$$s : \mathbb{R} \to \mathbb{R},\ t \to s(t) \tag{1}$$

$$\mathrm{i}\partial_t \Psi(x,t) = \left(\hat{H}_0 + V(x,t)\right)\Psi(x,t) \tag{2}$$

$$\hat{H}_0 = -\frac{1}{2m}\partial_x^2 - \frac{m\omega^2}{2}x^2 \tag{3}$$

$$V(x,t) = b(x)s(t) \tag{4}$$

Parameters:

name	symbol	def val.
reduced planck constant	$\hbar$	1
mass	m	1
frequency	ω	1
pertubation sigma	σ_b	1
pertubation amplitude	b_0	1
step roughness	s_r	1

$$\Psi(x,t) = \mathcal{T}\exp\left(-\mathrm{i}\hat{H}_0 t - \mathrm{i}b(x)\int_0^t \mathrm{d}t'\, s(t')\right)\Psi(x,0) \tag{5}$$

$$\Psi(x,t) = u(x,t) + \mathrm{i}v(x,t) \tag{6}$$

$$\partial_t u = -\frac{1}{2m}\partial_x^2 v - \frac{m\omega^2}{2}x^2 v + b(x)s(t)v \tag{7}$$

$$\partial_t v = \frac{1}{2m}\partial_x^2 u + \frac{m\omega^2}{2}x^2 u - b(x)s(t)u \tag{8}$$

$$\partial_t \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes \left(\frac{1}{2m}\partial_x^2 + \frac{m\omega^2}{2}x^2 - b(x)s(t)\right)\begin{pmatrix} u \\ v \end{pmatrix} \tag{9}$$

j indexes the time and i the space

$$\partial_t \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{1}{\Delta t}\begin{pmatrix} u_i^{j+1} - u_i^j \\ v_i^{j+1} - v_i^j \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\frac{1}{2m}\partial_x^2 \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{1}{4m\Delta x^2}\begin{pmatrix} -(v_{i+1}^{j+1} - 2v_i^{j+1} + v_{i-1}^{j+1}) - (v_{i+1}^j - 2v_i^j + v_{i-1}^j) \\ (u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1})(u_{i+1}^j - 2u_i^j + u_{i-1}^j) \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\frac{m\omega^2}{2}x^2 \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{m\omega^2 x_i^2}{4}\begin{pmatrix} -v_i^{j+1} - v_i^j \\ u_i^{j+1} + u_i^j \end{pmatrix} \tag{12}$$

$$-\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} b(x)s(t)\begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{b(x_i)}{2}\begin{pmatrix} s(t_{j+1})v_{j+1}^i + s(t_{j+1})v_j^i \\ -s(t_{j+1})u_{j+1}^i - s(t_{j+1})u_j^i \end{pmatrix} \tag{13}$$

$$u_i^{j+1} + \frac{\Delta t}{4m\Delta x^2}\left(v_{i+1}^{j+1} - 2v_i^{j+1} + v_{i-1}^{j+1}\right) + \frac{m\omega^2\Delta t}{4}x_i^2 v_i^{j+1} - \frac{b(x_i)\Delta t}{2}s(t_{j+1})v_{j+1}^i \tag{14}$$

$$= u_i^j + \frac{\Delta t}{4m\Delta x^2}\left(v_{i+1}^j - 2v_i^j + v_{i-1}^j\right) + \frac{m\omega^2\Delta t}{4}x_i^2 v_i^j - \frac{b(x_i)\Delta t}{2}s(t_j)v_j^i \tag{15}$$

$$v_i^{j+1} + \frac{\Delta t}{4m\Delta x^2}\left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1}\right) + \frac{m\omega^2\Delta t}{4}x_i^2 u_i^{j+1} - \frac{b(x_i)\Delta t}{2}s(t_{j+1})u_{j+1}^i \tag{16}$$

$$= v_i^j - \frac{\Delta t}{4m\Delta x^2}\left(u_{i+1}^j - 2u_i^j + u_{i-1}^j\right) - \frac{m\omega^2\Delta t}{4}x_i^2 u_i^j + \frac{b(x_i)\Delta t}{2}s(t_j)u_j^i \tag{17}$$

$$L\begin{pmatrix} u_1^{j+1} \\ \vdots \\ u_N^{j+1} \\ v_1^{j+1} \\ \vdots \\ v_N^{j+1} \end{pmatrix} = R\begin{pmatrix} u_1^j \\ \vdots \\ u_N^j \\ v_1^j \\ \vdots \\ v_N^j \end{pmatrix} \tag{18}$$