PS 42: Problem 4.20

Given the following:

$$\frac{\mu}{T} = -\left(\frac{\partial S}{\partial N}\right)_{E,V} \tag{1}$$

$$S(E,V,N) = Nk\left[\ln\frac{V}{N} + \frac{3}{2}\ln\left(\frac{mE}{3N\pi\hbar^2}\right) + \frac{5}{2}\right] \tag{2}$$

$$E = \frac{3}{2}NkT \tag{3}$$

Perform the necessary differentiation on (2) according to (1) and simplify:

$$\begin{aligned}
\frac{\mu}{T} &= -\frac{\partial}{\partial N}\left\{Nk\left[\ln\frac{V}{N} + \frac{3}{2}\ln\left(\frac{mE}{3N\pi\hbar^2}\right) + \frac{5}{2}\right]\right\} && (4)\\
&= -\frac{\partial}{\partial N}\left[kN\ln\frac{V}{N} + \frac{3}{2}kN\ln\left(\frac{mE}{3N\pi\hbar^2}\right) + \frac{5}{2}kN\right] && (5)\\
&= -\left[-k + k\ln\left(\frac{V}{N}\right) - \frac{3}{2}k + \frac{3}{2}k\ln\left(\frac{mE}{3N\pi\hbar^2}\right) + \frac{5}{2}k\right] && (6)\\
&= -k\left[-1 + \ln\left(\frac{V}{N}\right) + \frac{3}{2}\ln\left(\frac{mE}{3N\pi\hbar^2}\right) + 1\right] && (7)\\
&= -k\left[\ln\left(\frac{V}{N}\right) + \frac{3}{2}\ln\left(\frac{mE}{3N\pi\hbar^2}\right)\right] && (8)\\
&= -k\ln\left[\frac{V}{N}\left(\frac{mE}{3N\pi\hbar^2}\right)^{3/2}\right] && (9)
\end{aligned}$$

$$\boxed{\mu = -kT\ln\left[\frac{V}{N}\left(\frac{mE}{3N\pi\hbar^2}\right)^{3/2}\right]} \tag{10}$$

Plugging in (3),

$$\begin{aligned}
\mu(T,V,N) &= -kT\ln\left[\frac{V}{N}\left(\frac{m}{3N\pi\hbar^2}\frac{3}{2}NkT\right)^{3/2}\right] && (11)\\
&= -kT\ln\left[\frac{V}{N}\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}\right] && (12)
\end{aligned}$$

Using the relation $h = 2\pi\hbar$, rewrite as

$$\mu(T,V,N) = -kT\ln\left[\frac{V}{N}\left(\frac{4\pi^2 mkT}{2\pi h^2}\right)^{3/2}\right] \tag{13}$$

$$\boxed{\mu(T,V,N) = -kT\ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]} \tag{14}$$