Lead Compensation

A lead compensator is a plant placed in cascade with the open loop gain G(s) in a unity negative feedback loop. Lead compensators improve the transient response of a system.

Steps in designing a lead compensator:

1. Given the design specs, determine the desired pole location.
   a) Given peak time Tp and percent overshoot %OS compute ζ and ω_n .
   b) Calculate the desired pole location from $s_{d1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$.
2. Calculate the angle deficiency.
   a) Substitute $s_{d1,2}$ into G(s) and calculate the angle formed by $G(s_d)$, that is, $\measuredangle G(s_d) = \tan^{-1}\frac{\Im(G)}{\Re(G)}$ then calculate the angle deficiency given by, $\Phi_d = \pm 180^o - \measuredangle G(s_d)$.
3. Find z_c and p_c .
   a) Plot the desired pole location in s-space and draw a horizontal line through it.
   b) Draw a line from the origin to the desired pole location.
   c) Draw a bisecting line from the desired pole location to the real axis.
   d) Divide Φ_d by two and draw two lines with these angles clockwise and counterclockwise from the bisecting line.
   e) The intercept of these lines with the real axis are the p_c (more negative) and z_c (less negative).
4. Find K_c .
   a) Substitute p_c and z_c and in $K_c\frac{s+z_c}{s+p_c}$ and the desired pole location in $G(s) = \frac{a}{s(s+b)}$
   b) Solve for K_c in the equation $\left|K_c\frac{s_d+z_c}{s_d+p_c}\frac{a}{s_d(s_d+b)}\right| = 1$, that is,

If the compensator does not meet the design objectives, one can add another compensator in cascade.

Example:

In Lead Compensation the angle deficiency Φ_d is how much more angle is needed such that the angle of the transfer function evaluated at the desired pole location s_d becomes +/- 180°. Or

$$\measuredangle G(s_d)+\Phi_d=\pm 180^o \quad .$$

$$\text{Thus,} \quad \Phi_d=\pm 180-\measuredangle G(s_d)$$

Design a lead compensator for $G(s)=\dfrac{4}{s^2+2s}$ such that the peak time will occur at 0.5 seconds and the overshoot will not exceed 20%.

ANSWERS:

a) DESIRED POLE LOCATION

$$s_d=-\sigma_d\pm j\omega_d=-\zeta\omega_n+j\omega_n\sqrt{1-\zeta^2}$$

$$\zeta=\frac{-\ln(\%OS)}{\sqrt{\pi^2+\ln^2(\%OS)}}=\frac{-\ln(0.2)}{\sqrt{\pi^2+\ln^2(0.2)}}=0.456$$

$$\omega_n=\frac{\pi}{T_p\sqrt{1-\zeta^2}}=\frac{\pi}{0.5\sqrt{1-0.4559^2}}=7.06$$

$$\boxed{s_d=-3.22+j6.28}$$

b) ANGLE DEFICIENCY . Use formula above.

$$G(s_d)=\frac{4}{s_d^2+2s_d}=-0.07+j0.055$$

$$\measuredangle G(s_d)=141.89$$

$$\boxed{\Phi_d=180-\measuredangle G(s_d)=38^o}$$

c) COMPENSATOR POLES and ZEROS z_c and p_c

Shortcut result:

$$\alpha=\tan^{-1}\left(\sqrt{(1-\zeta^2)}/\zeta\right) \quad \text{(Exercise: Derive using trigonometry.)}$$

$$\boxed{z_c=-\omega_n\sqrt{(1-z^2)}*\tan\left(\frac{\alpha-\Phi_d}{2}\right)-\zeta\omega_n=-4.5985\approx-4.6} \quad \text{(5 pts)}$$

$$\boxed{p_c=-\omega_n\sqrt{(1-z^2)}*\tan\left(\frac{\alpha+\Phi_d}{2}\right)-\zeta\omega_n=-10.838\approx-10.84} \quad \text{(5 pts)}$$

d) K

Solve for K from

$$K\left|G(s_d)\frac{s_d+p_c}{s_d+z_c}\right|=1 \qquad K\left|-0.07+j0.055\frac{s_d+10.84}{s_d+4.6}\right|=1$$

$$\boxed{K=\frac{1}{\left|-0.07+j0.055\dfrac{(-3.22+j6.28+10.84)}{(-3.22+j6.28+4.6)}\right|}=17.341175\approx17.34}$$

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EXERCISE :

Calculations and simulations to be submitted on May 3, 2019, Friday via UVLE submission module.

Consider the open-loop transfer function $G=\frac{a}{s^2+bs}$ .Take the first two non-zero numbers of the 5-digit part of your student number. E.g. XX-07329 will use 7 and 3. The first digit will be your a and second digit will be your b. Using your G, create a closed loop unity feedback system and drive the system with a unit step.

Design a lead compensator $G_c=K_c\frac{s+z_c}{s+p_c}$ for the feedback system such that your system will peak 0.25 seconds after excitation with no more than 40% overshoot.

a) Compute the pole location for the desired output.

b) Compute the angle deficiency.

c) Find the zero and pole location.

d) Compute Kc.

e) Simulate in VisSim and show that the output plot meets the design objective.