Problem 1.3

(a) For $N = 8$ particles, there are $2^N = 256$ possible microstates, and $N + 1 = 9$ possible macrostates. Via Pascal's triangle, one can, for reasonable values of N and with a bit of work, determine the number of microstates for one macrostate:

N									
0	1								
1	1	1							
2	1	2	1						
3	1	3	3	1					
4	1	4	6	4	1				
5	1	5	10	10	5	1			
6	1	6	15	20	15	6	1		
7	1	7	21	35	35	21	7	1	
8	1	8	28	56	70	56	28	8	1
	0	1	2	3	4	5	6	7	8
					n				

Thus, for $n = 4$, $W(n) = 70$ and $P(n) = 70/256 \approx 27\%$. To generalize this for any N and n without having to explicitly write out Pascal's triangle, we can express the value for n from a selection of N using the combination notation ${}_N\mathrm{C}_n$, where

$$ {}_N\mathrm{C}_n = \binom{N}{n} = \frac{N!}{n!(N-n)!} \tag{1} $$

In other words, calculating for ${}_N\mathrm{C}_n$ yields the number of ways that n particles out of N can be in one partition of the box.

(b) The macrostate $n = N/2$ is much more probable than the macrostate $n = N$ because the macrostate $n = N/2$ is associated with more microstates that are indistinguishable from each other.