

PS 41: Problem 4.19

Given the following:

$$S(E, V, N) = Nk \left[\ln \frac{V}{N} + \frac{3}{2} \ln \left(\frac{mE}{3N\pi\hbar^2} \right) + \frac{5}{2} \right] \quad (1)$$

$$E = \frac{3}{2} NkT \quad (2)$$

Substitute (2) in (1):

$$S = Nk \left[\ln \frac{V}{N} + \frac{3}{2} \ln \left(\frac{m}{3N\pi\hbar^2} \frac{3}{2} NkT \right) + \frac{5}{2} \right] \quad (3)$$

$$= Nk \left[\ln \frac{V}{N} + \frac{3}{2} \ln \left(\frac{mkT}{2\pi\hbar^2} \right) + \frac{5}{2} \right] \quad (4)$$

Rewrite in terms of $h \equiv 2\pi\hbar$:

$$S = Nk \left[\ln \frac{V}{N} + \frac{3}{2} \ln \left(\frac{4\pi^2 mkT}{2\pi h^2} \right) + \frac{5}{2} \right] \quad (5)$$

$$= Nk \left[\ln \frac{V}{N} + \frac{3}{2} \ln \left(\frac{2\pi mkT}{h^2} \right) + \frac{5}{2} \right] \quad (6)$$

$$= Nk \left[\ln \left(\frac{V}{N} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} \right) + \frac{5}{2} \right] \quad (7)$$

$$\boxed{\frac{S}{Nk} = \ln \left[\frac{V}{N} \left(\frac{2\pi mkT}{h^2} \right)^{3/2} \right] + \frac{5}{2}} \quad (8)$$

which is the Sackur-Tetrode relation.