Behavior of a 2nd-order underdamped system

A general 2nd-order system G(s) w/ transfer function

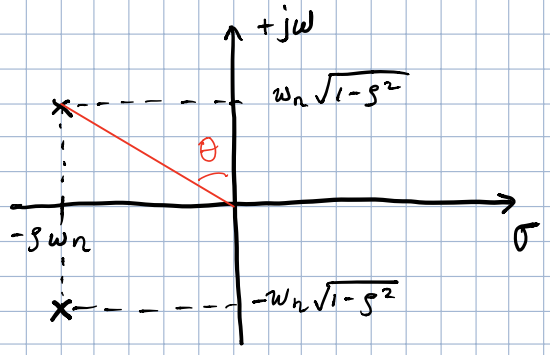
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

may be thought of as having two poles @

$$s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

In s-space the poles are located as shown:



QUESTION:

what is the hypotenuse?

$\sqrt{\zeta^2\omega_n^2 + \omega_n^2(1-\zeta^2)}$

$\omega_n = h$

What is the angle?

$\tan\theta = \frac{\zeta}{\sqrt{1-\zeta^2}}$

We calculated the 2nd-order underdamped response to a step function as

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos\left(\omega_n\sqrt{1-\zeta^2}\,t - \theta\right)$$

where $\theta = \tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}}$

From c(t) we see that

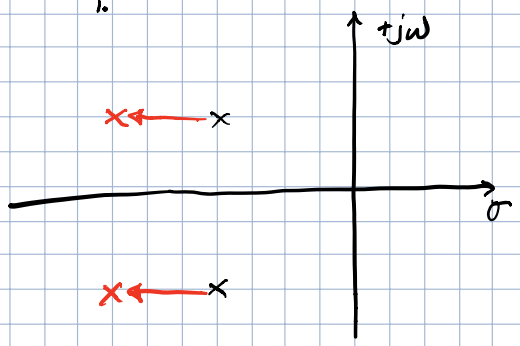
$(-\zeta\omega_n)$, the real part, dictates the shape of the exponential decay envelope.

$\omega_n\sqrt{1-\zeta^2}$, the imaginary part, dictates the frequency of oscillation inside the envelope.

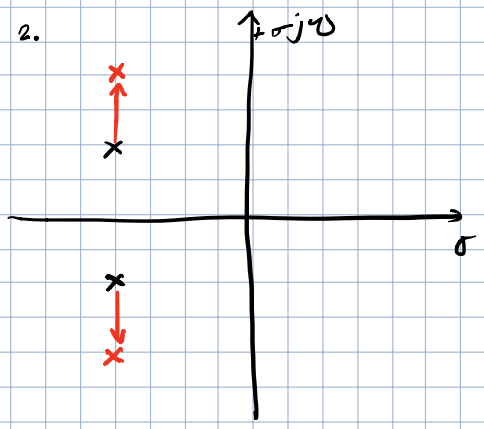
QUESTION:

what happens to the output if the poles move as shown:

1.



2.



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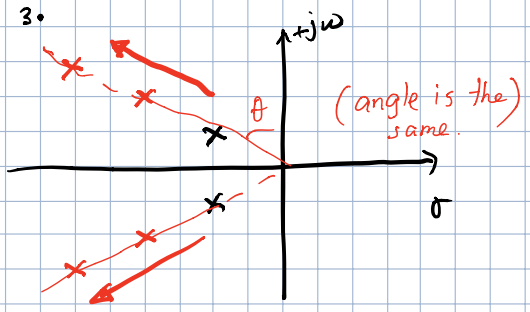

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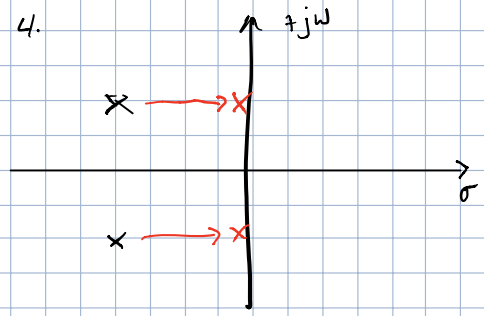
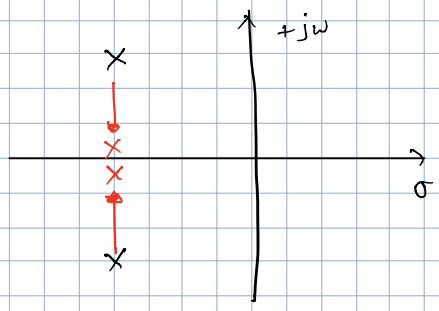
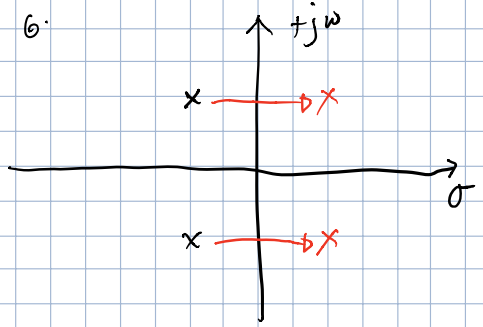


5.

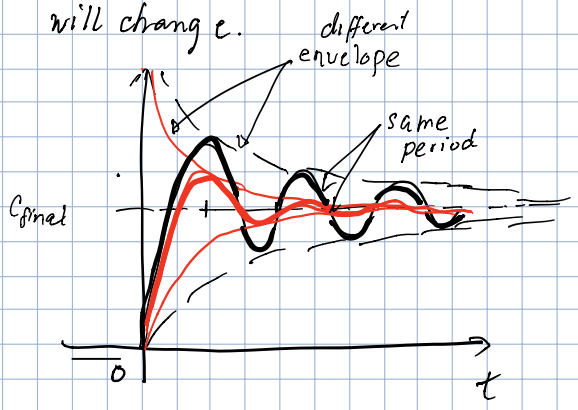


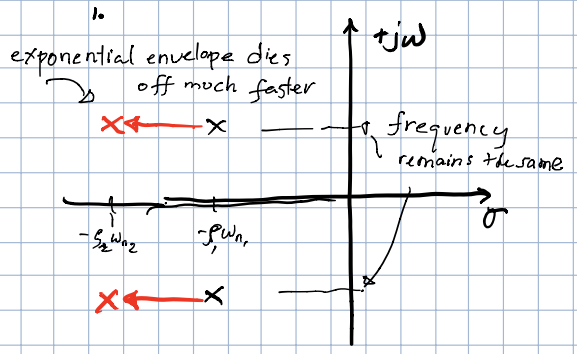
6.



Why are we doing this exercise? It gives you familiarity with how the system will behave if you are able to control its poles.

For example, the answer to No. 1 is: since the real part is becoming more negative and the imaginary part remains the same, only the exponential envelope will change.





EXERCISE:

PREDICT first what happens to the response. DO NOT PLOT anything yet. Justify your prediction.

OBSERVE what happens by plotting c(t) giving values for ζ and ω_n according to the pole plots. Use any plotting software.

EXPLAIN what you observe, noting if your prediction matched. If not, reflect on what you got wrong.

Submit your predictions, observations (plots) and explanations as a PDF in UVLE submission module.

(IF THAT DOESN'T WORK, email the pdf to msoriano@mip.upd.edu.ph.)

DUE WITHIN THE CLASS PERIOD.