Problem 2.20

From the given equation,

$$\text{COP} = \frac{Q_{hot}}{W} \tag{1}$$

Since we want to heat a room surrounded by a cooler environment, the work done is positive:

$$\begin{aligned} \text{COP} &= \frac{Q_{hot}}{Q_{hot} - Q_{cold}} \\ &= \frac{Nk_BT_{hot}\ln\left(\frac{V_f}{V_i}\right)}{Nk_BT_{hot}\ln\left(\frac{V_f}{V_i}\right) - Nk_BT_{cold}\ln\left(\frac{V_f}{V_i}\right)} \end{aligned} \tag{2}$$

$$\boxed{\text{COP} = \frac{T_{hot}}{T_{hot} - T_{cold}}} \tag{3}$$

Plugging the given conditions $T_{hot} = 296\text{K}$ and $T_{cold} = 273\text{K}$ into (3),

$$\begin{aligned} \text{COP} &= \frac{296}{296 - 273} \\ \boxed{\text{COP} &= 12.9} \end{aligned} \tag{4}$$

The denominator is larger for large temperature differences. The heater is more efficient in regions with mild winters.