Physics 151

Crib Sheet

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Consider the cyclic process as described in Example 2.1.
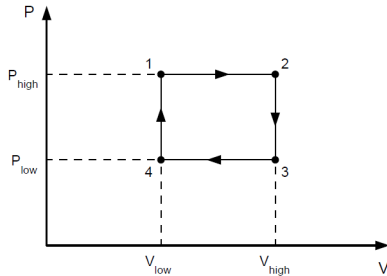

Figure 1: Cyclic process for this problem.

(a) Because the system was returned to its original pressure and volume, why is the net amount of work done on the system not zero?

(b) What would be the work done on the gas if the gas were taken from $1 \to 2 \to 3$ and then back to 1 along the diagonal path connecting 3 and 1?

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In Example 2.1 we showed that the net work done on the gas in the cyclic process shown in Figure 1 is nonzero. Assume that the gas is ideal with N particles and calculate the energy transfer by heating in each step of the process. Then explain why the net work done on the gas is negative and show that the net change of the internal energy is zero.

From Example 2.1, the net work done on a gas in a cyclic process was determined to be nonzero, with a value of

$$W_{net} = -\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \tag{1}$$

Assuming an ideal gas with N particles, the energy transfer due to heating for each step in the process is as follows:

$$Q_{1\to2} = -Q_{3\to4} = \int_{T_1}^{T_2} c_P \, \mathrm{d}T \tag{2}$$

$$= \nu c_P \int_{T_1}^{T_2} \mathrm{d}T \tag{3}$$

$$= \nu c_P \left(T_2 - T_1\right) \tag{4}$$

$$= \nu c_P \Delta T \tag{5}$$

$$Q_{2\to3} = -Q_{4\to1} = \int_{T_1}^{T_2} c_V \, \mathrm{d}T \tag{6}$$

$$= \nu c_V \int_{T_1}^{T_2} \mathrm{d}T \tag{7}$$

$$= \nu c_V \left(T_2 - T_1\right) \tag{8}$$

$$= \nu c_V \Delta T \tag{9}$$

Recalling the ideal gas equation,

$$PV = \nu R T \tag{10}$$

$$T = \frac{PV}{\nu R} \tag{11}$$

The net energy transfer due to heating is

$$W_{net} = Q_{1\to2} + Q_{2\to3} + Q_{3\to4} + Q_{4\to1} \tag{12}$$

Plugging (11) into each of the equations in (5) and (9), we have

$$Q_{1\to2} = \nu c_P P_{high} \frac{V_{high} - V_{low}}{\nu R} \tag{13}$$

$$Q_{2\to3} = \nu c_V V_{high} \frac{P_{low} - P_{high}}{\nu R} \tag{14}$$

$$Q_{3\to4} = \nu c_P P_{low} \frac{V_{low} - V_{high}}{\nu R} \tag{15}$$

$$Q_{4\to1} = \nu c_V V_{low} \frac{P_{high} - P_{low}}{\nu R} \tag{16}$$

Simplifying equations (13) through (16) and summing them as in (12), we have

$$\begin{aligned} Q_{net} = &\, c_P P_{high} \frac{V_{high} - V_{low}}{R} \\ &- c_V V_{high} \frac{P_{high} - P_{low}}{R} \\ &- c_P P_{low} \frac{V_{high} - V_{low}}{R} \\ &+ c_V V_{low} \frac{P_{high} - P_{low}}{R} \end{aligned} \tag{17}$$

$$\begin{aligned} Q_{net} = &\, \frac{c_P}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \\ &- \frac{c_V}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \end{aligned} \tag{18}$$

Recall that for an ideal gas,

$$c_P = \frac{5}{2}R \tag{19}$$

$$c_V = \frac{3}{2}R \tag{20}$$

Plugging these into (18),

$$\begin{aligned} Q_{net} = &\, \frac{5}{2}R\frac{1}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \\ &- \frac{3}{2}R\frac{1}{R}\left(P_{high} - P_{low}\right)\left(V_{high} - V_{low}\right) \end{aligned} \tag{21}$$

Which gives us the expected relation of $Q_{net} = -W_{net}$ (negative because in a cyclic process, there is

no change in internal energy; the work done is equal to the heat added):

$$Q_{net} = (P_{high} - P_{low})(V_{high} - V_{low}) \tag{22}$$

From the first thermodynamic law,

$$\Delta E = Q + W \tag{23}$$

Plugging equations (1) and (22) into this, we have

$$\Delta E = 0 \tag{24}$$

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Use (2.44) and the ideal gas pressure equation of state in (2.8) to show that in a quasistatic adiabatic processes P and V are related as

$$PV^\gamma = \text{constant} \tag{25}$$

Also show that T and P are related as

$$TP^{(1-\gamma)/\gamma} = \text{constant} \tag{26}$$

The given equations are:

$$TV^{\gamma-1} = C \tag{27}$$
$$PV = \nu RT \tag{28}$$

Isolating T in (28),

$$T = \frac{PV}{\nu R} \tag{29}$$

Plugging this into (27),

$$\frac{PV}{\nu R}V^{\gamma-1} = C$$
$$PV^\gamma = C\nu R = C \tag{30}$$

Therefore,

$$\boxed{PV^\gamma = \text{constant}} \tag{31}$$

Similarly, isolating V from (28),

$$V = \frac{\nu RT}{P} \tag{32}$$

Plugging this into (45),

$$P\left(\frac{\nu RT}{P}\right)^\gamma = C$$
$$(\nu RT)^\gamma P^{1-\gamma} = C$$
$$T^\gamma P^{1-\gamma} = \frac{C}{(\nu R)^\gamma}$$
$$T^\gamma P^{1-\gamma} = D \tag{33}$$

Raising both sides to $1/\gamma$,

$$\left(T^\gamma P^{1-\gamma}\right)^{\frac{1}{\gamma}} = D^{\frac{1}{\gamma}}$$
$$\boxed{TP^{(1-\gamma)/\gamma} = \text{constant}} \tag{34}$$

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(a) Air initially at 20°C is compressed by a factor of 15. Assuming adiabatic conditions, $V_i/15 = V_f$, $T_i = 293$K, and $\gamma = 1.4$,

$$TV^{\gamma-1} = C \tag{35}$$
$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$
$$293V_i^{0.4} = T_f\left(\frac{V_i}{15}\right)^{0.4}$$
$$293 = T_f\left(\frac{1}{15}\right)^{0.4}$$
$$T_f = 293\left(\frac{1}{15}\right)^{-0.4}$$
$$\boxed{T_f = 866 \text{ K}} \tag{36}$$

Assuming air behaves like an ideal gas,

$$\frac{P_iV_i}{T_i} = \frac{P_fV_f}{T_f} \tag{37}$$
$$\frac{P_i}{293}\frac{V_f}{15} = \frac{P_fV_f}{866}$$
$$\frac{P_i}{(293)(15)} = \frac{P_f}{866}$$
$$\boxed{P_i \approx 5P_f} \tag{38}$$

(b) If the compression is isothermal,

$$P_iV_i = P_fV_f \tag{39}$$
$$\boxed{P_i = 15P_f} \tag{40}$$

(c) The pressure increases more with adiabatic compression.

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(a) Use the result that we derived in (2.53) to obtain the alternative form (2.54).

From the given equations,

$$W = C_V(T_2 - T_1) \tag{41}$$
$$PV = \nu RT \tag{42}$$
$$C_V = \frac{3}{2}\nu k_B \tag{43}$$

Plugin (42) and (43) into (41):

$$W = \frac{3}{2}\nu k_B \left(\frac{P_2 V_2}{\nu k_B} - \frac{P_1 V_1}{\nu k_B} \right)$$
$$= \frac{3}{2} \left(P_2 V_2 - P_1 V_1 \right) \tag{44}$$

Since $\gamma = \frac{5}{3}$ for monatomic ideal gas,

$$W = \frac{(P_2 V_2 - P_1 V_1)}{\frac{2}{3}}$$
$$= \frac{(P_2 V_2 - P_1 V_1)}{\frac{5}{3} - \frac{3}{3}}$$
$$\boxed{W = \frac{(P_2 V_2 - P_1 V_1)}{\gamma - 1}} \tag{45}$$

(b) Show that another way to derive (2.54) is to use the relations (2.14) and (2.46).

From the given equations,

$$PV^\gamma = C \tag{46}$$
$$W = -\int_{V_1}^{V_2} P(T,V)\, \mathrm{d}V \tag{47}$$

Isolate P from (46) and plug into (47):

$$W = -\int_{V_1}^{V_2} CV^{-\gamma}\, \mathrm{d}V$$
$$= -C \frac{V^{1-\gamma}}{1-\gamma} \Big|_{V_1}^{V_2}$$
$$= \frac{1}{\gamma - 1} \left(CV_2^{1-\gamma} - CV_1^{1-\gamma} \right)$$
$$= \frac{1}{\gamma - 1} \left(CV_2^{-\gamma}V_2 - CV_1^{-\gamma}V_1 \right) \tag{48}$$

From (46),

$$P = CV^{-\gamma} \tag{49}$$

Therefore,

$$\boxed{W = \frac{1}{\gamma - 1} \left(P_2 V_2 - P_1 V_1 \right)} \tag{50}$$