Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Design and Correction of Optical Systems

Lecture 5: Wave aberrations

2017-05-19

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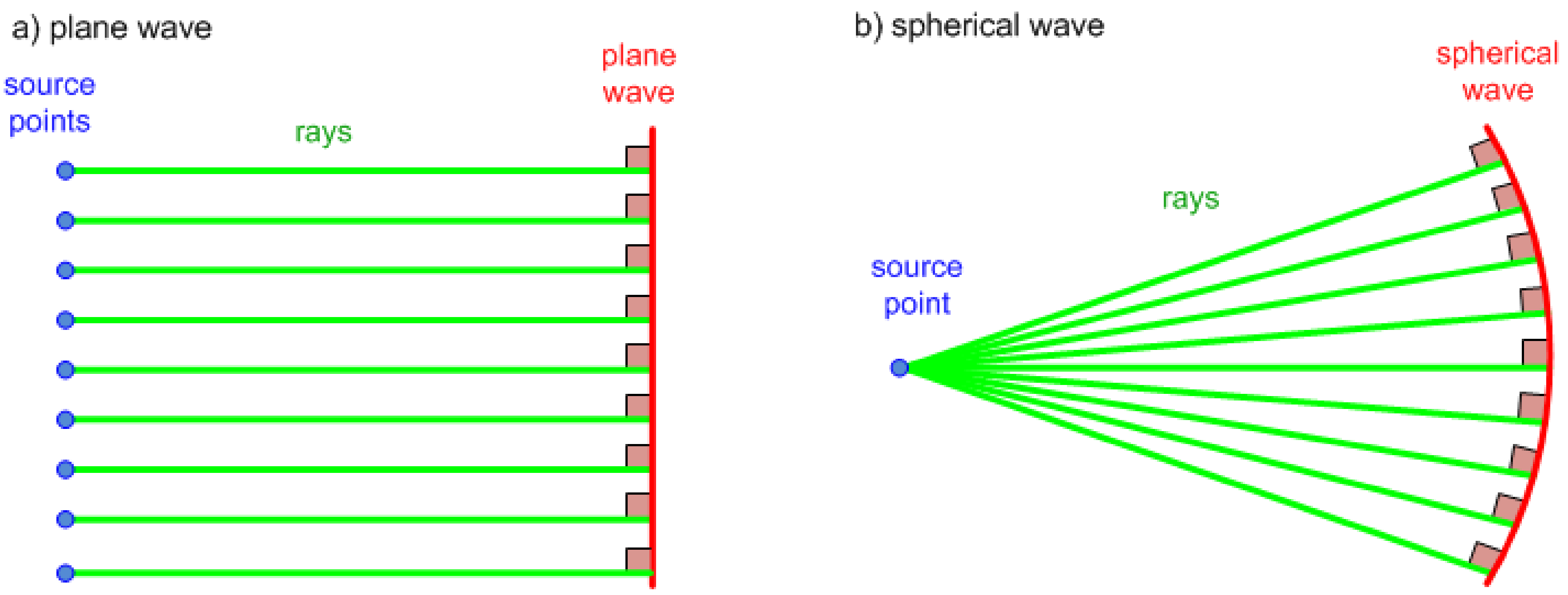
Preliminary Schedule - DCS 2017

1	07.04.	Basics	Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches
2	21.04.	Materials and Components	Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements
3	28.04.	Paraxial Optics	Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization
4	05.05.	Optical Systems	Pupil, ray sets and sampling, aperture and vignetting, telecentricity, symmetry, photometry
5	12.05.	Geometrical Aberrations	Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions
6	19.05.	Wave Aberrations	Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality
7	26.05.	PSF and Transfer function	Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model
8	02.06.	Further Performance Criteria	Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options
9	09.06.	Optimization and Correction	Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches
10	16.06.	Correction Principles I	Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres
11	23.06.	Correction Principles II	Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements
12	30.06.	Optical System Classification	Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes
13	07.07.	Special System Examples	Zoom systems, confocal systems

Contents

Ray-Wave Equivalent

- Rays and waves carry the same information
- Wave surface is perpendicular on the rays
- Wave is purely geometrical and has no diffraction properties



Law of Malus-Dupin

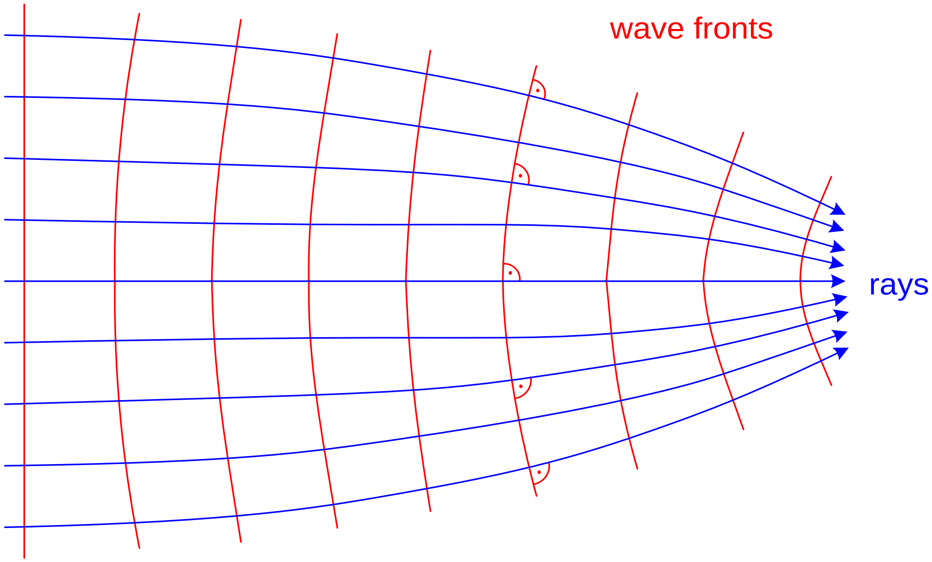
- Law of Malus-Dupin:
  - equivalence of rays and wavefronts
  - both are orthonormal
  - identical information

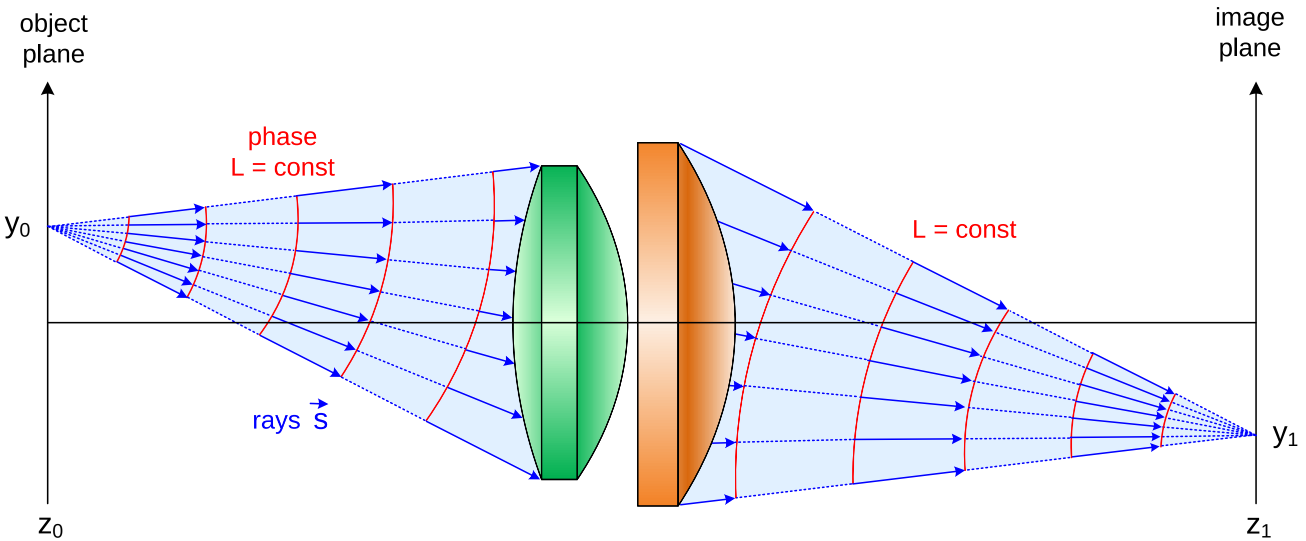
- Condition:
  No caustic of rays

- Mathematical:
  Rotation of Eikonal vanish

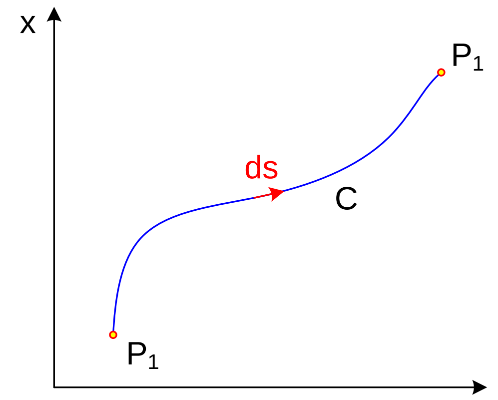
$$rot(n \cdot \vec{s}) = 0$$

- Optical system:
  Rays and spherical waves orthonormal





Fermat Principle



- Fermat principle:
  the light takes the ray path, which corresponds to the shortest time of arrival

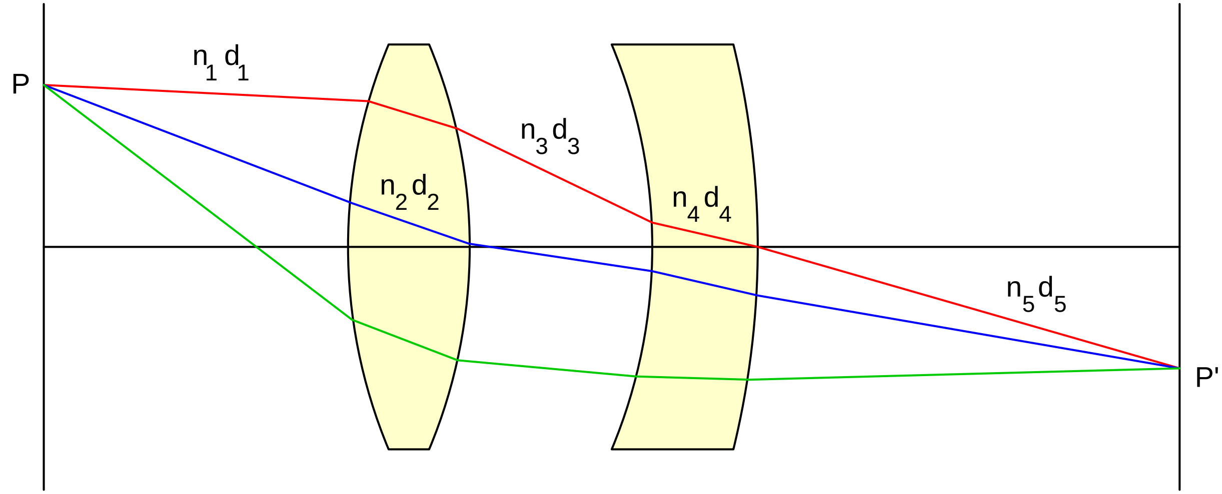
- The realized path is a minimum and therefore the first derivatives vanish

$$\delta L = \delta \int_{P_1}^{P_2} n(x,y,z)\, ds = 0$$

- Several realized ray pathes have the same optical path length

$$L = \int_{P1}^{P2} n \cdot \vec{s} \cdot d\vec{r} = const.$$

- The principle is valid for smooth and discrete index distributions



Relationships

- Concrete calculation of wave aberration: addition of discrete optical path lengths (OPL)

$$l_{OPL} = \int_{OE}^{AP} n \cdot d\vec{r}$$

- Reference on chief ray and reference sphere (optical path difference)

$$\Delta_{OPD}(x,y) = l_{OPL}(x,y) - l_{OPL}(0,0)$$

- Relation to transverse aberrations

$$\frac{\partial W}{\partial y_p} = -\frac{\Delta y'}{R-W} \approx -\frac{\Delta y'}{R}$$

- Conversion between longitudinal transverse and wave aberrations

$$\Delta s' = \frac{R}{y_p} \cdot \Delta y' = \frac{\Delta y'}{\sin u'} = -\frac{R^2}{y_p} \cdot \frac{\partial W(x_p, y_p)}{\partial y_p}$$

- Scaling of the phase / wave aberration:
  1. Phase angle in radiant
  2. Light path (OPL) in mm
  3. Light path scaled in λ

$$E(x) = A(x) \cdot e^{i \cdot \varphi(x)}$$

$$E(x) = A(x) \cdot e^{i \cdot k \Delta_{OPD}(x)}$$

$$E(x) = A(x) \cdot e^{2\pi i \cdot W(x)}$$

Relationship to Transverse Aberration

- Relation between wave and transverse aberration
- Approximation for small aberrations and small aperture angles u
- Ideal wavefront, reference sphere: W_{ideal}
- Real wavefront: W_{real}
- Finite difference

$$\Delta W = W = W_{real} - W_{ideal}$$

- Angle difference

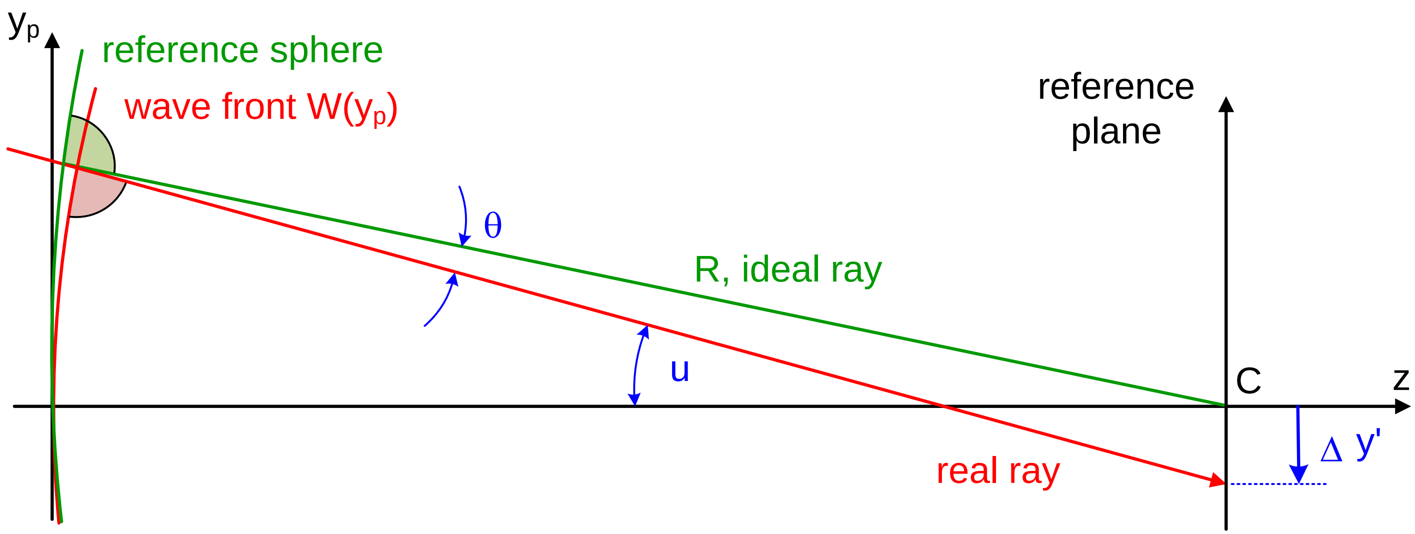
$$\varphi \approx \tan\varphi = \frac{\partial W}{\partial y_p}$$

- Transverse aberration

$$\Delta y' = -R \cdot \varphi$$

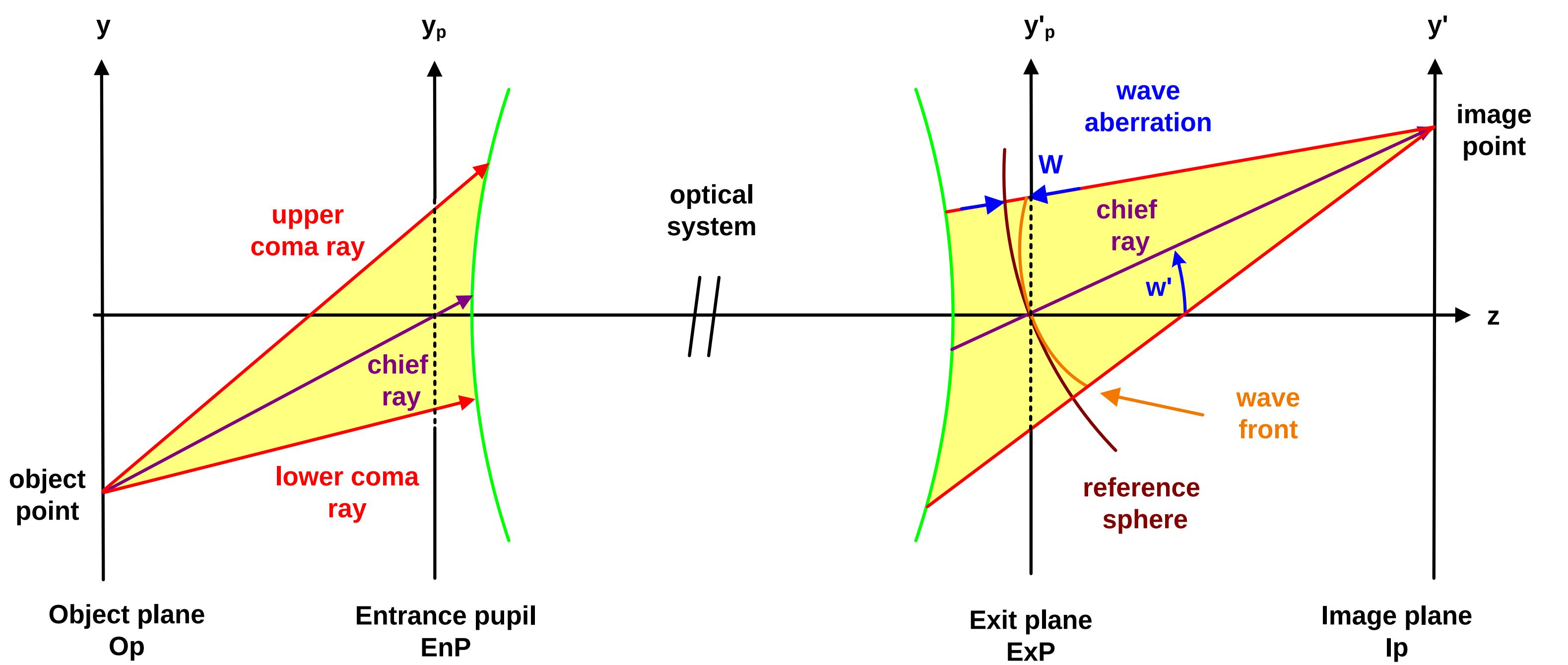
- Limiting representation

$$\frac{\partial W}{\partial y_p} = -\frac{\Delta y'}{R - W} \approx -\frac{\Delta y'}{R}$$



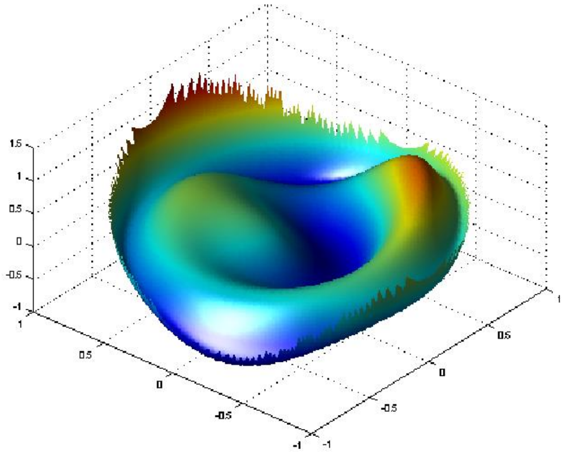
Wave Aberration in Optical Systems

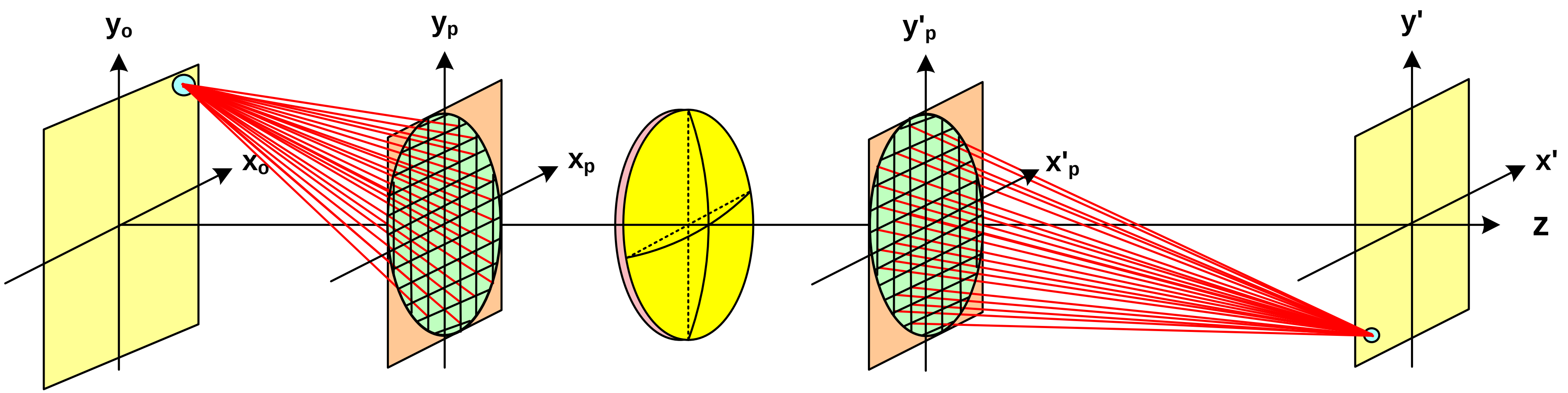
- Definition of optical path length in an optical system:
  Reference sphere around the ideal object point through the center of the pupil
- Chief ray serves as reference
  Difference of OPL : optical path difference OPD
- Practical calculation: discrete sampling of the pupil area,
  real wave surface represented as matrix



Pupil Sampling

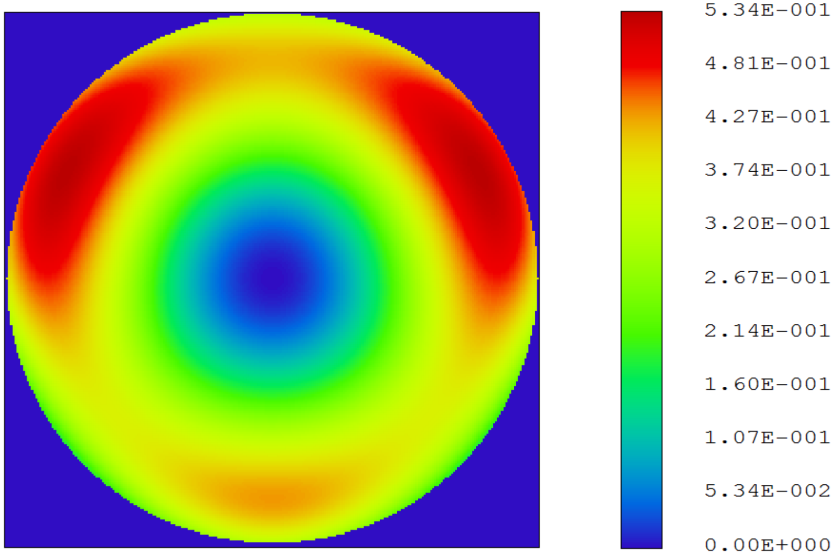
- All rays start in one point in the object plane
- The entrance pupil is sampled equidistant
- In the exit pupil, the transferred grid may be distorted
- In the image plane a spreaded spot diagram is generated

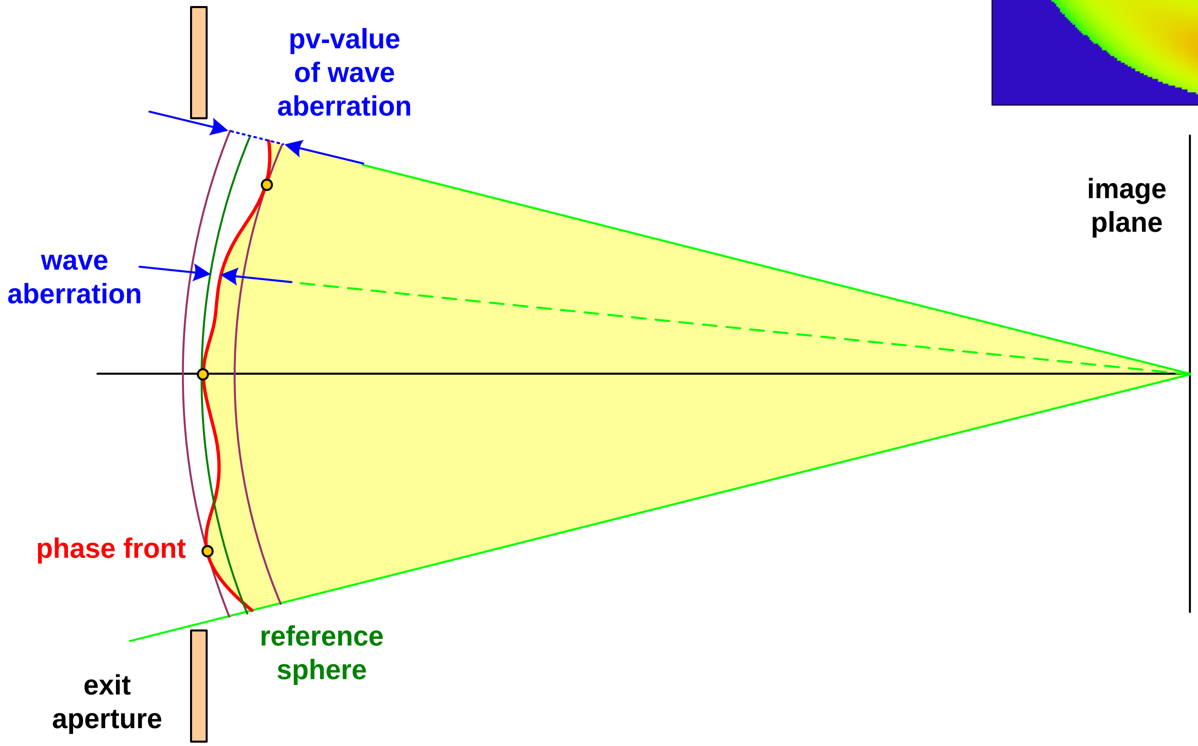




Wave Aberrations

- Quality assessment:
  - peak valley value (PV)
  - rms value, area average
  - Zernike decomposition for detailed analysis





Wave Aberrations – Sign and Reference

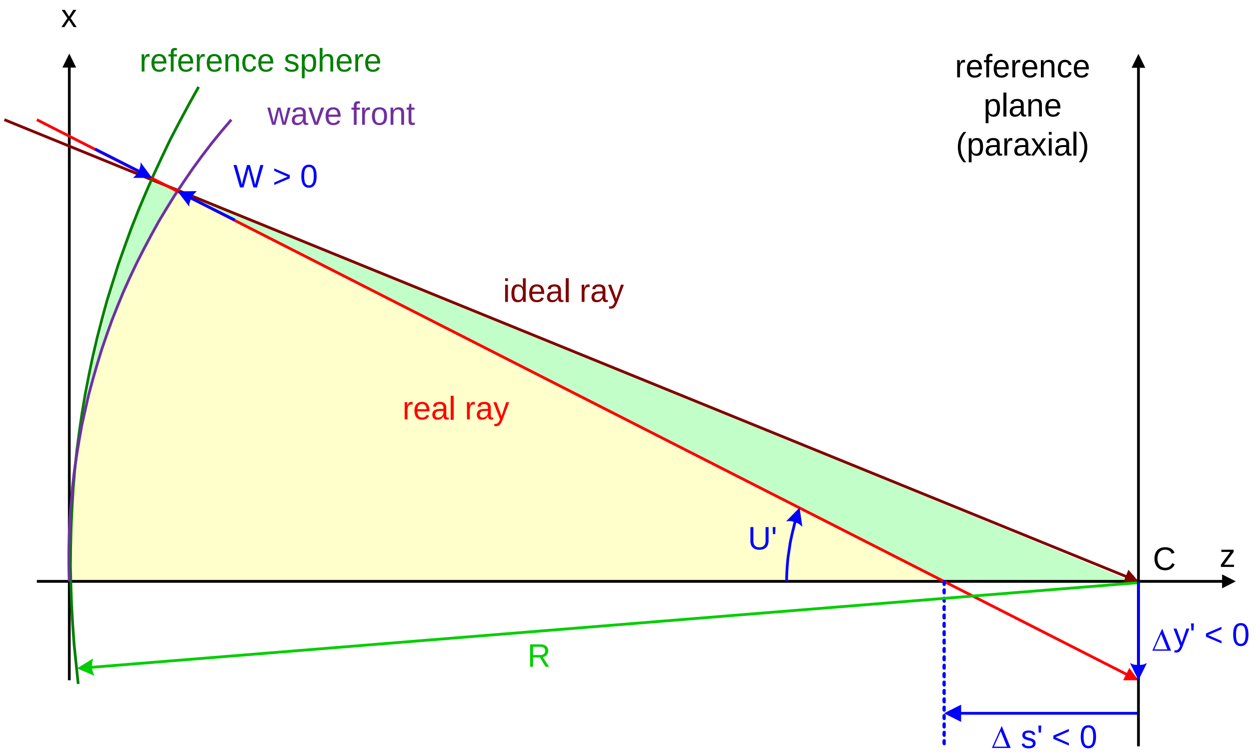
- Wave aberration: relative to reference sphere
- Choice of offset value: vanishing mean

$$\langle W(x,y)\rangle = \frac{1}{F_{ExP}} \iint W(x,y)\,dx\,dy = 0$$

- Sign of W :

  - W > 0 : stronger convergence intersection : s < 0

  - W < 0 : stronger divergence intersection : s < 0



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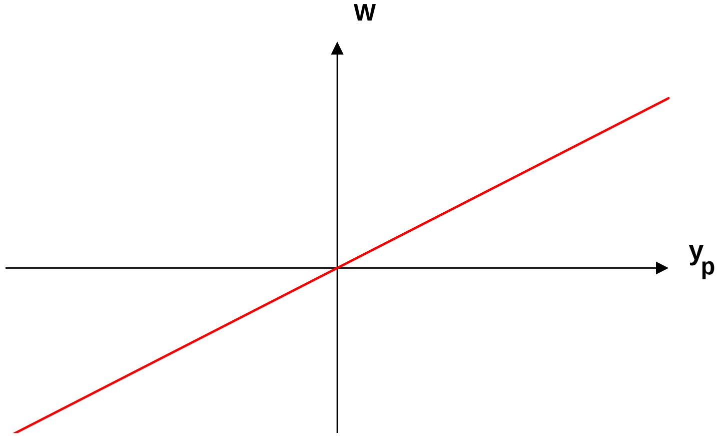
Tilt of Wavefront

- Change of reference sphere:
  tilt by angle θ
  linear in y_p

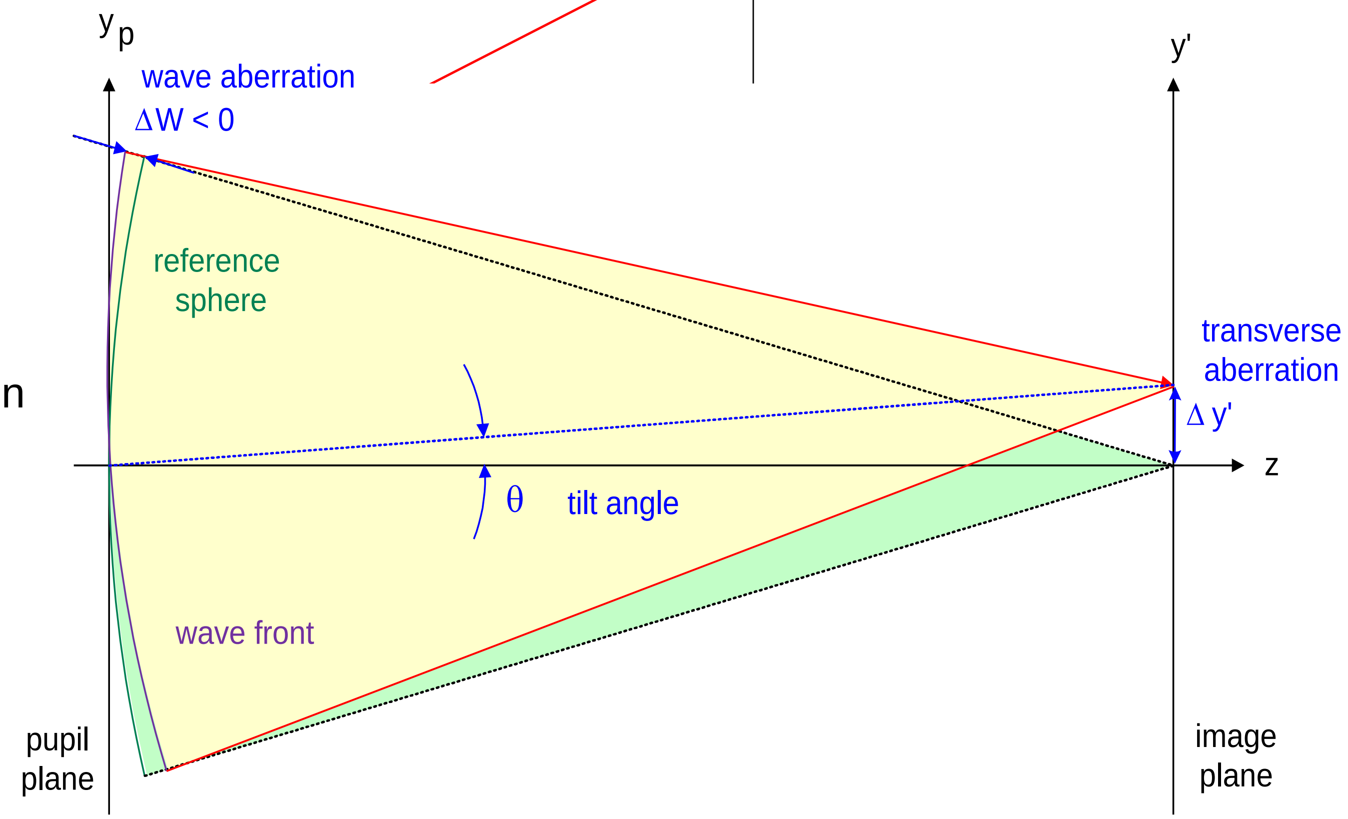
$$\Delta W_{tilt} = n \cdot y_p \cdot \theta$$



- Wave aberration
  due to transverse
  aberration $\Delta y'$

$$\Delta W_{tilt} = -\frac{y_p}{R_{Ref}} \cdot \Delta y'$$

- Is the usual description of distortion



Defocussing of Wavefront

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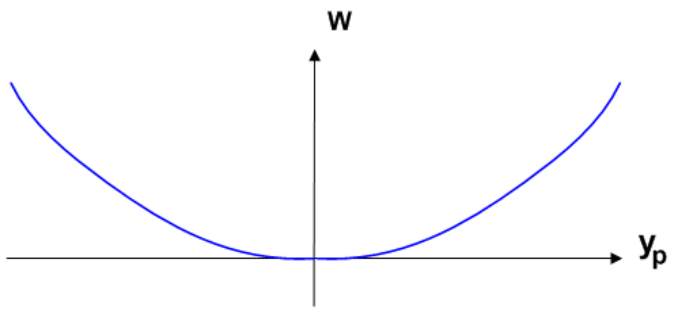
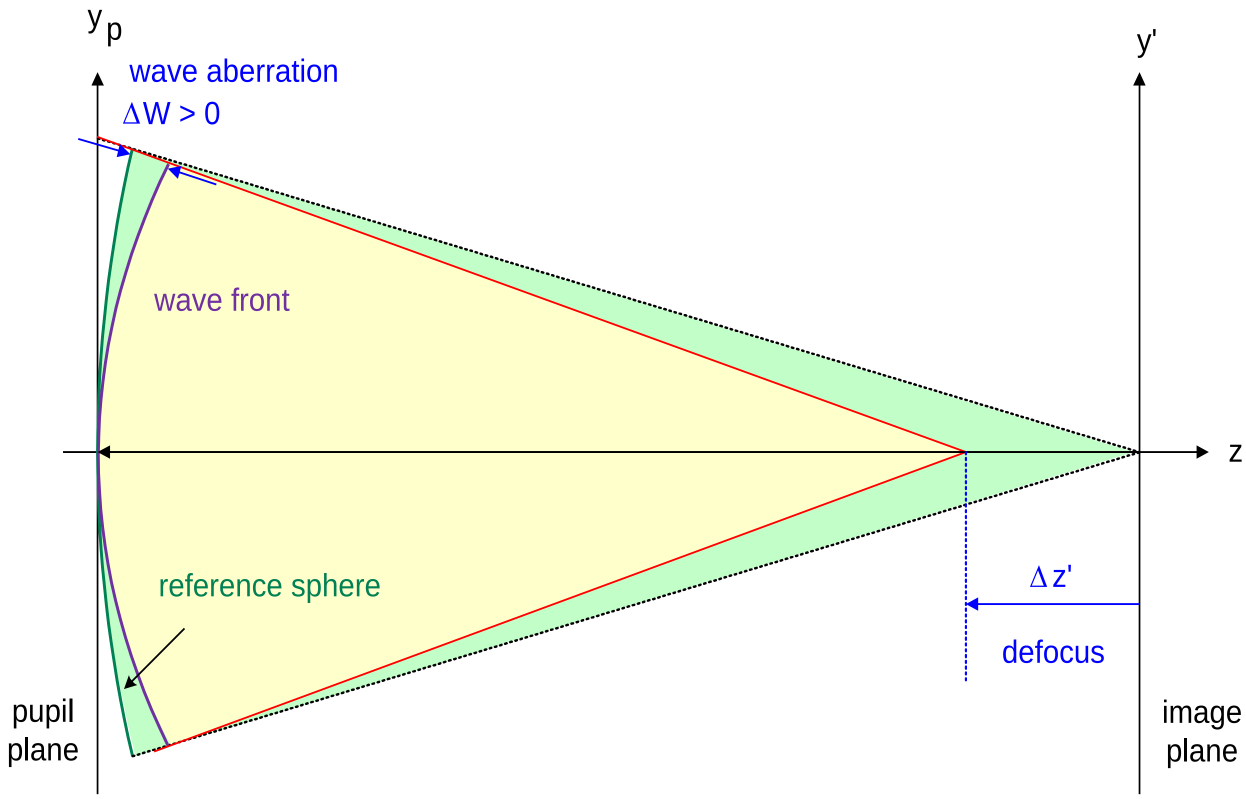
Paraxial defocussing by Δz:
Change of wavefront

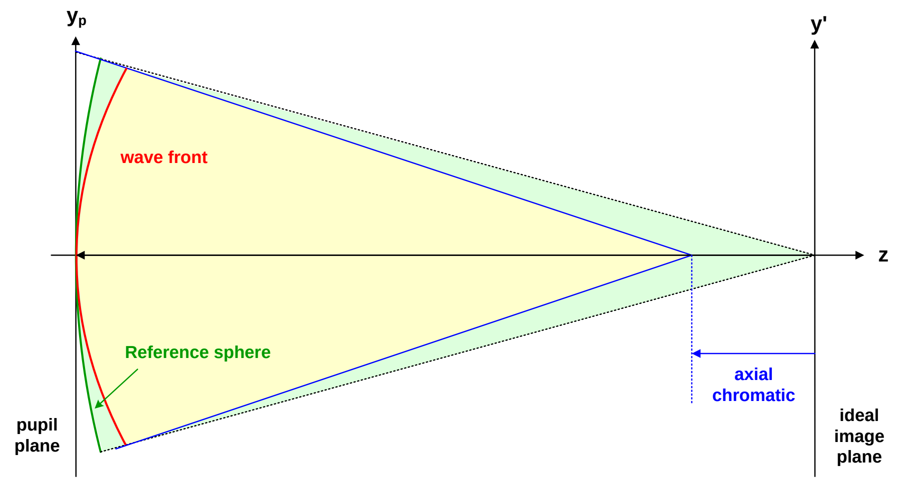
$$\Delta W_{Def} = -\frac{n \cdot r_p^2}{2R_{ref}^2} \cdot \Delta z' = -\frac{1}{2} n \cdot \Delta z' \cdot \sin^2 u$$



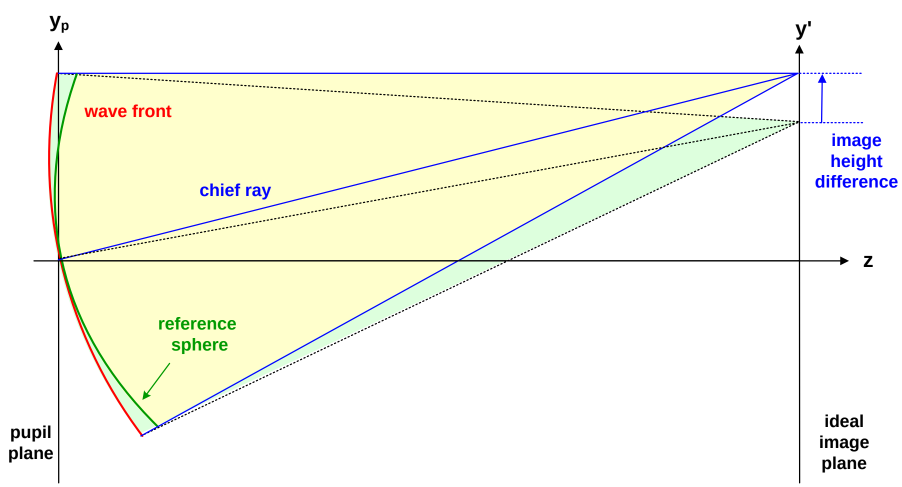


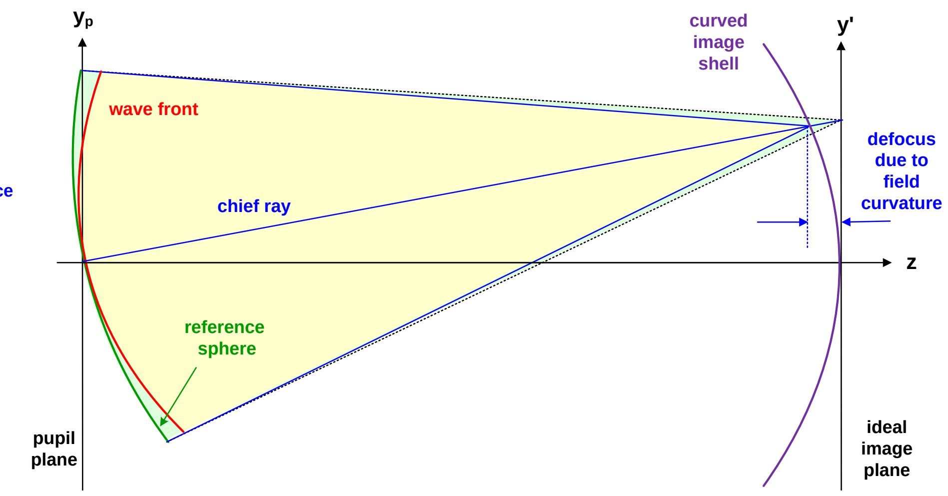
Special Cases of Wave Aberrations

- Wave aberrations are usually given as reduced aberrations:
  - wave front for only 1 field point
  - field dependence represented by discrete cases



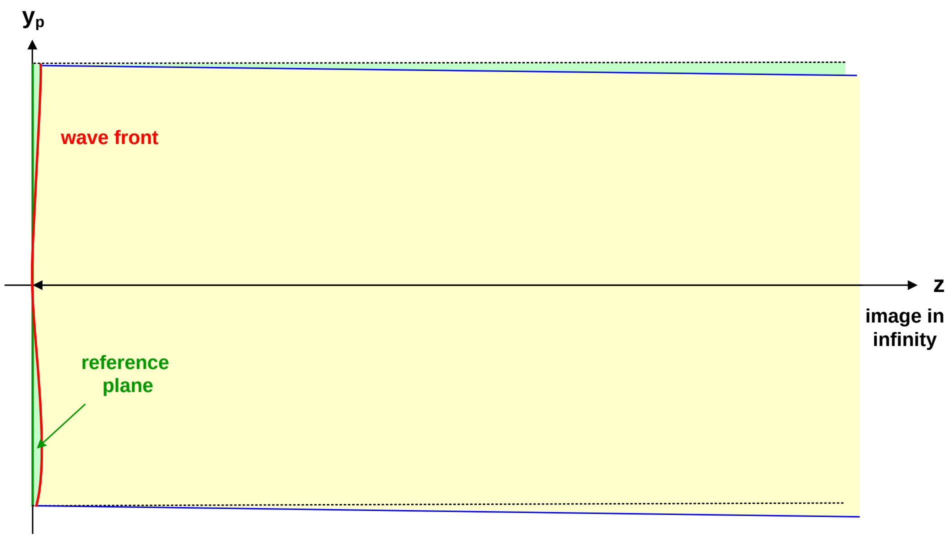
- Special case of aberrations:
  1. axial color and field curvature: represented as defocussing term, Zernike c_4
  2. distortion and lateral color: represented as tilt term, Zernike c_2, c_3



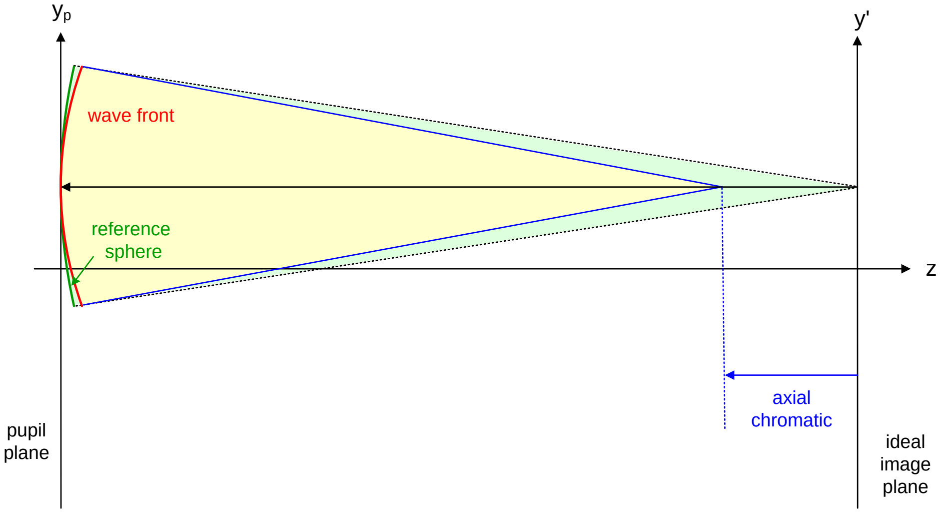


Special Cases of Wave Aberrations

3. afocal system
   - exit pupil in infinity
   - plane wave as reference



4. telecentric system
   chief ray parallel to axis



Expansion of the Wave Aberration

- Table as function of field and aperture
- Selection rules:
  checkerboard filling of the matrix

Image location

Primary aberrations / Seidel

Secondary aberrations

			Field y →					
			Spherical	Coma	Astigmatism			
			y^0	y^1	y^2	y^3	y^4	y^5
Aper-ture r ↓	Distortion	r^1		$y\,r\cos\theta$ Tilt		$y^3\,r\cos\theta$ Distortion primary		$y^5\,r\cos\theta$ Distortion secondary
		r^2	r^2 Defocus		$y^2\,r^2\cos^2\theta$ $y^2\,r^2$ Astig./Curvat.		$y^4\,r^2\cos^2\theta$ $y^4\,r^2$	
		r^3		$y\,r^3\cos\theta$ Coma primary		$y^3 r^3\cos^3\theta$ $y^3 r^3\cos\theta$		
		r^4	r^4 Spherical primary		$y^2\,r^4\cos^2\theta$ $y^2\,r^4$			
		r^5		$y\,r^5\cos\theta$ Coma secondary				
		r^6	r^6 Spherical secondary					

Polynomial Expansion of Wave Aberrations

- Taylor expansion of the wavefront:
  y' Image height — index k
  r_p Pupil height — index l
  θ Pupil azimuth angle — index m

$$W(y', r_p, \theta) = \sum_{k,l,m} W_{klm}\, y'^k\, r_p^l \cos^m\theta$$

- Symmetry invariance:
  1. Image height $\left|\vec{y}'\right|$
  2. Pupil height $\left|\vec{r}_p\right|$
  3. Scalar product between image and pupil vector $\vec{y}'\cdot\vec{r}_p = y'\cdot r_p \cdot \cos\theta$

- Number of terms
  sum of indices in the exponent i_{sum}

i_{sum}	N_i number of terms	Type of aberration
2	2	image location
4	5	primary aberrations, 4th order
6	9	secondary aberrations, 6th order
8	14	8th order

Taylor Expansion of the Primary Aberrations

- Expansion of the monochromatic aberrations
- First real aberration: primary aberrations, 4th order as wave deviation

$$W(y', r_p, y_p) = A_s r_p^4 + A_c y' r_p^2 y_p + A_a y'^2 y_p^2 + A_p y'^2 r_p^2 + A_d y'^3 y_p$$

- Coefficients of the primary aberrations:
  A_S : Spherical Aberration
  A_C : Coma
  A_A : Astigmatism
  A_P : Petzval curvature
  A_D : Distortion

- Alternatively: expansion in polar coordinates:
  Zernike basis expansion, usually only for one field point, orthogonalized

Zernike Polynomials

- Expansion of the wave aberration on a circular area

$$W(r,\varphi)=\sum_n \sum_{m=-n}^{n} c_{nm} Z_n^m(r,\varphi)$$

$$c_{nm}=\frac{2(n+1)}{\pi(1+\delta_{m0})}\cdot\int_0^1\int_0^{2\pi} W(r,\varphi) Z_n^{m*}(r,\varphi)\, d\varphi\, r\, dr$$

- Zernike polynomials in cylindrical coordinates:
  Radial function R(r), index n
  Azimuthal function φ, index m

$$Z_n^m(r,\varphi)=R_n^m(r)\cdot\begin{cases}\sin m\varphi & für\ \ m>0\\ \cos m\varphi & für\ \ m<0\\ 1 & für\ \ m=0\end{cases}$$

- Orthonormality

$$\int_0^1\int_0^{2\pi} Z_n^m(r,\varphi) Z_{n'}^{m'*}(r,\varphi)\, d\varphi\, r\, dr = \frac{\pi\cdot(1+\delta_{m0})}{2(n+1)}\cdot\delta_{nn'}\delta_{mm'}$$

- Advantages:
  1. Minimal properties due to W_{rms}
  2. Decoupling, fast computation
  3. Direct relation to primary aberrations for low orders

- Problems:
  1. Computation oin discrete grids
  2. Non circular pupils
  3. Different conventions concerning indeces, scaling, coordinate system

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Zernike Polynomials

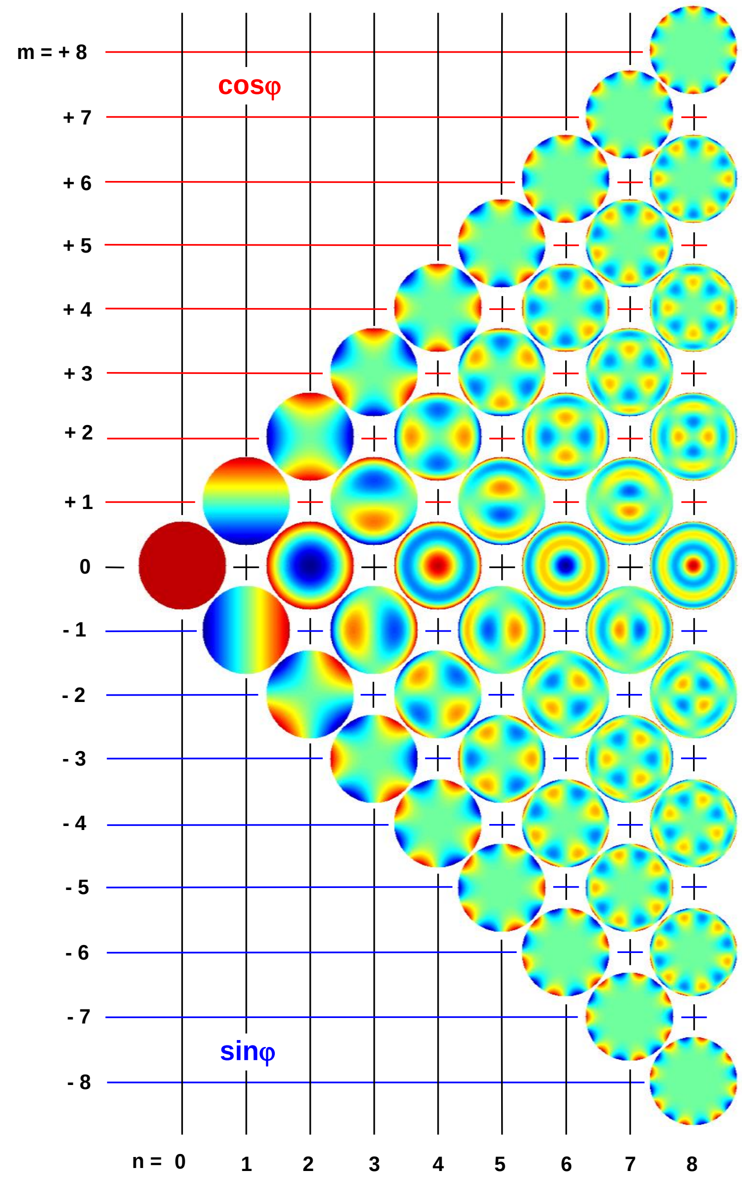
- Expansion of wave aberration surface into elementary functions / shapes

$$W(r,\varphi)=\sum_{n}\sum_{m=-n}^{n} c_{nm} Z_n^m(r,\varphi)$$

- Zernike functions are defined in circular coordinates r, φ

$$Z_n^m(r,\varphi)=R_n^m(r)\cdot\begin{cases}\sin(m\varphi) & for\ m<0\\ \cos(m\varphi) & for\ m>0\\ 1 & for\ m=0\end{cases}$$

- Ordering of the Zernike polynomials by indices:
  n : radial
  m : azimuthal, sin/cos
- Mathematically orthonormal function on unit circle for a constant weighting function
- Direct relation to primary aberration types



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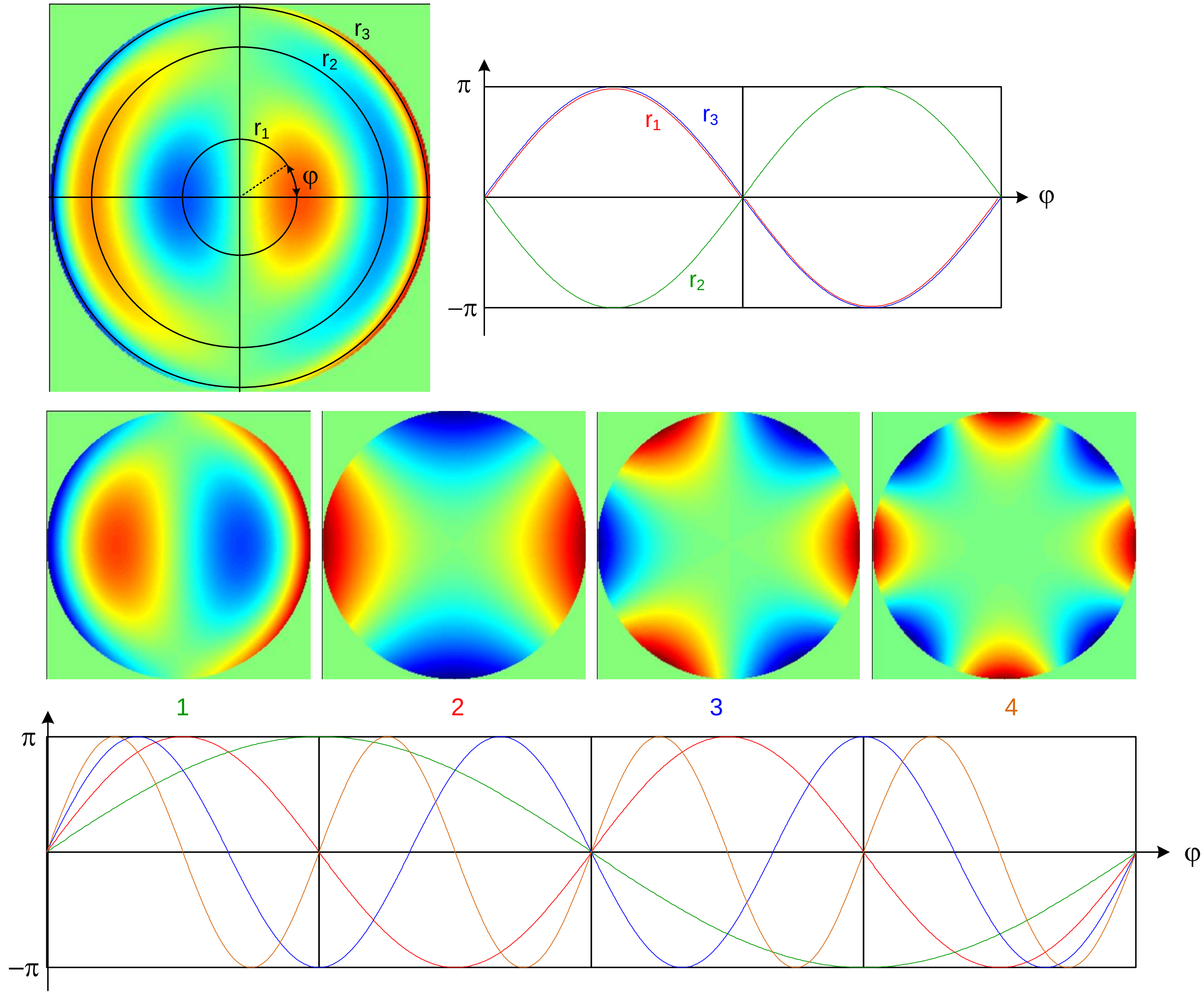
Azimuthal Dependence of Zernike Polynomials

- Azimuthal spatial frequency



Zernike Polynomials: Meaning of Lower Orders

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n	m	Polar coordinates	Cartesian coordinates	Interpretation
0	0	1	1	piston
1	-1	$r\sin\varphi$	x	tilt in x
1	-1	$r\cos\varphi$	y	tilt in y
2	-2	$r^2\sin 2\varphi$	$2xy$	Astigmatism 45°
2	0	$2r^2-1$	$2x^2+2y^2-1$	defocussing
2	2	$r^2\cos 2\varphi$	y^2-x^2	Astigmatism 0°
3	-3	$r^3\sin 3\varphi$	$3xy^2-x^3$	trefoil 30°
3	-1	$\left(3r^3-2r\right)\sin\varphi$	$3x^3-2x+3xy^2$	coma x
3	-1	$\left(3r^3-2r\right)\cos\varphi$	$3y^3-2y+3x^2y$	coma y
3	3	$r^3\cos 3\varphi$	y^3-3x^2y	trefoil 0°
4	-4	$r^4\sin 4\varphi$	$4xy^3-4x^3y$	Four sheet 22.5°
4	-2	$\left(4r^4-3r^2\right)\sin 2\varphi$	$8xy^3+8x^3y-6xy$	Secondary astigmatism
4	0	$6r^4-6r^2+1$	$6x^4+6y^4+12x^2y^2-6x^2-6y^2+1$	Spherical aberration
4	2	$\left(4r^4-3r^2\right)\cos 2\varphi$	$4y^4-4x^4+3x^2-3y^2-4x^2y^2$	Secondary astigmatism
4	4	$r^4\cos 4\varphi$	$y^4+x^4-6x^2y^2$	Four sheet 0°

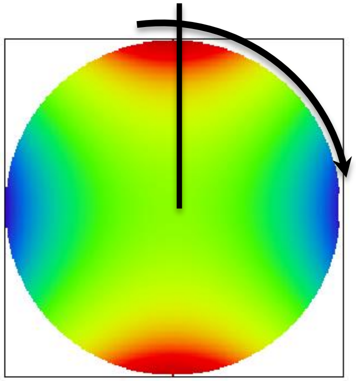
Zernike Fringe vs Zernike Standard Polynomials

ZERNIKE STANDARD POLYNOMIALS

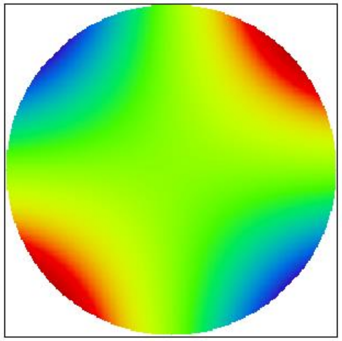
Term	$Z(\rho, \varphi)$
1	1
2	$\sqrt{4}\rho\cos\varphi$
3	$\sqrt{4}\rho\sin\varphi$
4	$\sqrt{3}(2\rho^2-1)$
5	$\sqrt{6}(\rho^2\sin 2\varphi)$
6	$\sqrt{6}(\rho^2\cos 2\varphi)$
7	$\sqrt{8}(3\rho^3-2\rho)\sin\varphi$
8	$\sqrt{8}(3\rho^3-2\rho)\cos\varphi$
9	$\sqrt{8}\rho^3\sin 3\varphi$
10	$\sqrt{8}\rho^3\cos 3\varphi$
11	$\sqrt{5}(6\rho^4-6\rho^2+1)$
12	$\sqrt{10}(4\rho^4-3\rho^2)\cos 2\varphi$
13	$\sqrt{10}(4\rho^4-3\rho^2)\sin 2\varphi$

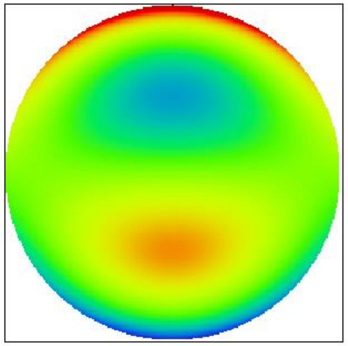
ZERNIKE FRINGE POLYNOMIALS

Term	$Z(\rho, \varphi)$
1	1
2	$\rho\cos\varphi$
3	$\rho\sin\varphi$
4	$2\rho^2-1$
5	$\rho^2\cos 2\varphi$
6	$\rho^2\sin 2\varphi$
7	$(3\rho^2-2)\rho\cos\varphi$
8	$(3\rho^2-2)\rho\sin\varphi$
9	$6\rho^4-6\rho^2+1$
10	$\rho^3\cos 3\varphi$
11	$\rho^3\sin 3\varphi$
12	$(4\rho^2-3)\rho^2\cos 2\varphi$
13	$(4\rho^2-3)\rho^2\sin 2\varphi$



In radial symmetric system for y-field (meridional) sinus-terms vanish





Fringe coefficients:

- Z 4,9,16 = n² spherical
- azimutal order ϕ grows if number increases

Standard coefficients - different term numbers

- $Z_N = 0.05$

→ RMS = 0.05λ

→ SR $\sim 1 - 40\, Z_N^2 = 0.9$

Elimination of tilt

No Elimination of defocus @ Zemax

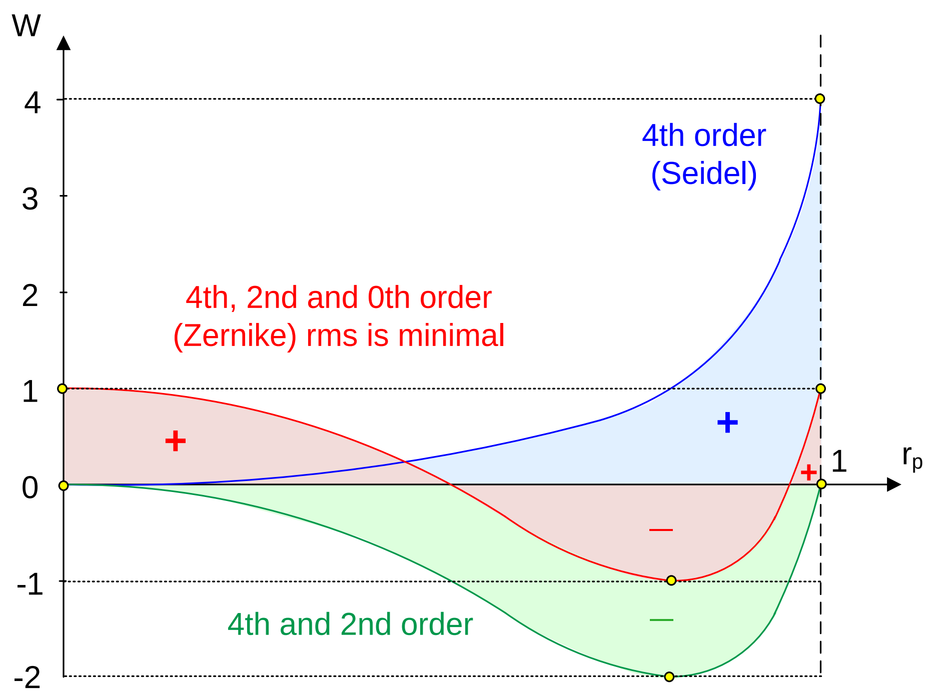
Balance of Lower Orders by Zernike Polynomials

- Mixing of lower orders to get the minimal W_{rms}

- Example spherical aberration:
  1. Spherical 4th order according to Seidel
  2. Additional quadratic expression: Optimal defocussing for edge correction
  3. Additional absolute term Minimale value of W_{rms}

- Special case of coma: Balancing by tilt contribution, corresponds to shift between peak and centroid

$$W(r_p) = 6r_p^4 - 6r_p^2 + 1$$



Zernike Polynomials

- Advantages of the Zernike polynomials:
  - de-coupling due to orthogonality
  - stable numerical computation
  - direct relation of lower orders to classical aberrations
  - optimale balancing of lower orders (e.g. best defocus for spherical aberration)
  - fast calculation of W_{rms} and Strehl ratio in approximation of Marechal

- Necessary requirements for orthogonality:
  - pupil shape circular
  - uniform illumination of pupil (corresponds to constant weighting)
  - no discretization effects (finite number of points, boundary)

- Different standardizations used concerning indexing, scaling, sign of coordinates (orientation for off-axis field points):
  - Fringe representation: peak value 1, normalized
  - Standard representation W_{rms} normalized
  - Original representation according to Nijbor-Zernike

  Norm ISO 10110 allows Fringe and Standard representation

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Calculation of Zernike Polynomials

- Assumptions:
  1. Pupil circular
  2. Illumination homogeneous
  3. Neglectible discretization effects /sampling, boundary)

- Direct computation by double integral:
  1. Time consuming
  2. Errors due to discrete boundary shape
  3. Wrong for non circular areas
  4. Independent coefficients

$$c_j = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} W(r,\varphi) Z_j^*(r,\varphi)\, d\varphi\, r dr$$

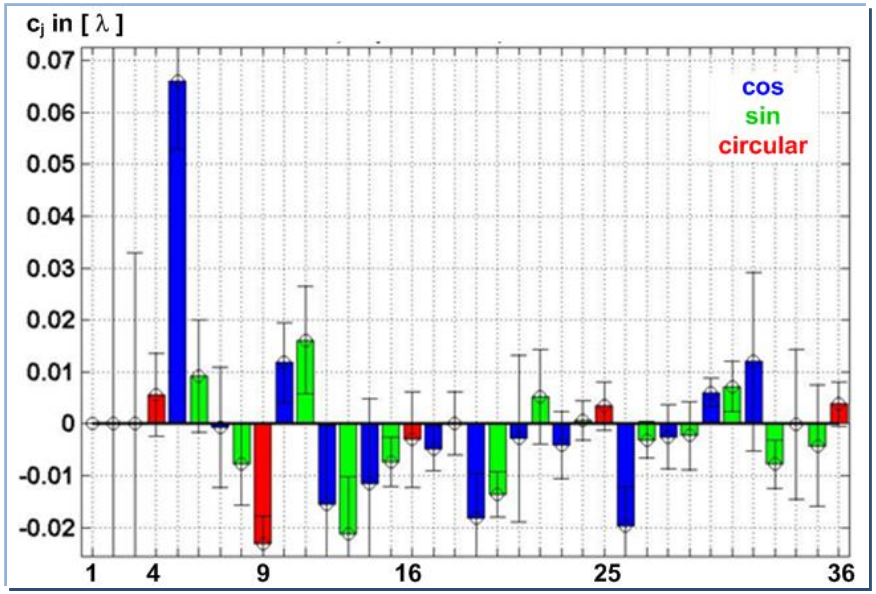
- LSQ-fit computation:
  1. Fast, all coefficients c_j simultaneously
  2. Better total approximation
  3. Non stable for different numbers of coefficients, if number too low
- Stable for non circular shape of pupil

$$\sum_{i=1} \left[W_i - \sum_{j=1}^{N} c_j Z_j(r_i) \right]^2 = \min$$

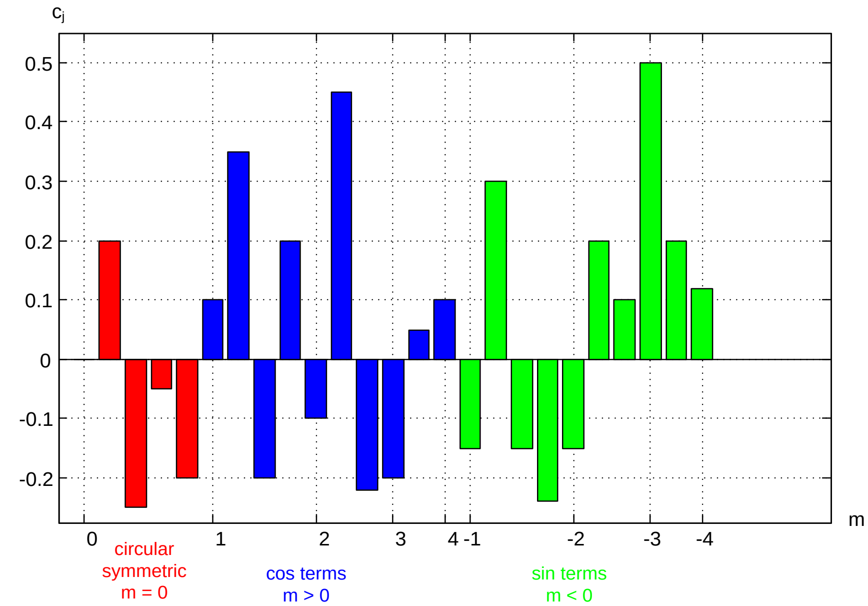
$$\vec{c} = \left(\underline{Z}^T \underline{Z} \right)^{-1} \underline{Z}^T \vec{W}$$

Performance Description by Zernike Expansion

- Vector of c_j
  linear sequence with runnin g index



- Sorting by symmetry



Zernikepolynomials: Different Conventions

- Different standardizations used concerning:
  1. indexing
  2. scaling / normalization
  3. sign of coordinates (orientation for off-axis field points)

- Fringe - representation
  1. CodeV, Zemax, interferometric test of surfaces
  2. Standardization of the boundary to ± 1
  3. no additional prefactors in the polynomial
  4. Indexing according to m (Azimuth), quadratic number terms have circular symmetry
  5. coordinate system invariant in azimuth

- Standard - representation
  - CodeV, Zemax, Born / Wolf
  - Standardization of rms-value on ± 1 (with prefactors), easy to calculate Strehl ratio
  - coordinate system invariant in azimuth

- Original - Nijboer - representation
  - Expansion:

$$W(r,\varphi) = a_{00} + \frac{1}{\sqrt{2}} \sum_{n=0}^{k} a_{0n} R_n^0 + \sum_{n=0}^{k} \sum_{\substack{m=1 \\ n-m \\ gerade}}^{n} a_{nm} R_n^m \cos(m\varphi) + \sum_{n=0}^{k} \sum_{\substack{m=1 \\ n-m \\ gerade}}^{n} b_{nm} R_n^m \sin(m\varphi)$$

  - Standardization of rms-value on ± 1
  - coordinate system rotates in azimuth according to field point

Wave Aberration Criteria

- Mean quadratic wave deviation (W_{Rms} , root mean square)

$$W_{rms} = \sqrt{\langle W^2 \rangle - \langle W \rangle^2} = \sqrt{\frac{1}{A_{ExP}} \iint \left[W\left(x_p, y_p\right) - W_{mean}\left(x_p, y_p\right) \right]^2 dx_p dy_p}$$

with pupil area

$$A_{ExP} = \iint dxdy$$

- Peak valley value W_{pv} : largest difference

$$W_{pv} = \max\left[W_{\max}\left(x_p, y_p\right) - W_{\min}\left(x_p, y_p\right) \right]$$

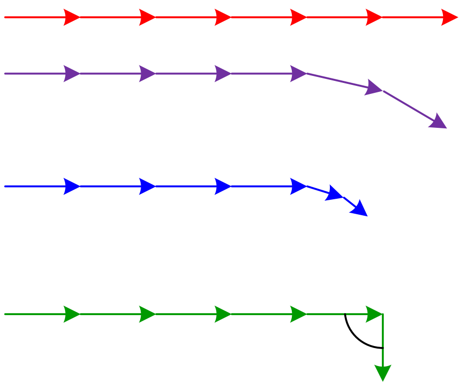
- General case with apodization:
  weighting of local phase errors with intensity, relevance for psf formation

$$W_{rms} = \sqrt{\frac{1}{A_{ExP}^{(w)}} \iint I_{ExP}\left(x_p, y_p\right) \cdot \left[W\left(x_p, y_p\right) - W_{mean}^{(w)}\left(x_p, y_p\right) \right]^2 dx_p dy_p}$$

Rayleigh Criterion

- The Rayleigh criterion $\left|W_{PV}\right| \leq \frac{\lambda}{4}$

  gives individual maximum aberrations coefficients,
  depends on the form of the wave



a) optimal constructive interference

b) reduced constructive interference due to phase aberrations

c) reduced effect of phase error by apodization and lower energetic weighting

d) start of destructive interference for 90° or $\lambda/4$ phase aberration begin of negative z-component

- Examples:

aberration type		coefficient
defocus	Seidel	$a_{20} = 0.25$
defocus	Zernike	$c_{20} = 0.125$
spherical aberration	Seidel	$a_{40} = 0.25$
spherical aberration	Zernike	$c_{40} = 0.167$
astigmatism	Seidel	$a_{22} = 0.25$
astigmatism	Zernike	$c_{22} = 0.125$
coma	Seidel	$a_{31} = 0.125$
coma	Zernike	$c_{31} = 0.125$

Criteria of Rayleigh and Marechal

- Rayleigh criterion:
  1. maximum of wave aberration: $W_{pv} < \lambda/4$
  2. beginning of destructive interference of partial waves
  3. limit for being diffraction limited (definition)
  4. as a PV-criterion rather conservative: maximum value only in 1 point of the pupil
  5. different limiting values for aberration shapes and definitions (Seidel, Zernike,...)

- Marechal criterion:
  1. Rayleigh crierion corresponds to $W_{rms} < \lambda/14$ in case of defocus

$$W_{rms}^{Rayleigh} \leq \frac{\lambda}{\sqrt{192}} = \frac{\lambda}{13.856} \approx \frac{\lambda}{14}$$

  2. generalization of $W_{rms} < \lambda/14$ for all shapes of wave fronts
  3. corresponds to Strehl ratio $D_s > 0.80$ (in case of defocus)
  4. more useful as PV-criterion of Rayleigh

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PV and W_{rms}-Values

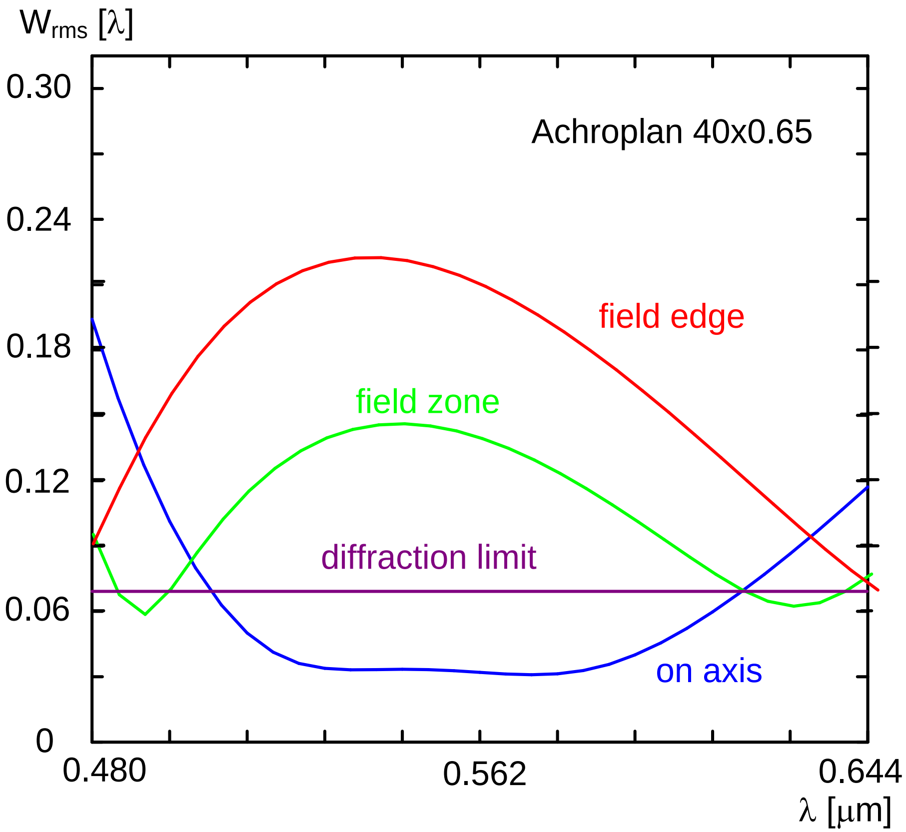
- PV and W_{rms} values for different definitions and shapes of the aberrated wavefront

- Due to mixing of lower orders in the definition of the Zernikes, the W_{rms} usually is smaller in comparison to the corresponding Seidel definition

aberration type		definition	mean W_{mean}	peak-valley W_{PV}	root mean square W_{rms}
defocus	Seidel	$a_{20} \cdot r_p^2$	$\frac{a_{20}}{2}$	a_{20}	$\frac{a_{20}}{2\sqrt{3}} = 0.289 \cdot a_{20}$
defocus	Zernike	$c_{20} \cdot \left(2r_p^2 - 1\right)$	0	$2c_{20}$	$\frac{c_{20}}{\sqrt{3}} = 0.577 \cdot c_{20}$
spherical aberration	Seidel	$a_{40} \cdot r_p^4$	$\frac{a_{40}}{3}$	a_{40}	$\frac{2a_{40}}{3\sqrt{5}} = 0.298 \cdot a_{40}$
spherical aberration with defocus		$b_{40} \cdot \left(r_p^4 - r_p^2\right)$	$-\frac{b_{40}}{6}$	$\frac{b_{40}}{4}$	$\frac{b_{40}}{6\sqrt{5}} = 0.075 \cdot b_{40}$
spherical aberration	Zernike	$c_{40} \cdot \left(6r_p^4 - 6r_p^2 + 1\right)$	0	$\frac{3c_{40}}{2}$	$\frac{c_{40}}{\sqrt{5}} = 0.447 \cdot c_{40}$
astigmatism	Seidel	$a_{22} r_p^2 \cos^2\theta$	$\frac{a_{22}}{4}$	a_{22}	$\frac{a_{22}}{4} = 0.25 \cdot a_{22}$
astigmatism with defocus		$b_{22}\left(r_p^2 \cos^2\theta - \frac{1}{2} r_p^2\right)$	0	b_{22}	$\frac{b_{22}}{2\sqrt{6}} = 0.204 \cdot b_{22}$
astigmatism	Zernike	$c_{22}\left(2r_p^2 \cos^2\theta - r_p^2\right)$	0	$2c_{22}$	$\frac{c_{22}}{\sqrt{6}} = 0.408 \cdot c_{22}$
coma	Seidel	$a_{31} r_p^3 \cos\theta$	0	$2a_{31}$	$\frac{a_{31}}{2\sqrt{2}} = 0.353 \cdot a_{31}$
coma with tilt		$b_{31}\left(r_p^3 - \frac{2}{3} r_p\right)\cos\theta$	0	$\frac{2b_{31}}{3}$	$\frac{b_{31}}{6\sqrt{2}} = 0.118 \cdot b_{31}$
coma	Zernike	$c_{31}\left(3r_p^3 - 2r_p\right)\cos\theta$	0	$2c_{31}$	$\frac{c_{31}}{2\sqrt{2}} = 0.353 \cdot c_{31}$

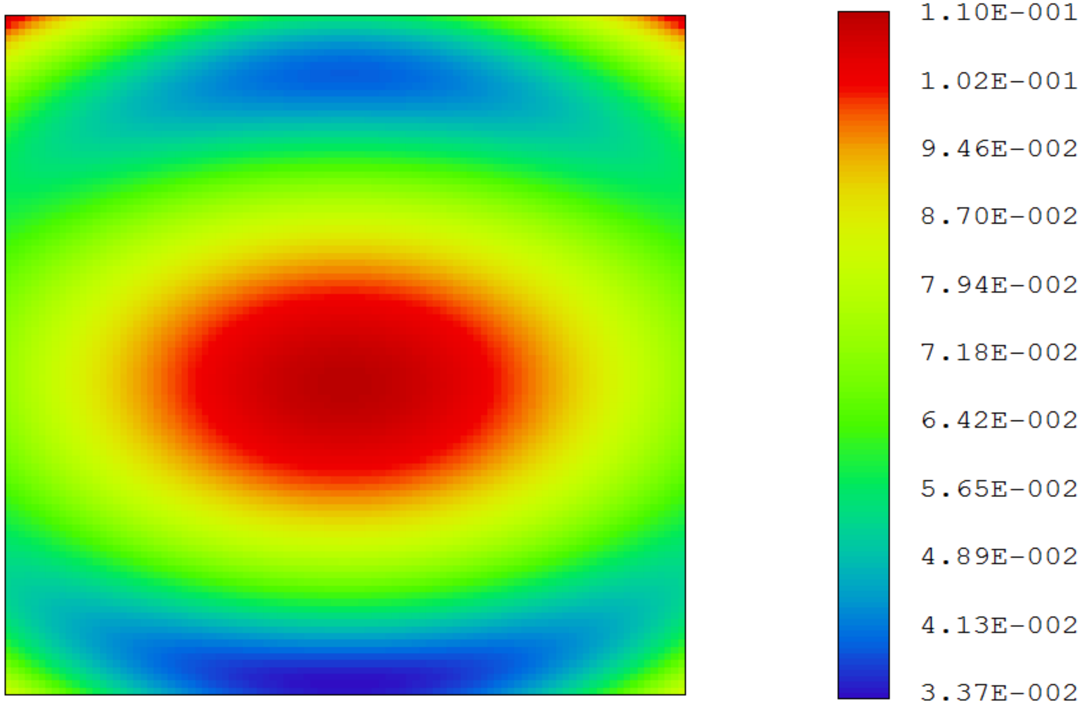
Typical Variation of Wave Aberrations

- Microscopic objective lens
- Changes of rms value of wave aberration with
  1. wavelength
  2. field position
- Common practice:
  1. diffraction limited on axis for main part of the spectrum
  2. Requirements relaxed in the outer field region
  3. Requirement relaxed at the blue edge of the spectrum



Rms of Wavefront as Function of the Field

- Rms of the wavefront changes with field position scaled in λ
- Only one scalar number per fiel point:
  fine structure suppressed



Zernike Calculation on distorted grids

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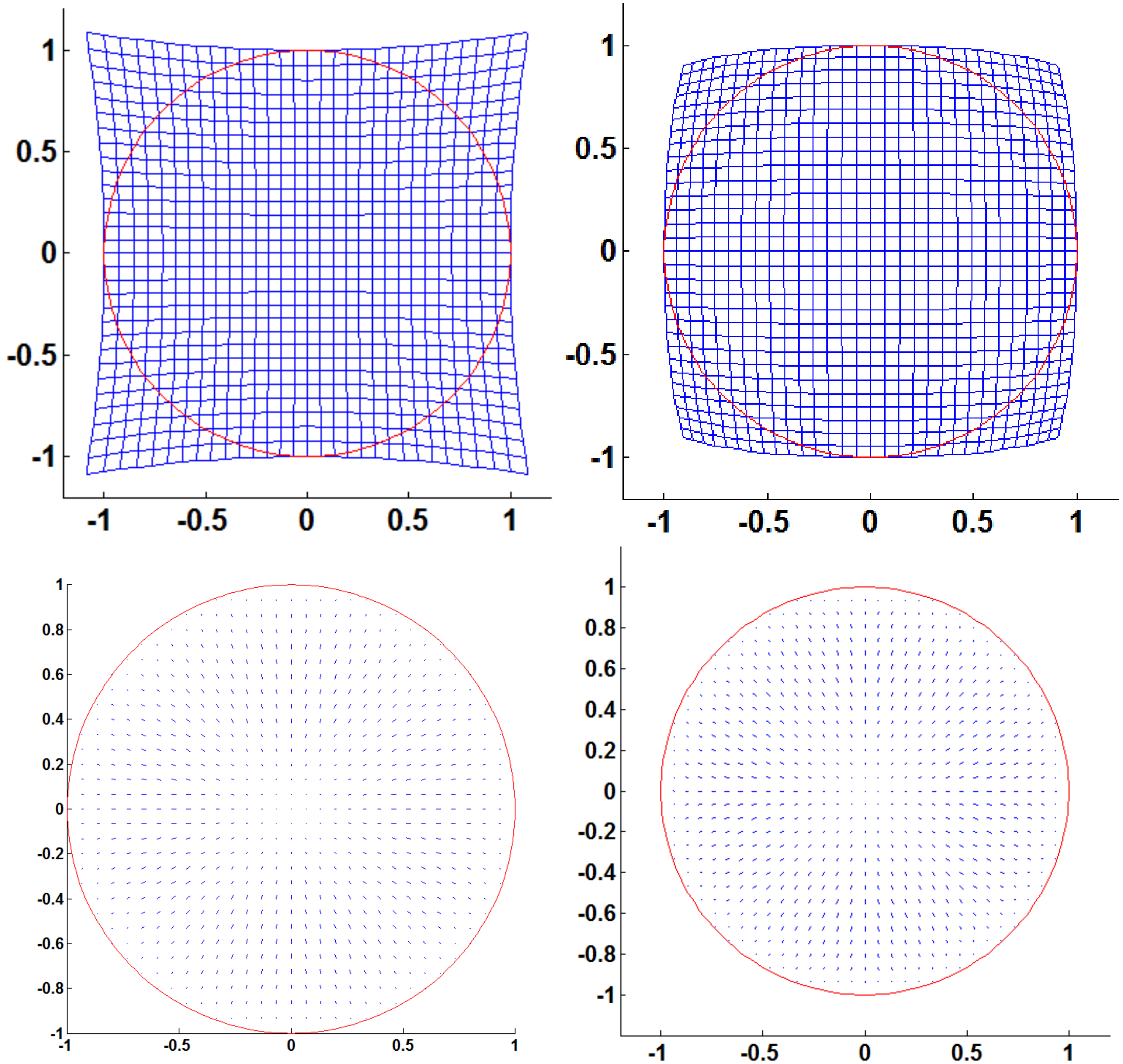
- Conventional calculation of the Zernikes:
  equidistant grid in the entrance pupil
- Real systems:
  Pupil aberrations and distorted grid in the exit pupil
- Deviating positions of phase gives errors in the Zernike calculation
- Additional effect:
  re-normalization of maximum radius



Zernike Calculation on distorted grids

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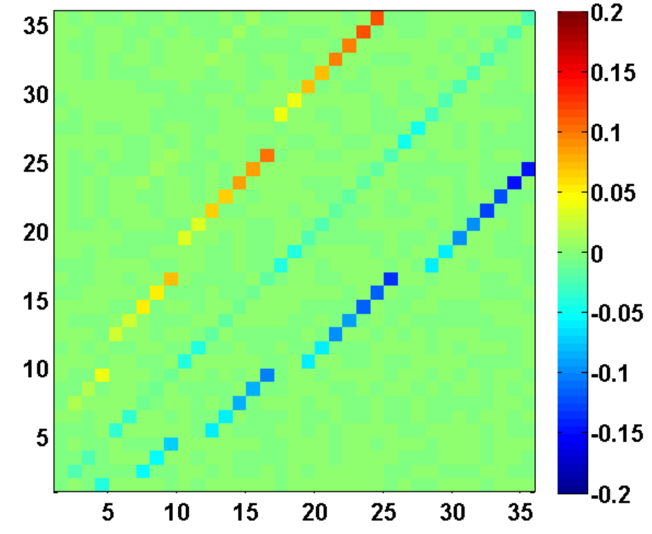
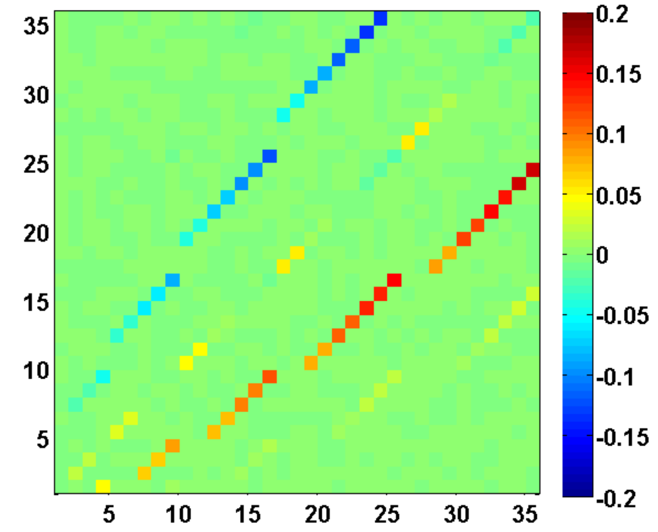
- 5% radial distorted grid with both signs:
  +5%: pincushion
  -5% barrel
- Wavefront of only one selected Zernike is re-analyzed on distorted re-normalized grid:
- Effects of distortion:
  - errors in the range of some percent
  - cross-mixing to other zernikes of same symmetry
  - neighboring Zernikes partly influenced until 20%
  - larger effects of higher order Zernikes
  - slightly larger effects for pincushion distortion





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Zernike Coefficients for Change of Radius

- Change of normalization radius,
  Problem, if pupil edge is not well known or badly defined

- Deviation in the radius of normalization of the pupil size:
  1. wrong coefficients
  2. mixing of lower orders during fit-calculation, symmetry-dependent

- Example primary spherical aberration:
  polynomial:

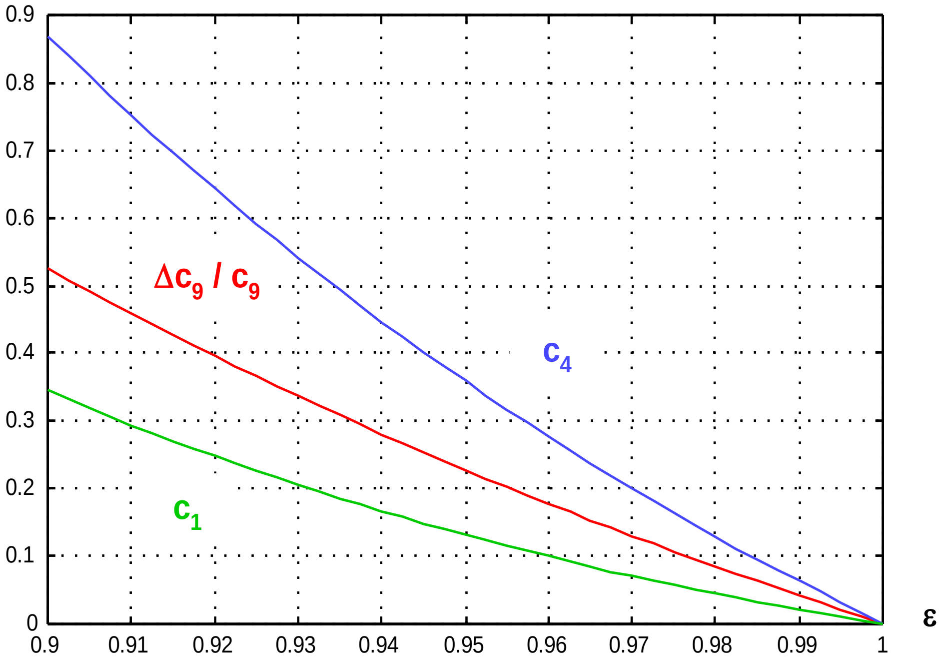
$$Z_9(\rho) = 6\rho^4 - 6\rho^2 + 1$$

Stretching factor ε of the radius

$$r = \varepsilon \cdot \rho$$

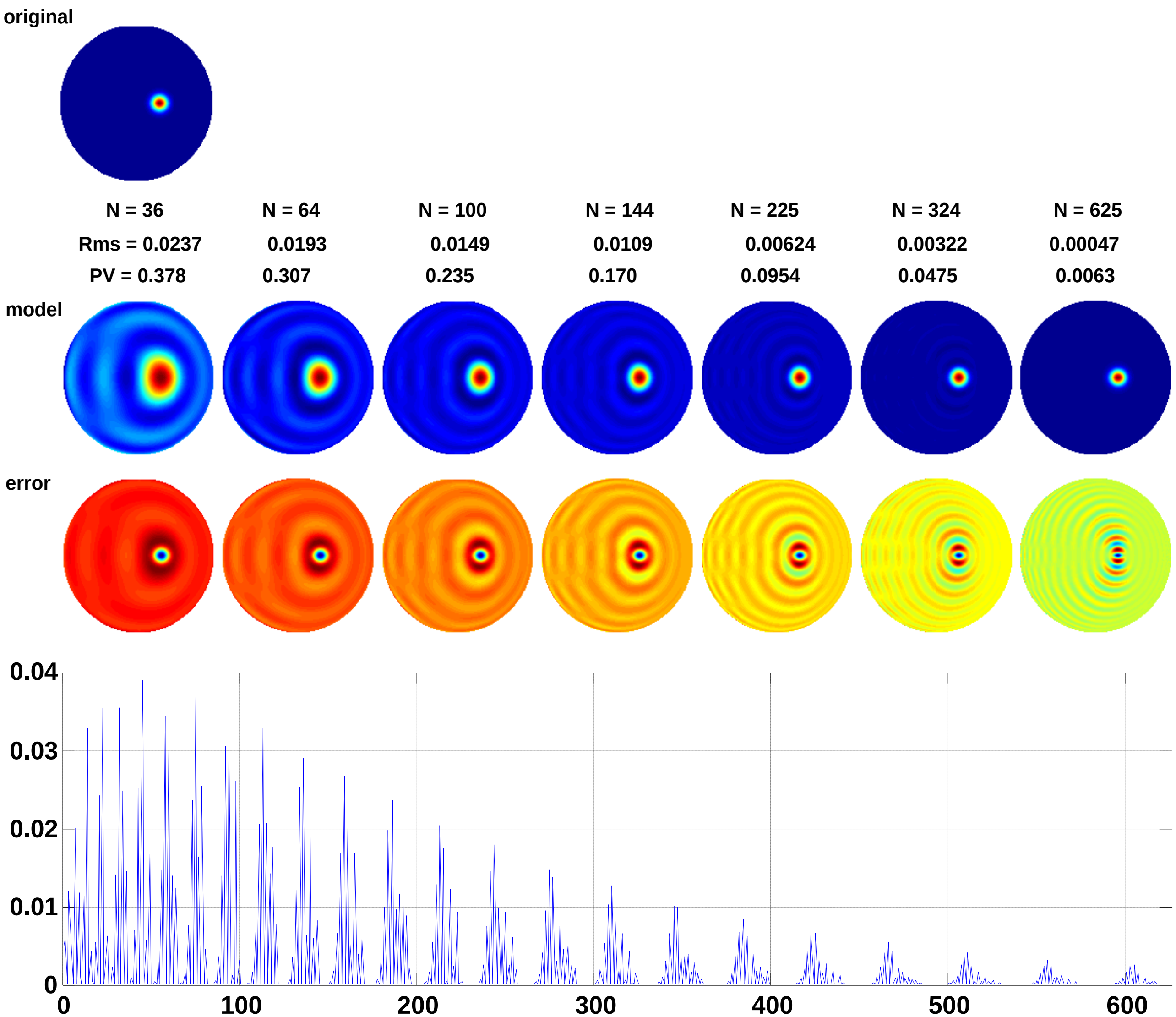
New Zernike expansion on basis of r

$$Z_9(\rho) = \frac{1}{\varepsilon^4} \cdot \overline{Z}_9(r) + \frac{3(1-\varepsilon^2)}{\varepsilon^4} \cdot \overline{Z}_4(r) + \frac{\varepsilon^4 - 3\varepsilon^2 + 2}{\varepsilon^4} \cdot \overline{Z}_1$$



Zernike Expansion of Local Deviations

Small Gaussian bump in the topology of a surface



Spectrum of coefficients for the last case

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Measurement of Wave Aberrations

- Wave aberrations are measurable directly

- Good connection between simulation/optical design and realization/metrology

- Direct phase measuring techniques:
  1. Interferometry
  2. Hartmann-Shack
  3. Hartmann sensor
  4. Special: Moire, Holography, phase-space analyzer

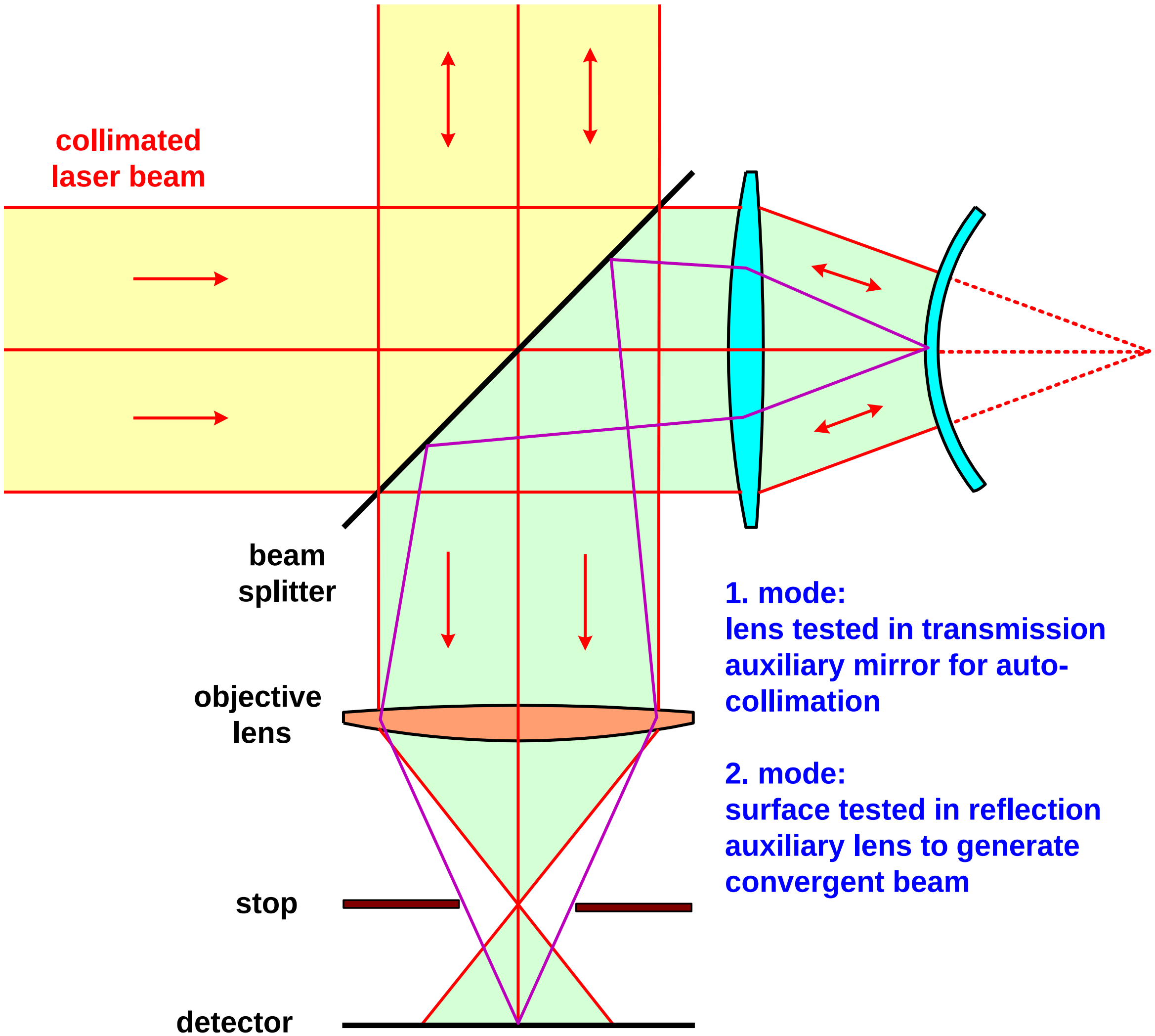
- Indirect measurement by inversion of the wave equation:
  1. Phase retrieval of PSF z-stack
  2. Retrieval of edge or line images

- Indirect measurement by analyzing the imaging conditions:
  from general image degradation

- Accuracy:
  1. $\lambda/1000$ possible, $\lambda/100$ standard for rms-value
  2. Rms vs. individual Zernike coefficients

Testing with Twyman-Green Interferometer

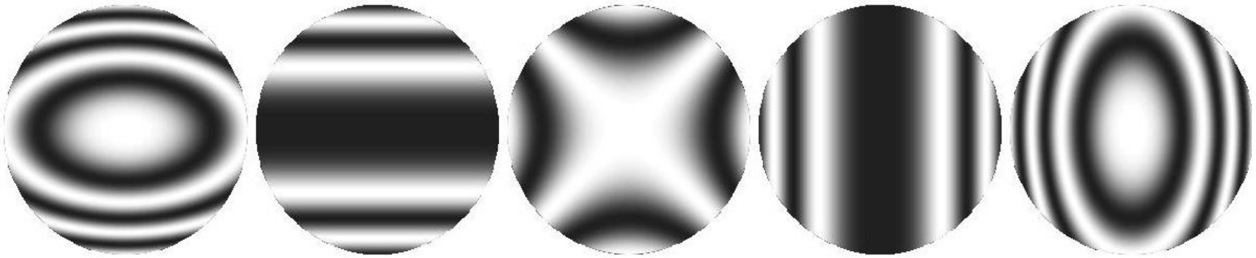
- Short common path, sensible setup
- Two different operation modes for reflection or transmission
- Always factor of 2 between detected wave and component under test



Interferograms of Primary Aberrations

Spherical aberration 1 λ

Astigmatism 1 λ



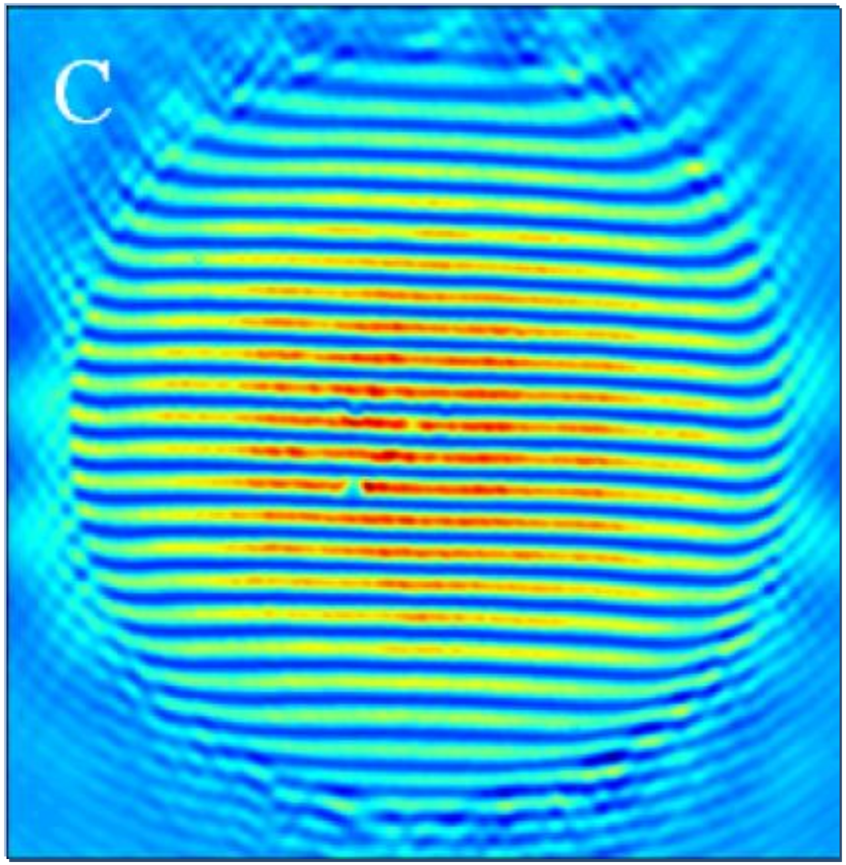
Coma 1 λ

-1	-0.5	0	+0.5	+1

Defocussing in λ

Interferogram - Definition of Boundary

- Critical definition of the interferogram boundary and the Zernike normalization radius in reality



Hartmann Shack Wavefront Sensor

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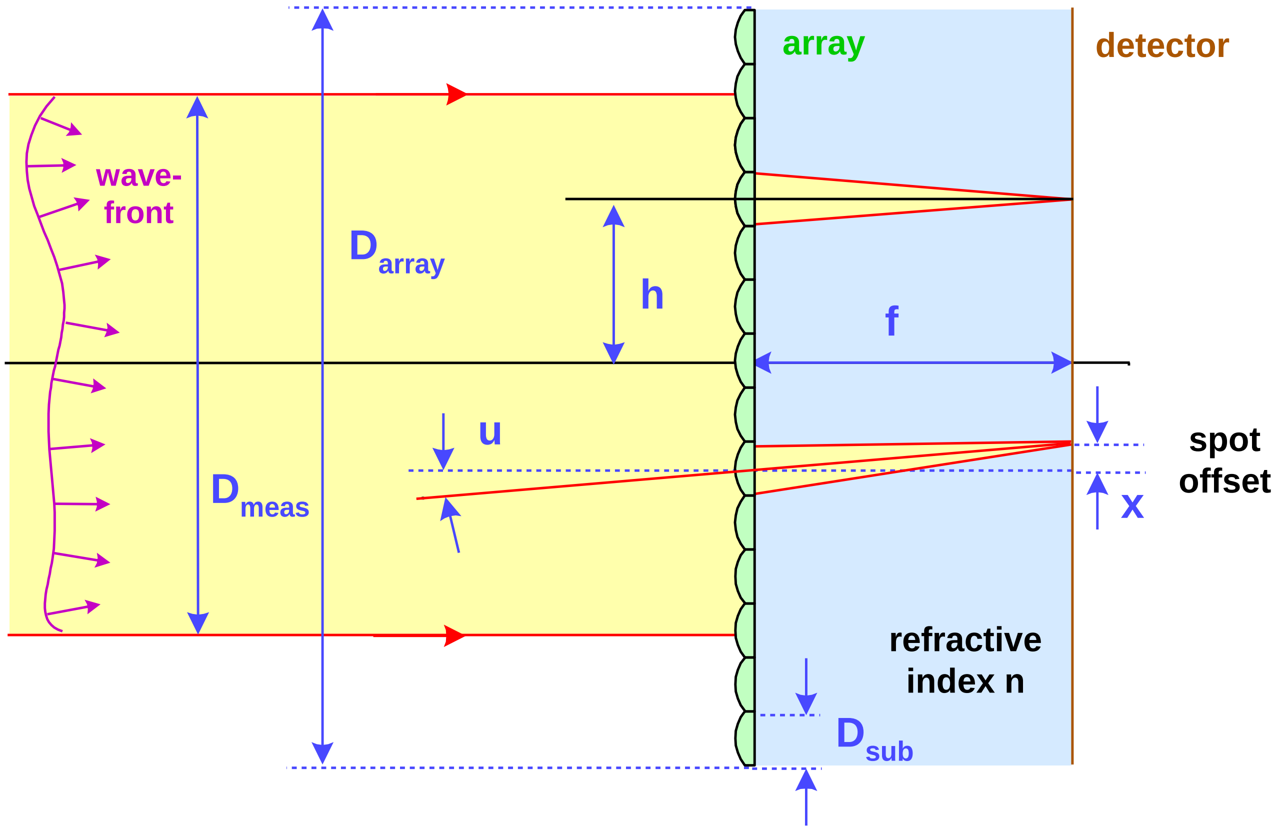
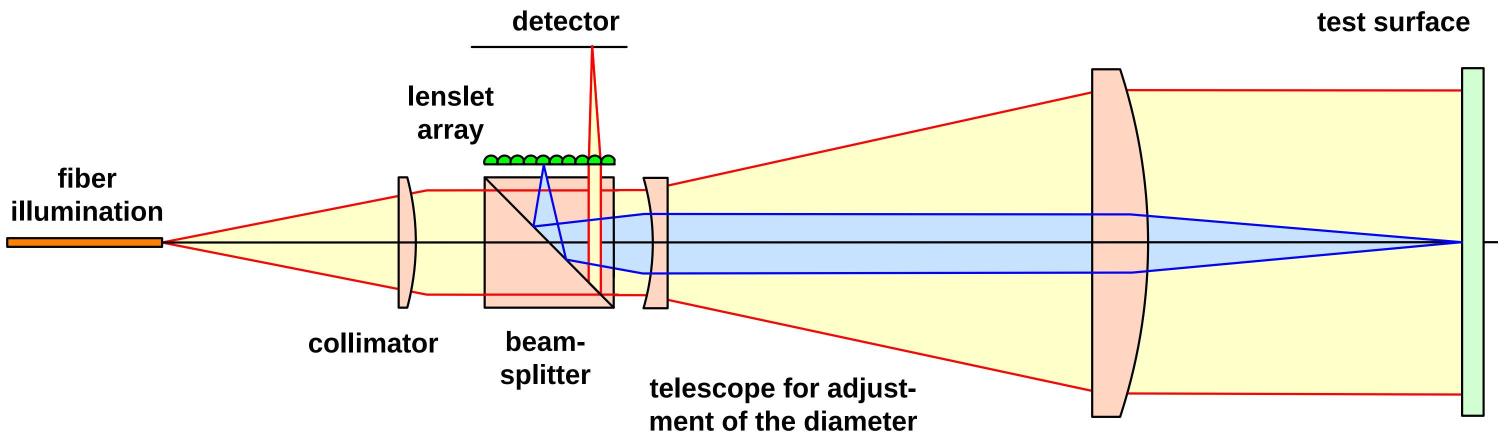
- Lenslet array divides the wavefront into subapertures
- Every lenslet generates a simgle spot in the focal plane
- The averaged local tilt produces a transverse offset of the spot center
- Integration of the derivative matrix delivers the wave front W(x,y)



Hartmann Shack Wavefront Sensor

- Typical setup for component testing



- Lenslet array