

Activity 15 - Phase measurement using Digital Holography

Basic Principles:

Holography

Holography is a technique that allows one to record the amplitude and phase of an object wave. This is done by interfering the object wave with a reference wave, which effectively encodes phase information into the intensity.

Consider the object wave O and the reference wave R :

$$O = o e^{i\phi_o}$$

$$R = r e^{i\phi_r}$$

If we interfere (add) them, and then take the intensity (amplitude squared):

$$U = O + R$$

$$I = U^* U = (O + R)^* (O + R)$$

$$I = (|O|^2 + |R|^2) + O^* R + R^* O$$

$(|O|^2 + |R|^2)$: DC term

$R^* O$: scaled object field (if R is constant)

$O^* R$: scaled conjugate of object field (if R is constant)

The total intensity is thus the sum of the intensities of O and R plus cross-terms. The object phase can be extracted from these cross-terms.

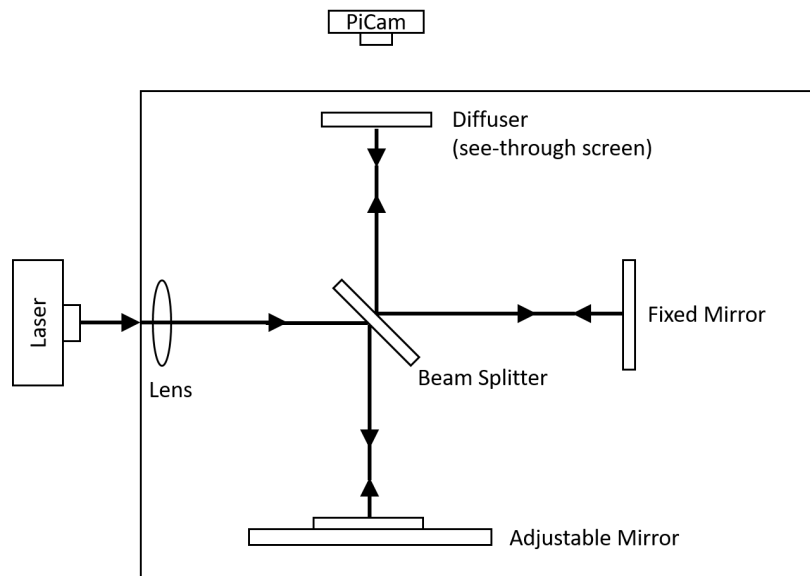
Materials:

PASCO Interferometer Apparatus, He-Ne laser, Polarizers, PiCam, tripod

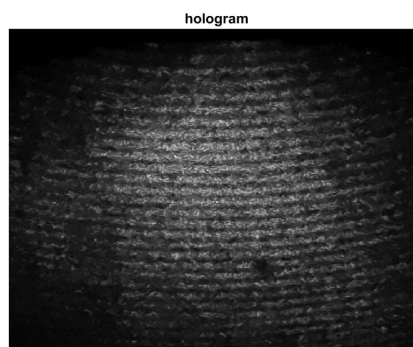
Procedure:

Recording

1. Setup the PASCO interferometer as shown below. Make sure that the laser hits the centers of both the fixed and adjustable mirrors.



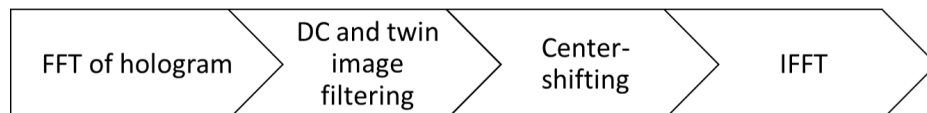
2. Remove the lens temporarily. You should see two sets of bright spots on the screen/diffuser. These represent the light rays coming from the fixed mirror and the adjustable mirror. Tweak the adjustable mirror to make these bright spots coincide
3. Reattach the lens, making sure that the laser beam still hits the centers of the mirrors. You should see a set of circular fringes on the screen/diffuser.
4. Tweak one knob on the adjustable mirror until you observe parallel fringes on your PiCam. Make the fringes as small as possible, but still detectable by your camera. Thinner fringes will make it much easier to reconstruct the image later on.
5. You can take an image of the fringes by placing your camera behind the screen/diffuser. Make sure that there is no saturation in your image, and that you maximize the number of pixels used in capturing the image. Insert two polarizers before the lens if you need to adjust the intensity of the laser. Congratulations! You have just taken an image of a hologram. See an enlarged example below.



Reconstruction

Armed with your holograms, you can now perform image processing to recover the amplitude and phase of the object (in this case your object is one of the mirrors). By analyzing the object reconstruction, you can see the defects on your mirror.

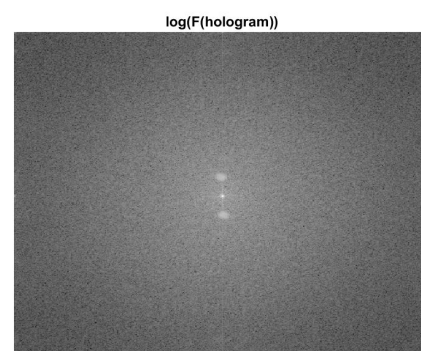
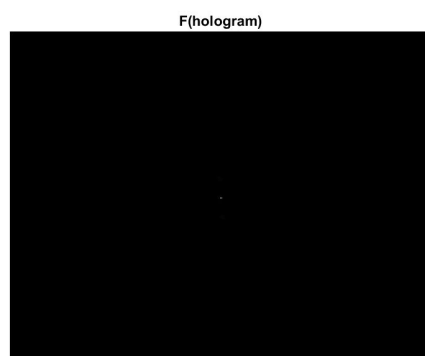
Below is a flowchart of the process to reconstruct your object wave.



FFT of hologram

1. Convert your hologram to grayscale and crop it such that the fringes occupy most if not all of the image.
2. Take the FFT of your hologram. For MATLAB users, you may try using:

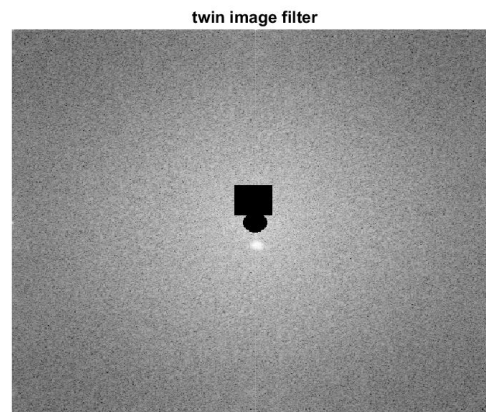
$$\text{fspect} = \text{fftshift}(\text{fft2}(\text{hologram}))$$
 where `fftshift` and `fft2` are MATLAB functions, and **fspect** is the Fourier spectrum of your hologram.
3. You should observe a bright spot at the center (zero-frequency) of your Fourier spectrum. This is the very bright DC term, which you will later filter out.
 Note: The Fourier spectrum is a complex matrix, so you need to take the absolute value or magnitude to display it as an image.
4. To observe the weak signals of your Fourier spectrum, display it in log scale.



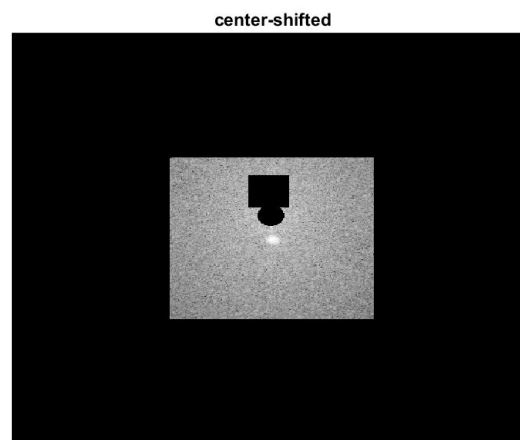
5. You should observe three distinct regions. These regions correspond to the Fourier signal of the twin images and the DC term. The thinner the fringes of your hologram (i.e. higher frequency), the farther apart the three signals will appear in your hologram.

DC and twin image filtering

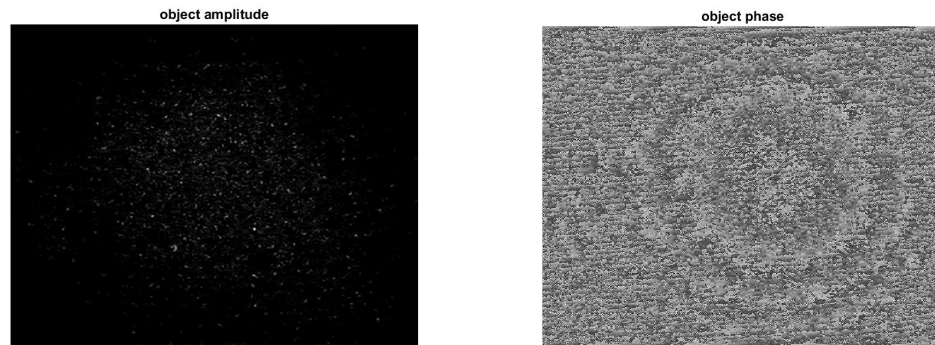
1. The goal here is to remove the DC term and either one of the twin images, so that we're left with the Fourier signal of the desired image. This will be accomplished via filtering in the Fourier domain.
2. Create a mask (matrix of 1's and 0's) of a circle with radius **R_circ** centered at the origin. Multiply with **fspect** to remove the DC term. Make sure that you do not filter out the twin images.
3. Next, choose one of the twin images and estimate the coordinates of its center, **xc** and **yc**. Make a mask(circle or square) with length **R_twin** centered at **(xc, yc)**, and multiply with **fspect** to remove the twin image signal.

Center-shifting and IFFT

1. Estimate the coordinates of the center of the remaining twin image, **xc2** and **yc2**. Crop out a section (circle or square) with length **R_cent** centered at **(xc2, yc2)**, and place it at the center of a zero matrix. With this, you have obtained the reconstructed object wave, and removed the effect of the tilted beam



2. Take the inverse FFT of your center-shifted spectrum. Congratulations! That's your object wave in the hologram plane.
3. Display the amplitude and phase of your object wave using the **abs** and **angle** functions.



4. Comment on the quality of your reconstruction and investigate the effects of changing:
- DC mask radius **R_circ**
 - Twin mask side length **R_twin**
 - Center-shifting size **R_cent**

References:

Cuche, E., Marquet, P. and Depeursinge, C. (2000). Spatial filtering for zero-order and twin-image elimination in digital off-axis holography. *Applied Optics*, 39(23), p.4070. 10.1364/AO.39.004070

Belashov, Andrey & Petrov, Nikolay & Lai, Xin Ji & Cheng, Chau-Jern. (2014). Comparison of phase reconstruction algorithms accuracy in off-axis digital holography. 10.13140/2.1.1839.0088.

More on digital holography

Tatsuki Tahara, Xiangyu Quan, Reo Otani, Yasuhiro Takaki, Osamu Matoba, Digital holography and its multidimensional imaging applications: a review, *Microscopy*, Volume 67, Issue 2, April 2018, Pages 55–67, <https://doi.org/10.1093/jmicro/dfy007>