PS 36: Problem 4.5

E_A	$\Omega_A(E_A)$	$\Omega_B(6-E_A)$	$\Omega_A\Omega_B$	$P_A(E_A)$
6	7	1	7	7/84
5	6	2	12	12/84
4	5	3	15	15/84
3	4	4	16	16/84
2	3	5	15	15/84
1	2	6	12	12/84
0	1	7	7	7/84

Table 1: The probability $P_A(E_A)$ that subsystem A has energy E_A given $N_A = N_B = 2$ and $E_{tot} = E_A + E_B = 6$.

The standard deviation of the energy of subsystem A is given by

$$\sigma = \sqrt{\langle E_A^2 \rangle - \langle E_A \rangle^2} \tag{1}$$

The first moment (mean) $\langle E_A \rangle$ is obtained by

$$\begin{aligned}\langle E_A \rangle &= \sum_{i=1}^{N} (E_A)_i (P_A)_i \qquad (2)\\ &= 6\left(\frac{7}{84}\right) + 5\left(\frac{12}{84}\right) + 4\left(\frac{15}{84}\right)\\ &+ 3\left(\frac{16}{84}\right) + 2\left(\frac{15}{84}\right) + 1\left(\frac{12}{84}\right) + 0\left(\frac{7}{84}\right)\\ &= 12\end{aligned}$$

The second moment (raw variance) $\langle E_A^2 \rangle$ is obtained by

$$\begin{aligned}\langle E_A^2 \rangle &= \sum_{i=1}^{N} (E_A)_i^2 (P_A)_i \qquad (3)\\ &= 36\left(\frac{7}{84}\right) + 25\left(\frac{12}{84}\right) + 16\left(\frac{15}{84}\right)\\ &+ 9\left(\frac{16}{84}\right) + 4\left(\frac{15}{84}\right) + 1\left(\frac{12}{84}\right) + 0\left(\frac{7}{84}\right)\\ &= 3\end{aligned}$$

So the standard deviation is

$$\begin{aligned}\sigma &= \sqrt{12 - 3^2}\\ &= \sqrt{3}\\ \boxed{\sigma &\approx 1.73} \qquad (4)\end{aligned}$$