PS 26: Problem 3.20

(a) For a coin toss, assuming the coin is unbiased, the probabilities of each side turning up are equal. Hence,

$$S = \ln \Omega \tag{1}$$

$$\boxed{S = \ln 2 \approx 0.69} \tag{2}$$

(b) For unequal uncertainties $P_1 = 1/5$ and $P_2 = 4/5$,

$$S = -\sum_i P_i \ln P_i \tag{3}$$

$$= -\left(\frac{1}{5} \ln \frac{1}{5} + \frac{4}{5} \ln \frac{4}{5} \right)$$

$$\boxed{S = \frac{1}{5} \ln 5 + \frac{4}{5} \ln 4 \approx 0.67} \tag{4}$$

The uncertainty is lower since one of the outcomes is now more likely to occur.

(c) For third and fourth experiments, all $\{P_i\}_{i=1}^4 = 1/4$ and $\{P_i\}_{i=1}^6 = 1/6$, respectively. Their uncertainties are

$$\boxed{S_3 = \ln 4 \approx 1.39} \tag{5}$$

$$\boxed{S_4 = \ln 6 \approx 1.79} \tag{6}$$

The uncertainty is greater when there are more states since there is a greater number of unique possible outcomes.