PS 20: Problem 2.27

The differentials of the thermodynamic potentials are

$$\mathrm{d}F = -S\,\mathrm{d}T - P\,\mathrm{d}V \tag{1}$$
$$\mathrm{d}G = -S\,\mathrm{d}T + V\,\mathrm{d}P \tag{2}$$
$$\mathrm{d}H = T\,\mathrm{d}S + V\,\mathrm{d}P \tag{3}$$

The natural variables of (1) are

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \tag{4}$$
$$P = -\left(\frac{\partial F}{\partial V}\right)_T \tag{5}$$

Taking the partial derivative of (4) wrt V under constant T,

$$\begin{aligned}\left(\frac{\partial S}{\partial V}\right)_T &= -\frac{\partial}{\partial V}\left[\left(\frac{\partial F}{\partial T}\right)_V\right]_T \\ &= -\frac{\partial}{\partial T}\left[\left(\frac{\partial F}{\partial V}\right)_T\right]_V\end{aligned}$$
$$\boxed{\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V} \tag{6}$$

The natural variables of (2) are

$$S = -\left(\frac{\partial G}{\partial T}\right)_P \tag{7}$$
$$V = \left(\frac{\partial G}{\partial P}\right)_T \tag{8}$$

Taking the partial derivative of (7) wrt P under constant T,

$$\begin{aligned}\left(\frac{\partial S}{\partial P}\right)_T &= -\frac{\partial}{\partial P}\left[\left(\frac{\partial G}{\partial T}\right)_P\right]_T \\ &= -\frac{\partial}{\partial T}\left[\left(\frac{\partial G}{\partial P}\right)_T\right]_P\end{aligned}$$
$$\boxed{\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P} \tag{9}$$

The natural variables of (3) are

$$T = \left(\frac{\partial H}{\partial S}\right)_P \tag{10}$$
$$V = \left(\frac{\partial H}{\partial P}\right)_S \tag{11}$$

Taking the partial derivative of (10) wrt P under constant S,

$$\begin{aligned}\left(\frac{\partial T}{\partial P}\right)_S &= \frac{\partial}{\partial P}\left[\left(\frac{\partial H}{\partial S}\right)_P\right]_S \\ &= \frac{\partial}{\partial S}\left[\left(\frac{\partial H}{\partial P}\right)_S\right]_P\end{aligned}$$
$$\boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P} \tag{12}$$