PS 49: Problem 4.29

(a)

(b) Letting the program run with parameters $d = 3$, $N = 40$, and $E = 40$ for a time $> 100,000$ mcs, we obtain the mean energy of the demon $\langle E_d \rangle = 0.65$, and the mean energy per particle $\langle E \rangle / N = 0.98$. For varying N, we have the following:

Table 1: Energy values for $E = 40$.

N	$\langle E_d \rangle$	$\langle E \rangle$	$\langle E \rangle / N$
40	0.65	39.35	0.984
60	0.438	0.329	0.659
80	0.329	39.671	0.496
100	0.265	39.735	0.397

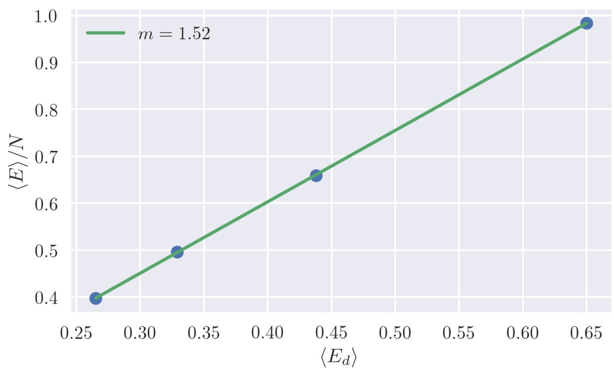

Figure 1: Relationship between $\langle E_d \rangle$ and $\langle E \rangle / N$ for $N = 40$.

From linear regression, we observe a direct relationship between $\langle E_d \rangle$ and $\langle E \rangle / N$, with a proportionality constant $m = 1.52$ or $m = \frac{31}{20} \approx \frac{3}{2}$. This implies the relation

$$\boxed{\frac{\langle E \rangle}{N} \approx \frac{3}{2} \langle E_d \rangle} \tag{1}$$

(c) The mean energy of an ideal classical gas is 3 dimensions is

$$\langle E \rangle = \frac{3}{2} NkT \tag{2}$$

Setting units of $k = 1$, and rearranging terms,

$$\frac{\langle E \rangle}{N} = \frac{3}{2} T \tag{3}$$

But (1) implies

$$\frac{3}{2} \langle E_d \rangle \approx \frac{3}{2} T \tag{4}$$

or

$$\boxed{\langle E_d \rangle \approx T} \tag{5}$$

which means that the temperature of the gas approximates the mean energy of the demon at any N.

(d) The exponential form of $p(E_d)$ is verified using the Curve Fits section of the Data Tool in the simulator. Using the same initial parameters in (b), we fit it to an equation of the form $Ae^{-\beta x}$, where x are the E_d values, and A and β are the parameters. For the initial parameters above, the obtain the fit $A = 3.250 \times 10^6$ and $\beta = 1.514$. For varying N, we have

Table 2: Comparison of β and T values for $E = 40$.

N	β	β^{-1}	T
40	1.514	0.66	0.65
60	2.266	0.44	0.438
80	3.016	0.33	0.329
100	3.779	0.264	0.265

We see that the β^{-1} values are very close to the T values.

(e) The simulation results verify our prediction in (5).

(f) In 2 dimensions, $\langle E_d \rangle$ stabilizes at around 0.973 for $E, N = 40$, which implies a 1:1 relation between $\langle E_d \rangle$ and $\langle E \rangle / N$. The mean energy of an ideal gas in 2D is

$$\langle E \rangle = \frac{2}{2} NkT \tag{6}$$

Letting units of $k = 1$ and isolating T,

$$T = \frac{\langle E \rangle}{N} \tag{7}$$

which shows that the mean demon energy still approximates the temperature well in 2D.