Problem 2.18

(a) From the given equations,

$$W = C_V (T_2 - T_1) \tag{1}$$
$$PV = \nu RT \tag{2}$$
$$C_V = \frac{3}{2}\nu k_B \tag{3}$$

Plugin (2) and (3) into (1):

$$\begin{aligned} W &= \frac{3}{2}\nu k_B \left(\frac{P_2V_2}{\nu k_B} - \frac{P_1V_1}{\nu k_B}\right) \\ &= \frac{3}{2}(P_2V_2 - P_1V_1) \end{aligned} \tag{4}$$

Since $\gamma = \frac{5}{3}$ for monatomic ideal gas,

$$\begin{aligned} W &= \frac{(P_2V_2 - P_1V_1)}{\frac{2}{3}} \\ &= \frac{(P_2V_2 - P_1V_1)}{\frac{5}{3} - \frac{3}{3}} \\ \boxed{W} &= \boxed{\frac{(P_2V_2 - P_1V_1)}{\gamma - 1}} \end{aligned} \tag{5}$$

(b) From the given equations,

$$PV^\gamma = C \tag{6}$$
$$W = -\int_{V_1}^{V_2} P(T, V)\,\mathrm{d}V \tag{7}$$

Isolate P from (6) and plug into (7):

$$\begin{aligned} W &= -\int_{V_1}^{V_2} CV^{-\gamma}\,\mathrm{d}V \\ &= -C\frac{V^{1-\gamma}}{1-\gamma}\bigg|_{V_1}^{V_2} \\ &= \frac{1}{\gamma - 1}\left(CV_2^{1-\gamma} - CV_1^{1-\gamma}\right) \\ &= \frac{1}{\gamma - 1}\left(CV_2^{-\gamma}V_2 - CV_1^{-\gamma}V_1\right) \end{aligned} \tag{8}$$

From (6),

$$P = CV^{-\gamma} \tag{9}$$

Therefore,

$$\boxed{W = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)} \tag{10}$$