

PS 35: Problem 3.72

- (a) The first moment of the Gaussian distribution is given by

$$\langle n \rangle = \int_{-\infty}^{+\infty} n P(n) \, dn \quad (1)$$

where

$$P(n) = A e^{-\frac{1}{2} B (n - \tilde{n})^2} \, dn \quad (2)$$

Let $x \equiv n - \tilde{n}$, $dx \equiv dn$,

$$\langle n \rangle = \int_{-\infty}^{+\infty} (x + \tilde{n}) A e^{-\frac{1}{2} B x^2} \, dx \quad (3)$$

(3) is a Gaussian integral, and can be evaluated as

$$\langle n \rangle = A \sqrt{\frac{2\pi}{B}} \tilde{n} \quad (4)$$

If we assume that (2) has been normalized in the first place such that $A = 1$ and $B = 2\pi$,

$$\boxed{\langle n \rangle = \tilde{n}} \quad (5)$$