LE3 Problem 3

The describing function calculated from Problem 2 was

$$N(M, \omega) = -\frac{4jA}{\pi M} \tag{1}$$

And we cascade this with a process

$$G(s) = \frac{5}{s(1+s)^2} \tag{2}$$

a) **Calculation for ω**

We express $G(s)$ as a frequency response

$$\begin{align}
G(j\omega) &= \frac{5}{j\omega(1+j\omega)^2} \tag{3}\\
&= \frac{5}{j\omega(1+2j\omega-\omega^2)} \tag{4}\\
&= \frac{5}{-2\omega^2 + j(\omega-\omega^3)} \tag{5}\\
&= \frac{5}{-2\omega^2 + j(\omega-\omega^3)} \cdot \frac{-2\omega^2 + j(\omega-\omega^3)}{-2\omega^2 + j(\omega-\omega^3)} \tag{6}\\
&= \frac{-10\omega^2 + 5j(\omega-\omega^3)}{4\omega^4 - (\omega-\omega^3)^2} \tag{7}
\end{align}$$

We remove possible phase differences by equating the imaginary part to zero

$$\begin{align}
0 &= \omega - \omega^3 \tag{8}\\
&= \omega(1-\omega^2) \tag{9}\\
\omega_{z_1} = 0 \qquad \omega_{z_{2+}} &= 1 \qquad \omega_{z_{2-}} = -1 \tag{10}
\end{align}$$

However, we can discard $\omega = 0$ since it causes $G(j\omega)$ to blow up. We can also discard $\omega = -1$ since all the remaining terms will have even exponents. Thus, $\boxed{\omega = 1}$. Substituting $\omega = 1$ into G,

$$G(j1) = -\frac{10}{4} = -\frac{5}{2} = -2.5 \tag{11}$$

b) **Calculation for M**

We solve for M from $G = -N^{-1}$:

$$G = \frac{-1}{N} \tag{12}$$

$$-\frac{5}{2} = \frac{\pi M}{4jA} \tag{13}$$

$$\boxed{M = -\frac{10jA}{\pi}} \tag{14}$$

c) **Conclusion**

The input amplitude M is purely imaginary, so there is no limit cycle at $\omega = 1$ and $M = -10jA/\pi$.

For completeness, the Simulink implementation of the system is shown in Fig. 1 and its phase plot in Fig. 2. We can observe that there is indeed no limit cycle but instead, an attractor is present.

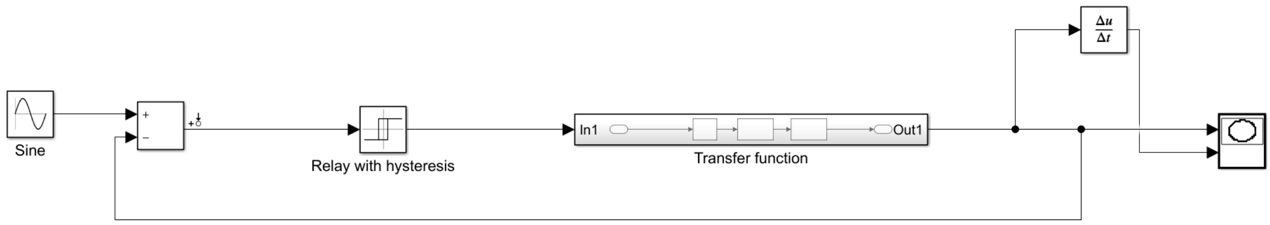

Figure 1: Non-linear system implementation in MATLAB Simulink.

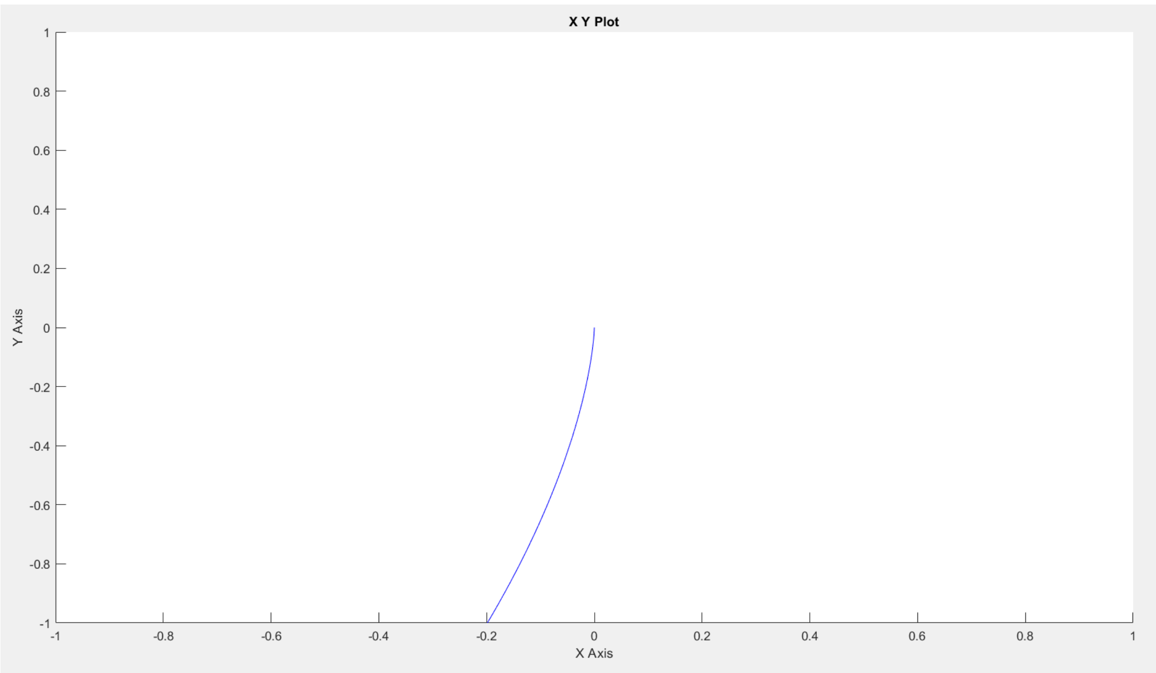

Figure 2: Phase plot of the system in Fig. 1.