Experimental determination of Rydberg constant for hydrogen via diffraction spectroscopy

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Abstract

Rydberg constant appears in a formulae where wave numbers of lines is being determined. It describes light's wavelengths and frequencies and being used usually for Hydrogen atoms in Balmer series. The goal is to obtain experimental values of Rydberg constants. It was done using student spectrometer, diffraction grating, magnifying glass, and discharge tube that was set up for the experiment. Percent deviations of the left and right reading of the first and second order were 23.80%, 10.48%, 0.89% and 11.87%, respectively. Some sources of error were usually from the difficulty in reading the measurement.

Keywords: Rydberg constant, diffraction, spectroscopy

1 Introduction

The present theory of quantum mechanics started out with the crude model of the electron orbiting a nucleus inspired by the motion of celestial bodies. However, such a theory would imply that, since accelerating charges radiate, the electron would lose energy and spiral towards the nucleus [1]. To reconcile this problem, Bohr postulated that electrons can only assume a discrete set of nonzero angular momenta, thereby only having access to certain orbits. An electron can only jump to another orbit level by gaining or losing energy, the latter usually being in the form of electromagnetic energy, whose wavelength λ is specified by

$$E_n - E_m = \frac{hc}{\lambda} \tag{1}$$

where h is Planck's constant, and c is the speed of light in a vacuum. Bohr then introduced the principal quantum number n, signifying the discrete nature of the orbital levels

$$E = \frac{Z^2}{8\pi\epsilon_0}\frac{e^2}{r} \tag{2}$$

(for a hydrogen atom), where Z is the atomic number, ϵ_0 is the vacuum electric permittivity, e is the elementary charge, and r is the orbital radius [2]. (1) can be rewritten in terms of (2)

$$\frac{hc}{\lambda} = -\frac{Z^2 e^2}{8\pi\epsilon_0}\left(\frac{1}{r_m} - \frac{1}{r_n}\right) \tag{3}$$

Imposing Bohr's postulate that the angular momentum of an electron is an integer multiple of Planck's constant [3], we have

$$m_e v r = n \frac{h}{2\pi} = n\hbar \tag{4}$$

where $\hbar$ is the reduced Planck's constant, and m_e is the electron rest mass. Using this to express the velocity of the electron in terms of its orbital radius and subjecting it to the Coulomb potential, we have

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \tag{5}$$

We can express r in terms of the principal quantum number n, given by

$$r = \frac{4\pi\epsilon_0^2 n^2 \hbar^2}{m_e} \tag{6}$$

With this, (3) can be rewritten as

$$\begin{aligned} \frac{hc}{\lambda} &= \frac{m_e e^4}{8\epsilon_0^3 h^2} \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \\ \frac{1}{\lambda} &= \frac{m_e e^4}{8\epsilon_0^3 h^3 c} \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \end{aligned} \tag{7}$$

where m and n are positive integers. Rearranging terms and expressing the factored term as a constant,

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right) \tag{8}$$

A special set of spectral lines observable in the visible spectrum can be accessed by letting $m = 2$, and is known as the Balmer series:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right) \tag{9}$$

which describes the electromagnetic emissions for an electron jumping from an energy level $n \geq 3$ to $n = 2$. The quantity R_H is known as Rydberg's constant (for hydrogen), and is equal to $R_H = 1.097 \times 10^7 \text{ m}^{-1}$ [4].

In this experiment, R_H is experimentally determined by observing the diffraction angle of the hydrogen spectral lines and calculating the wavelength from the measured value using the diffraction equation

$$\lambda = \frac{d \sin\theta}{m} \tag{10}$$

where m indicates the order of the diffraction line, d is the diffraction grating slit width, and θ is the diffraction angle. This reciprocal of this value is plotted against the term in parenthesis in (9) using reference values for n based on the spectral line color. The slope of the best-fit line for the data gives the experimental value for the Rydberg constant.

This paper is organized as follows: In Section 2, we illustrate the experimental setup and materials used. In Section 3, we describe and enumerate the parameters considered and methods used in performing the experiment. In Section 4, we analyze and discuss the results. We conclude and summarize the paper and give recommendations to improve similar experiments in Section 5.

2 Experimental Setup and Materials

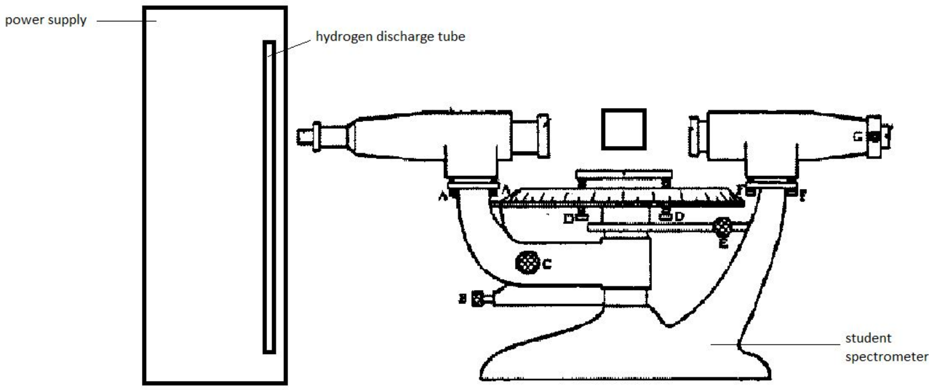

Figure 1: Figure 2.1: Experimental set up [5]

The experiment mainly used student spectrometer, diffraction grating, power supply of the discharge tube and the hydrogen tube. The set up can be seen from Fig 1. Other materials such as magnifying glass and flashlight was used for better readings.

3 Parameters and Methodology

The experiment started by preparing the set up shown in Fig 2.1. The student spectrometer was calibrated by focusing the telescope for parallel light and adjusting the necessary knobs of the spectrometer until axes are coincident. This should also be perpendicular to the axis of rotation. The diffraction grating of 300 lines/mm was mounted on the prism table of the spectrometer. This was then aligned such as the eyepiece will be showing a slit of the hydrogen discharge tube. We measured the position of the visible vertical lines, for the first order in our case. Left and right readings were taken. These repeated for colors that are visible and clear. Wavelengths were computed using (10). A plot of λ^{-1} versus $2^{-2} - n^{-2}$ was graphed. By fitting the best fit line, we determined the R_H from the slope of this line.

4 Results and Discussion

The dashed lines in the plots of λ^{-1} versus $2^{-2} - n^{-2}$ are the best fit lines for the set of data points gathered from the data. The slope of each dashed line is the experimental Rydberg constant.

The experimental value of the Rydberg constant for the first order are 0.87×10^7 m^{-1} and 1.21×10^7 m^{-1} for the left and right readings, respectively. For the second order, the experimental values are 1.09×10^7 m^{-1} and 1.23×10^7 m^{-1} for the left and right readings, respectively. The percent deviation of the left and right reading for the first order, and the left and right readings for the second order are 23.80%, 10.48%, 0.89% and 11.87%, respectively.

Possible sources of error include reading the measuring instrument because of the difficulty in discerning the coinciding lines, and disturbances that might affect the reference angle on which the measured angles for each color of a specific order would depend.

5 Conclusions

Two graphs were plotted, one for the first order, and another one for the 2nd order spectral lines. These figures show the experimental Rydberg constant through the slope of each dashed line.

We were able to get values and yielded percent deviations of the left and right reading of the first and second order: 23.80%, 10.48%, 0.89% and 11.87%, respectively.

Experiment like this is prone to error. One of the usual error is the difficulty in reading the measurement. The deficient lighting and the rusty used spectrometer made the process harder. Experimenters could read the wrong readings.

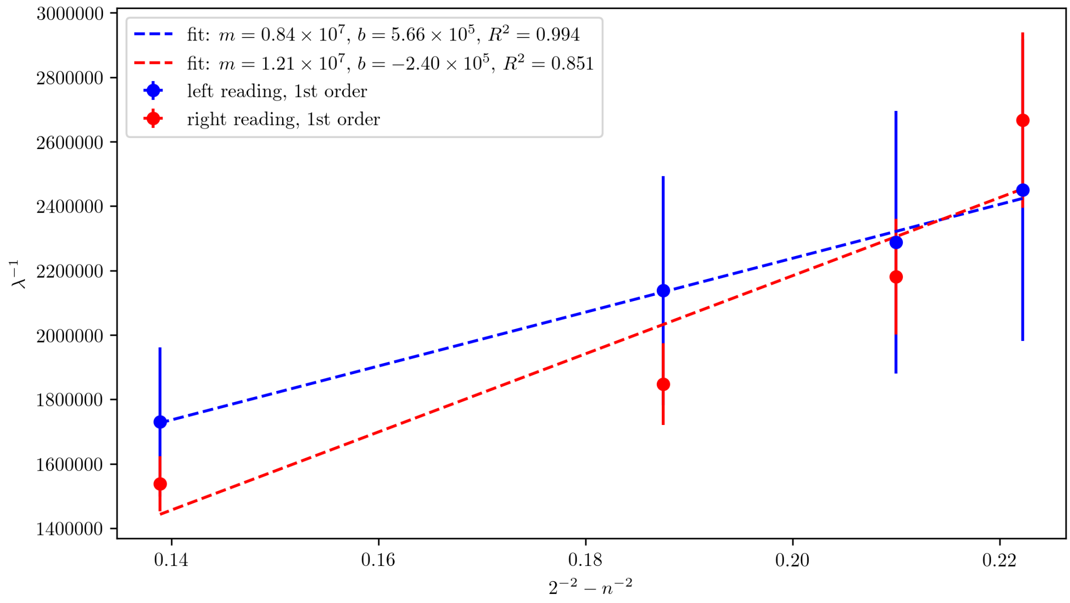

Figure 2: Balmer series of hydrogen (1st order spectral lines).

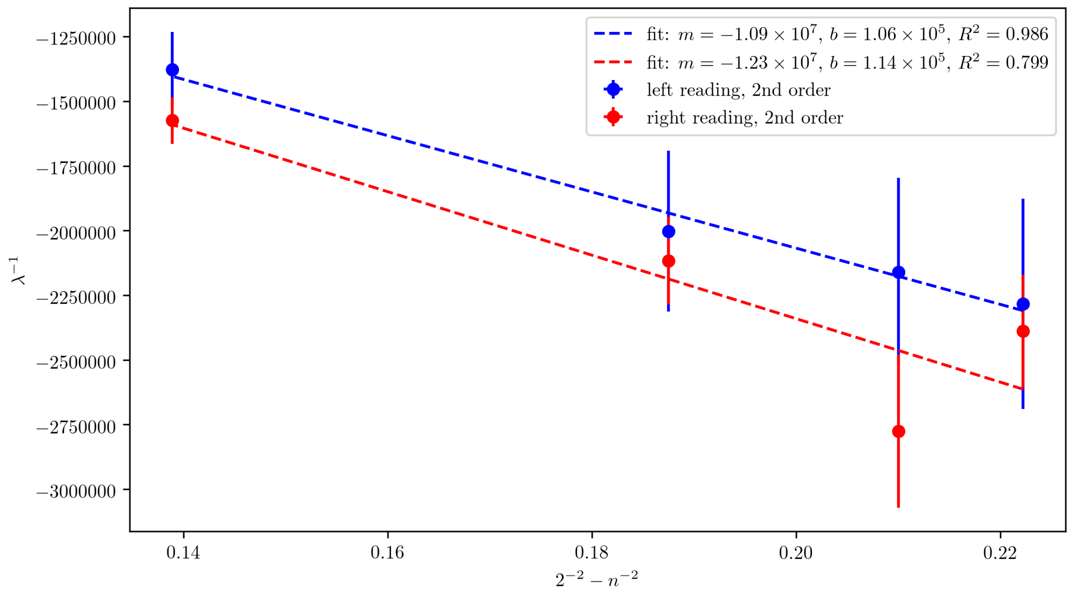

Figure 3: Balmer series of hydrogen (2nd order spectral lines).

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