Towards Multi-objective Mixed Integer Evolution Strategies

Koen van der Blom[1,a)], Kaifeng Yang[1,b)], Thomas Bäck[1,c)] and Michael Emmerich[1,d)]

[1]*LIACS, Leiden University, Niels Bohrweg 1, 2333CA Leiden, Netherlands*

[a)]Corresponding author: k.van.der.blom@liacs.leidenuniv.nl
[b)]k.yang@liacs.leidenuniv.nl
[c)]t.h.w.baeck@liacs.leidenuniv.nl
[d)]m.t.m.emmerich@liacs.leidenuniv.nl

Abstract. Many problems are of a mixed integer nature, rather than being restricted to a single variable type. Although mixed integer algorithms exist for the single-objective case, work on the multi-objective case remains limited. Evolution strategies are stochastic optimisation algorithms that feature step size adaptation mechanisms and are typically used in continuous domains. More recently they were generalised to mixed integer problems. In this work, first steps are taken towards extending the single-objective mixed integer evolution strategy for the multi-objective case. First results are promising, but step size adaptation for the multi-objective case can likely be improved.

INTRODUCTION

Multi-objective optimisation for either only continuous or only integer variables is widely studied, the mixed integer case is however largely neglected. In single-objective optimisation the mixed integer case was successfully tackled by algorithms such as the mixed integer evolution strategy (MIES) [1]. This paper takes first steps to extend the MIES algorithm for the multi-objective case. It should be noted that other multi-objective mixed integer approaches exist, such as the enhanced directed search method [2]. However, these do not distinguish between integer and nominal discrete (encoded by integers) variables like MIES does. Naturally, handling these variables separately may be advantageous.

As in canonical evolution strategies (ES) [3, 4], one of the core principles of the MIES algorithm is automatic step size adaptation, i.e., the online adaptation of the strength of the stochastic perturbations. However, step size adaptation mechanisms for the single-objective case do not necessarily directly transfer to the multi-objective case. Furthermore, in [5] the authors analysed step size adaptation in evolutionary multi-objective optimisation for continuous problems, and reported the best performance when recombination is not used.

This paper analyses mutation only approaches and step size adaptation in multi-objective mixed integer evolution strategies. To this end comparisons are made on both the performance in terms of diversity and convergence (combined in the hypervolume) to the Pareto front, as well as in terms of step size adaptation for the different variable types.

ALGORITHMS

Firstly, an algorithm is considered that combines the canonical mixed integer evolution strategy (MIES) as proposed in [1] with $\mathcal{S}$-metric selection and non-dominated sorting as used in SMS-EMOA [6], as well as the $(\mu + 1)$ strategy considered there. In the multi-objective case recombination may have a disruptive effect on the step size adaptation mechanism. This is the result of different individuals navigating towards different parts of the Pareto front. However, this first variant will retain recombination for comparison purposes.

Secondly, an alternative is considered that only uses mutation, but is otherwise equivalent to the first algorithm. Instead of mutating the offspring resulting from recombination, mutation is applied to a randomly selected individual.

Thirdly, an approach is considered that uses a tournament between mutants of the same parent as a local selection mechanism, but is otherwise equivalent to the second algorithm. In the tournament m mutants are generated for the

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TABLE 1. Settings of the benchmark functions

	n_r	range	n_z	range	n_d	range
f_{sphere}	5	$[0, 20]$	5	$[0, 20]$	5	$[0, 20]$
$f_{barrier}$	5	$[0, 20]$	5	$[0, 20]$	5	$[0, 20]$
$f_{optfilt}$	11	$[0, 1]$	N/A	N/A	11	$\{0, 1\}$

selected individual, rather than one. The mutant with the greatest hypervolume contribution is chosen as the winner, and enters $\mathcal{S}$-metric selection as usual. The idea is that competition between offspring may benefit step size adaptation.

In [1] no bounds were considered for continuous and integer step sizes. However, since these step size adaptation mechanisms were not designed with multi-objective optimisation in mind, step sizes might behave erratically and grow excessively. This could be one of the negative effects of recombining individuals that are navigating towards different parts of the Pareto front. To prevent this, step sizes for continuous and integer variables are given an upper bound equal to half the used variable range. Step sizes of nominal discrete variables were already bounded in [1].

EXPERIMENTAL SETUP

To evaluate the algorithms three problems are considered: the multi-sphere (msphere) function in Equation 1, the multi-barrier (mbarrier) function in Equation 2, and the multi-objective optical filter (moptfilt) problem [7, 8]. For the multi-objective case both the sphere and barrier functions can be adjusted with an offset for each term, such that continuous, integer, and nominal discrete optima are different in the second objective. The settings considered for each of these problems are shown in Table 1. Here $\mathbf{r}, \mathbf{z}, \mathbf{d}$ represent continuous, integer, and nominal discrete variables respectively, and n_r, n_i, n_d the dimensionality for each of them.

$$f_{sphere_1}(\mathbf{r}, \mathbf{z}, \mathbf{d}) = \sum_{i=1}^{n_r} r_i^2 + \sum_{i=1}^{n_z} z_i^2 + \sum_{i=1}^{n_d} d_i^2 \rightarrow min \qquad f_{sphere_2}(\mathbf{r}, \mathbf{z}, \mathbf{d}) = \sum_{i=1}^{n_r} (r_i - 2)^2 + \sum_{i=1}^{n_z} (z_i - 2)^2 + \sum_{i=1}^{n_d} (d_i - 2)^2 \rightarrow min \quad (1)$$

$$\begin{aligned} f_{barrier_1}(\mathbf{r}, \mathbf{z}, \mathbf{d}) &= \sum_{i=1}^{n_r} \left(r_i^2 + \theta \sin(r_i)^2 \right) + \sum_{i=1}^{n_z} A\,[z_i]^2 + \sum_{i=1}^{n_d} B_i\,[d_i]^2 \rightarrow min \\ f_{barrier_2}(\mathbf{r}, \mathbf{z}, \mathbf{d}) &= \sum_{i=1}^{n_r} \left((r_i - 2)^2 + \theta \sin(r_i - 2)^2 \right) + \sum_{i=1}^{n_z} (A\,[z_i] - 2)^2 + \sum_{i=1}^{n_d} (B_i\,[d_i] - 2)^2 \rightarrow min \end{aligned} \quad (2)$$

For the barrier function $\theta = 1$, A is generated by Algorithm 6 from [1] with the parameter $C = 20$, and $B_{i \in 1,\ldots,n_d}$ is a set of n_d random permutations of the integer sequence $\{0, 1, \ldots, 20\}$. Both A and B remain fixed for all experiments. Unlike in [1], here smooth wave-like barriers are used in the continuous part, rather than staircase-like barriers.

A variant of the optical filter problem from [7, 8] with mixed variables and a second objective is considered here. Pairs of continuous and (binary) nominal discrete variables are used. When a binary variable is active, the corresponding continuous variable is used in the objective functions, otherwise it is ignored. If all bits are inactive a penalty of $(250, 1250)$ is returned. The original objective considers the transformation of a light wave by means of a filter that consists of layers of different materials from a limited set of materials (discrete variables). The layers can have different widths (continuous variables). The transformed waveform is compared to a target waveform and the root mean square error is measured and to be minimised. In addition we consider a second objective, the minimisation of the filter thickness: $f_{optfilt_2}(\mathbf{r}, \mathbf{d}) = \sum_{i=1}^{n_r} r_i d_i \rightarrow min$.

Algorithm Settings

The canonical approach uses dominant recombination for the variables, and intermediate recombination for the step sizes. All three approaches use single step size mode in all domains (continuous, integer, and nominal discrete), meaning a single step size per domain. Furthermore, $\mu = 10$, and a reference point $(2500, 2500)$ are considered for all approaches and objective functions. The tournament based approach uses a tournament of size 2. Step sizes are initialised to 25% of the variable range for continuous and integer variables, and $\frac{1}{n_d}$ for nominal discrete variables.

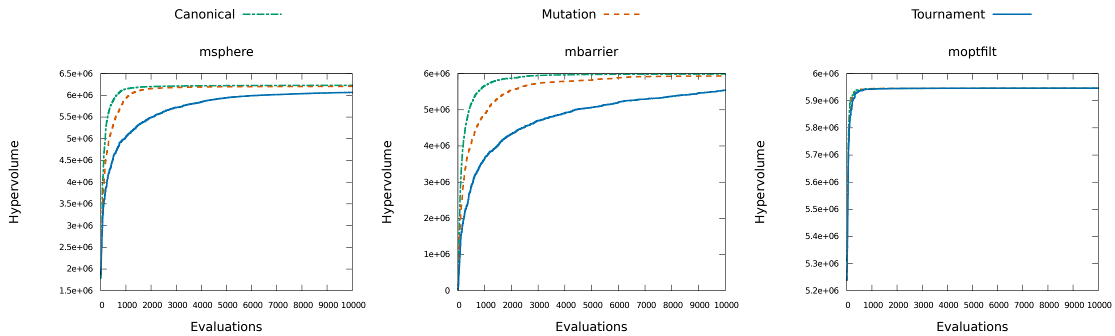

FIGURE 1. Mean hypervolume convergence over 25 repetitions.

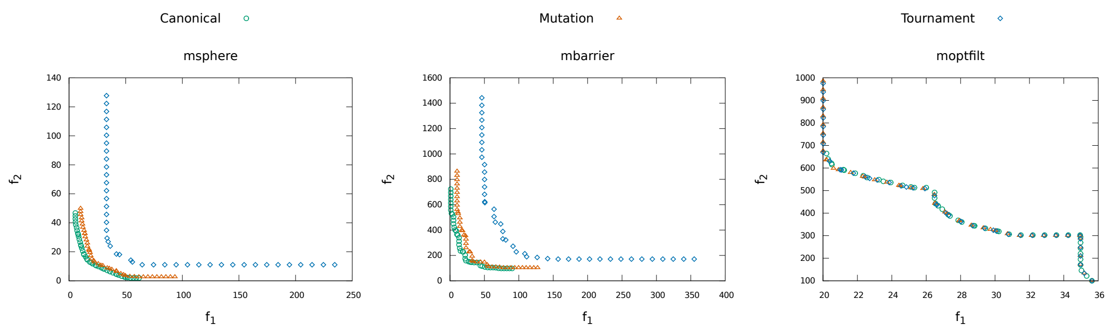

FIGURE 2. Median attainment curves over 25 repetitions.

Step sizes are bounded to $[0, 10]$ for continuous, $[1, 10]$ for integer, and $[\frac{1}{n_d}, 0.5]$ for nominal discrete variables. To be able to analyse the hypervolume and step size convergence during various phases in the optimisation process an evaluation budget of 10,000 is used.

RESULTS

For both the msphere and mbarrier problems the canonical MIES with $\mathcal{S}$-metric selection shows the fastest convergence in Figure 1, and outperforms the other approaches throughout the optimisation process. The mutation only approach has a slower start, but ultimately reaches only slightly worse hypervolume values. Any possible advantages of the additional selection pressure in the tournament approach are clearly mitigated by the larger number of evaluations used per generation. On the moptfilt problem all three approaches quickly converge to a stable situation.

The median attainment curves [9] in Figure 2 show that the canonical and mutation approaches find similar Pareto front approximations on the msphere and mbarrier functions, with the canonical approach remaining slightly better, as expected given the observed hypervolume convergence. All three approaches find very similar Pareto front approximations for the moptfilt problem, which suggests that they are close to the true Pareto front.

From Figure 3 it appears that step sizes σ for continuous and ζ for integer variables stabilise reasonably well, whereas step sizes p for the nominal discrete variables show more erratic behaviour. However, Figure 4 shows that step sizes generated for the offspring vary widely. The exception is the step size for continuous variables where the mutation only and tournament approaches do seem to stabilise. Although the tournament approach does so much later, this is likely due to its slower convergence. Thus, it appears only using mutation does indeed contribute to step size adaptation, but integer and nominal discrete step size adaptation have to be adjusted for the multi-objective case. Further, despite step sizes not adapting well when using recombination, it does result in better algorithm performance.

The experiments in this work show that a multi-objective mixed integer evolution strategy converges to accurate Pareto front approximations on two test problems, and a real world problem. However, step sizes do not adapt well in most considered situations. Only step sizes for continuous variables stabilise, and only without using recombination.

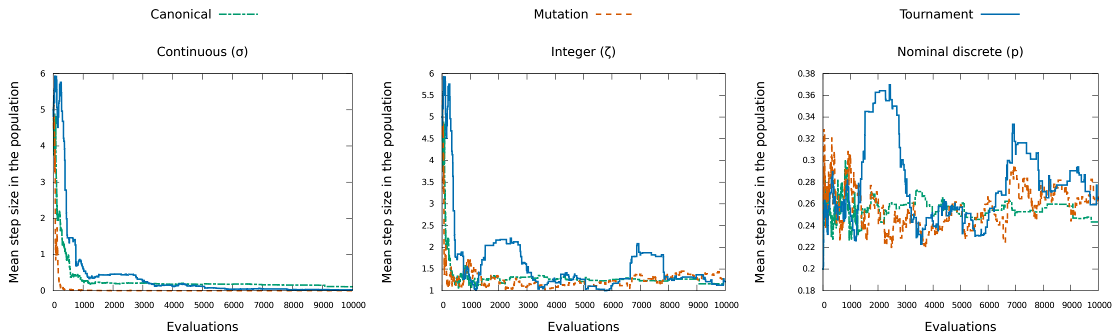

FIGURE 3. Mean step size in the population for each variable type, single run on the msphere problem

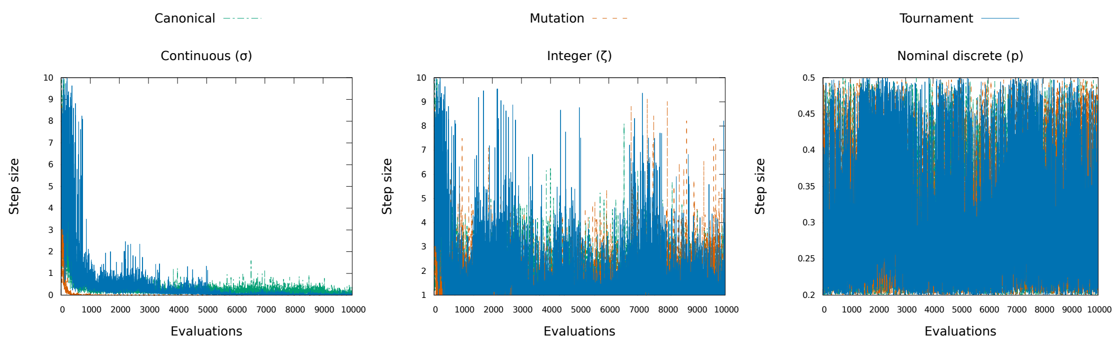

FIGURE 4. Step size per generated individual for each variable type, single run on the msphere problem

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