Root Locus

Rules to sketching the root locus

1. **Number of branches** – the number of branches of the root locus equal the number of closed-loop poles.
2. **Symmetry** – The root locus is symmetrical about the real axis.
3. **Real-axis segments** – On the real axis, for K >0 the root locus exists to the left of an odd number of real axis, finite open loop poles and /or finite open-loop zeros.
4. **Starting and ending points** – The root locus begins at the finite and infinite poles of G(s)H(s) and ends at the finite and infinite zeros of G(s)H(s) (open loop)
5. **Behaviour at infinity** – The root locus approaches straight lines as symptotes as the locus approaches infinity. The equation of the asymptotes is given by the real-axis intercept, σ_0 , and angle, θ , as follows:
$$\sigma_o = \frac{\sum finite\ poles - \sum finite\ zeros}{\#finite\ poles - \#finite\ zeros} \qquad \theta = \frac{(2k+1)\pi}{\#finite\ poles - \#finite\ zeros}$$
6. **Breakaway points** – when the poles are real and the zeros are complex, it is possible for the poles to eventually breakaway from the real to the imaginary axis. The point at which this happens can be found by
   a. Let 1 + KG(s)H(s) = 0 or KG(s)H(s) = -1.
   b. Solve for K, that is K = -1/G(s)H(s)
   c. Find the solution for s which satisfies dK/ds = 0.
7. **Break-in points** – when the poles are complex and the zeros are real, the poles can possibly break into the real axis. The break-in point is found by solving for s in the equation
$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i}$ where z_i and p_i are the negative of the zeros and poles of the open loop transfer function, respectively.

Exercise

Sketch the root locus of a unity feedback loop with G(s) given by:

1. $G = \frac{K(s^2+4)}{s^2+1}$
2. $G = \frac{K(s+2)(s+6)}{s^2+8s+25}$
3. $G = \frac{(s+2)(s+3)}{(s+1)(s+5)(s+4)}$
4. $G = \frac{K(s^2+1)}{s^2}$
5. $G = \frac{K}{(s+1)^3(s+4)}$