PS 47: Problem 4.26

The Boltzmann probability is given by

$$P(E) = \frac{1}{Z} e^{-\beta E_s} \tag{1}$$

where $Z \equiv \sum_s e^{-\beta E_s}$ is the partition function. Substituting this in (1) gives

$$P(E) = \frac{e^{-\beta E_s}}{\sum_s e^{-\beta E_s}} \tag{2}$$

The probability that a system has energy between E and $E + \Delta E$ is

$$p(E)\Delta E = P(E + \Delta E) - P(E) \tag{3}$$

$$\approx \frac{\mathrm{d}P(E)}{\mathrm{d}E} \Delta E \tag{4}$$

$$\approx \mathrm{d}E \, \frac{e^{-\beta E_s}}{\sum_s e^{-\beta E_s}} \Delta E$$

$$\boxed{p(E)\Delta E \approx \frac{-\beta e^{-\beta E}}{\sum_s e^{-\beta E_s}} \Delta E} \tag{5}$$