## Problem 1.8

(a) Using parameters $N = 40$, $E = 40$, and $d = 3$, the mean energy of the demon $\langle E_d \rangle$, after running the program for some time, starts to approach a steady value.

(b) After running the program for a reasonable time ($> 100000$ mcs), the mean energy of the demon $\langle E_d \rangle$ appears to have stabilized at a value of 0.655, while system mean energy $\langle E \rangle$ appears to have stabilized at a value of 39.345, which corresponds to a $\langle E \rangle / N$ value of 0.984, indicating that the mean energy per particle is greater than the demon's mean energy. The ratio $\nu = \frac{\langle E_d \rangle}{\langle E \rangle / N}$ is 0.667.

(c) Doubling the value of $E$ and running the program for the same amount of time, $\langle E_d \rangle$ and $\langle E \rangle / N$ respectively approach values of 1.308 and 1.967, corresponding to the ratio $\nu = 0.665$.

Table 1: Energy values for different $N$ and $E$.

| $N$ | $E$ | $\langle E_d \rangle$ | $\langle E \rangle$ | $\langle E \rangle / N$ | $\nu$ |
|---|---|---|---|---|---|
| 40 | 40 | 0.655 | 39.345 | 0.984 | 0.667 |
| 60 | 40 | 0.436 | 39.564 | 0.659 | 0.662 |
| 80 | 40 | 0.331 | 39.669 | 0.496 | 0.667 |
| 100 | 40 | 0.265 | 39.735 | 0.397 | 0.667 |
| 40 | 80 | 1.308 | 78.692 | 1.967 | 0.665 |
| 60 | 80 | 0.879 | 79.121 | 1.319 | 0.667 |
| 80 | 80 | 0.66 | 79.34 | 0.992 | 0.665 |
| 100 | 80 | 0.532 | 79.468 | 0.795 | 0.669 |

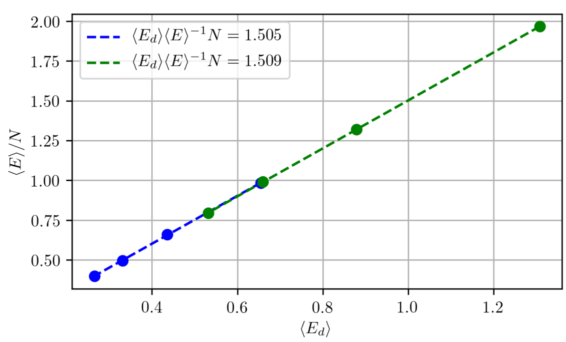


Figure 1: Plot showing the relation of the mean demon energy and mean system energy.

By varying the values of $N$ and $E$, a linear relation between the mean demon energy and mean system energy can be observed, as shown in Table 1 and Figure 1. The ratio $\nu$ remains fairly constant, and via linear regression, this value appears to be in the neighborhood of $1.507 \approx 3/2$, which corresponds to a familiar relation

$$\frac{\langle E \rangle}{N} = \frac{3}{2} \langle E_d \rangle. \tag{1}$$

(d) From the mean energy of an ideal gas in 3 dimensions and rearranging some terms,

$$\frac{\langle E \rangle_{ideal}}{N} = \frac{3}{2} k_B T \tag{2}$$

Comparing this with (1), it is evident that

$$k_B T \propto \langle E_d \rangle \tag{3}$$

but then, we have set units of $k_B = 1$. Therefore,

$$T \propto \langle E_d \rangle \tag{4}$$

(e)