Properties and Applications of the 2D Fourier Transform

Introduction

The Fourier Transform (FT) of a signal gives the spatial frequency distribution of that signal. Essentially, the Fourier Theorem states that any image or signal can be expressed as a superposition of sinusoids. Unique to the 2D FT (as compared to the 1D FT) is the fact that rotation of the sinusoids result to the rotation of their FT's.

Activity 1. Anamorphic Property of FT of different 2D patterns

Since the FT space is in inverse dimension what is wide in one axis will be narrow in the spatial frequency axis, this is known as anamorphism. You observed this with your circular apertures, the smaller the aperture image, the larger the FT Airy pattern and vice versa. If the patterns have different dimensions along each axis you will expect anamorphism along each axis independently.

1. Create the following patterns, take their Fourier Transform and display their FT modulus. You may either generate them in Scilab for better symmetry or hand draw them in Paint.
   a) Tall rectangle aperture
   b) Wide rectangle aperture
   c) Two dots along x-axis symmetric about the center
   d) Same as (c) with different spacing between the dots.

Activity 2. Rotation Property of the FT

1. Create a 2D sinusoid using Scilab in the X direction (similar to a corrugated roof).

```
nx = 100; ny = 100;
x = linspace(-1,1,nx);
y = linspace(-1,1,ny);
[X,Y] = ndgrid(x,y);
f = 4 //frequency – you may change this later.
z = sin(2*%pi*f*X);
h = scf();
grayplot(x,y,z');
h.color_map = hotcolormap(32);
```

2. Take the Fourier Transform and display the FT modulus.
3. Change the frequency and discuss what happens to the FT.
4. Digital images have no negative values. Simulate a real image by adding a constant bias to the sinusoid you generated in Step 1. Take the FT. What do you observe? Suppose you took an image of an interferogram in a Young's Double Slit experiment. What can you do to find the actual frequencies? Suppose a non-constant bias is added to the sinusoid (e.g. very low frequency sinusoids), how can you get the frequencies of the interferogram?
5. Rotate the sinusoid and discuss what happens to its FT.

```
theta = 30;
z = sin(2*%pi*f*(Y*sin(theta) + X*cos(theta)));
```

6. Create a pattern which is a combination of sinusoids in X and Y and observe its FT. For example, the code below is a product of two corrugated roofs, one running in the X-direction, the other in Y.

```
z = sin(2*%pi*4*X).*sin(2*%pi*4*Y);
```

7. Add several rotated sinusoids to the pattern in 6. Predict the FT image **<u>BEFORE</u>** you calculate and observe its FT. Did your predictions match the actual output?

Filtering in Fourier Space

Unwanted repetitive patterns in an image can be removed by masking their frequencies in the Fourier domain. Alternatively, desired frequencies in the image may also be enhanced.

We can create filter masks to block out unwanted frequencies. In creating a filter mask, careful consideration of the convolution theorem is necessary. Remember that :

1. The FT of a convolution of two functions in space is the product of the two functions' FT , i.e., $FT[f*g]=FG$.

2. The convolution of a dirac delta and a function f(t) results in a replication of f(t) in the location of the dirac delta, i.e., $\int f(\tau)\delta(t_o-\tau)d\tau=f(t_o)$.

Activity 3. Convolution Theorem Redux

1. Create a binary image of two dots (one pixel each) along the x-axis symmetric about center. Take the FT and display the modulus.
2. Replace the dots with circles of some radius. Discuss what you observe in the FT modulus as you vary the radius.
3. Replace the dots with squares of some width. Discuss what you observe in the FT modulus as you vary the width.
5. Replace the dots with Gaussians, i.e., $\exp\left(\frac{-(x\pm\mu_o)^2}{\sigma^2}\right)$ of varying variance σ^2. Here, $\pm\mu_o$ are the peak location. You will need to create this image in Scilab. Observe the FT modulus as you vary the variance.
6. Create a 200×200 array of zeros. Put 10 1's in random locations in the array. These ones will approximate dirac deltas. Call this array **A**. Create an arbitrary 9×9 pattern, call it **d**. Convolve **A** and **d**. What do you observe?
7. Create another 200×200 array of zeros but this time put equally spaced 1's along the x- and y-axis in the array. Get the FT and display the modulus. Change the spacing the 1's and repeat. Explain what you observe.

Activity 4. Fingerprints : Ridge Enhancement

1. Prepare an image of your own fingerprint in grayscale. You may do this by taking a picture of your stamped-ink fingerprint on paper. If you fail to

prepare your own fingerprint, download raw images from the web. Make sure the image is NOT YET BINARIZED like Figure 1 below. Remember to cite your sources.

Figure 1: *Fingerprint in grayscale.*

2. Open in Scilab as grayscale image.
3. Obtain the FT of the image and investigate where the frequencies of the fingerprint ridges lie. **Tip:** because the range of values of the modulus of the FT image will span several orders of magnitude, use the log scale to display the FT image. Example :

```
imshow(uint8(log(abs(fftshift(FT_A)))));
```

4. Design a filter mask to enhance the appearance of these ridges at the same time remove the blotches.

Activity 5. Lunar Landing Scanned Pictures : Line removal

1. Download **hi_res_vertical_lg.gif** from the website listed in the caption of Figure 2 below. Open in Scilab as grayscale image. Below is a description of the file.

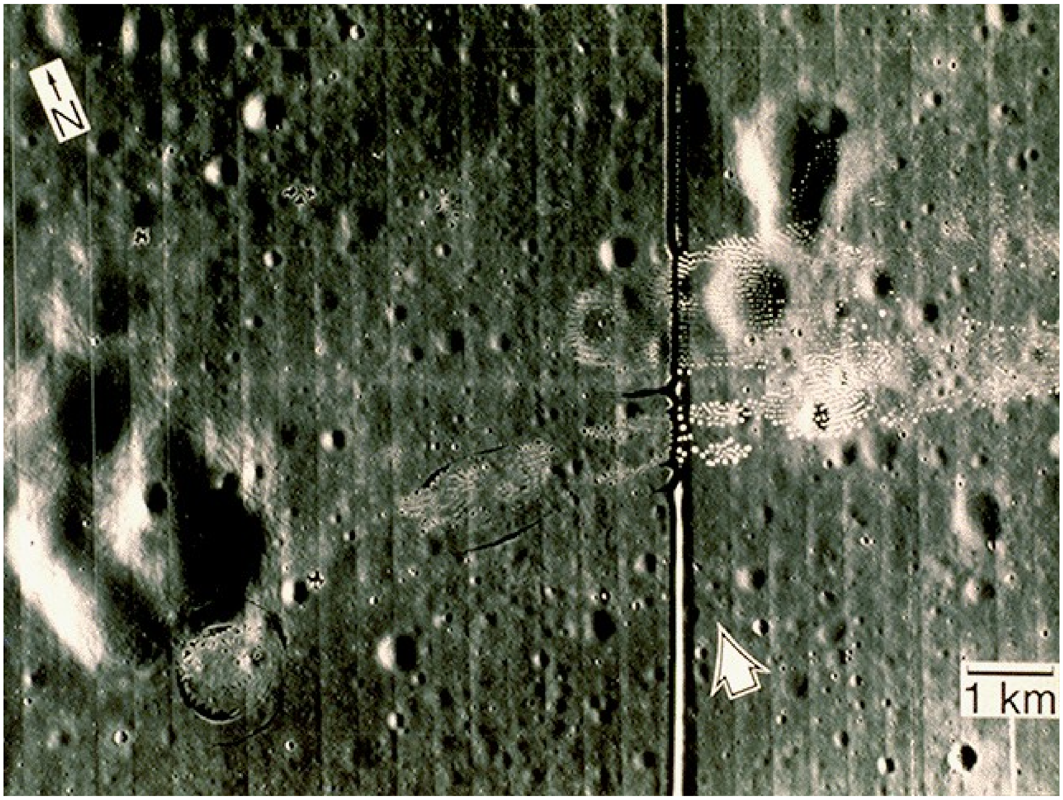

Figure 2: *Composite image from http://www.lpi.usra.edu/lunar/missions/apollo/apollo_11/images/hi_res_vertical_lg.gif*

> *" The two groups of irregularly shaped craters north and west of the landing site are secondaries from Sabine Crater. This view was obtained by the unmanned Lunar Orbiter V spacecraft in 1967 prior to the Apollo missions to the Moon. The black and white film was automatically developed onboard the spacecraft and subsequently digitized for transmission to Earth. The regularly spaced vertical lines are the result of combining individually digitized 'framelets' to make a composite photograph and the irregularly-shaped bright and dark spots are due to nonuniform film development. [NASA Lunar Orbiter photograph]"*

2. Remove the vertical lines in the image by filtering in the Fourier Domain.

Activity 6. Canvas Weave Modeling and Removal

1. Download the image **`canvasweave.jpg`** from the email sent and open as a grayscale image.

Figure 3: *Detail of an oil painting from the UP Vargas Museum Collection.*

2. Create a filter mask to remove the canvas weave patterns. Are the brushstrokes in the painting enhanced?

3. Invert the filter mask (0's become 1's and vice versa) and take the inverse fourier transform. Observe the generated modulus image. Is it close to the appearance of the canvas weave?

