

PS 21: Problem 2.29

The Van der Waals energy equation of state is given by

$$E = \frac{3}{2}Nk_B T - N\frac{N}{V}a \quad (1)$$

The Joule coefficient is given by

$$\left(\frac{\partial T}{\partial V}\right)_E = -\frac{\left(\frac{\partial E}{\partial V}\right)_T}{\left(\frac{\partial E}{\partial T}\right)_V} \quad (2)$$

Performing the necessary derivatives indicated in (2) on (1), we have

$$\begin{aligned} \left(\frac{\partial T}{\partial V}\right)_E &= -\frac{N\frac{N}{V^2}a}{\frac{3}{2}Nk_B} \\ \boxed{\left(\frac{\partial T}{\partial V}\right)_E} &= -\frac{2Na}{3k_B V^2} \end{aligned} \quad (3)$$

The change in temperature w.r.t. volume is negative, which indicates cooling.