PS 39: Problem 4.15

Given the following:

$$\Gamma(E,V,N) \approx \frac{1}{N!}\Gamma_1\left(\frac{E}{N},V\right)^N \tag{1}$$

$$\Gamma_1(E) = \frac{4\pi V}{3h^3}(2mE)^{3/2} \tag{2}$$

We wish to find the form of $\Gamma(E,V,N)$ for Γ_1. (2) becomes

$$\Gamma_1\left(\frac{E}{N}\right) = \frac{4\pi V}{3h^3}\left(\frac{2mE}{N}\right)^{3/2} \tag{3}$$

Plugging this into (1), we have

$$\Gamma(E,V,N) = \frac{1}{N!}\left[\frac{4\pi V}{3h^3}\left(\frac{2mE}{N}\right)^{3/2}\right]^N$$

$$\boxed{\Gamma(E,V,N) = \frac{1}{N!}\left(\frac{4\pi V}{3h^3}\right)^N\left(\frac{2mE}{N}\right)^{3N/2}} \tag{4}$$

From (4), we see that $\Gamma \propto V^N$, $\Gamma \propto E^{3N/2}$, and $\Gamma \propto \left(N!N^{3N/2}\right)^{-1}$. In contrast, the form of $\Gamma(E,V,N)$ given in GT 4.49 is

$$\Gamma(E,V,N) = \frac{1}{N!}\left(\frac{V}{h^3}\right)^N \frac{(2\pi mE)^{3N/2}}{(3N/2)!} \tag{5}$$

where $\Gamma \propto V^N$, $\Gamma \propto E^{3N/2}$, and $\Gamma \propto [N!(3N/2)!]^{-3N/2}$.