Problem 2.13

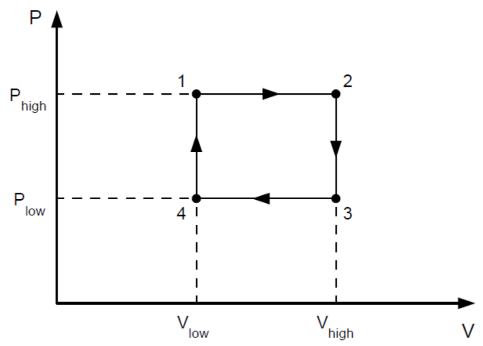

Figure 1: Cyclic process for this problem.

From Example 2.1, the net work done on a gas in a cyclic process was determined to be nonzero, with a value of

$$W_{net} = -\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \tag{1}$$

Assuming an ideal gas with N particles, the energy transfer due to heating for each step in the process is as follows:

$$Q_{1\to 2} = -Q_{3\to 4} = \int_{T_1}^{T_2} c_P \, \mathrm{d}T \tag{2}$$
$$= \nu c_P \int_{T_1}^{T_2} \mathrm{d}T \tag{3}$$
$$= \nu c_P \left(T_2 - T_1\right) \tag{4}$$
$$= \nu c_P \Delta T \tag{5}$$
$$Q_{2\to 3} = -Q_{4\to 1} = \int_{T_1}^{T_2} c_V \, \mathrm{d}T \tag{6}$$
$$= \nu c_V \int_{T_1}^{T_2} \mathrm{d}T \tag{7}$$
$$= \nu c_V \left(T_2 - T_1\right) \tag{8}$$
$$= \nu c_V \Delta T \tag{9}$$

Recalling the ideal gas equation,

$$PV = \nu RT \tag{10}$$
$$T = \frac{PV}{\nu R} \tag{11}$$

The net energy transfer due to heating is

$$W_{net} = Q_{1\to 2} + Q_{2\to 3} + Q_{3\to 4} + Q_{4\to 1} \tag{12}$$

Plugging (11) into each of the equations in (5) and (9), we have

$$Q_{1\to 2} = \nu c_P P_{high} \frac{V_{high} - V_{low}}{\nu R} \tag{13}$$
$$Q_{2\to 3} = \nu c_V V_{high} \frac{P_{low} - P_{high}}{\nu R} \tag{14}$$
$$Q_{3\to 4} = \nu c_P P_{low} \frac{V_{low} - V_{high}}{\nu R} \tag{15}$$
$$Q_{4\to 1} = \nu c_V V_{low} \frac{P_{high} - P_{low}}{\nu R} \tag{16}$$

Simplifying equations (13) through (16) and summing them as in (12), we have

$$\begin{aligned} Q_{net} = {} & c_P P_{high} \frac{V_{high} - V_{low}}{R} \\ & - c_V V_{high} \frac{P_{high} - P_{low}}{R} \\ & - c_P P_{low} \frac{V_{high} - V_{low}}{R} \\ & + c_V V_{low} \frac{P_{high} - P_{low}}{R} \end{aligned} \tag{17}$$

$$\begin{aligned} Q_{net} = {} & \frac{c_P}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \\ & - \frac{c_V}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \end{aligned} \tag{18}$$

Recall that for an ideal gas,

$$c_P = \frac{5}{2}R \tag{19}$$
$$c_V = \frac{3}{2}R \tag{20}$$

Plugging these into (18),

$$\begin{aligned} Q_{net} = {} & \frac{5}{2}R\frac{1}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \\ & - \frac{3}{2}R\frac{1}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \end{aligned} \tag{21}$$

Which gives us the expected relation of $Q_{net} = -W_{net}$:

$$Q_{net} = (P_{high} - P_{low})(V_{high} - V_{low}) \tag{22}$$

From the first thermodynamic law,

$$\Delta E = Q + W \tag{23}$$

Plugging equations (1) and (22) into this, we have

$$\Delta E = 0 \tag{24}$$