PS 28: Problem 3.34

(a) Assuming that the gas is dilute and that position of a molecule is independent of the position of the others', the probability of finding a particular molecule in V_1 is

$$\boxed{p = \frac{1}{2}} \tag{1}$$

(b) Given that the total number of particles is $N = N_1 + N_2$, the probability of finding N_1 particles in V_1 can be calculated using the binomial distribution:

$$P_N(n) = \frac{N!}{n!(N-n)!}2^{-N} \tag{2}$$

$$\boxed{P_N(N_1) = \frac{N!}{N_1!(N-N_1)!}2^{-N}} \tag{3}$$

$$\begin{aligned} P_N(N_2) &= \frac{N!}{N_2!(N-N_2)!}2^{-N} \\ &= \frac{N!}{(N-N_1)![N-(N-N_1)]!}2^{-N} \\ &= \frac{N!}{(N-N_1)!N_1!}2^{-N} \end{aligned}$$

$$\boxed{P_N(N_2) = P_N(N_1)} \tag{4}$$

(c) The average number of molecules in each part is

$$\bar{n} = pN \tag{5}$$

$$\boxed{\bar{n} = \frac{1}{2}N} \tag{6}$$

(d) The relative fluctuation is

$$\begin{aligned} \overline{(\Delta n)^2} &= p(1-p)N \\ &= \frac{1}{2}\left(1 - \frac{1}{2}\right)N \end{aligned} \tag{7}$$

$$\boxed{\overline{(\Delta n)^2} = \frac{1}{4}N} \tag{8}$$