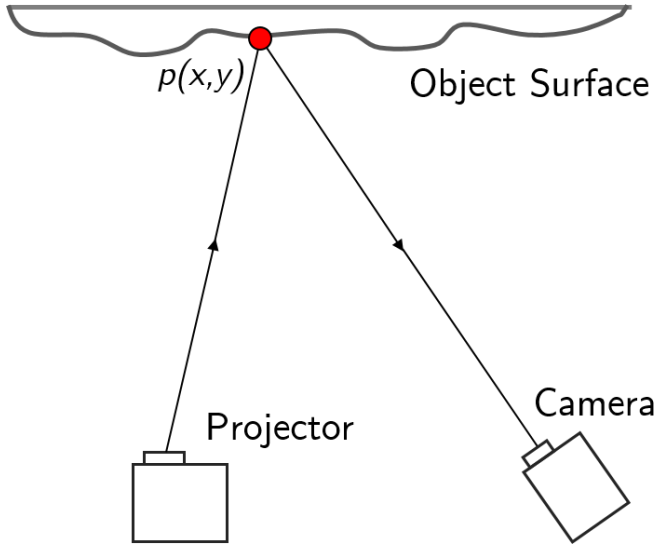
Shape from Structured Illumination

AP 187

Credits: Janno Vergara and Ritz Ann Aguilar thesis slides

Phase Shift Profilometry Algorithm

1. Phase Wrapping
2. Phase Unwrapping
3. Phase-To-Height Conversion



PSP experimental setup.

Phase Wrapping

Four phase-shifted sinusoidal fringes:

$$I_1(x,y) = I_0(x,y) + I_{mod}(x,y)\cos\left(\phi(x,y)\right),$$
$$I_2(x,y) = I_0(x,y) + I_{mod}(x,y)\cos\left(\phi(x,y) + \frac{\pi}{2}\right),$$
$$I_3(x,y) = I_0(x,y) + I_{mod}(x,y)\cos\left(\phi(x,y) + \pi\right),$$
$$I_4(x,y) = I_0(x,y) + I_{mod}(x,y)\cos\left(\phi(x,y) + \frac{3\pi}{2}\right),$$

[1]

We express equation 2 as:

$$I_1(x,y) = I_0(x,y) + I_{mod}(x,y)\cos\left(\phi(x,y)\right),$$
$$I_2(x,y) = I_0(x,y) - I_{mod}(x,y)\sin\left(\phi(x,y)\right),$$
$$I_3(x,y) = I_0(x,y) - I_{mod}(x,y)\cos\left(\phi(x,y)\right),$$
$$I_4(x,y) = I_0(x,y) + I_{mod}(x,y)\sin\left(\phi(x,y)\right).$$

[2]

- $I_o(x,y)$ = average background intensity value
- $I_{mod}(x,y)$ = intensity value of fringe pattern
- $\phi(x,y)$ = phase value

Phase Wrapping

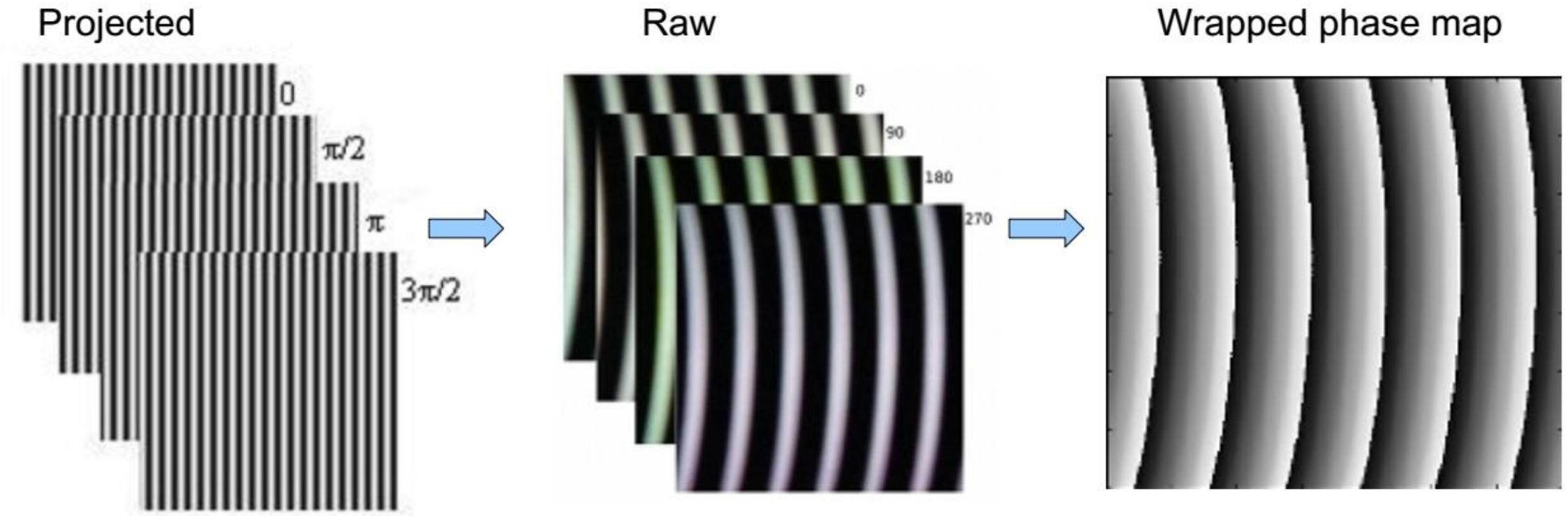
Subtracting like terms:

$$I_4(x,y) - I_2(x,y) = 2I_{mod}\sin\left(\phi(x,y)\right),$$
$$I_1(x,y) - I_3(x,y) = 2I_{mod}\cos\left(\phi(x,y)\right). \quad [3]$$

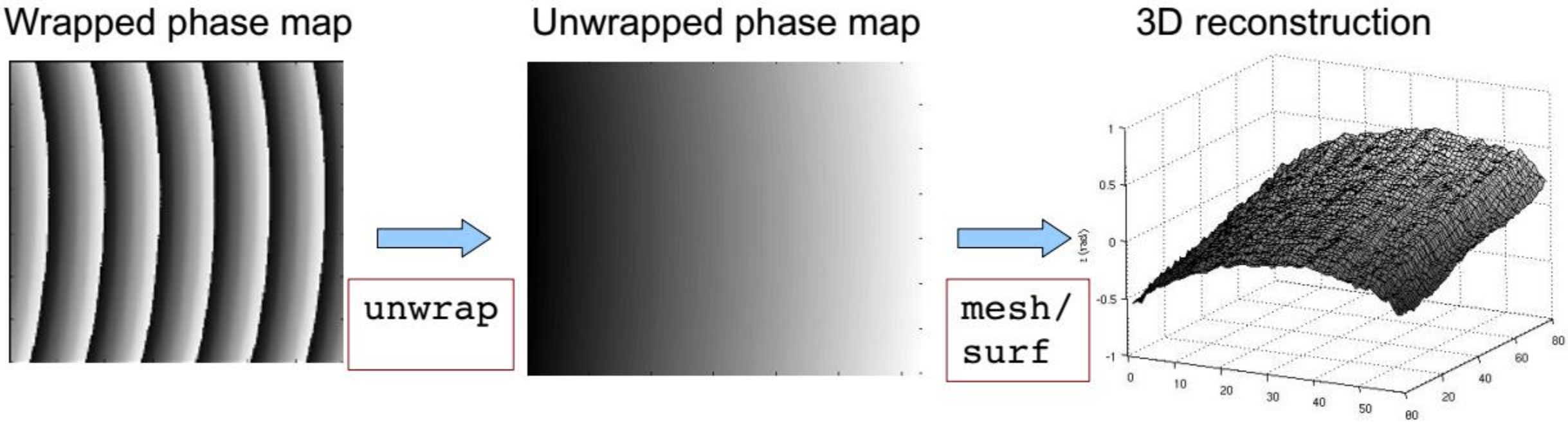
Phase-value equation:

$$\phi(x,y) = \tan^{-1}\left(\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right) \quad [4]$$

Phase Calculation



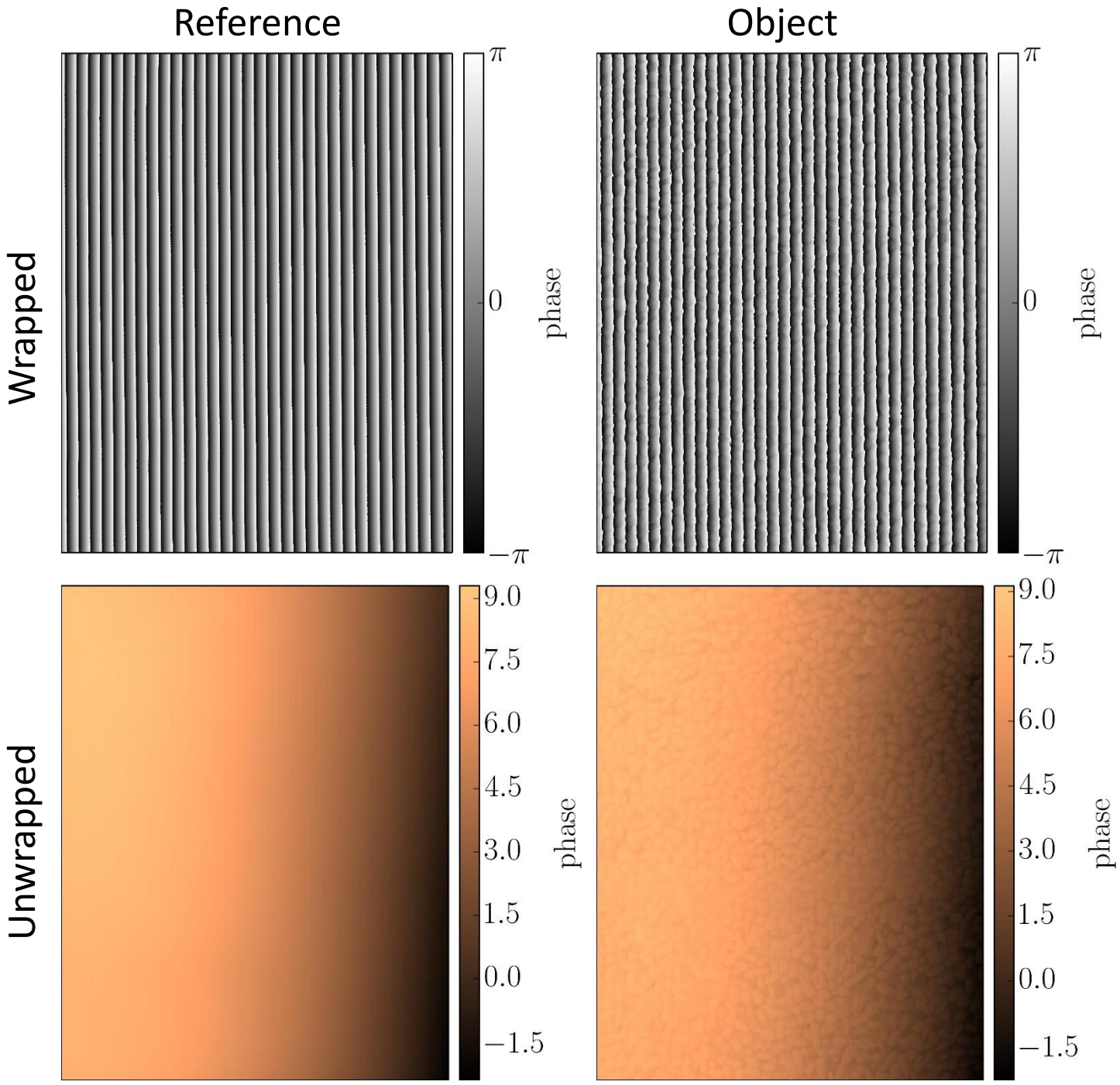
Phase unwrapping

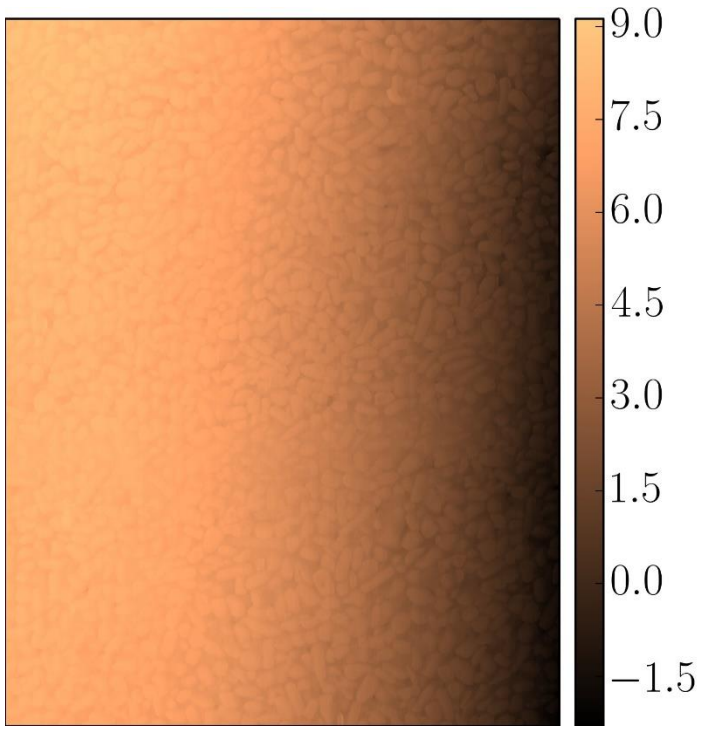


- Unwrap the phase by adding multiples of 2π to discontinuities
- Subtract unwrapped phase of flat reference.
- Display phase difference as mesh.

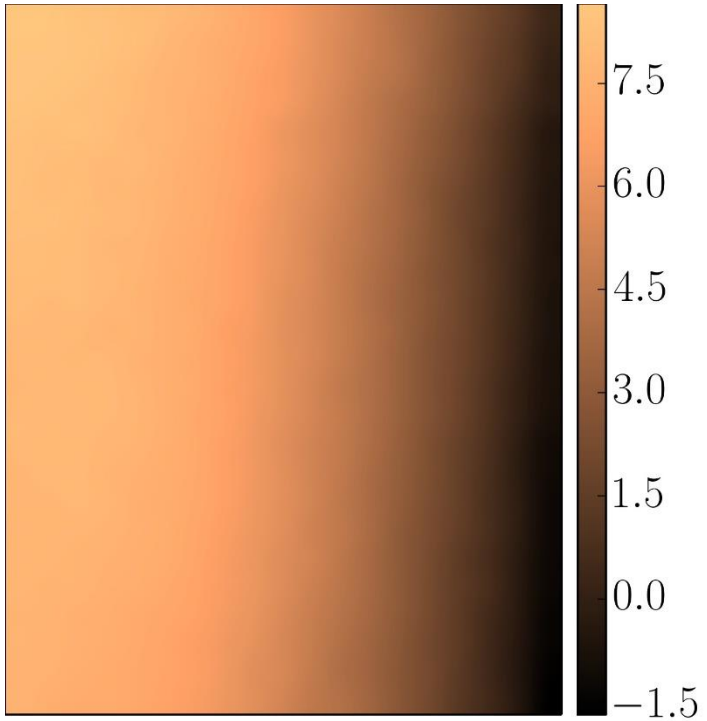
Phase-to-Height Conversion

- In the simplest approximation $z = k\Delta\phi$

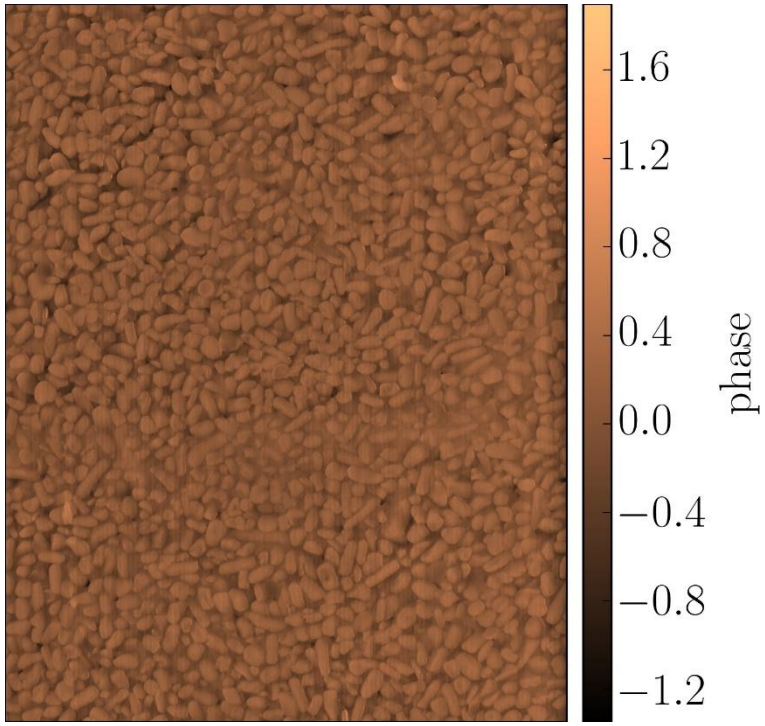

Reference
Object
Wrapped
Unwrapped
π
0
−π
phase
9.0
7.5
6.0
4.5
3.0
1.5
0.0
−1.5



Object’s phase map



Reference’s phase map



Output phase map