Experiment-342 A

SIMULATION OF RADIOACTIVE DECAY USING ROLLING OF DICE

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Abstract

Using six-faced cubic dice, radioactive decay is simulated by rolling them and removing a certain number from them after each round of rolling on the basis of statistical probability. The decay constant is determined for a single species as well as for two different species. The uncertainties in the various measurements are determined using Poisson statistics.

Introduction

Natural radioactivity or nuclear decay is a radioactive decay process in which at a given time a certain number of atoms disintegrate spontaneously. In this process unstable atoms emit ionizing particles or radiations and eventually reach a stable state. Basically there are three types of nuclear decay, namely

1. Alpha decay
2. Beta decay
3. Gama decay

The particle radiations resulting from radioactive decay are rather easy to detect experimentally. In fact, their discovery in 1890's accelerated research in nuclear physics. As the radioactive decay processes are of statistical nature, understanding of the decay at the + 2 level would be rather presumptuous. Secondly, the radioactive sources cannot be handled with bare hands without taking adequate protection. Further their handling requires special permission from relevant Government authorities concerned with radiation monitoring/protection. Special housing is required to store radioactive sources and materials, involving a small isolated room lined on its interior walls as well as ceiling with slabs of lead that act as shield protecting the experimenter against exposure to harmful radiation.

Moreover, the syllabus in higher classes (graduate/post-graduate levels) includes nuclear physics experiments. In view of these considerations, in this experiment it is aimed to give a theoretical background and familiarize students with the analysis of the data involved in nuclear physics experiments, which also includes estimation of errors by means of appropriate statistical methods.

Theory of radioactive decay

In a radioactive decay process, the rate at which atoms undergo decay is proportional to the number of atoms initially present at a given instant of time; the governing equation being

$$\frac{dN}{dt} = -\lambda N \qquad ...1$$

where the decay constant λ is related to the probability of decay of an atom of a given radioactive element. One can easily solve the above first order equation and obtain the well known exponential law of radioactive decay as

$$N(t) = N_o\, e^{-\lambda t} \qquad ...2$$

where N_o is initial number of atoms present
N(t) is number of atom present at a certain time, t ; and
λ is a measure of the rate of decay, called as the decay constant, which is constant for a given radioactive element.

The half life of an element undergoing radioactive decay is the time for the amount to become half of its initial value. Thus, half life of a radioactive element is given by

$$T_{1/2} = \frac{0.693}{\lambda} \qquad ...3$$

The nuclear decay process can be simulated by rolling of dice repeatedly, taking out a particular species of dice each time from the lot. In this simulation, N_o represents the initial number of dice taken (100 or 200 number of similar dice), *N(t)* is the number dice remaining after each round of rolling (decay) and *t* corresponds to the number of rounds rolled (equivalent to time).

The decay Equation-2 can be further simplified to obtain the decay constant. Taking natural logarithm of both the sides, Equation-2 becomes

$$\ln N(t) = \ln N_o - \lambda t \qquad ...4$$

This equation represents a straight line in ln N (t) versus t, with slope λ and Y intercept as ln (N_o). Hence both λ and N_o can be determined by studying the rolling of dice.

If a six faced die is used to simulate the decay process, then the probability of finding one side up (say for example with number-6) is 1/6.

Probability, p, of finding the side with number six up is given by

$$\text{Probability of finding one side up } p = \frac{1}{f} \qquad ...5$$

where f is number of faces in the die.

Decay through two species

Some radionuclides may follow several different paths of decay. For example, approximately 36% of bismuth-212 decays by alpha-emission to thallium-208 while the remaining 64% decays by beta-emission to polonium-212. Both thallium-208 and polonium-212 are radioactive daughter products of bismuth-212 and both decay chains ultimately lead to lead-208 which are stable. Such a process can be simulated by taking two different numbers of dice after each roll. In such a case the probability becomes 2/6 for a six faced cube die. In this mode the decay is faster.

If λ_1 is decay constant of species-1 and λ_2 is the decay constant of species-2, the total decay constant of the process is the sum of the two independent decay constants, as given by

$$\lambda = \lambda_1 + \lambda_2 \qquad \ldots 6$$

In general, for radioactive decay of n species, one can write

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \ldots + \lambda_N \qquad \ldots 7$$

Half life of a decay process involving multiple species is given by

$$\frac{1}{T_{1/2}} = \frac{1}{(T_1)_{1/2}} + \frac{1}{(T_2)_{1/2}} + \ldots \frac{1}{(T_N)1/2} \qquad \ldots 8$$

where $(T_1)_{1/2}$ is the half life of species-1
$(T_2)_{1/2}$ is the half life of species-2, etc.

Such a decay process can be tested using the same set of six faced dice. In this case instead of looking for a particular number up in each round of rolling of dice, one can select two numbers, say the face with number-2 orresponding to species-1 and number-3 up corresponding to species-2 in each round of rolling of dice.

Error in the simulation process

Most of the counting experiments follow the Poisson statistics in which events occur randomly, but at a definite rate on an average. Examples in physics, besides radioactive decay, include particle lifetime measurement, and photon counting. Similar to the Gaussian distribution, the Poisson distribution is a special case of the Binomial distribution (which is employed to determine probability of successes in tossing of a coin, whether one gets a head or a tail as the outcome). In fact, when the mean of a Poisson distribution is large, it approximates a Gaussian distribution. The important fact to note about the Poisson distribution, however, is that its standard error equals the square root of the mean

$$\sigma_x = \sqrt{< x >} \qquad \ldots 3$$

where σ_x is the standard error in the simulation. It can be the number of atoms, N (t), remaining after each rolling of dice or the number of atoms (n) that have decayed in the process. In this experiment, an exponential decay may be expected for the trend whose individual data points are distributed about the mean (in which one measurement is made for each point) by employing the Poisson statistics.

Dice used in the experiment

One can make use of a two faced coin (probability 1/2), or a four faced die (1/4) or a six faced (1/6) die to perform this simulation. We have taken six- faced cubical dice for our experiment. It can be used to simulate 6 different species of decay. Figure-1 shows the die used in this experiment.

If six faced dice are used for simulation, and decay occurs if the face with number six is "up", then from probability considerations

Decay constant, $\lambda = p = \frac{1}{6}$ =0.166/ round of rolling of dice

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.166} = 4.17$$

Figure-1: The six faced cubic die used in the experiment

Materials used

Six-faced dice, a large board for rolling of the dice, as shown in Figure-2, and two buckets for collecting the dice

Figure-2: The dice and a large board used for rolling the dice on it

Experimental procedure

The experiments consists of two parts, namely

Part-A: Decay with one species
Part-B: Decay with two species

Part-A: Decay with one species

1. About 100 numbers of six faced cubic dice (similar to the one shown in Figure-1) are taken in a bucket. The exact number of dice taken is counted. This is the initial number of atoms

   N_o =95

2. The dice are rolled on a large board so that none of them overlap or interact (i.e., touch) with each other on the board after rolling. This is the first round of rolling of dice, i.e. r =1, for which all the dice with the face 'six-up' are collected and counted. Their number (n) gives the number of decayed atoms.

3. All the dice with the face six-up are collected, counted and placed in a separate bucket

   Number of dice with the face six-up, n =15

   Hence number of dice (atoms) remaining after the first round, N (t) = 95-15=80.

   As given by Equation-2, the number of atoms remaining is given by

   $$N(t) = N_o\ e^{-\lambda t} \quad = 95e^{-\lambda.1}$$

   λ for six faced dice =0.166

   $N(t) = 95e^{-0.166} = 80.46$

   This shows that 80 atoms remain after the first round of rolling of dice or 80 atoms remained after a time of t. These numbers match with the number obtained (80) after the first round of rolling, as shown in Table-1.

4. The number of dice remaining on the board is now collected and rolled again. In the second round of rolling the number of dice with their six-up face is removed from the board, counted and kept separately. The number of remaining dice is counted and rolled on the board again. The number of dice with the face six-up and the number of remaining dice are recorded in Table-1.

5. Rolling of dice is continued for 10-15 rounds (until about 10% of the dice initially taken are left on the board). To determine the decay constant experimentally, a graph

is drawn between the natural logarithm of the experimental values of atoms remaining versus the number of rounds dice have been rolled, as shown in Figure-3.

Table-1

No. of roll (t)	No. of atoms decayed (n)		No. of atoms remaining N(t)		lnN(t)
	Expt.	Thet.	Expt.	Thet.	Expt.
0	-		95	-	
1	15	14.54	80	80.46	4.38
2	10	12.30	70	68.16	4.24
3	15	10.43	55	57.73	4.00
4	7	8.83	48	48.90	3.87
5	6	7.48	42	41.42	3.73
6	11	6.34	31	35.08	3.43
7	6	5.36	25	29.72	3.21
8	1	4.55	24	25.17	3.17
9	7	3.85	17	21.32	2.83
10	2	3.26	15	18.06	2.70
11	1	2.76	14	15.30	2.63
12	5	2.34	9	12.96	2.19
13	1	1.99	8	10.97	2.07
14	0	1.68	8	9.29	2.07
15	3	1.42	5	7.87	1.60

Number of atoms decayed and remaining after each round of rolling

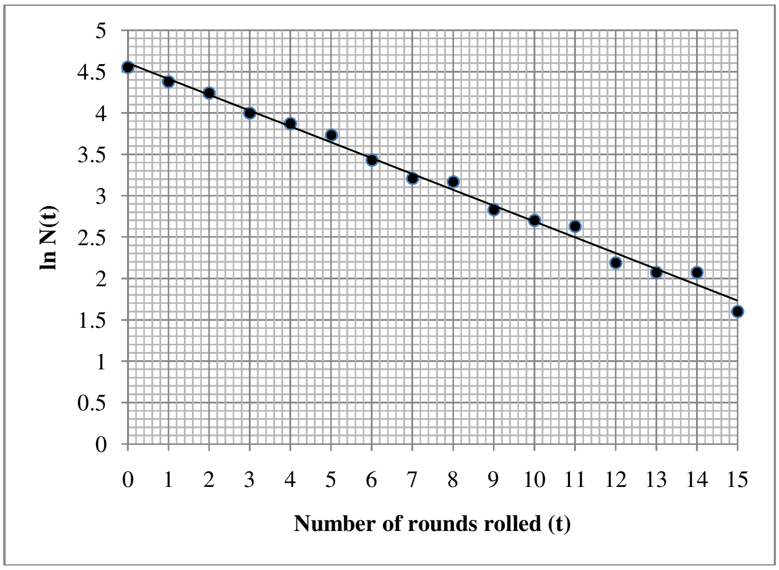

Figure-3: The linear decay equation

Slope of the straight line,

$$\lambda = \frac{3 \ x \ 0.5}{8} = 0.1875$$

The experimental value of the decay constant 0.187 is quite close to the theoretical value which is 0.166.

The half life of the decay can be calculated using Equation-3

$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.187} = 3.70$

This is close to the theoretical value of half life which is 4.17.

Poisson's error

The Poisson's error is calculated for the number of atoms remaining after each decay as

$\sum N(t) = 482.41$

Average $N(t) = 482.41/15 = 32.16$

$\sigma_x = \sqrt{< x >} = \sqrt{32.16} = 5.67$ atoms per decay

This is within 5% of the total number of atoms taken.

Part-B: Decay with two species

In this case the original atom can decay to give rise to two daughter atoms with different decay constants, λ_1 and λ_2. To identify the two different species, we shall label the face with number-2 up as species-1 with decay constant λ_1; and the number-3 face up as species-2 with decay constant λ_2. The theoretical values of the decay constants for these two species are the same, viz.

$\lambda_1 = \lambda_2 = 1/6 = 0.167$
$\lambda_1 + \lambda_2 = 2/6 = 0.33$

$\frac{1}{(T_1)_{1/2}} = \frac{\lambda_1}{0.693} = \frac{0.167}{0.693} = 0.24$

$(T_1)_{1/2} = 4.14 = (T_2)_{1/2}$

$\frac{1}{T_{1/2}} = \frac{1}{(T_1)_{1/2}} + \frac{1}{(T_2)_{1/2}} = \frac{1}{4.16} + \frac{1}{4.16} = \frac{2}{4.16} = 0.480$

$(T)_{1/2} = 2.08$

This is half of the value for the previous case; hence it decays at a faster rate.

Total number of dice taken, $N_0 = 95$

6. The dice are rolled once again. From the rolled dice, those with number-2 face up are collected and counted. This gives the number of species-1 that have decayed.

7. Similarly from the rolled dice, dice with number-3 face up are collected and counted. This is the number of species-2 that have decayed.

8. The dice remaining on the board after picking out species-1 and species-2 atoms are counted. This number is

   N(1)=58

9. The remaining dice are rolled again. The decayed atoms are separated out as before and recorded in Table-2. The trial is continued up to 5-6 rounds of rolling until the number of atoms that remain on the board is about 10.

   A graph showing the number of rounds of rolling versus the natural logarithm of number of atoms remaining is shown in Figure-4. From the graph, the slope of the straight line is calculated which gives the experimental decay constant for the case with two species.

   $\lambda=\lambda_1+\lambda_2$ = Slope of the line = (1/3) =0.333

10. Similarly, the half life is calculated and compared with the corresponding theoretical value

    $(T)_{1/2}$= 0.693/0.333=2.08

    The half life of the decay process with two species is exactly equal to its corresponding theoretical value.

Table-2

No. of rolls (t)	No. of atoms decayed (n)				No. atoms remaining (N_t)		ln N(t)
	Expt.	Thet.*	Expt.	Thet.*	Expt.	Thet*	Expt.
0	Species-1		Species-2		95	95	4.55
1	19	14.54	18	14.54	58	68.07	4.22
2	11	12.30	15	12.30	32	48.77	3.88
3	4	10.43	5	10.43	23	34.94	3.55
4	1	8.83	2	8.83	20	25.04	3.22
5	1	7.48	3	7.48	16	17.94	2.88

Simulation of decay for the case of the two species

Poisson's error

The error on the number of atoms remaining after each decay is calculated, employing the Poisson statistics, as

$\sum$N(t) =244, Average N(t) = 48.8

$\sigma_x = \sqrt{< x >} = \sqrt{48.8}$ = 6.98 atoms per decay

This is within 7% of the number of atoms initially present.

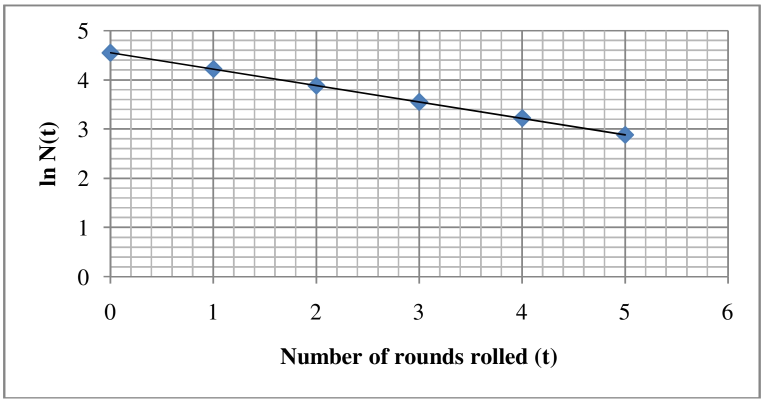

Figure-4: Variation of ln N(t) with the number of rounds dice are rolled in two-species decay

Results

The results obtained are tabulated in Table-3.

Table-3

Parameter	Decay with			
	One species		Two species	
	Expt.	Thet.	Expt.	Thet.
Decay constant (λ)	0.187	0.166	0.333	0.333
Half life (roll)	3.70	4.17	2.08	2.08
Poisson's error (%)	5.6	-	7	-

Experimental results

Discussion

This experimental simulation of nuclear decay is a simple, yet illustrative experiment. One can perform this simple experiment before actually performing any other complex nuclear physics experiment. The accuracy in the determination of decay constant and half life can be further improved by taking larger number of dice (about 200). The same simulation can also be done on a computer using appropriate software programs.

Reference

[1] Simulated radioactive decay using dice,
http:// helios.augustana.edu/~jvh/courses/350/Dice10.pdf