PS 33: Problem 3.43

(a) The Lorentz distribution is given by

$$p(x) = \frac{1}{\pi} \frac{\gamma}{(x-a)^2 + \gamma^2} \tag{1}$$

Comparing it with the Gaussian distribution:

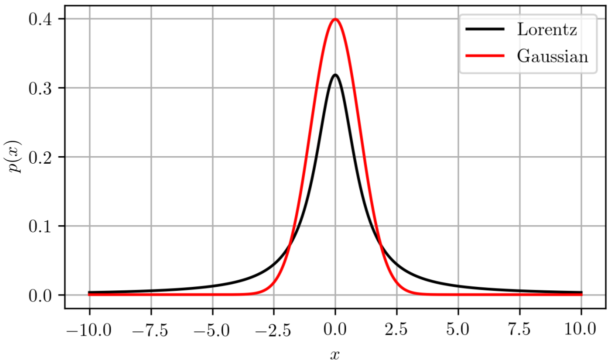

Figure 1: Comparison of Lorentzian and Gaussian distributions.

This shows that for the same set of parameters, the Gaussian has a higher peak, and quickly falls off and approaches zero as one moves away from the mean/central value.

(b) For $a = 0$ and $\gamma = 1$, the first moment of the Lorentzian is given by

$$\begin{aligned} \langle x^1 \rangle &= \int_{-\infty}^{+\infty} x p(x) \, \mathrm{d}x \\ &= \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x}{x^2+1} \, \mathrm{d}x \end{aligned} \tag{2}$$

Let $u \equiv x^2 + 1$, $\mathrm{d}u \equiv 2x \, \mathrm{d}x$,

$$\begin{aligned} \langle x^1 \rangle &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{u} \, \mathrm{d}u \\ &= \frac{1}{2\pi} \left[\ln(u) \right|_{-\infty}^{+\infty} \\ &= \frac{1}{2\pi} \left[\ln\left(x^2+1\right) \right|_{-\infty}^{+\infty} \end{aligned}$$