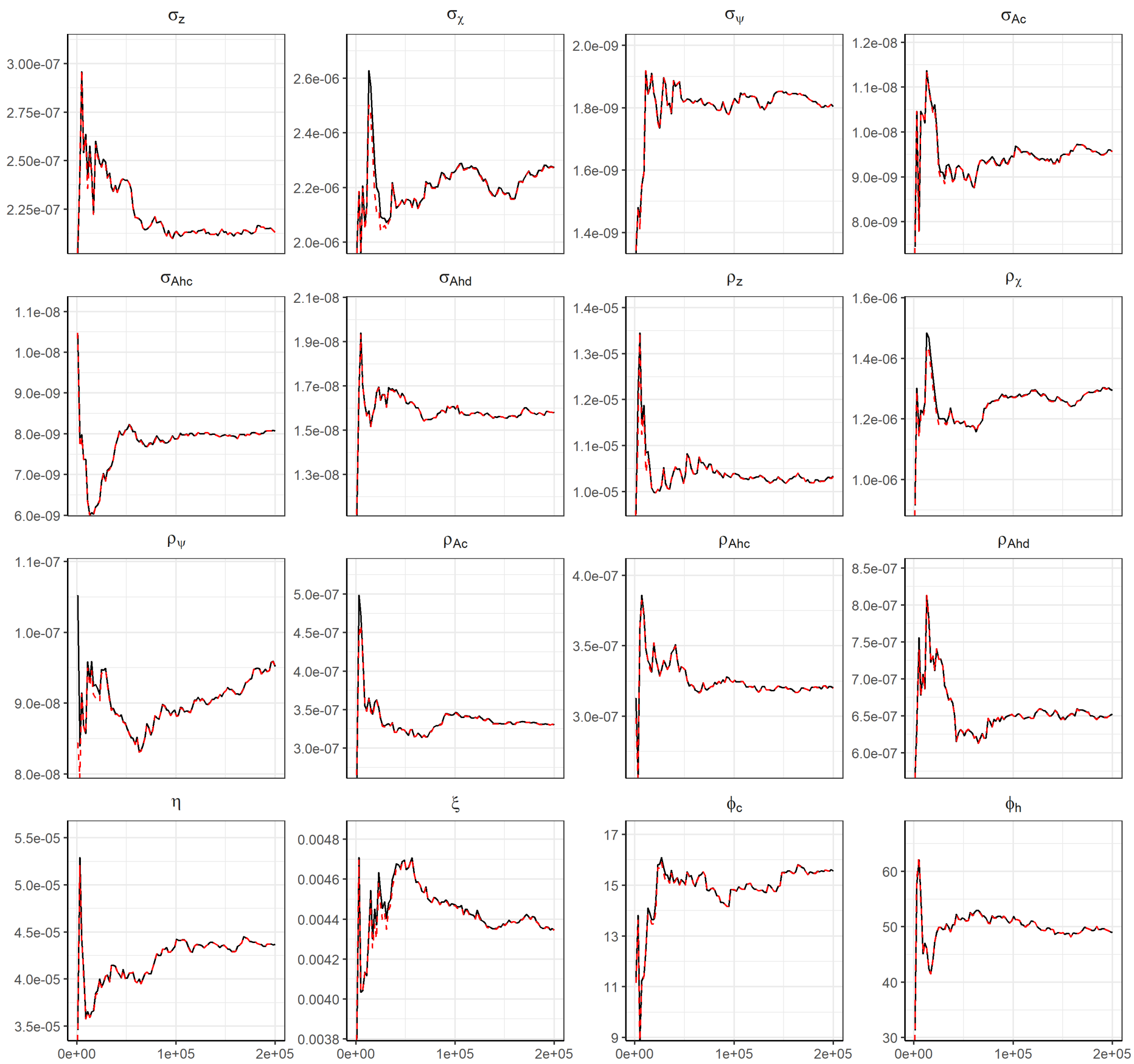


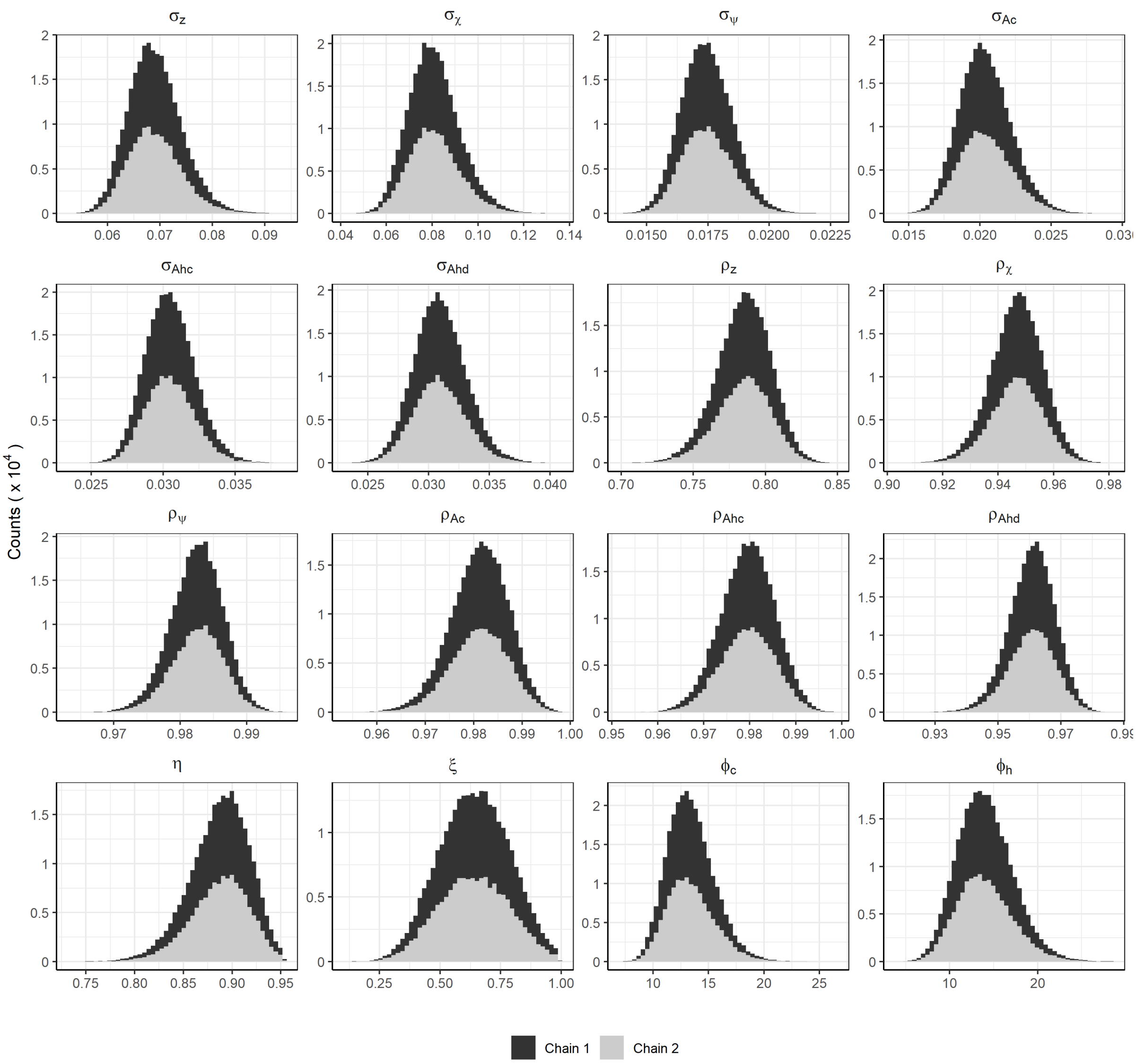
Figure 17 – Univariate Diagnostic - m2



D.3 Posterior Densities

In the following graphs we report the posterior densities of selected parameters for both chains. The posterior ones are based on 200,000 draws from the Metropolis algorithm.

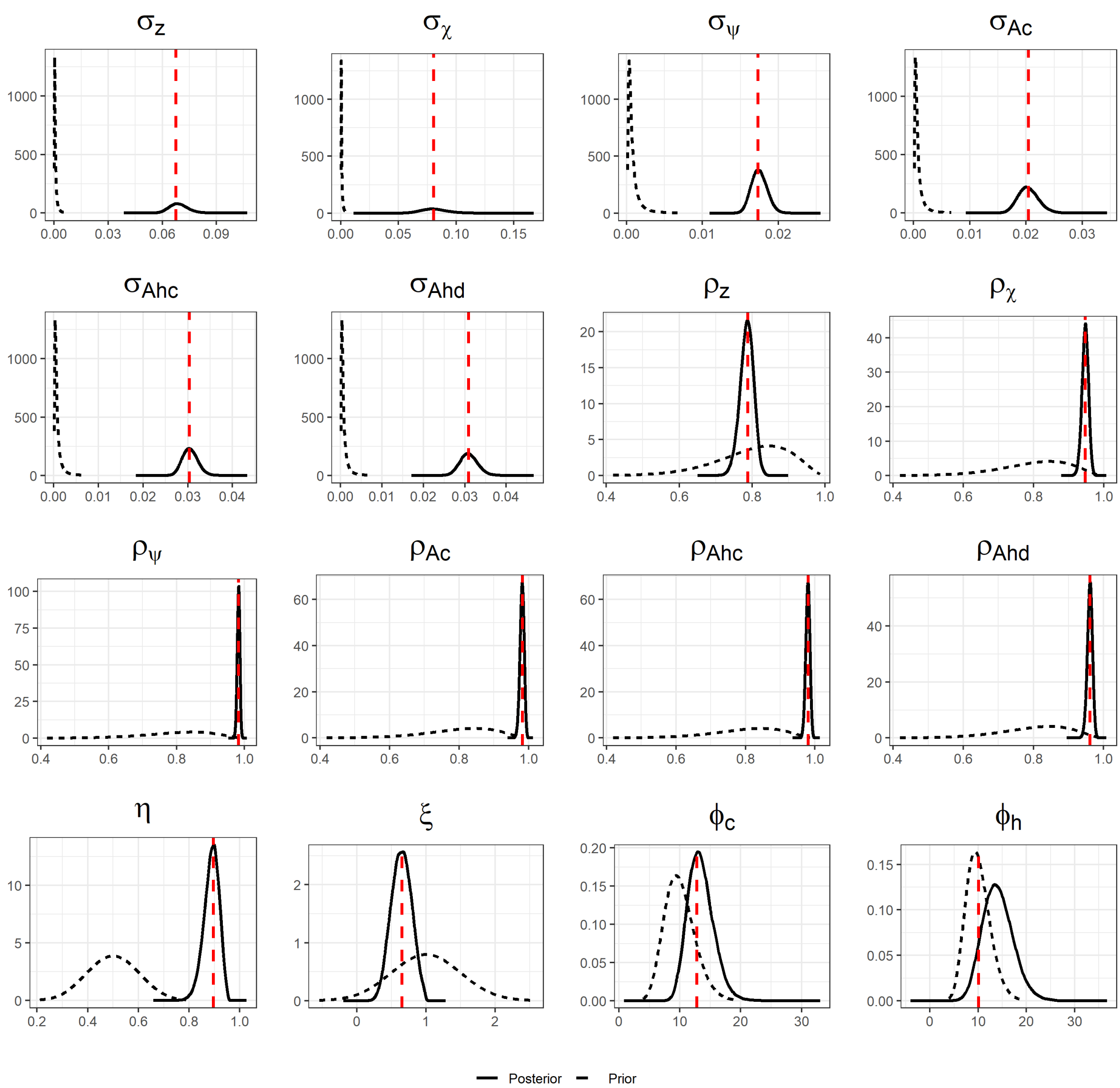
Figure 18 – Posterior Densities



D.4 Prior and posterior densities

In the following graphs we report the prior and posterior densities of selected parameters. The posterior ones are based on 200,000 draws from the Metropolis algorithm and are estimated using a Gaussian kernel. Vertical red lines correspond to the posterior mode.

FIGURE 19 – PRIOR & POSTERIOR DENSITIES



D.5 Recursive Mean

Figure 20 – Recursive Mean - Chain 1

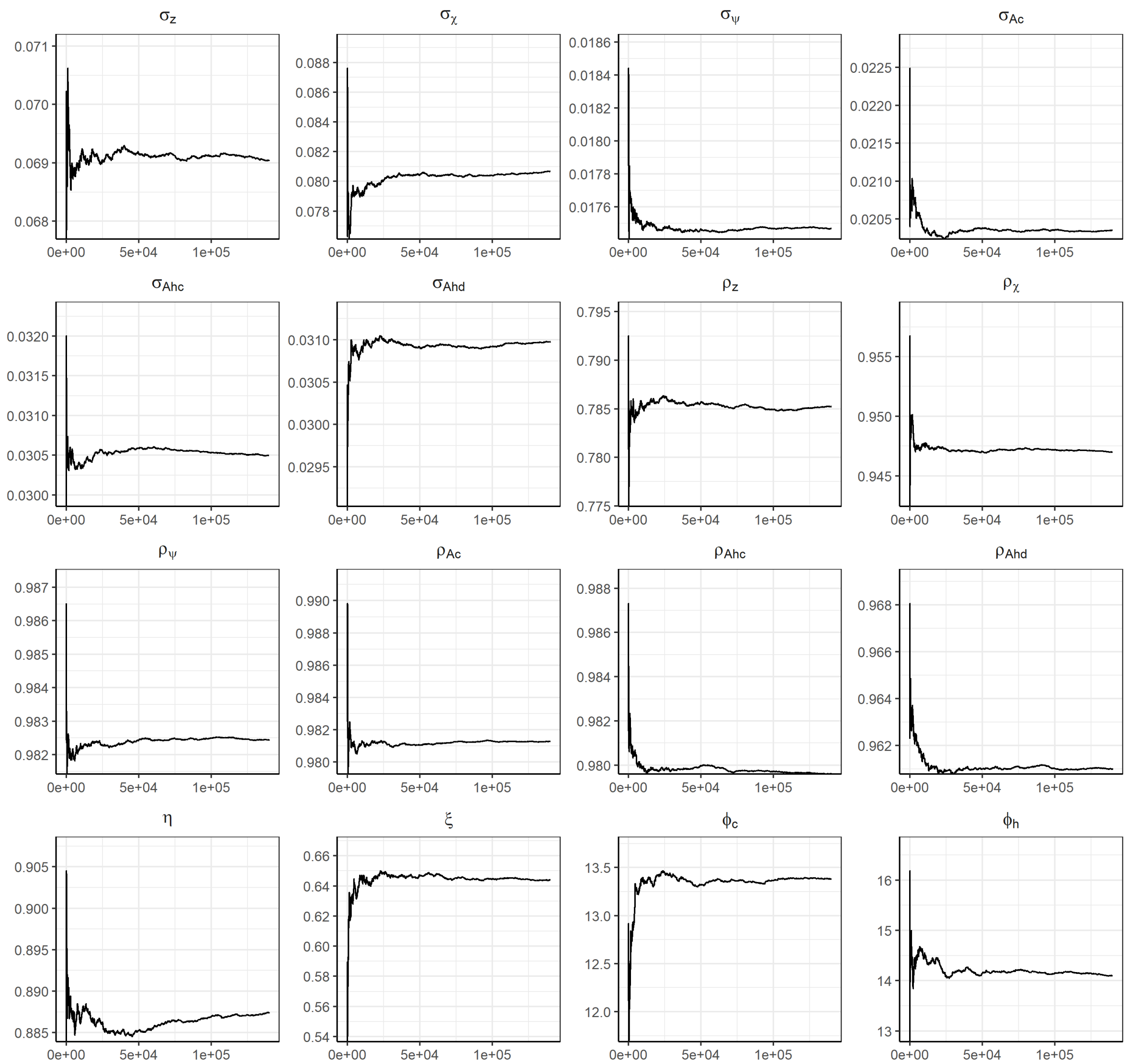


Figure 21 – Recursive Mean - Chain 2