PS 37: Problem 4.10

(a) For an Einstein solid of $N = 20$ distinguishable particles, the total number of accessible microstates $\Omega(E)$ is given by

$$\Omega(E) = \frac{(E+N-1)!}{E!(N-1)!} \tag{1}$$

For various E, we have

$$\Omega(E=10) = \frac{(10+20-1)!}{10!(20-1)!}$$
$$\boxed{\Omega(E=10) = 20,030,010} \tag{2}$$
$$\Omega(E=100) = \frac{(100+20-1)!}{100!(20-1)!}$$
$$\boxed{\Omega(E=100) \approx 4.91 \times 10^{21}} \tag{3}$$
$$\Omega(E=1000) = \frac{(1000+20-1)!}{1000!(20-1)!}$$
$$\boxed{\Omega(E=1000) \approx 9.93 \times 10^{39}} \tag{4}$$

Continuing this process for larger E for fixed N, we obtain the graph in Figure 1.

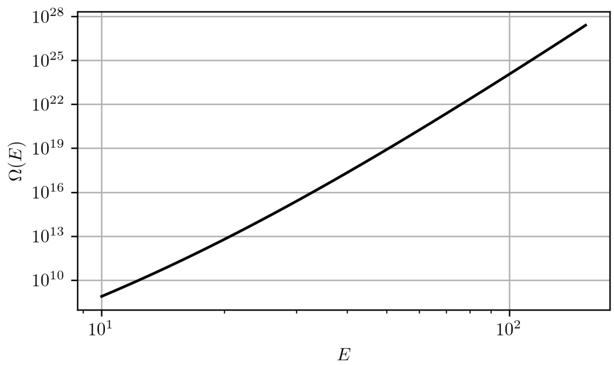

Figure 1: $\Omega(E) \forall E \in \left[10^1, 10^3\right]$

Thus, $\Omega(E)$ is an exponentially increasing function of E for fixed N.

(b) For fixed $E = 10$ and varying N, we obtain the graph in Figure 2.

Thus, $\Omega(E)$ is also an exponentially increasing function of N for fixed E.

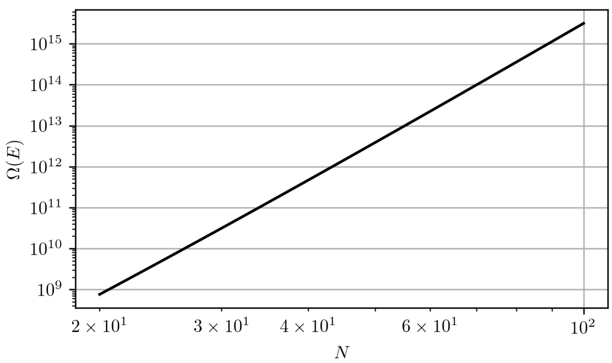

Figure 2: $\Omega(E) \forall N \in [20, 100]$