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Advanced Time Series and Financial Econometrics

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Forecasting the Term Spread of Treasury Yields

Introduction

The "term-spread" of interest rates refers to the difference in interest rates between long-term and short-term securities and is commonly calculated as the difference between the interest rates on treasuries of 10-year maturity and either the interest rate on treasuries of 2-year maturity or the interest rates on treasury bills (less than 1-year in maturity). This spread between rates has important implications for the economy. For example, a high 10-year minus 2-year rate reflects investor beliefs of low short-term uncertainty. On the other hand, a low or negative 10-year minus 2-year rate reflects investor uncertainty in the short-term and/or the impact of short-term interest rate hikes. A low or negative 10-year minus 2-year rate reflects an "inversion" of the yield curve and is commonly regarded as a signal of incoming recession. For these reasons, a sufficient method of forecasting term spreads is necessary for predicting future financial instability.

For this study, different forecasting methods will be utilized to find a sufficient method of forecasting the difference between 10-year market yields and 2-year market yields. A high 10-year minus 2-year treasury yield reflects economic stability, while a low or negative 10-year minus 2-year treasury yield reflects short-term uncertainty and an inversion of the yield curve. Because treasury yields move "in tandem" with interest rates according to economic theory, the difference between these yields reflects changes in the term-spread of interest rates.

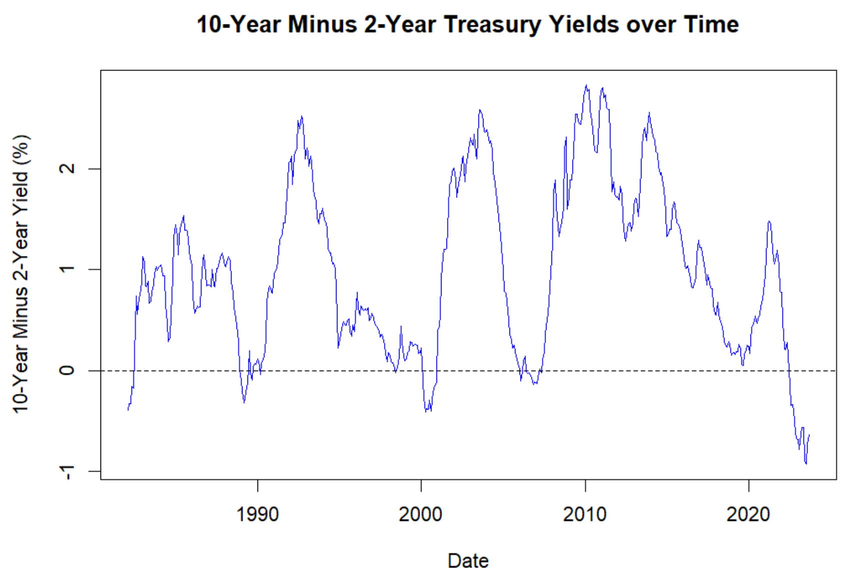
My personal interest in this project stems from observing the current inversion of the yield curve amidst the uncertainty of the current financial environment. In economics coursework I have learned of the importance of the yield curve to indicating recession and would like to analyze which method is best for forecasting this behavior.

The datasets used in this study come from the Federal Reserve Bank of St. Louis' FRED database. The main datasets used are:

- (GS10) Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Quoted on an Investment Basis, Monthly Average
- (GS2) Market Yield on U.S. Treasury Securities at 2-Year Constant Maturity, Quoted on an Investment Basis, Monthly Average
- Term-spread of yields, calculated as the GS10 – GS2

The ARIMA forecasts will include data from September 1981 to September 2023. This time span ensures that these forecasting models are built on treasury yields from a wide range of economic environments in American history, such as the end of stagflation in 1982, economic crises of the 1990s, stability of the early 2000s, financial crisis of 2008, and COVID-19 pandemic post-2020. The section on cointegration will use data from June 1976 (the earliest data from the GS2 series) to September 2023; this includes the volatility of the Volcker era Fed, which is excluded from the ARIMA modeling as not skew that model. Volatility in this manner is not likely to occur again, as the Fed has instituted differing policy directions from those of the Volcker era.

The time series of the difference in 10-Year and 2-Year Treasury Yields is as such:



Forecasting 10-Year minus 2-Year Yields – ARIMA approach

An ARIMA(p,d,q) method will be used to forecast the difference in 10-Year and 2-Year Treasury Yields from September 1981 to September 2023, using expanding window forecasts.

First, an Augmented Dickey-Fuller Test is conducted to determine if this series is stationary. From the graph, it seems that this series is not drifting and does not contain a trend. Therefore, the DF Test conducted will not consider drift or trend in the series. The value of the resulting test statistic is too large to reject the null hypothesis that the series is non-stationary, at the 95% confidence level.

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Value of test-statistic is: -1.8049

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
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The first difference of the series is taken, and the Augmented Dickey-Fuller Test concludes that this series is stationary, as the test statistic is far lower than the 95% critical value. The graph of the differenced series justifies this as it appears to be mean-reverting.

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Value of test-statistic is: -16.1197

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
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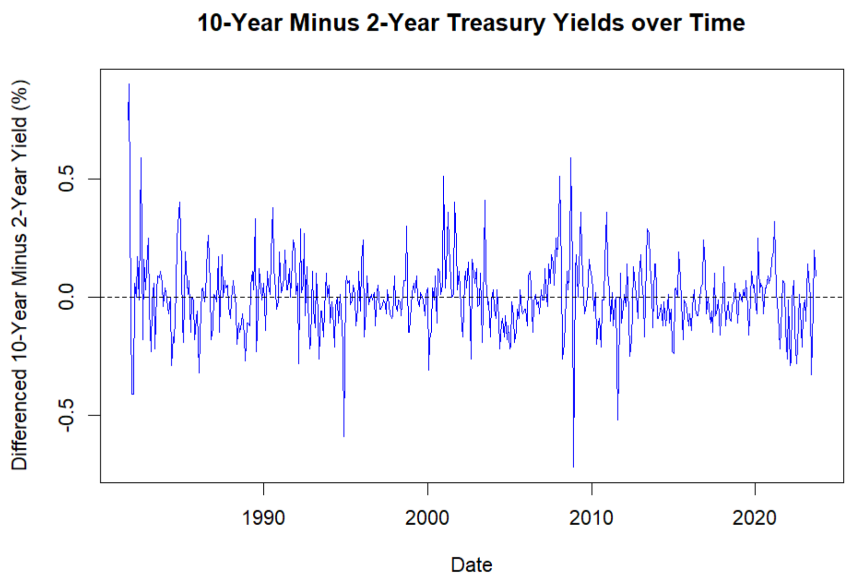
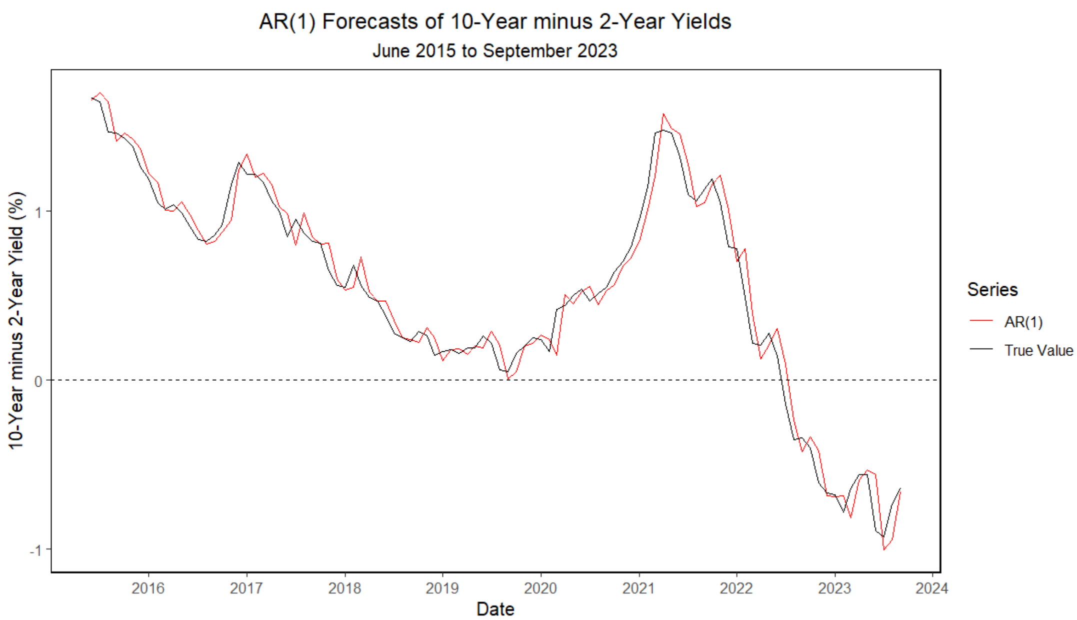


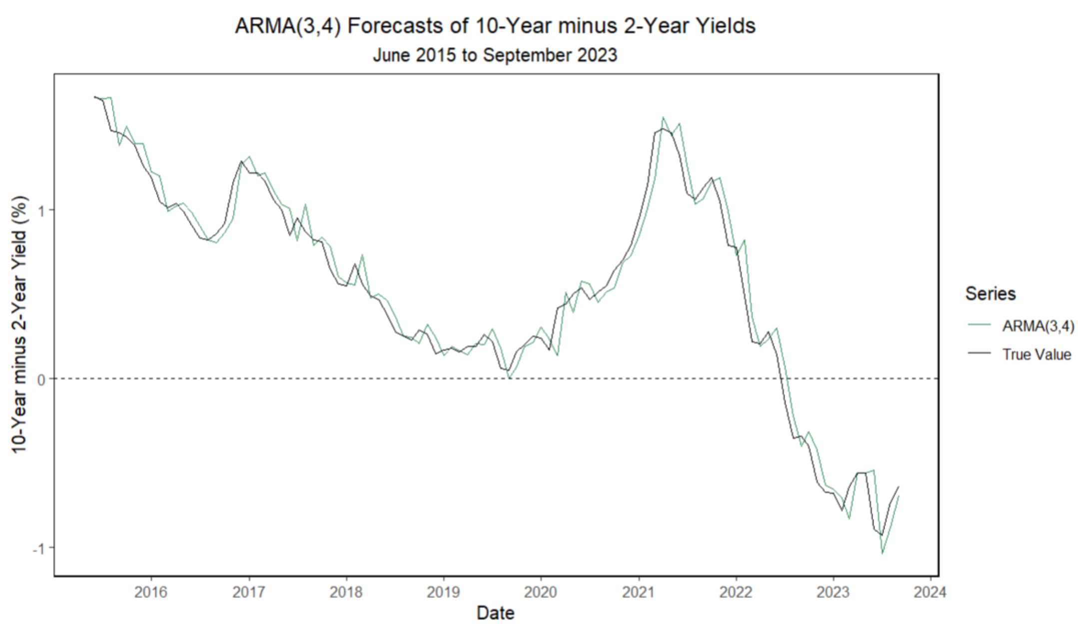
For robustness, a random walk model, an AR(1) model, and an ARMA(3,4) model (the model selected by the auto.arima function) will be compared to evaluate which model has lowest forecast errors. The out-of-sample period will be from June 2015 to September 2023, which constitutes roughly 20% of the observations in the series. The resulting MSPE errors are as follows:

Model	Random Walk	AR(1)	ARMA(3,4)
MSPE	0.01720	0.01194	0.01235

To test if one of these forecasts is better than the other, Diebold-Mariano Tests will be conducted on the forecast errors to determine if any one forecast is better than the other two. Both the AR(1) model and ARMA(3,4) models have significantly lower forecasting errors than the Random Walk Model, with p-values returned from the DM Test between these forecasts and the Random Walk Model being 0.009 and 0.022 respectively. However, the two models do not have significantly different forecasting errors from each other, as the p-value returned by the DM Test between these two models is 0.486, which is larger than the value of 0.05 needed to reject the assumption of indifference between these two forecasts.

The AR(1) and ARMA(3,4) forecasts are very close to each other across the out-of-sample period. From these graphs and the results of the DM Test, the AR(1) model and ARMA(3,4) models are acceptable forecasts of 10-Year minus 2-Year Treasury Yields.

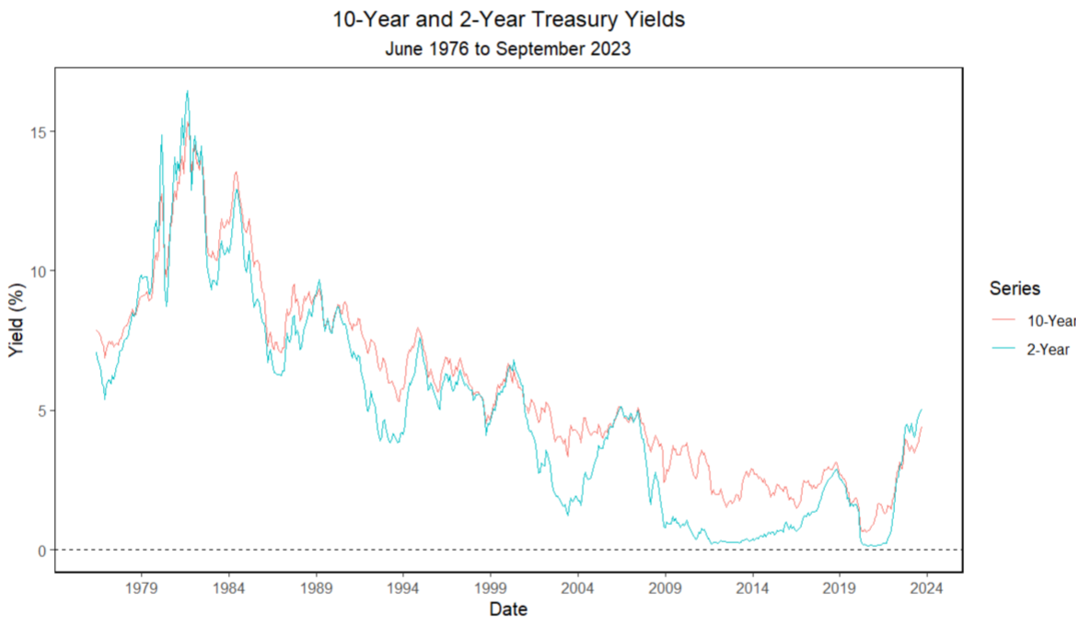




Forecasting 10-Year minus 2-Year Yields – Cointegration approach

A second approach to forecasting 10-Year minus 2-Year Treasury Yields is to utilize a cointegration approach between the two yields. For two time series to be cointegrating, they must follow a long-term equilibrium relationship, i.e. following the same long-run stochastic trend. According to economic theory, the 10-Year yields and the 2-Year yields *should* be cointegrating rates. This is because they both return yields equal to the yields on "risk-free" bonds (such as the 3-month treasury bill), plus a risk premium. Therefore, in equilibrium the 10-Year Treasury Yield should be equal to the 2-Year Treasury Yield plus a constant.

Data from June 1976 to September 2023 will be considered in this cointegration approach. The same out-of-sample period will be used to evaluate the model generated using this approach.



To test if these two series are cointegrating, a linear regression model between the two series is conducted. If the residuals from this regression are stationary, it can be concluded that the series are cointegrating.

The linear regression $Y_{10} \sim Y_2$ is conducted, with Y_{10} being the 10-Year Treasury Yield, and Y_2 being the 2-Year Treasury Yield. Then, an Augmented Dickey-Fuller Test (with drift) is used to determine if the residuals from this model are stationary. The result is as follows:

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Value of test-statistic is: -3.3777 5.7601

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.43 -2.86 -2.57
phi1  6.43  4.59  3.78
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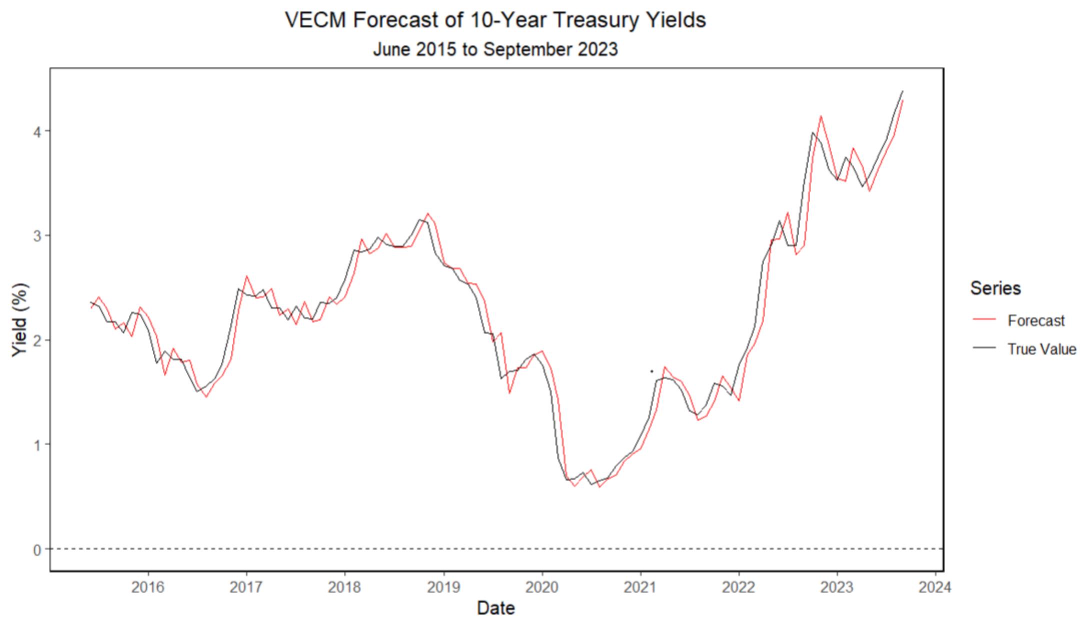
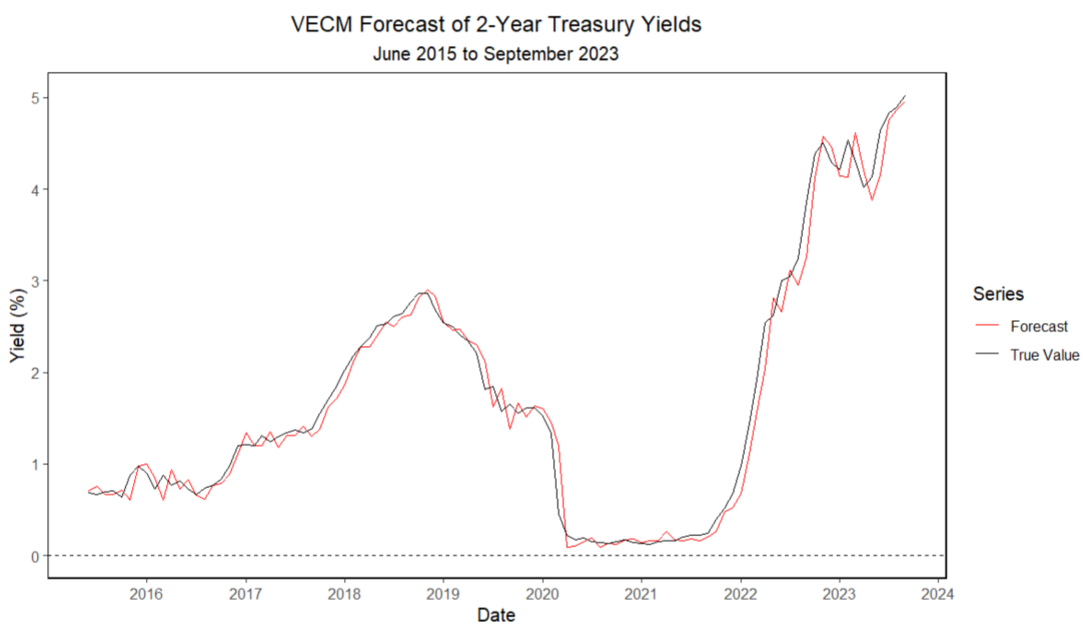
Since the test statistic of -3.378 is smaller than the critical value of -2.86 to reject the null hypothesis that the residuals are non-stationary, it can be concluded that the series are cointegrating. With this conclusion, a VECM model is constructed to return a VAR model that can be used to forecast the relationship between 10-Year Treasury Yields and 2-Year Treasury Yields. The following VECM model with a lag-order of 2 is constructed, with L representing the 10-Year Treasury Yield and S representing the 2-Year Treasury Yield:

$$L_t = 1.396L_{t-1} - 0.074S_{t-1} - 0.411L_{t-2} + 0.088S_{t-2} + 0.014$$

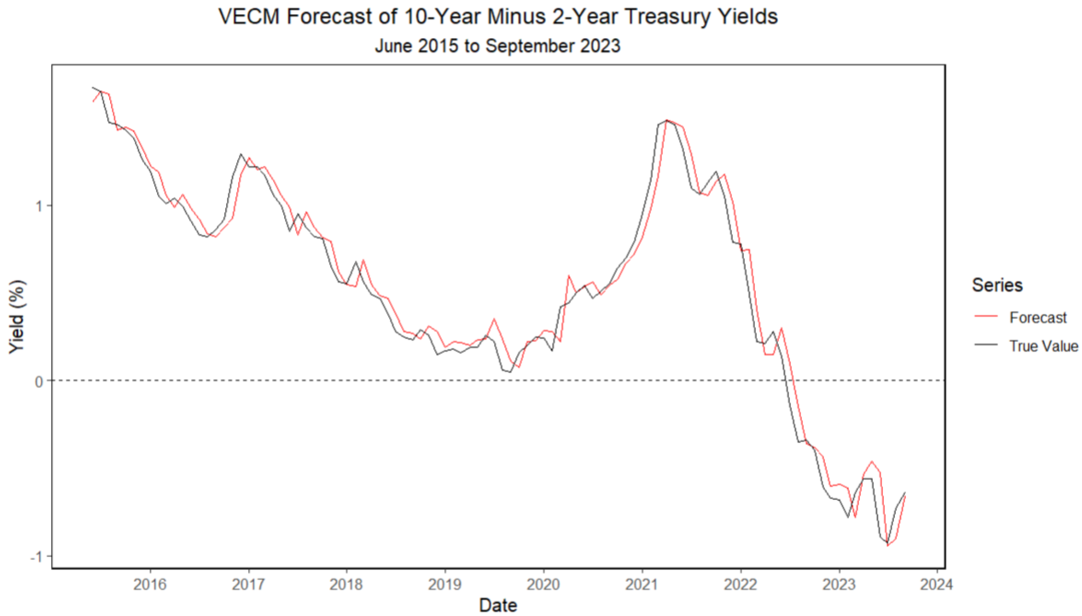
$$S_t = 0.300L_{t-1} + 1.164S_{t-1} - 0.279L_{t-2} - 0.184S_{t-2} - 0.026$$

Using this model, forecasts are generated for both the 10-Year and 2-Year Treasury Yields within the out-of-sample period of June 2015 to September 2023.





The forecast of the 10-Year minus 2-Year Treasury Yield is simply the difference between the forecast of the 10-Year Yield and the 2-Year Yield:

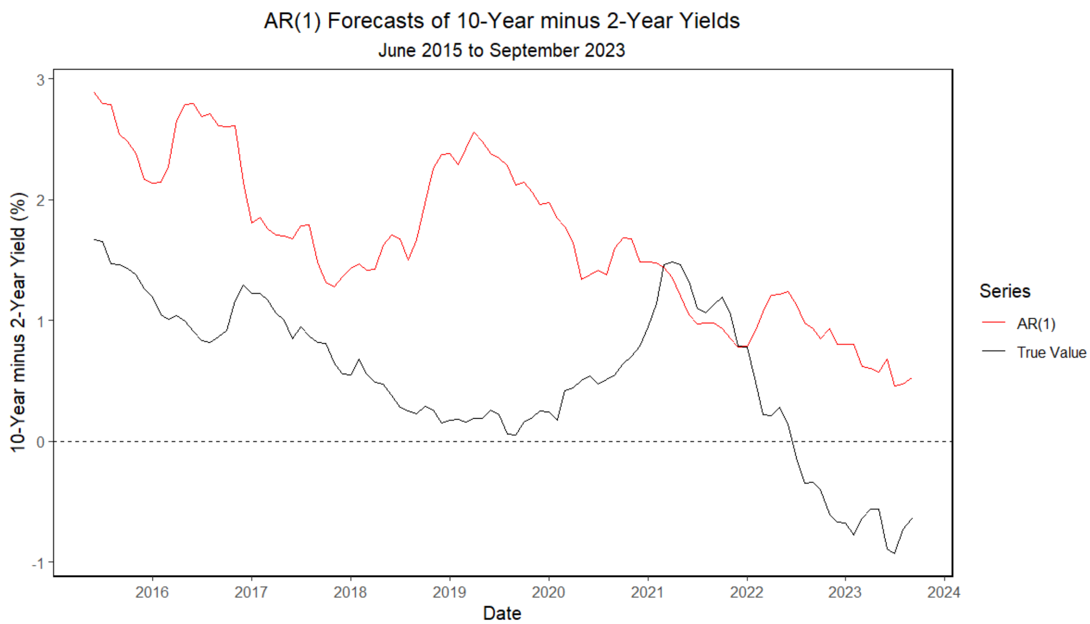


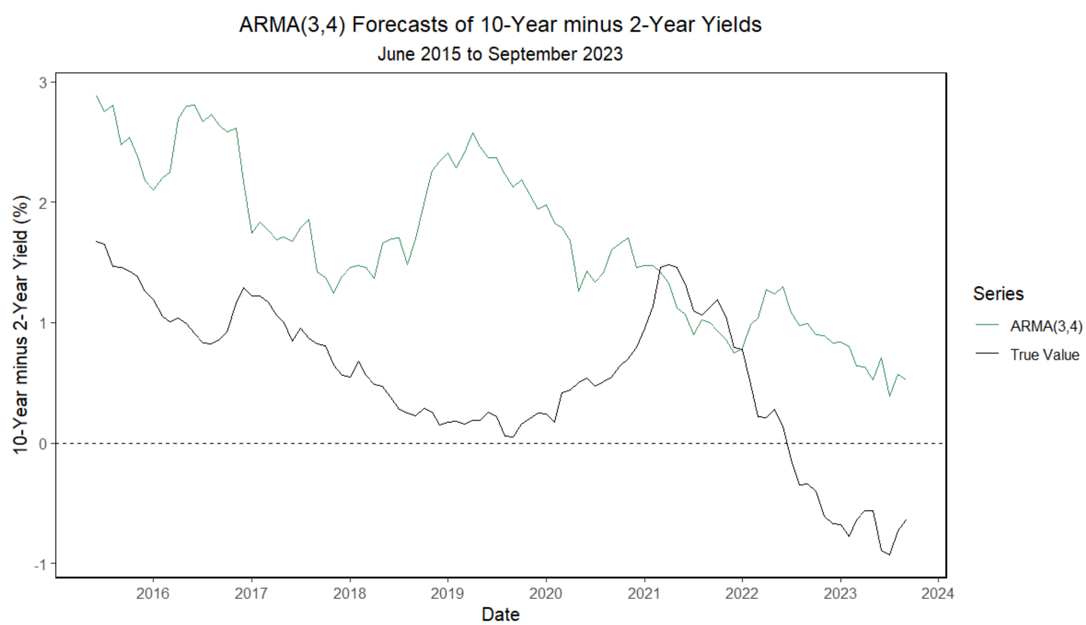
Visually this model does not appear any more or less accurate than the AR(1) or ARMA(3,4) models. The MSPE of this model is 0.01262. When running DM Tests to compare the forecast errors of the VECM model to the AR(1) or ARMA(3,4) models, the p-values are respectively 0.412 and 0.78. As these p-values are greater than 0.05, the DM Test concludes that the forecast errors between the VECM model and the AR(1) model, and the errors between the VECM model and the ARMA(3,4) model, are not significantly different, i.e., the VECM model is as good as both of these models at predicting the path of the 10-Year minus 2-Year Treasury Yield.

Conclusion:

Forecasting the monthly spread of the difference in yields between 10-Year Treasury Bonds and 2-Year Treasury Bonds is important to understanding instability in the financial system. Based on the techniques analyzed in this paper, it seems that both an AR(1) and ARMA(3,4) process sufficiently capture the behavior of 10-Year minus 2-Year Treasury Bond Yields. Using a cointegration approach generated another accurate forecast of this spread; additionally, this approach proved that 10-Year Treasury Yields and 2-Year Treasury Yields are cointegrating with 95% confidence, and the VECM model generated with this approach also found methods of forecasting the individual 10-Year and 2-Year Treasury Yields. As economic theory suggests, these two series have some long-term equilibrium relationship.

One method of improving this project could be to find a method of including data from January 1976 to September 2023, as opposed to data from September 1981 to the present. Including data from the Volcker era skews the AR(1) and ARMA(3,4) forecasts, as it shown below:





A way to approach this data could be to use a rolling forecasting window, as opposed to an expanding window as is used in this project. However, data from June 1976 was used in the cointegration approach, as the cointegration approach analyzes the movement of 10-Year and 2-Year Treasury Yield series with each other as opposed to the value of the 10-Year minus 2-Year series.

Data Sources:

GS10: *Market yield on U.S. Treasury securities at 10-year constant maturity, quoted on an investment basis*. FRED. (2023, December 15). https://fred.stlouisfed.org/series/GS10

GS2: *Market yield on U.S. Treasury securities at 2-year constant maturity, quoted on an investment basis*. FRED. (2023, December 15). https://fred.stlouisfed.org/series/GS2