Determination of the resonant frequency of a driven pendulum

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Abstract

The frequency at which a driven pendulum system experiences resonance was measured in this experiment. A massive bob was attached to a speaker using a string. By varying the input frequency, the movement of the bob was recorded by the motion sensor. It was observed that at its natural frequency of 0.6 Hz, resonance occurs wherein the ma amplitude was observed.

Keywords: resonance, driven oscillation, pendulum.

1 Introduction

In elementary physics courses, we are often introduced to the concepts of simple harmonic motion and the differential form of the equations of motion via the simple harmonic oscillator. Such a system, if left to oscillate starting from an arbitrary location x_0, has an equation of motion of the form [1]

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \zeta\omega_0 \frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = f(t) \tag{1}$$

where ζ is the damping coefficient, ω_0 is the natural frequency of the system, and $f(t)$ is some forcing function. A simple pendulum is an example of a simple harmonic oscillator and is visualized as a massive bob attached to a massless string, whose other end is fixed at some point, conventionally at the origin. Because the pendulum bob is a fixed distance from the origin and moves around it, it is convenient to express its equation of motion in polar coordinates [2]

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L}\sin\theta = f(t) \tag{2}$$

where g is the acceleration due to gravity, and L is the length of the pendulum. For demonstration purposes, we assume $f(t) \equiv \cos(\omega t)$, such that

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L}\sin\theta = \cos(\omega t) \tag{3}$$

(3) has no analytic solution, but can be solved using the elliptic integral. For small θ however, we can apply the small-angle approximation $\sin\theta \approx \theta$, so that the differential equation's solution becomes

$$\theta(t) \approx \frac{\theta_0\omega^2}{L(\omega_0^2 - \omega^2)}(\cos\omega t - \cos\omega_0 t) \tag{4}$$

where $\omega_0 \equiv (g/L)^{1/2}$ is the natural frequency of oscillation. Evidently, one period of oscillation is

$$T = \frac{1}{f_0} = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{L}{g}} \tag{5}$$

From (4), we see that θ_0 is maximum at $\lim_{\omega\to\omega_0} \theta(t)$. Therefore, resonance occurs when $\omega/\omega_0 \to 1$.

2 Methodology

Figure 1 shows the setup and materials used in the experiment. The pendulum with a massive bob was attached to the speaker by the metal rod and a string with length of 0.64 m. The speaker was then connected at the back of the amplifier while the function generator was connected to the female jack of the amplifier. The other setup consisted of the motion sensor connected to the Lab Quest and was 0.50 m away from the massive bob. We started with a frequency of 0.1 Hz and increased in increments of 0.1 Hz. Using the motion sensor, the position and velocity vs time graphs were obtained and the maximum amplitude per frequency was identified which was plotted against the driving angular frequency ω_d divided by the natural angular frequency ω_0.

3 Results and Discussion

In this experiment we studied the effect of supplying a sinusoidal driving force with angular frequency ω to the damped harmonic oscillation.

Figure 2 shows the resonance curve of the setup under the influence of a square forcing function. We have observed that the system exhibits the highest amplitude at a frequency of 0.6 Hz, while at different frequency a relatively low amplitude is observed. The peak observed is the point where the system experiences resonance. The theoretical natural frequency of the pendulum system of length 0.64 m was computed to be 0.62 Hz, which is consistent with the observed frequency at which resonance occur. A more precise measurement could not be conducted due to the limited resolution of the function generator.

4 Conclusions

A damped system can be forced to oscillate, but gets the most oscillation at its resonant frequency. In this experiment wherein a pendulum system of length 0.64 m was used, resonance was observed to occur at 0.6 Hz which is consistent with the theoretical value. For an in-depth study of different kinds of oscillation, we recommend studying the effects of damping in a system in simple, damped, and forced harmonic oscillation.

References

[1] Boundless. (2013). Damped and driven oscillations. Retrieved 13 March 2019, from *LumenLearning*: https://courses.lumenlearning.com/boundless-physics/chapter/damped-and-driven-oscillations/.

[2] Russel, D.A. (2018). The simple pendulum. Retrieved 13 March 2019, from *Acoustics and vibration animations*: https://www.acs.psu.edu/drussell/Demos/Pendulum/Pendula.html.

Appendix

Figures and Diagrams

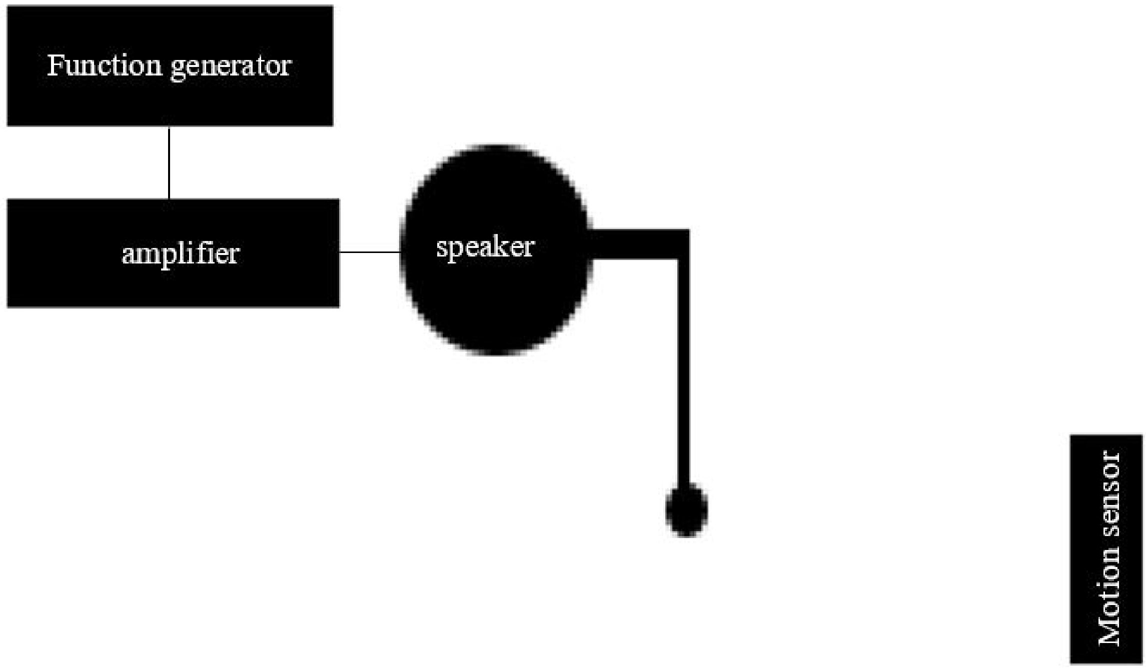

Figure 1: Experimental setup.

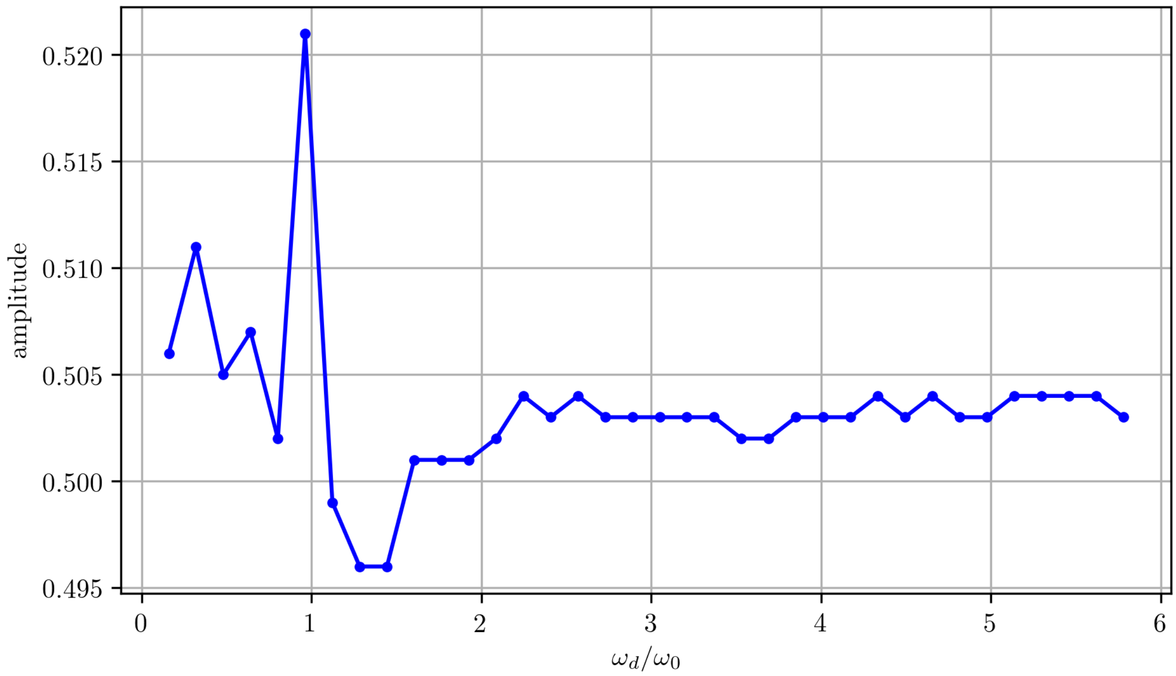

Figure 2: Resonance curve for driven oscillation.