LE3 Problem 1

Last 4 digits of phone number: 2256. $G(s)$ is

$$G(s) = \frac{3}{2s^2 + 5s + 6} \tag{1}$$

Putting this into a unity-gain negative feedback gives

$$H(s) = \frac{G(s)}{1 + G(s)} \tag{2}$$

Its response to a unit step function is shown in Fig. 1, with %OS = 10% and $T_p = 1.83$ s. There is also a steady state error of 67%.

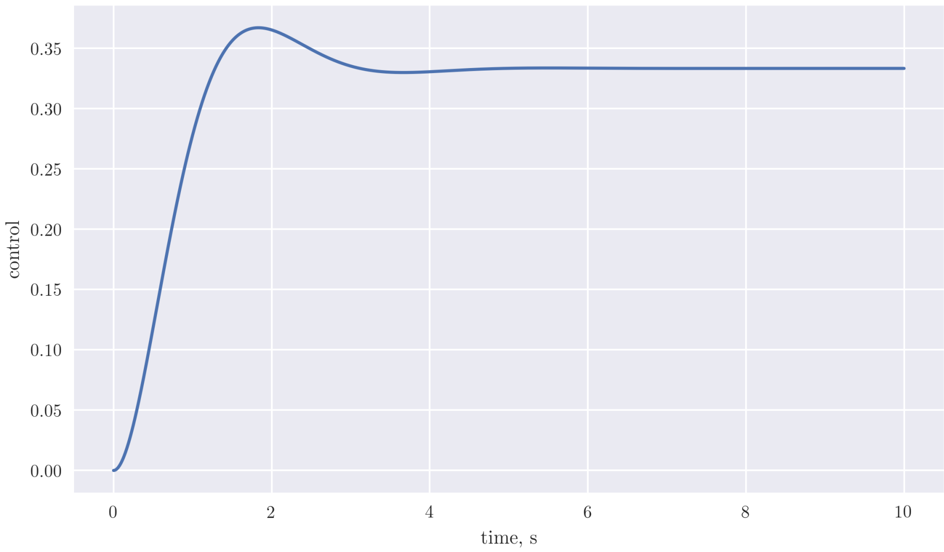

Figure 1: Response of $H(s)$ to a unit step function.

We want a step response with a percent overshoot of at most 10% and a peak time of 0.1 s.

a) Desired pole location

$$s_d = -\sigma_d \pm j\omega_d \tag{3}$$

$$= \zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} \tag{4}$$

$$\zeta = \frac{-\ln(\%OS)}{\sqrt{\pi^2 + \ln^2(\%OS)}} \tag{5}$$

$$= \frac{-\ln(0.1)}{\sqrt{\pi^2 + \ln^2(0.1)}} \tag{6}$$

$$= 0.59 \tag{7}$$

$$\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}} \tag{8}$$
$$= \frac{\pi}{0.1\sqrt{1-0.59^2}} \tag{9}$$
$$= 38.95 \text{ rad/s} \tag{10}$$
$$s_d = -(0.59)(38.95) + j(38.95)\sqrt{1-(0.59)^2} \tag{11}$$
$$\boxed{s_d = -23.03 + 31.42j} \tag{12}$$

b) **Angle deficiency**

$$G(s_d) = \frac{3}{2s_d^2 + 5s_d + 6} \tag{13}$$
$$= -3.60\times 10^{-4} + 9.62\times 10^{-4}j \tag{14}$$
$$\measuredangle G(s_d) = \arctan\left(\frac{\mathrm{Im}[G]}{\mathrm{Re}[G]}\right) \tag{15}$$
$$= 1.92 \text{ rad} = 110.49^\circ \tag{16}$$
$$\Phi_d = \pi - \measuredangle G(s_d) \tag{17}$$
$$\boxed{\Phi_d = 1.21 \text{ rad}} \tag{18}$$

c) **Compensator poles and zeros**

$$\alpha = \arctan\left(\frac{1-\zeta^2}{\zeta}\right) \tag{19}$$
$$= \arctan\left(\frac{1-0.59^2}{0.59}\right) \tag{20}$$
$$= 0.94 \tag{21}$$
$$z_c = -\omega_n\sqrt{1-\zeta^2}\tan\left(\frac{\alpha-\Phi_d}{2}\right) - \zeta\omega_n \tag{22}$$
$$= -(38.95)\sqrt{1-(0.59)^2}\tan\left(\frac{0.94-3.42}{2}\right) - (0.59)(38.95) \tag{23}$$
$$\boxed{z_c = -18.68} \tag{24}$$

$$p_c = -\omega_n\sqrt{1-\zeta^2}\tan\left(\frac{\alpha+\Phi_d}{2}\right) - \zeta\omega_n \tag{25}$$
$$= -(38.95)\sqrt{1-(0.59)^2}\tan\left(\frac{0.94+3.42}{2}\right) - (0.59)(38.95) \tag{26}$$
$$\boxed{p_c = -81.21} \tag{27}$$

d) **Compensator gain**

$$K = \frac{1}{\left|G(s_d)\frac{s_d + z_c}{s_d + p_c}\right|} \tag{28}$$

$$= \frac{1}{\left|(-3.60\times10^{-4} + 9.62\times10^{-4}j)\frac{-23.03+31.42j-18.68}{-23.03+31.42j-81.21}\right|} \tag{29}$$

$$\boxed{K = 2030.30} \tag{30}$$

e) **Simulink implementation**

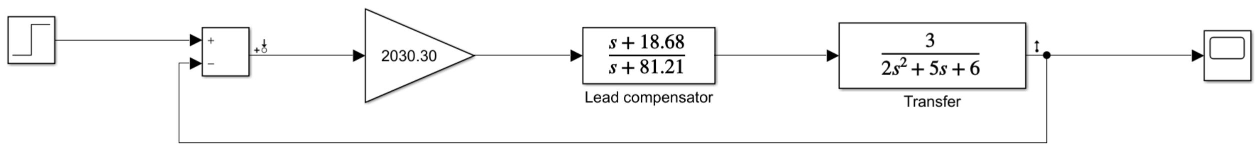

Figure 2: Lead compensator implementation in MATLAB Simulink.

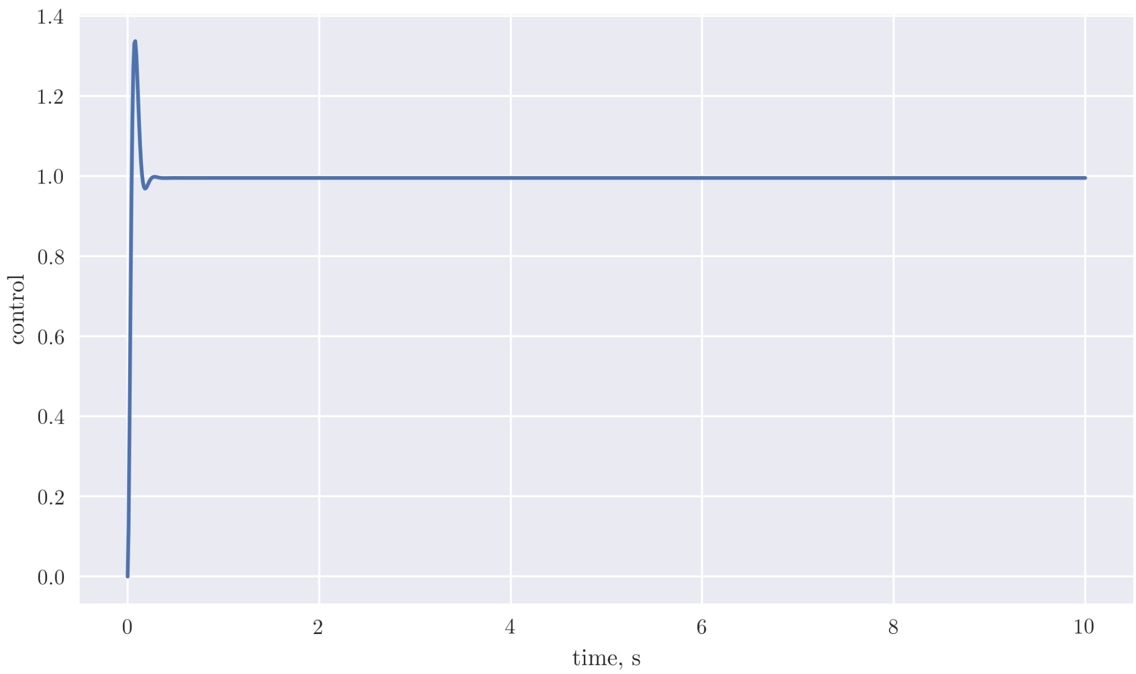

Figure 3: Unit step response of the system in Fig. 2.

From Fig. 3, we have %OS = 34% and $T_p = 0.08$ s. The lead compensator was able to bring the peak time slightly below 0.1 s, but actually increased the overshoot.