PS 18: Problem 2.26

(a) The useful work is defined as

$$W_{useful} = -\left(\Delta E + P\Delta V - T_{bath}\Delta S\right) \tag{1}$$

If $\Delta V = 0$,

$$W_{useful} = -\left(\Delta E - T_{bath}\Delta S\right) \tag{2}$$

Recall the Helmholtz free energy:

$$F = E - TS \tag{3}$$

The availability is defined as

$$\Delta A = \Delta F = \Delta E - T_{bath}\Delta S \tag{4}$$

Substituting (4) into (2),

$$\boxed{W_{useful} = -\Delta F} \tag{5}$$

(b) Recall the Gibbs free energy:

$$G = E - PV + TS \tag{6}$$

Taking the delta differentials,

$$\Delta G = \Delta E - (P\Delta V + V\Delta P) + (T\Delta S + S\Delta T) \tag{7}$$

If $\Delta P = \Delta T = 0$,

$$\Delta G = \Delta E - P\Delta V + T\Delta S \tag{8}$$

Substituting (8) into (1),

$$\boxed{W_{useful} = -\Delta G} \tag{9}$$