

$$s : \mathbb{R} \rightarrow \mathbb{R}, \quad t \rightarrow s(t) \quad (1)$$

$$s(t) = \frac{1}{2} \tanh(s_r(t - t_0)) + \frac{1}{2} \quad (2)$$

$$b(x) = \frac{b_0}{\sigma_b \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_b^2}\right) \quad (3)$$

$$i\partial_t \Psi(x, t) = \left(\hat{H}_0 + V(x, t)\right) \Psi(x, t) \quad (4)$$

$$\hat{H}_0 = -\frac{1}{2m} \partial_x^2 - \frac{m\omega^2}{2} x^2 \quad (5)$$

$$V(x, t) = b(x)s(t) \quad (6)$$

Parameters:

name	symbol	def val.
reduced planck constant	\hbar	1
mass	m	1
frequency	ω	1
pertubation sigma	σ_b	1
pertubation amplitude	b_0	1
step roughness	s_r	1

$$\Psi(x, t) = \mathcal{T} \exp\left(-i\hat{H}_0 t - ib(x) \int_0^t dt' s(t')\right) \Psi(x, 0) \quad (7)$$

$$\Psi(x, t) = u(x, t) + iv(x, t) \quad (8)$$

$$\partial_t u = -\frac{1}{2m} \partial_x^2 v - \frac{m\omega^2}{2} x^2 v + b(x)s(t)v \quad (9)$$

$$\partial_t v = \frac{1}{2m} \partial_x^2 u + \frac{m\omega^2}{2} x^2 u - b(x)s(t)u \quad (10)$$

$$\partial_t \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes \left(\frac{1}{2m} \partial_x^2 + \frac{m\omega^2}{2} x^2 - b(x)s(t) \right) \begin{pmatrix} u \\ v \end{pmatrix} \quad (11)$$

j indexes the time and i the space

$$\partial_t \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{1}{\Delta t} \begin{pmatrix} u_{i+1}^{j+1} - u_i^j \\ v_i^{j+1} - v_i^j \end{pmatrix} \quad (12)$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{1}{2m} \partial_x^2 \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{1}{4m\Delta x^2} \begin{pmatrix} -(v_{i+1}^{j+1} - 2v_i^{j+1} + v_{i-1}^{j+1}) - (v_{i+1}^j - 2v_i^j + v_{i-1}^j) \\ (u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1})(u_{i+1}^j - 2u_i^j + u_{i-1}^j) \end{pmatrix} \quad (13)$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \frac{m\omega^2}{2} x^2 \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{m\omega^2 x_i^2}{4} \begin{pmatrix} -v_{i+1}^{j+1} - v_i^j \\ u_i^{j+1} + u_i^j \end{pmatrix} \quad (14)$$

$$- \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} b(x)s(t) \begin{pmatrix} u \\ v \end{pmatrix} \sim \frac{b(x_i)}{2} \begin{pmatrix} s(t_{j+1})v_{j+1}^i + s(t_{j+1})v_j^i \\ -s(t_{j+1})u_{j+1}^i - s(t_{j+1})u_j^i \end{pmatrix} \quad (15)$$

$$u_i^{j+1} + \frac{\Delta t}{4m\Delta x^2} \left(v_{i+1}^{j+1} - 2v_i^{j+1} + v_{i-1}^{j+1} \right) + \frac{m\omega^2 \Delta t}{4} x_i^2 v_i^{j+1} - \frac{b(x_i) \Delta t}{2} s(t_{j+1}) v_{j+1}^i \quad (16)$$

$$= u_i^j + \frac{\Delta t}{4m\Delta x^2} \left(v_{i+1}^j - 2v_i^j + v_{i-1}^j \right) + \frac{m\omega^2 \Delta t}{4} x_i^2 v_i^j - \frac{b(x_i) \Delta t}{2} s(t_j) v_j^i \quad (17)$$

$$v_i^{j+1} + \frac{\Delta t}{4m\Delta x^2} \left(u_{i+1}^{j+1} - 2u_i^{j+1} + u_{i-1}^{j+1} \right) + \frac{m\omega^2 \Delta t}{4} x_i^2 u_i^{j+1} - \frac{b(x_i) \Delta t}{2} s(t_{j+1}) u_{j+1}^i \quad (18)$$

$$= v_i^j - \frac{\Delta t}{4m\Delta x^2} \left(u_{i+1}^j - 2u_i^j + u_{i-1}^j \right) - \frac{m\omega^2 \Delta t}{4} x_i^2 u_i^j + \frac{b(x_i) \Delta t}{2} s(t_j) u_j^i \quad (19)$$

$$L \begin{pmatrix} u_1^{j+1} \\ \vdots \\ u_N^{j+1} \\ v_1^{j+1} \\ \vdots \\ v_N^{j+1} \end{pmatrix} = R \begin{pmatrix} u_1^j \\ \vdots \\ u_N^j \\ v_1^j \\ \vdots \\ v_N^j \end{pmatrix} \quad (20)$$