PS 17: Problem 2.24

(a) From the given

$$\Delta S = \frac{3}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{V_2}{V_1}\right) \tag{1}$$

The change in entropy is zero for a quasistatic adiabatic process:

$$\begin{aligned} 0 &= \frac{3}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{V_2}{V_1}\right) && (2)\\ \frac{3}{2}Nk\ln\left(\frac{T_2}{T_1}\right) &= -Nk\ln\left(\frac{V_2}{V_1}\right) && (3)\\ Nk\ln\left(\frac{T_2}{T_1}\right) &= -\frac{2}{3}Nk\ln\left(\frac{V_2}{V_1}\right) && (4)\\ \ln\left(\frac{T_2}{T_1}\right) &= -\frac{2}{3}\ln\left(\frac{V_2}{V_1}\right) && (5) \end{aligned}$$

Recall that for a monatomic ideal gas, $\gamma = \frac{5}{3}$. Expressing the constant coefficient on the RHS of (5) in terms of γ,

$$\gamma - 1 = \frac{5}{3} - 1 = \frac{2}{3} \tag{6}$$

Plugging this into (5),

$$\ln\left(\frac{T_2}{T_1}\right) = -(\gamma-1)\ln\left(\frac{V_2}{V_1}\right) \tag{7}$$

By the power rule of exponentials,

$$\begin{aligned} \ln\left(\frac{T_2}{T_1}\right) &= -\ln\left[\left(\frac{V_2}{V_1}\right)^{(\gamma-1)}\right] && (8)\\ \ln\left(\frac{T_2}{T_1}\right) &= \ln\left[\left(\frac{V_1}{V_2}\right)^{(\gamma-1)}\right] && (9)\\ \left(\frac{T_2}{T_1}\right) &= \left[\left(\frac{V_1}{V_2}\right)^{(\gamma-1)}\right] && (10)\\ \frac{T_2}{T_1} &= \frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} && (11)\\ T_1V_1^{\gamma-1} &= T_2V_2^{\gamma-1} && (12) \end{aligned}$$

Thus,

$$\boxed{TV^{\gamma-1} = C} \tag{13}$$

(b) If the container is thermally insulated, $T_1 = T_2 = T$, and (5) becomes

$$\begin{aligned} \Delta S &= \frac{3}{2}Nk\ln\left(\frac{T}{T}\right) + Nk\ln\left(\frac{V_2}{V_1}\right)\\ \Delta S &= \frac{3}{2}Nk\ln(1) + Nk\ln\left(\frac{V_2}{V_1}\right)\\ &\boxed{\Delta S = Nk\ln\left(\frac{V_2}{V_1}\right)} && (14) \end{aligned}$$

(c) From the ideal gas law,

$$\begin{aligned} PV &= NkT && (15)\\ V &= \frac{NkT}{P} && (16) \end{aligned}$$

Plugging this into (5),

$$\begin{aligned} \Delta S &= \frac{3}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{\frac{NkT_2}{P_2}}{\frac{NkT_1}{P_1}}\right)\\ &= \frac{3}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{T_2P_1}{T_1P_2}\right)\\ &= \frac{3}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{T_2}{T_1}\right)\\ &\quad + Nk\ln\left(\frac{P_1}{P_2}\right)\\ &= \frac{5}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{P_1}{P_2}\right) \end{aligned}$$

$$\boxed{\Delta S(T,P) = \frac{5}{2}Nk\ln\left(\frac{T_2}{T_1}\right) + Nk\ln\left(\frac{P_1}{P_2}\right)} \tag{17}$$