Determining the distribution of the focal lengths of different lenses using chi-square test

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Abstract

The thin lens equation relates the focal length of the lens to the object distance and image distance. In this work, the thin lens equation was used for lenses of various focal lengths and the experimental measurements were investigated using the Chi-squared test. The average focal lengths calculated for 250mm, 200mm and 100mm deviated by 0.28%, 3.8% and 2.25% and the distribution of the trials fit the expected normal distribution.

Keywords: thin lens equation, focal length, Chi-squared test

1 Introduction

Lenses are refracting optical devices that focuses or disperses light. They are classified by the curvature of its two optical surfaces which can either be convex (bulging outward from the lens), concave (depressed into the lens) or planar (flat) [1]. As seen in Fig. 1, a combination of these surfaces produce different kinds of lenses such as biconvex or convex (two convex surfaces), biconcave or concave (two concave surfaces), plano-convex (planar and convex surface) and etc. which can be used for different purposes. A relation used to describe lenses is the lensmaker's equation given by

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right] \tag{1}$$

where f is the focal length of the lens, n is the refractive index of the material, R_1 and R_2 are the radius of curvature of the two lens surfaces and d is the thickness of the lens [1].

Thin lenses are a type of lens wherein the lens thickness is negligible compared to the radius of curvature of the lens surfaces [1]. When the thickness approaches zero (d→0), the last term of the lensmaker's equation vanishes and the relation simplifies into [1]

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{2}$$

Another equation that relates the focal length of the lens (f) to the object distance (s_o) and image distance (s_i) is the thin lens equation provided by [1]

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}. \tag{3}$$

This equation is only valid for rays which follow the paraxial ray approximation passing through thin lenses [1]. In this experiment, the thin lens equation was tested for lenses of different focal lengths and the experimental measurements were investigated using the Chi-Squared test.

2 Methodology

In a setup shown in figure 1, a lamp and an optical bench was used to verify the thin lens equation using a convex lens. The distance of the viewing screen was fixed while the distance of the convex lens was varied until the light from the lamp is focused on the viewing screen. For each trial, the distance of the object or the lamp from the lens and the distance of the image on the viewing screen from the lens were recorded. This procedure was repeated for 40 trials using three different focal lengths of the lenses. After recording the distances, the thin lens equation was verified and used to determine the focal length of the lens used. The distribution of the calculated focal length of the lenses was determined and examined using chi-squared test.

3 Results and Discussion

Chauvenet's criterion shows no outlier from the datasets collected. From the dataset of the distance of the object, p and distance of the image, q, the mean and standard deviation is calculated and is shown in Table 2. To determine the fit of the datasets, chi-square test was used. Based on the results, a near zero chi-square value was calculated. This means that the datasets fits with the distribution of the expected value.

To calculate the focal length for each lens, the thin lens equation, eqn. 3 is used where the parameters used were the object distance and image distance. Based from the data gathered, the average focal length calculated for the 250mm, 200mm and 100mm are 249.3mm, 207.6mm and 97.75mm, respectively. This produced a deviation from the expected value equal to 0.28%, 3.8% and 2.25%. This deviation from the expected value may be attributed to the consistency on the alignment of the lens. Another reason is due to parallax of the observers when viewing the image on the screen.

4 Conclusion

The average focal lengths calculated for the 250mm, 200mm, and 100mm lenses are 249.3mm, 207.6mm, and 97.75mm, respectively. Using chi-squared test, the distribution of the 40 trials of the calculated focal lengths fits with the expected normal distribution. The experimental data followed the thin lens equation.

References

[1] E. Hecht, *Optics* (Addison-Wesley, CA, USA, 2017).

5 Appendix

5.1 Figures

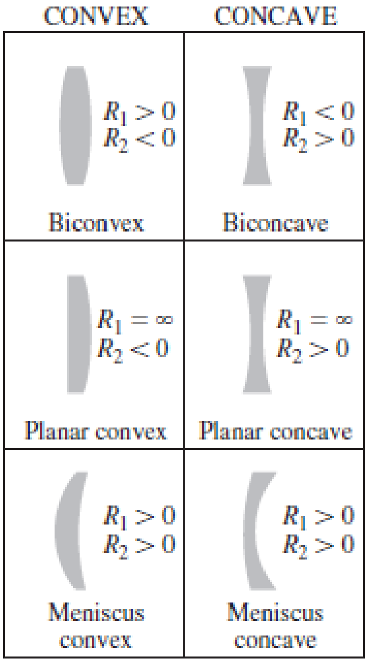

Figure 1: Cross sections of various centered spherical simple lenses. [1]

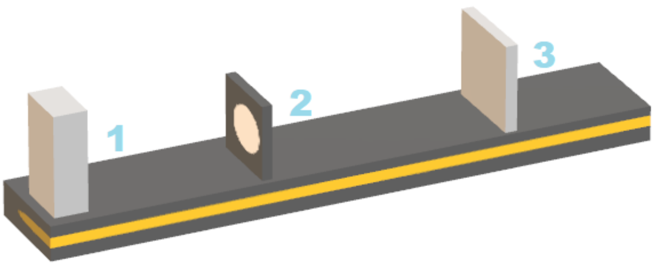

Figure 2: Setup for thin lens equation verification: (1)lamp, (2)lens, (3)screen

5.2 Tables

Table 1: Mean and standard deviation of lenses with focal length equals 250 mm, 200 mm and 100 mm.

mean of focal length, $\overline{f}$	mean of p, $\overline{p}$	mean of q, $\overline{q}$	standard deviation σ
249.3 mm	52.31	47.68	1.08
207.6 mm	70.59	29.41	0.23
97.75 mm	36.67	13.33	0.16

Table 2: Mean and standard deviation of lenses with focal length equals 250 mm, 200 mm and 100 mm.

expected focal length	χ^2
250 mm	0.013
200 mm	1.18
100 mm	0.224