Determination of the frequency response of low-pass and high-pass RC filters

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Abstract

The frequency response of first and second-order low and high-pass RC filters were determined using fixed resistances, capacitances, and AC source. The magnitude Bode plots were obtained for each circuit and compared to the plots determined from the theoretically calculated transfer functions. It was shown using 5.1 kΩ resistors and 22 μF capacitors with a 5 V sinusoid input that the first-order circuits have a turning point at around 10^3 Hz. For the second-order low-pass filter, the first turning point was at around 10^2 Hz, and the second at around 10^4 Hz, while for the second-order high-pass filter, the first turning point was at around 10^1 Hz, and the second at 10^3 Hz.

Keywords: frequency response, Bode plot, transfer function.

1 Introduction

Mathematical modelling aims to facilitate our understanding and design of control systems. Here we try to derive the system's complex mathematical representation known as the "transfer function", which is simply an input-output explanation, by looking at its frequency response [1, 2].

The frequency response of a device or a circuit is a dynamic characterization of a circuit and it can be shown by plotting the gain and phase over a wide range of operating frequencies in a semi-logarithmic scaled Bode plot [2, 3]. The gain pertains to the ratio of the output and input signals, while the phase refers to the phase shift introduced to an input AC signal [2]. Gain and phase can be calculated using the equations

$$\text{Gain(dB)} = 20 \log \frac{V_{out}}{V_{in}} \tag{1}$$

$$\phi = -\arctan(2\pi f RC) \tag{2}$$

where V_{in} and V_{out} are the magnitude of the input and output signals respectively and f is the operating frequency at which the aforementioned values were recorded [4].

In this experiment, a control system out of passive components (resistor and capacitor) shall be constructed in order to create a low-pass filter and high-pass filter, which filters out unwanted high-frequencies and low-frequencies respectively. We also compare the bode plots of the first-order and second order filters by comparing their bode plots, cut-off frequencies, and phase shift ϕ. For both low-pass and high-pass filters, f_c is given by

$$f_c^{\text{1st-order}} = \frac{1}{2\pi RC} \tag{3}$$

$$f_c^{\text{2nd-order}} = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \tag{4}$$

where R is the resistor's value and C is the capacitors value [4, 5]. Ultimately, the corresponding transfer function $H(s)$ of the circuit shall be described analytically.

2 Methodology

The circuits in Figures 1 and 3 were constructed using 5.1 kΩ resistor and 22 μF capacitor powered with a 5V sinusoid signal input. Input and output waveforms were displayed on an oscilloscope and their maximum voltage values were recorded. This was done for various frequencies ranging from 10 Hz to 100,000 Hz. Bode plots were carried out by computing for the gain and phase shifts as per Eqs. (1) & (2). Near the theoretical cut-off frequencies, we reduced the increments of operating frequency to obtain a good resolution of our bode plots in the transition (cut-off) region. The same methodology was carried our for the analysis of second-order filters where we used the configurations in Figures 2 & 4

3 Results and Discussion

The gain of the system was calculated using (1) and was plotted against frequency in a semi-logarithmic scale as shown in Figure 5b. This was then compared to the theoretical frequency response of a low pass filter, shown in Figure 5a, where gain was calculated as

$$H(s)_{LP} = -20\log\frac{X_c}{\sqrt{R^2 + X_c^2}} \tag{5}$$

where R is the resistance of resistor R_1 and X_c is the capacitive reactance. Plotting them in one axis, as in Figure 6 we observe the same trend for a low-pass filter for the observed frequency response wherein low-frequency signal is allowed to pass through while high-frequency signals aren't. A more linear response is expected after the corner frequency however this was not the case for the observed plot for the gain.

What set the high-pass filter different with the low-pass filter is that the gain is calculated as

$$H(s)_{HP} = -20\log\frac{R}{\sqrt{R^2 + X_c^2}} \tag{6}$$

In this case, high frequency signals are allowed to pass through, while low frequency signals aren't. Again, the theoretical and observed bode plots are plotted as in Figure 7a and 7b respectively. As compared to the low-pass filter, the bode plot of the observed frequency response of a high-pass filter was more close to that of the theoretical, only having changes in the magnitude, but having the same trend of being linear until the corner frequency and then allowing the higher-value frequency signals to pass through.

For the second-order filters, the analytic transfer function is then dependent on the circuit itself. For a second-order low-pass filter, we were able to obtain a bode plot as seen in Figure 9 with gain having an equation of gain as follows

$$H(s)_{LP}^{\text{2nd-order}} = -20\log\left[\frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}\right] \tag{7}$$

where R_1 and R_2 are the resistance of the two resistors, and C_1 and C_2 are the capacitance of the two capacitors, in our case, $R = R_1 = R_2$ and $C = C_1 = C_2$ and as such the gain can be then simplified into

$$H(s)_{LP}^{2\text{nd}-\text{order}} = -20\log\left[\frac{\frac{1}{R^2C^2}}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}\right] \tag{8}$$

When compared to the bode plot of the first order low pass filter, as in Figure 10, it is observed that the second-order low pass filter gives of a steeper roll-off rate as compared to the first-order low pass filter. The same trend, however, was observed, wherein instead of a linear response after the cut-off frequency, the plot instead curves of after some frequency, reaching a steady state gain.

Finally, for a second-order low pass filter, the gain can be calculated as

$$H(s)_{HP}^{2\text{nd}-\text{order}} = -20\log\left[\frac{s^2}{s^2 + s\left(\frac{1}{C_1R_1} + \frac{1}{C_1R_2} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1C_2R_1R_2}}\right] \tag{9}$$

Given the same conditions where the resistance and capacitance have the same values, the gain can be simplified to the following equation

$$H(s)_{HP}^{2\text{nd}-\text{order}} = -20\log\left[\frac{s^2}{s^2 + \frac{3}{RC}s + \frac{1}{C^2R^2}}\right] \tag{10}$$

Figure 11 shows the observed bode plot of the second-order high-pass filter, where the same trend is observed, low-frequency signals are not allowed to pass through but high-frequency signals are. Plotting them in the same axis, as seen in Figure 12, we also observe that the second-order high-pass filter gives off a bode plot having a steeper roll-off rate as compared to the first-order high-pass filter. Instead of a more linear response in the region before the corner frequency however, a more curved response is observed in the second order.

4 Conclusions

The frequency response of first-order and second-order low-pass and high-pass RC filters were determined. Due to equipment error, only the magnitude Bode plots were obtained. For the first-order low-pass filter, it was observed that low frequencies were allowed to pass through the circuit with minimal attenuation, and at around 10^3 Hz, the output magnitude starts to fall off logarithmically. The reverse was observed for the high-pass filter. For the second-order filters, the same behavior can be observed except that there are two turning points present in the Bode plot. The experiment may be improved upon by using proper equipment in order to obtain both magnitude and phase Bode plots.

References

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Appendix

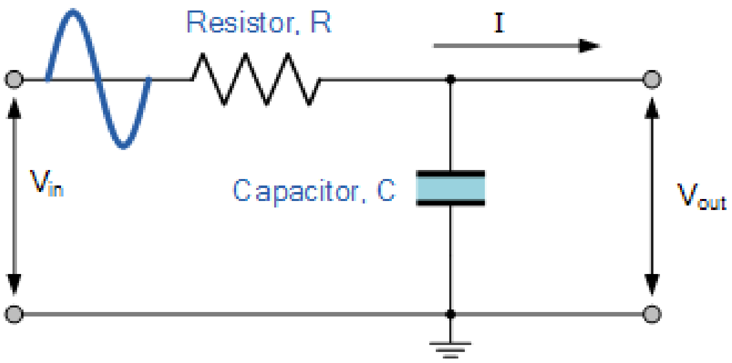


Figure 1: First-order low pass filter circuit [4].

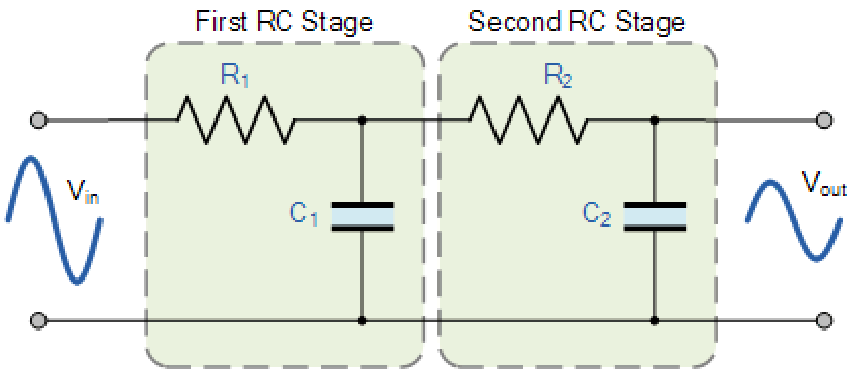


Figure 2: Second-order low pass filter circuit [5].

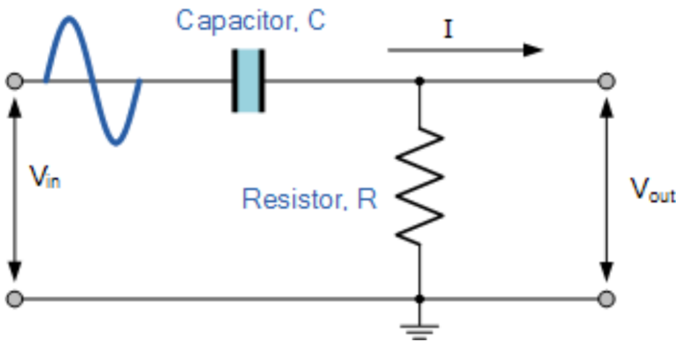


Figure 3: First-order high pass filter circuit [4].

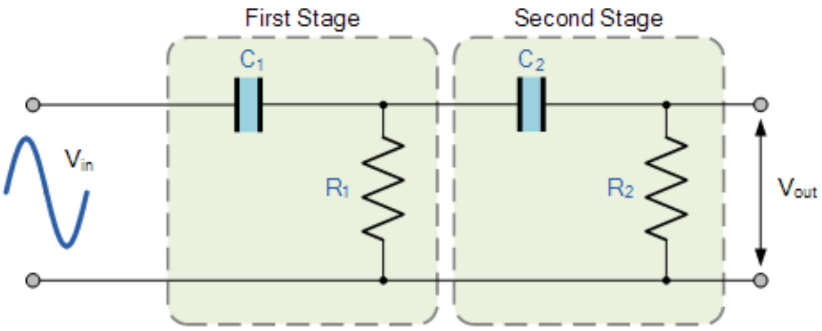
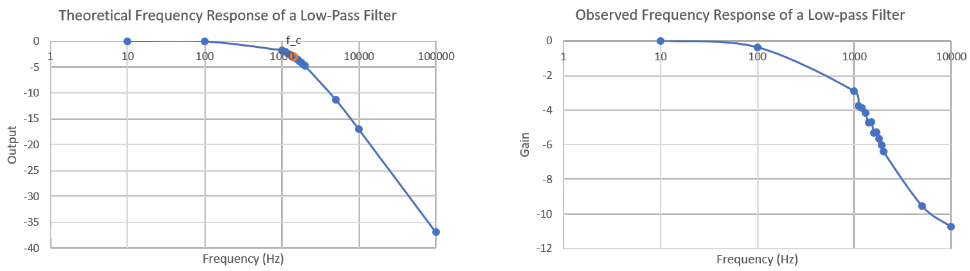


Figure 4: Second-order high pass filter circuit [5].



(a) Theoretical

(b) Observed

Figure 5: Frequency response of a low-pass filter

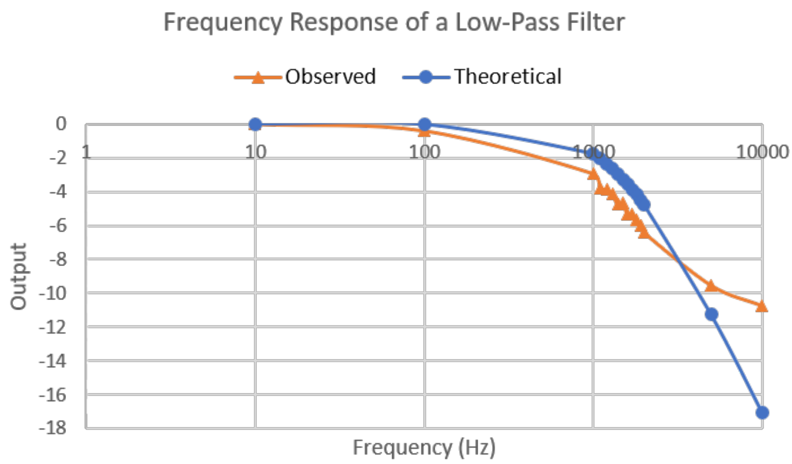
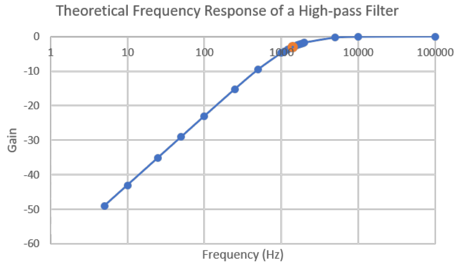
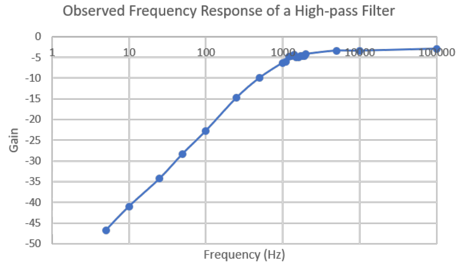


Figure 6: Comparison of theoretical and observed frequency responses.



(a) Theoretical



(b) Observed

Figure 7: Frequency response of a high-pass filter

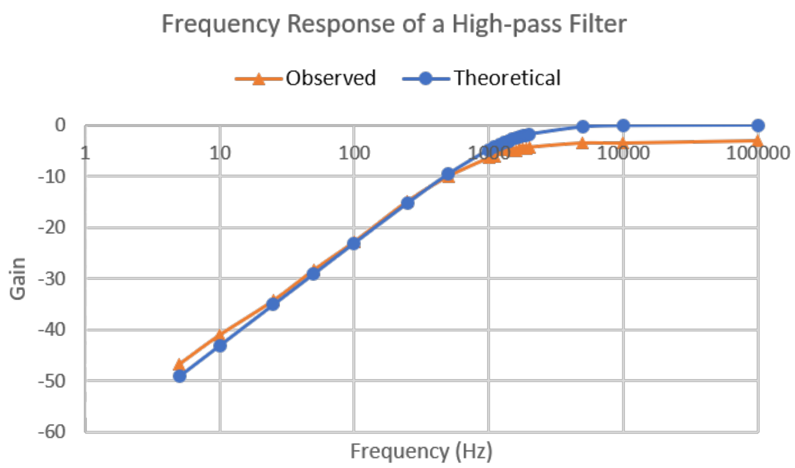


Figure 8: Comparison of theoretical and observed frequency responses.

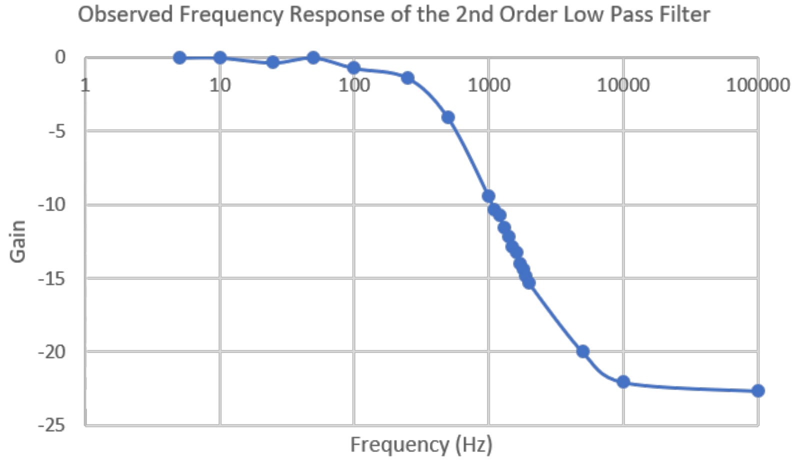


Figure 9: Frequency response of a second order low-pass filter.

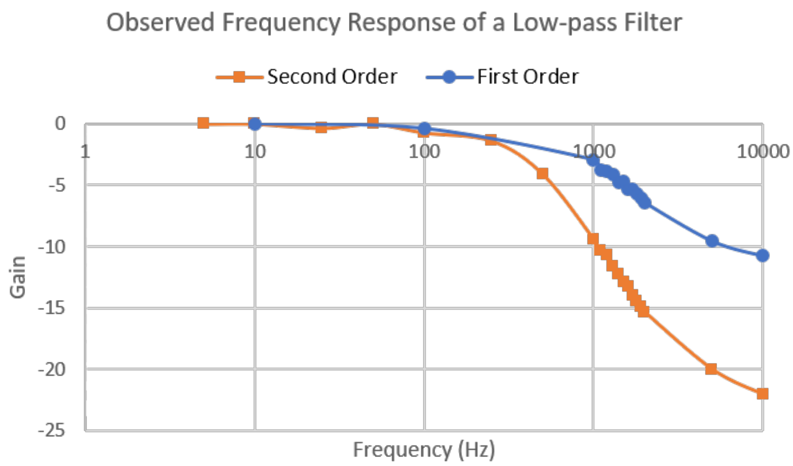


Figure 10: Comparison of the first order and second order low-pass RC filter.

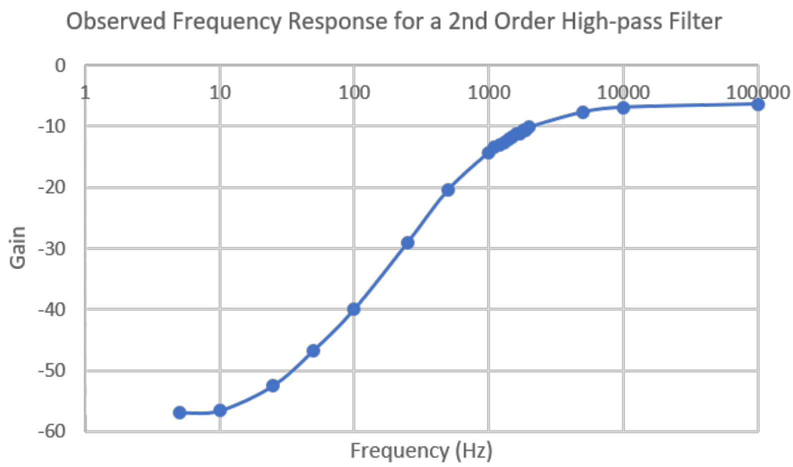


Figure 11: Frequency response of a second order high-pass filter.

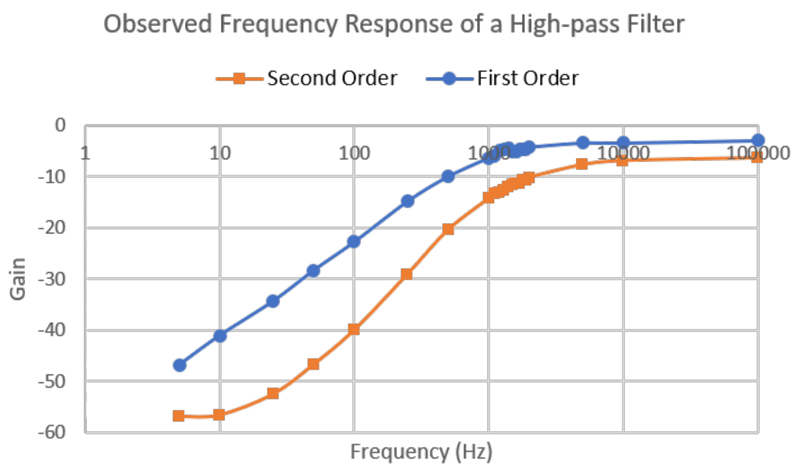


Figure 12: Comparison of the first order and second order high-pass RC filter.