$$s:\mathbb{R}\to\mathbb{R},\ t\to s(t) \tag{1}$$

$$s(t)=\frac{1}{2}\tanh\left(s_r(t-t_0)\right)+\frac{1}{2} \tag{2}$$
$$s(t)=\frac{1}{2}\operatorname{erf}\left(s_r(t-t_0)\right)+\frac{1}{2} \tag{3}$$
$$s(t)=\frac{1}{\pi}\arctan\left(s_r(t-t_0)\right)+\frac{1}{2} \tag{4}$$

$$b(x)=\frac{b_0}{\sigma_b\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma_b^2}\right) \tag{5}$$
$$b(x)=\frac{b_0}{\pi}\frac{\sigma_b}{\sigma_b^2+x^2} \tag{6}$$

$$\mathrm{i}\partial_t\Psi(x,t)=\left(\hat{H}_0+V(x,t)\right)\Psi(x,t) \tag{7}$$
$$\hat{H}_0=-\frac{1}{2m}\partial_x^2-\frac{m\omega^2}{2}x^2 \tag{8}$$
$$V(x,t)=b(x)s(t) \tag{9}$$

Parameters:

name	symbol	def val.
reduced planck constant	$\hbar$	1
mass	m	1
frequency	ω	1
pertubation sigma	σ_b	1
pertubation amplitude	b_0	1
step roughness	s_r	1

$$\Psi(x,t)=\mathcal{T}\exp\left(-\mathrm{i}\hat{H}_0t-\mathrm{i}b(x)\int\limits_0^t \mathrm{d}t'\,s(t')\right)\Psi(x,0) \tag{10}$$

$$\Psi(x,t)=u(x,t)+\mathrm{i}v(x,t) \tag{11}$$
$$\partial_t u=-\frac{1}{2m}\partial_x^2 v-\frac{m\omega^2}{2}x^2v+b(x)s(t)v \tag{12}$$
$$\partial_t v=\frac{1}{2m}\partial_x^2 u+\frac{m\omega^2}{2}x^2u-b(x)s(t)u \tag{13}$$
$$\partial_t\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}0&-1\\1&0\end{pmatrix}\otimes\left(\frac{1}{2m}\partial_x^2+\frac{m\omega^2}{2}x^2-b(x)s(t)\right)\begin{pmatrix}u\\v\end{pmatrix} \tag{14}$$

j indexes the time and i the space

$$\partial_t \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{1}{\Delta t} \begin{pmatrix} u_i^{j+1} - u_i^j \\ v_i^{j+1} - v_i^j \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{1}{2m} \partial_x^2 \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{1}{4m\Delta x^2} \begin{pmatrix} -(v_{i+1}^{j+1} - 2v_i^{j+1} + v_{i-1}^{j+1}) - (v_{i+1}^j - 2v_i^j + v_{i-1}^j) \\ (u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1})(u_{i+1}^j - 2u_i^j + u_{i-1}^j) \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{m\omega^2}{2} x^2 \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{m\omega^2 x_i^2}{4} \begin{pmatrix} -v_i^{j+1} - v_i^j \\ u_i^{j+1} + u_i^j \end{pmatrix} \tag{17}$$

$$-\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} b(x)s(t) \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{b(x_i)}{2} \begin{pmatrix} s(t_{j+1}) v_{j+1}^i + s(t_{j+1}) v_j^i \\ -s(t_{j+1}) u_{j+1}^i - s(t_{j+1}) u_j^i \end{pmatrix} \tag{18}$$

$$u_i^{j+1} + \frac{\Delta t}{4m\Delta x^2}\left(v_{i+1}^{j+1} - 2v_i^{j+1} + v_{i-1}^{j+1}\right) + \frac{m\omega^2 \Delta t}{4} x_i^2 v_i^{j+1} - \frac{b(x_i)\Delta t}{2} s(t_{j+1}) v_{j+1}^i \tag{19}$$

$$= u_i^j + \frac{\Delta t}{4m\Delta x^2}\left(v_{i+1}^j - 2v_i^j + v_{i-1}^j\right) + \frac{m\omega^2 \Delta t}{4} x_i^2 v_i^j - \frac{b(x_i)\Delta t}{2} s(t_j) v_j^i \tag{20}$$

$$v_i^{j+1} + \frac{\Delta t}{4m\Delta x^2}\left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}\right) + \frac{m\omega^2 \Delta t}{4} x_i^2 u_i^{j+1} - \frac{b(x_i)\Delta t}{2} s(t_{j+1}) u_{j+1}^i \tag{21}$$

$$= v_i^j - \frac{\Delta t}{4m\Delta x^2}\left(u_{i+1}^j - 2u_i^j + u_{i-1}^j\right) - \frac{m\omega^2 \Delta t}{4} x_i^2 u_i^j + \frac{b(x_i)\Delta t}{2} s(t_j) u_j^i \tag{22}$$

$$L \begin{pmatrix} u_1^{j+1} \\ \vdots \\ u_N^{j+1} \\ v_1^{j+1} \\ \vdots \\ v_N^{j+1} \end{pmatrix} = R \begin{pmatrix} u_1^j \\ \vdots \\ u_N^j \\ v_1^j \\ \vdots \\ v_N^j \end{pmatrix} \tag{23}$$