

PS 19: Problem 2.25

(a) The enthalpy is defined as

$$H = E + PV \quad (1)$$

Taking the differential,

$$dH = dE + P dV + V dP \quad (2)$$

From the fundamental thermodynamic relation,

$$dE = T dS - P dV + \mu dN \quad (3)$$

Substituting (3) into (2),

$$dH = T dS - P dV + P dV + \mu dN + V dP$$

$$\boxed{dH = T dS + V dP + \mu dN} \quad (4)$$

Therefore, the natural variables are

$$\boxed{T = \left(\frac{\partial H}{\partial S} \right)_{P,N}} \quad (5)$$

$$\boxed{V = \left(\frac{\partial H}{\partial P} \right)_{S,N}} \quad (6)$$

$$\boxed{\mu = \left(\frac{\partial H}{\partial N} \right)_{S,P}} \quad (7)$$