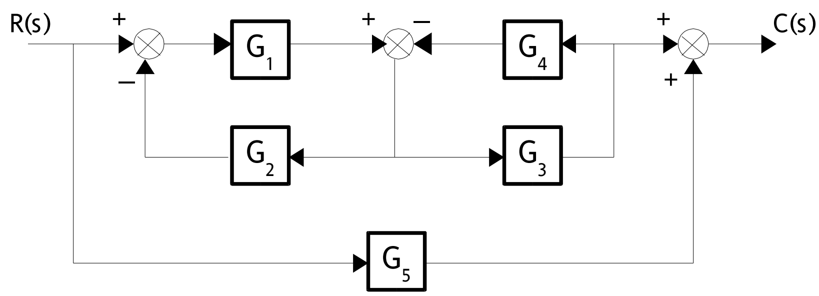
Solution to Second Long Exam Applied Physics 195

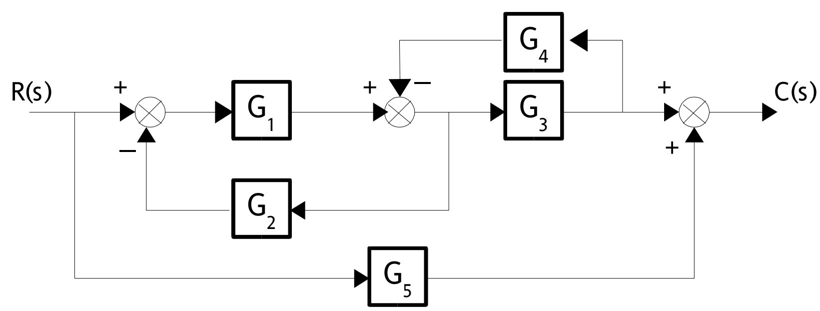
MHV 1st semester AY 2003-2004
11:00-1:00 September 15, 2003

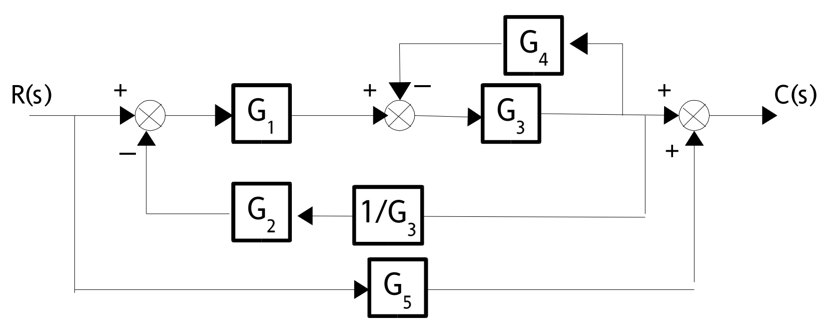
Analysis

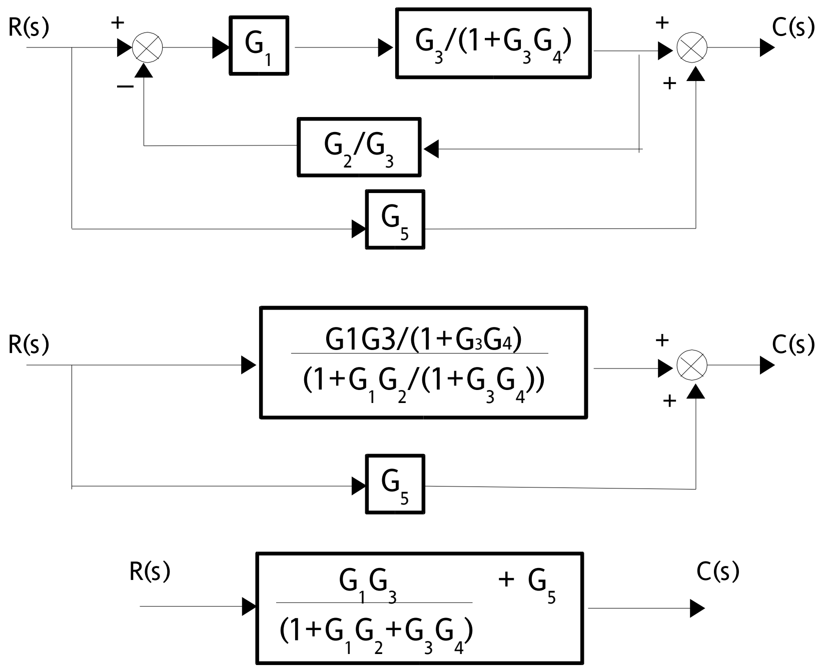
1. Compute the overall transfer function by reducing the block diagram shown below: (20 pts)



Answer:
Redraw by pushing block G4 and G3 up.





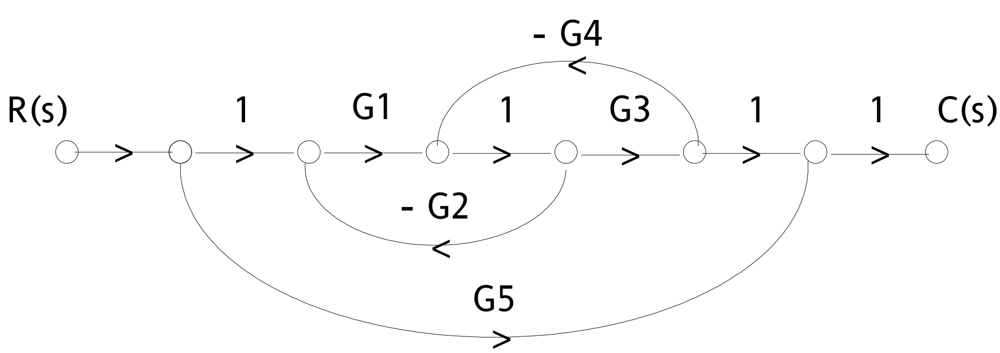


The Transfer Function after simplifying is

$$G(s)=\frac{G_1G_3+G_5+G_5G_1G_2+G_5G_3G_4}{1+G_1G_2+G_3G_4}$$

2. Redraw the system above as a signal flow graph and compute the transfer function using Mason's Gain Formula. (15 pts)

Answer:



The Forward Path Gains are:

$$P_1 = G_1 G_3$$
$$P_2 = G_5$$

The Loop Gains are:

$$L_1 = -G_3 G_4$$
$$L_2 = -G_1 G_2$$

The Determinants for the forward paths are:

$\Delta = 1 - (L_1 + L_2) = 1 + G_1 G_2 + G_3 G_4$

$\Delta_1 = 1$

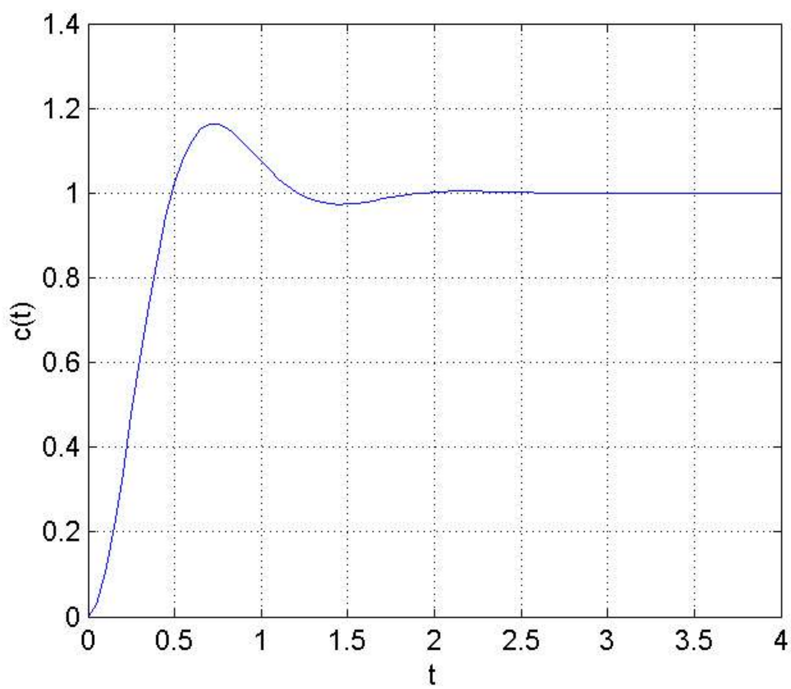
$\Delta_2 = 1 + G_1 G_2 + G_3 G_4$ (since both loops do not touch P_2)

Thus, the Transfer Function of the system is

$$T = \frac{\sum P_k \Delta_k}{\Delta}$$
$$= \frac{G_1 G_3 + G_5 + G_5 G_1 G_2 + G_5 G_3 G_4}{1 + G_1 G_2 + G_3 G_4}$$

which is the same as that obtained thru block-diagram reduction.

3. Estimate the transfer function of a system whose unit step response is shown below: (15 pts)



Answer:

From the graph, estimate peak time, Tp and %OS. By inspection, Tp is approximately 0.75 and %OS is roughly 17%.

Substitute %OS into

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(OS/100)}}$$

which gives $\zeta = 0.4913$. (This symbol pala is zeta.)
From

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

We can get

$$\omega_n = \frac{\pi}{T_p \sqrt{1-\zeta^2}}$$

which upon substitution of T_p and ζ we get

$$\omega_n = 4.8091$$

Substituting

ω_n and ζ into

$G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ we get

$$G(s) = \frac{23.1279}{s^2 + 4.7252 s + 23.1279}$$

The function I used to generate the graph in MATLAB was

$$G(s) = \frac{25}{s^2 + 5s + 25}.$$

4. Solve the following state equation and output equation for y(t) given a unit step input, u(t). (20 pts)

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & -1 \\ 0 & -1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = [0 \quad 1]\boldsymbol{x}; \qquad \boldsymbol{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Answer:

$$s\boldsymbol{I} - \boldsymbol{A}^{-1} = \begin{bmatrix} s+2 & 1 \\ 0 & s+1 \end{bmatrix} = \frac{\begin{bmatrix} s+1 & -1 \\ 0 & s+2 \end{bmatrix}}{(s+2)(s+1)}$$

$$L^{-1}\begin{bmatrix} \dfrac{1}{s+2} & \dfrac{-1}{(s+2)(s+1)} \\ 0 & \dfrac{1}{s+1} \end{bmatrix} = \begin{bmatrix} e^{-2t} & e^{-2t} - e^{-t} \\ 0 & e^{-t} \end{bmatrix} = \Phi$$

$$x=\Phi(t)\boldsymbol{x}(0)+\int_0^t \Phi(t-\tau)\boldsymbol{B}u(\tau)d\tau$$

first term :

$$\Phi(t)\boldsymbol{x}(0)=\begin{bmatrix} e^{-2t} & e^{-2t}-e^{-t} \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}2e^{-2t}-e^{-t}\\ e^{-t}\end{bmatrix}$$

second term:

$$\Phi(t-\tau)\boldsymbol{B}u(\tau)=\begin{bmatrix} e^{-2(t-\tau)} & e^{-2(t-\tau)}-e^{-(t-\tau)} \\ 0 & e^{-(t-\tau)} \end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}e^{-2(t-\tau)}-e^{-(t-\tau)}\\ e^{-(t-\tau)}\end{bmatrix}$$

$$\int_0^t \Phi(t-\tau)\boldsymbol{B}u(\tau)=\begin{bmatrix} e^{-2t}\int_0^t e^{2\tau}d\tau-e^{-t}\int_0^t e^{\tau}d\tau \\ e^{-t}\int_0^t e^{\tau} \end{bmatrix}=\begin{bmatrix}\frac{-1}{2}\\ 1\end{bmatrix}$$

$$\boldsymbol{x}=\begin{bmatrix}2e^{-2t}-e^{-t}\\ e^{-t}\end{bmatrix}+\begin{bmatrix}-1/2\\ 1\end{bmatrix}=\begin{bmatrix}2e^{-2t}-e^{-t}-1/2\\ e^{-t}+1\end{bmatrix}$$

Thus,

$y=[0 \quad 1]\boldsymbol{x}=e^{-t}+1$ (Mali pala yung solution ko sa board.)

Concepts

1. Without solving for the analytic form of the system response, sketch the output of a system whose transfer function is given by:

$$T(s)=\frac{16}{s^2+4s+16}\ .$$

(Kindly use your graphing paper. Don't forget to write your name on it.) (15 pts)

Answer:

This is the reverse of problem 3. First, solve for Tp and %OS from

$$T(s)=\frac{b}{s^2+as+b}$$

$$\omega_n=\sqrt{b}=4$$

$$\zeta=\frac{a}{2\sqrt{b}}=0.5$$

$$T_p=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}=0.9069$$

$$\%OS=e^{-\zeta\pi/\sqrt{1-\zeta^2}}\times 100=16.4733$$

From the peak time we can get the period of oscillation T from

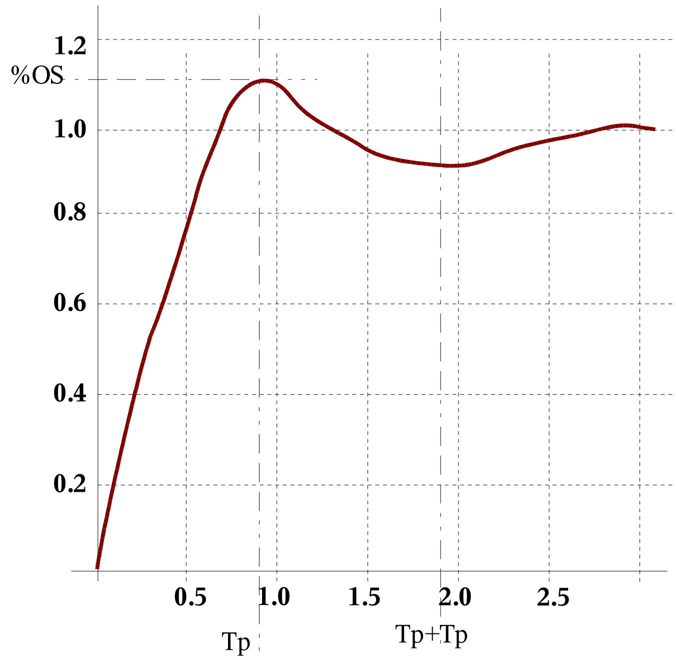
$$T_p=\frac{\pi}{\omega_d}=\frac{\pi}{2\pi/T}=\frac{T}{2}$$

$$T=2T_p=1.8138$$

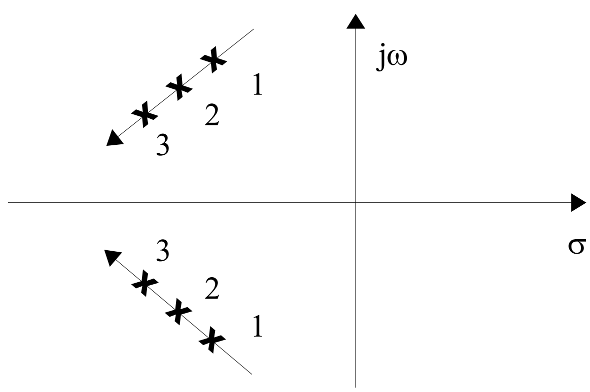
which means that the next peak will occur at Tp + T (= 2.7207) but the first trough will be at Tp +

Tp (=1.8138)

Sketch:



2. Given the pole motion in s-space draw the step response. (15 pts)



Answer:

If the pole motion continues, both poles will eventually coincide at the real axis. When this happens, the system is critically damped. Therefore, from 1 to 3 the system output will be from high frequency, slow exponential decay to low frequency fast exponential decay. Overshoot will decrease from 1 to 3.

Sketch: