My Title

psvar

Methodology

Let Y_t be an n×1 vector of observables. We assume that the dynamics of the observables are described by a system of linear simultaneous equations

$$AY_t = \sum_{j=1}^{p} \alpha_j Y_{t-j} + \varepsilon_t$$

$$Y_t = \sum_{j=1}^{p} \delta_j Y_{t-j} + B\varepsilon_t$$

$$u_t = B\epsilon_t$$

$$E(u_t u_t') = BB' = \Sigma_u$$

$$\begin{bmatrix} u_{1,t} \\ (k \times 1) \\ u_{2,t} \\ (N-k \times 1) \end{bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} \\ (k \times k) & (k \times N-k) \\ \beta_{21} & \beta_{22} \\ (N-k \times k) & (N-k \times N-k) \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ (k \times 1) \\ \epsilon_{2,t} \\ (N-k \times 1) \end{bmatrix}$$

Consider the following partitioning of B:

$$B = \begin{bmatrix} \beta_1 & \beta_2 \\ n \times k & n \times (n-k) \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} \beta_{11}' & \beta_{21}' \\ k \times k & k \times (n-k) \end{bmatrix}', \quad \beta_2 = \begin{bmatrix} \beta_{12}' & \beta_{22}' \\ (n-k) \times k & (n-k) \times (n-k) \end{bmatrix}'$$

$$\beta_{21} = (\Sigma_{mu_1}^{-1} \Sigma_{mu_2'})' \beta_{11}$$

$$\begin{aligned}
&\beta_{21}\beta_{11}^{-1} = (\Sigma_{mu1}^{-1}\Sigma_{mu_2'})' \\
&\beta_{12}\beta_{22}^{-1} = (\beta_{12}\beta_{12}'(\beta_{21}\beta_{11}^{-1})' + (\Sigma_{21} - \beta_{21}\beta_{11}^{-1}\Sigma_{11})')(\beta_{22}\beta_{22}'^{-1}) \\
&\beta_{12}\beta_{12}' = (\Sigma_{21} - \beta_{21}\beta_{11}^{-1}\Sigma_{11})' Z^{-1} (\Sigma_{21} - \beta_{21}\beta_{11}^{-1}\Sigma_{11}) \\
&\beta_{22}\beta_{22}' = \Sigma_{22} + \beta_{21}\beta_{11}^{-1}(\beta_{12}\beta_{12}' - \Sigma_{11})(\beta_{21}\beta_{11}^{-1})' \\
&\beta_{11}\beta_{11}' = \Sigma_{11} - \beta_{12}\beta_{12}'
\end{aligned}$$

$$Z = \beta_{21}\beta_{11}^{-1}\Sigma_{11}(\beta_{21}\beta_{11}^{-1})' - (\Sigma_{21}(\beta_{21}\beta_{11}^{-1})' + \beta_{21}\beta_{11}^{-1}\Sigma_{21}') + \Sigma_{22}$$