PS 46: Problem 4.25

The Boltzmann distribution is given by

$$P_S = \frac{1}{Z} e^{-\beta E_s} \tag{1}$$

where $Z \equiv \sum_s e^{-\beta E_s}$ is the partition function. The probability that a system is in any microstate with energy E is

$$p(E) = \frac{\Omega(E) e^{-\beta E}}{\sum\limits_{\text{levels}} \Omega(E) e^{-\beta E}} \tag{2}$$

In the limit N, $V \to \infty$, the gap between adjacent energy levels becomes infinitesimal, so E can be considered a continuous variable. The probability that a system is in any microstate with energy between E and $E + \mathrm{d}E$ is $p(E)\,\mathrm{d}E$. Let $g(E)\,\mathrm{d}E$ be the number of microstates between E and $E + \mathrm{d}E$. The form of the probability distribution of the energy of a system in the canonical ensemble is

$$\boxed{p(E)\,\mathrm{d}E = \frac{g(E) e^{-\beta E}\,\mathrm{d}E}{\int_0^\infty g(E) e^{-\beta E}\,\mathrm{d}E}} \tag{3}$$

where $\beta \equiv \frac{1}{kT}$.