$$\mathbf{E} = -2\phi_0 x \mathbf{e}_x \tag{1}$$

$$\phi = -\mathbf{r}\cdot\int_0^1 \mathrm{d}u\, \mathbf{E}(u\mathbf{r},t) = 2\phi_0 x^2 \int_0^1 \mathrm{d}u\, u = \phi_0 x^2 \tag{2}$$

$$\mathbf{B} = B e_z \tag{3}$$

$$\mathbf{A} = -\mathbf{r}\times\int_0^1 \mathrm{d}u\, u\mathbf{B}(u\mathbf{r},t) = \frac{B}{2}\begin{pmatrix}-y\\ x\\ 0\end{pmatrix} \tag{4}$$

$$\mathbf{A} \to \mathbf{A} + \nabla\Lambda = B\begin{pmatrix}0\\ x\\ 0\end{pmatrix} \tag{5}$$

$$\Lambda = \frac{B}{2}xy \tag{6}$$

$$\phi' \to \phi - \partial_t \Lambda = \phi \tag{7}$$

$$\hat{H} = \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_z^2) + \frac{1}{2m}(\hat{p}_y - qB\hat{x})^2 + \phi_0 \hat{x}^2 \tag{8}$$

$$= \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2) - \frac{qB}{m}\hat{x}\hat{p}_y + \left(\frac{q^2B^2}{2m} + \phi_0\right)\hat{x}^2 \tag{9}$$

$$\left[\hat{H},\hat{p}_z\right] = \left[\hat{H},\hat{p}_y\right] = 0 \tag{10}$$

$$\hat{p}_z \to p_z, \quad \hat{p}_y \to p_y, \quad \hat{p}_x \to -\mathrm{i}\hbar\partial_x \tag{11}$$

$$\psi(x,y,z) = e^{\frac{\mathrm{i}}{\hbar}(yp_y + zp_z)}\chi(x) \tag{12}$$

$\omega_c = \sqrt{\frac{q^2B^2}{m^2} + 2\frac{\phi_0}{m}}$

$$\hat{H} = \frac{p_y^2 + p_z^2}{2m} - \frac{\hbar^2}{2m}\partial_x^2 - \frac{qBp_y}{m}x + \frac{1}{2}m\omega_c^2 x^2 \tag{13}$$

$$= \frac{p_y^2 + p_z^2}{2m} - \frac{\hbar^2}{2m}\partial_x^2 + \frac{1}{2}m\omega_c^2\left(x - \frac{qBp_y}{m^2\omega_c^2}\right)^2 - \frac{q^2B^2p_y^2}{2m^3\omega_c^2} \tag{14}$$

$\xi = x - x_0 = x - \frac{qBp_y}{m^2\omega_c^2}$, $C = \frac{q^2B^2p_y^2}{2m^3\omega_c^2}$

$$\hat{H} = \frac{p_y^2 + p_z^2}{2m} - C - \frac{\hbar^2}{2m}\partial_\xi^2 + \frac{1}{2}m\omega_c^2\xi^2 \tag{15}$$

$$\hat{H}_1 = \hat{H} - \hat{H}_0 = -\frac{\hbar^2}{2m}\partial_\xi^2 + \frac{1}{2}m\omega_c^2\xi^2 \tag{16}$$

$$E_1\chi(\xi) = \hat{H}_1\chi(\xi) \tag{17}$$

$$\chi_n(\xi) = N_n e^{-\frac{m\omega_c}{2\hbar}\xi^2} H_n\left(\sqrt{\frac{m\omega_c}{\hbar}}\xi\right) \tag{18}$$

$N_n = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega_c}{\pi\hbar}\right)^{\frac{1}{4}}$

$$\psi_{n,p_y,p_z} = e^{\frac{\mathrm{i}}{\hbar}(yp_y + zp_z - Ct)}\chi_{n,p_y}(x - x_0) \tag{19}$$

$$\chi_{n,p_y}(x - x_0) = N_n \exp\left(-\frac{m\omega_c}{2\hbar}\left(x - \frac{qBp_y}{m^2\omega_c^2}\right)^2\right) H_n\left(\sqrt{\frac{m\omega_c}{\hbar}}\left(x - \frac{qBp_y}{m^2\omega_c^2}\right)\right) \tag{20}$$

$$\psi_{0,p_y,p_z}(x,y,z) = \left(\frac{m\omega_c}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left(-\frac{m\omega_c}{2\hbar}\left(x - \frac{qBp_y}{m^2\omega_c^2}\right)^2 + \frac{\mathrm{i}}{\hbar}p_y y + \frac{\mathrm{i}}{\hbar}p_z z - \frac{\mathrm{i}}{\hbar}Ct\right) \tag{21}$$

$$\left|\psi_{0,p_y,p_z}(x,y,z)\right|^2 = \sqrt{\frac{m\omega_c}{\pi\hbar}} \exp\left(-\frac{m\omega_c}{\hbar}\left(x - \frac{qBp_y}{m^2\omega_c^2}\right)^2\right) \tag{22}$$

$$E = E_0 + E_1^{(n)} = \frac{p_y^2 + p_z^2}{2m} - C + \hbar\omega_c(n + \frac{1}{2}) \tag{23}$$

$$= \frac{p_y^2 + p_z^2}{2m} - \frac{q^2B^2p_y^2}{2m^3\left(\frac{q^2B^2}{m^2} + 2\frac{\phi_0}{m}\right)} + \hbar\sqrt{\frac{q^2B^2}{m^2} + 2\frac{\phi_0}{m}}\left(n + \frac{1}{2}\right) \tag{24}$$

$$\mathbf{A}' = \mathbf{A} + \nabla\Lambda = B\begin{pmatrix}0\\x\\0\end{pmatrix} - B\begin{pmatrix}0\\x_1\\0\end{pmatrix} \tag{25}$$

$$\Lambda = -Bx_1 y \tag{26}$$

$$\phi' = \phi - \partial_t\Lambda = \phi \tag{27}$$

$$\hat{H}' = \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_z^2) + \frac{1}{2m}(\hat{p}_y - qB\hat{x} + qBx_1)^2 + \phi_0\hat{x}^2 \tag{28}$$

$$= \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2) - \left(\frac{qB}{m}\hat{p}_y + \frac{q^2B^2}{m}x_1\right)\hat{x} + \left(\frac{q^2B^2}{2m} + \phi_0\right)\hat{x}^2 + \frac{qB}{m}\hat{p}_y x_1 + \frac{q^2B^2}{2m}x_1^2 \tag{29}$$

$\omega_c = \sqrt{\frac{q^2B^2}{m^2} + 2\frac{\phi_0}{m}}$

$$\hat{H}' = \frac{p_y^2 + p_z^2}{2m} - \frac{\hbar^2}{2m}\partial_x^2 - \left(\frac{qB}{m}p_y + \frac{q^2B^2}{m}x_1\right)x + \frac{1}{2}m\omega_c^2 x^2 + \frac{qB}{m}p_y x_1 + \frac{q^2B^2}{2m}x_1^2 \tag{30}$$

$$= \frac{p_y^2 + p_z^2}{2m} - \frac{\hbar^2}{2m}\partial_x^2 + \frac{1}{2}m\omega_c^2\left(x - \frac{qBp_y}{m^2\omega_c^2} - \frac{q^2B^2}{m^2\omega_c^2}x_1\right)^2 - \frac{qBp_y}{2m^2\omega_c^2} - \frac{q^2B^2}{2m^2\omega_c^2}x_1 + \frac{qB}{m}p_y x_1 + \frac{q^2B^2}{2m}x_1^2 \tag{31}$$

$\zeta = x - x_0' = x - \frac{qBp_y}{m^2\omega_c^2} - \frac{q^2B^2}{m^2\omega_c^2}x_1$, $C' = \frac{qBp_y}{2m^2\omega_c^2} + \frac{q^2B^2}{2m^2\omega_c^2}x_1 - \frac{qB}{m}p_y x_1 - \frac{q^2B^2}{2m}x_1^2$

$$\hat{H}' = \frac{p_y^2 + p_z^2}{2m} - C' - \frac{\hbar^2}{2m}\partial_\zeta^2 + \frac{1}{2}m\omega_c^2\zeta^2 \tag{32}$$

$$\hat{H}_1 = \hat{H} - \hat{H}_0 = -\frac{\hbar^2}{2m}\partial_\zeta^2 + \frac{1}{2}m\omega_c^2\zeta^2 \tag{33}$$

$N_n = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega_c}{\pi\hbar}\right)^{\frac{1}{4}}$

$$\psi'_{n,p_y,p_z} = e^{\frac{\mathrm{i}}{\hbar}(yp_y + zp_z - C't)}\chi'_{n,p_y}(x - x_0') \tag{34}$$

$$\chi'_{n,p_y}(x - x_0') = N_n \exp\left(-\frac{m\omega_c}{2\hbar}\left(x - \frac{qBp_y}{m^2\omega_c^2} - \frac{q^2B^2}{m^2\omega_c^2}x_1\right)^2\right)H_n\left(\sqrt{\frac{m\omega_c}{\hbar}}\left(x - \frac{qBp_y}{m^2\omega_c^2} - \frac{q^2B^2}{m^2\omega_c^2}x_1\right)\right) \tag{35}$$

$$\psi'_{0,p_y,p_z}(x,y,z) = \left(\frac{m\omega_c}{\pi\hbar}\right)^{\frac{1}{4}}\exp\left(-\frac{m\omega_c}{2\hbar}\left(x - \frac{qBp_y}{m^2\omega_c^2} - \frac{q^2B^2}{m^2\omega_c^2}x_1\right)^2 + \frac{\mathrm{i}}{\hbar}p_y y + \frac{\mathrm{i}}{\hbar}p_z z - \frac{\mathrm{i}}{\hbar}C't\right) \tag{36}$$

$$\left|\psi'_{0,p_y,p_z}(x,y,z)\right|^2 = \sqrt{\frac{m\omega_c}{\pi\hbar}}\exp\left(-\frac{m\omega_c}{\hbar}\left(x - \frac{qBp_y}{m^2\omega_c^2} - \frac{q^2B^2}{m^2\omega_c^2}x_1\right)^2\right) \neq \left|\psi_{0,p_y,p_z}(x,y,z)\right|^2 \tag{37}$$

$$p_y \neq p_y' \tag{38}$$

$$\mathbf{E} = -2\phi_{0x}x\mathbf{e}_x - 2\phi_{0y}y\mathbf{e}_y \tag{39}$$

$$\phi = -\mathbf{r} \cdot \int_0^1 \mathrm{d}u\, \mathbf{E}(u\mathbf{r}, t) = 2(\phi_{0x}x^2 + \phi_{0y}y^2) \int_0^1 \mathrm{d}u\, u = \phi_{0x}x^2 + \phi_{0y}y^2 \tag{40}$$

$$\mathbf{B} = Be_z \tag{41}$$

$$\mathbf{A} = -\mathbf{r} \times \int_0^1 \mathrm{d}u\, u\mathbf{B}(u\mathbf{r}, t) = \frac{B}{2} \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \tag{42}$$

$$\mathbf{A} \to \mathbf{A} + \nabla\Lambda = B \begin{pmatrix} 0 \\ x \\ 0 \end{pmatrix} \tag{43}$$

$$\Lambda = \frac{B}{2}xy \tag{44}$$

$$\phi' \to \phi - \partial_t\Lambda = \phi \tag{45}$$

$$\hat{H} = \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_z^2) + \frac{1}{2m}(\hat{p}_y - qB\hat{x})^2 + \phi_{0x}\hat{x}^2 + \phi_{0y}\hat{y}^2 \tag{46}$$

$$= \frac{1}{2m}(\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2) - \frac{qB}{m}\hat{x}\hat{p}_y + \left(\frac{q^2B^2}{2m} + \phi_{0x}\right)\hat{x}^2 + \phi_{0y}\hat{y}^2 \tag{47}$$

$$\left[\hat{H}, \hat{p}_z\right] = 0 \tag{48}$$

$$\hat{p}_z \to p_z, \quad \hat{p}_y \to -\mathrm{i}\hbar\partial_y, \quad \hat{p}_x \to -\mathrm{i}\hbar\partial_x \tag{49}$$

$$\psi(x, y, z) = e^{\frac{\mathrm{i}}{\hbar}zp_z}\chi(x, y) \tag{50}$$

$\omega_c = \sqrt{\frac{q^2B^2}{m^2} + 2\frac{\phi_{0x}}{m}}$

$$\hat{H} = \frac{p_z^2}{2m} - \frac{\hbar^2}{2m}\partial_y^2 + \phi_{0y}y^2 - \frac{\hbar^2}{2m}\partial_x^2 - \frac{qB}{m}x\partial_y + \frac{1}{2}m\omega_c^2x^2 \tag{51}$$