

Measuring Viscosity

Advanced Higher Physics Investigation



Jakub Srsen SCN: 050950891

Boroughmuir High School

Center Number: _____

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Introduction

Summary

In my investigation I will attempted calculate viscosity of liquids. I used and compared two approaches. In the first approach I used Terminal Velocity of a free-falling object and Stokes' law to calculate the viscosity of the liquid. In the second approach I have used the traditional capillary method of viscosity measurement. I found that the capillary method was more accurate as it produced a value closer to the known value. However, this method was more experimentally demanding and required specialised instruments. On the other hand the first method had a very simple experimental setup but there was a bigger difference between the calculated viscosity and the known values.

Underlying Physics

Viscosity

Some fluids flow more easily than others, i.e. water flows more easily than honey. In general the flow of a fluid involves different parts of the fluid moving at different velocities, usually in layers. The viscosity depends on how easily they can slide past each other. In more viscous fluids like honey, where flow is hard to maintain, the layers slide past each other with difficulty due to the friction between the layers. On the other hand water flows much easier as there is little friction between the layers. In brief, the viscosity of a fluid is a measure of its resistance to flow.

Determining Viscosity using Terminal velocity & Stokes' Law

Terminal velocity is achieved when the force of gravity ($+F_g$) equals the force of drag ($-F_d$) on a free falling objects. When the object is first dropped it will accelerate at g until the object's weigh (mg) equals the force of drag. The object reaches a terminal velocity because the drag force resisting motion is directly proportional to the square of its speed. At low speeds, the drag is much lower than the gravitational force so the object accelerates. As it accelerates, the drag increases, until it equals the weight of the object. Drag also depends on the projected area. This is why objects with a large projected area, such as parachutes, have a lower terminal velocity than smaller objects such as cannon balls.

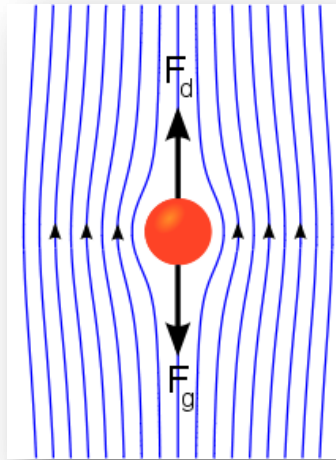


Fig 1. $F_d = F_g$

In 1851, George Gabriel Stokes devised an equation for frictional force, which is now known as Stokes' Law:

$$F_d = 6\pi\eta RV$$

(Ref 1.)

This equation can be used to derive an expression for terminal velocity:

$$V_s = \frac{2(\rho_p - \rho_f)}{9\eta} gR^2$$

(Ref 2.)

Stokes' Law states, when a particle reaches terminal velocity its "frictional force", viscosity, combined with buoyancy exactly balances with the gravitational force.

I will adapt the equation to:

$$V_s = \frac{2(\rho_p - \rho_f)}{9\eta} gR^2$$

From which η could be calculated:

$$\eta = \frac{gD^2(\rho_p - \rho_f)}{18V_s}$$

Where:

η is the fluid's dynamic viscosity (Kg.m^{-1})

g is the gravitational acceleration (m.s^{-2})

D is the diameter of the spherical object (ball bearing) (m)

ρ_p is the Density of the spherical object (ball bearing) (kg.m^{-3})

ρ_f is the Density of the fluid (kg.m^{-3})

V_s is the spherical object's (ball bearing's) terminal velocity, settling velocity (m.s^{-2})

Determining Viscosity using Capillary Method & Poiseuille's Law

Poiseuille's method uses the rate of flow of through a capillary tube, the pressure difference across the tube and the dimensions of the tube to determine the viscosity of a fluid.

The Poiseuille's Law was derived by Jean Louis Marie Poiseuille in 1838. (Ref 3.)

$$\Delta P = \frac{8\eta LQ}{\pi r^4}$$

(Ref 3.)

Where:

ΔP is the change in pressure

L is the length of capillary tube

η is the viscosity

Q is the volumetric flow rate

r is the radius of the capillary tube

π is pi

Procedures

Method 1:

Measuring Viscosity of Glycerol Using Terminal Velocity & Stokes' Law

Aim:

To calculate the viscosity of glycerol through the relationship between terminal velocity and viscosity and then compare results with known values for viscosity.

Method:

Apparatus was set up as shown below in the image below (Fig. 2).

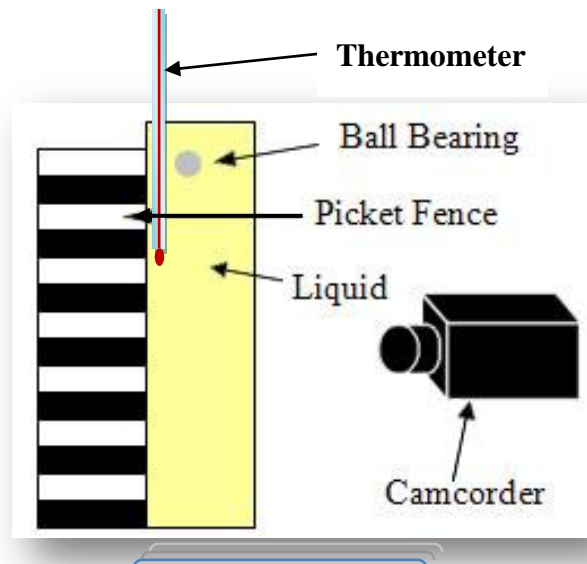


Fig 2 Experimental Setup

Steel bearing balls used during experiment had their diameter measured using micrometer gauge 3 times and values averaged to minimise error. The average diameter of the ball was **3.47mm** and a density **7.85 g/cm³** (Ref 5.). The balls were dropped from level into glycerol (density **1.262 g/cm³** (Ref 6.)) in a glass cylinder 17cm high at temperature **20°C, 25°C & 30°C**.

To obtain the terminal velocity of the ball bearing, I filmed the ball bearing falling through the glycerol. To work out its speed at different stages of the free fall I used a piece of graphic software (Adobe Photoshop (Ref 7.)) to examine the video frame by frame and measure the distance between 2 points on the video. First of all I calibrated the measuring tool using the picket fence, which I created by colouring in black strips 1cm wide and 1cm apart on a piece of card. I found that 50 pixels were equal to 1 cm, so a distance of 213 pixels was 4.26cm. The camcorder I used had shutter speed of 30 frames per second. Using the software I marked a spot where the bearing was every 30 frames (1 sec). I then used the calibrated measuring tool to find the distance between the points to obtain the speed of the ball bearing in cm.s^{-1} . (Fig 3.)

The temperature was continuously monitored throughout the experiment to make sure the temperature stayed near to the target temperature i.e. **20°C/25°C/30°C**.

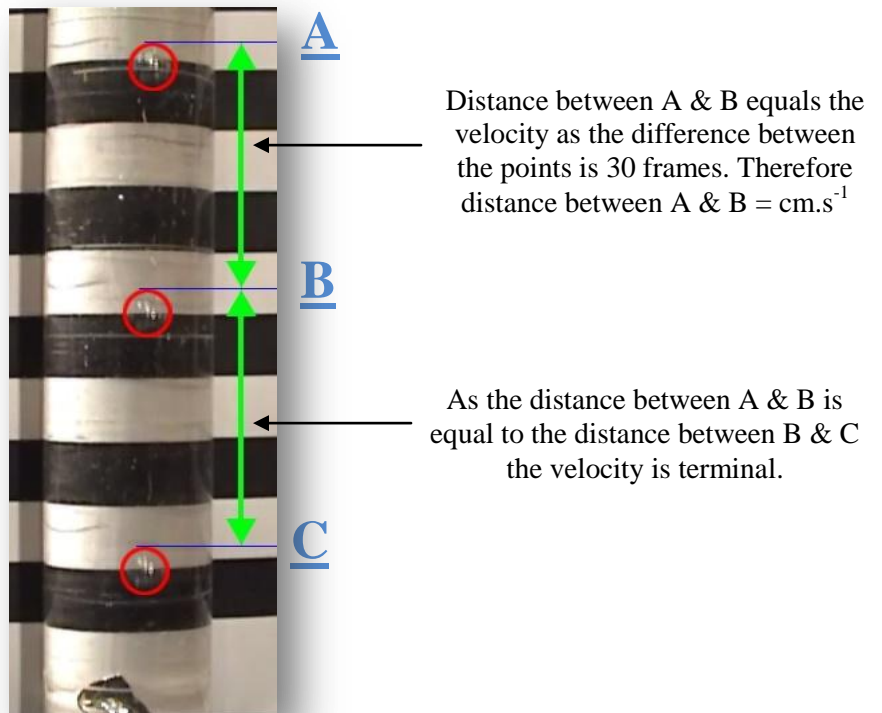


Fig 3. Working out Terminal Velocity

Results:

Method 1 - Experiment 1 (20°C)

Ball Bearing	Distance Ball Bearing has traveled during Frame 0 -> 30 (cm)	Distance Ball Bearing has traveled during Frame 30 -> 60 (cm)
1	2.66	2.80
2	2.78	2.85
3	2.76	2.88
4	2.59	2.75
5	2.81	2.91
	Total (cm) :	27.79
Average Terminal Velocity (cm.s ⁻¹) :		2.78

Uncertainties

The uncertainties of taking measurements in this experiment were:

Reading uncertainty of the micrometer gauge = $\pm 0.005\text{mm}$

Calibration uncertainty of the micrometer gauge = $\pm 0.002\text{mm}$

Reading uncertainty of the velocity image = $\pm 3\text{ mm}$

Reading uncertainty of electronic thermometer = $\pm 0.1^\circ\text{C}$

Calibration uncertainty of electronic thermometer = $\pm 0.5^\circ\text{C}$

Temperature of water = $20 \pm 0.5^\circ\text{C}$

Calculations:

Stokes' Equation:

$$\eta = \frac{gD^2(\rho_p - \rho_f)}{18V_T}$$

Acceleration due to Gravity (g)	= 9.81	m.s ⁻²
Particle Diameter (D)	= 3.47 mm = 0.00347 m	
Density of Particle (ρ_p)	= 7.85 g.cm ³ = 7850000	g.m ³
Density of Fluid (ρ_f)	= 1.262 g.cm ³ = 1262000	g.m ³
Terminal Velocity (V_T)	= 2.78 cm.s ⁻¹ = 0.0278	m.s ⁻¹

$$\eta = \frac{9.81 \times 0.00347^2 (7850000 - 1262000)}{18 \times 0.0278}$$

$$\eta = \frac{9.81 \times (1.20409 \times 10^{-5}) \times 6588000}{0.5004}$$

$$\eta = \frac{9.81 \times (1.20409 \times 10^{-5}) \times 6588000}{0.5004}$$

$$\eta = \frac{778.1826567}{0.5004}$$

$$\eta = 1555.12121633$$

Solution:

$$\text{Viscosity of Fluid } (\eta) = 1555.12 \text{ g.m.s}^{-1}$$

Other Units:

$$\text{Viscosity of Fluid } (\eta) = 5598.43637879 \text{ Kg.m.h}^{-1}$$

$$\text{Viscosity of Fluid } (\eta) = 1.56 \text{ Kg.m.s}^{-1}$$

$$\text{Viscosity of Fluid } (\eta) = 1.56 \text{ Pa.s}$$

Calculation of Uncertainties

$$\Delta D_T = \pm \sqrt{[(\Delta D_r)^2 + (\Delta D_c)^2]}$$

$$= \pm \sqrt{[(0.005)^2 + (0.002)^2]}$$

$$= \pm 0.005 \text{ mm}$$

Where ΔD_T = Total uncertainty in particle diameter value

ΔD_r = Reading uncertainty of the micrometer gauge
 ΔD_c = Calibration uncertainty of the micrometer gauge

Therefore the percentage error:

$$D = 3.47 \times 10^{-3} \pm 5 \times 10^{-6} m$$

$$D = 3.47 \times 10^{-3} m \pm 0.14\%$$

$$\Delta V_T = \pm 3 mm$$

Where ΔV_T = Total uncertainty in terminal velocity value

Therefore the percentage error:

$$V = 2.78 \times 10^{-2} \pm 3 \times 10^{-3} m$$

$$V = 2.78 \times 10^{-2} m \pm 10.79\%$$

Total Uncertainty & Resulting Value

$$\Delta \eta = \pm \sqrt{\left[\left(\frac{0.14}{100} \right)^2 + \left(\frac{10.79}{100} \right)^2 \right]}$$

$$= \pm \sqrt{1.16 \times 10^{-2}}$$

$$= \pm 0.1079$$

$$= \pm 10.79\%$$

Therefore

$$\Delta \eta = 0.168 Pa.s$$

The final value for viscosity of water using this method was:

$$\eta = 1.56 \pm 0.17 Pa.s$$

Method 1 - Experiment 2 (25°C)

Ball Bearing	Distance Ball Bearing has traveled during Frame 0 -> 30 (cm)	Distance Ball Bearing has traveled during Frame 30 -> 60 (cm)
1	4.26	4.32
2	4.01	4.18
3	4.02	4.18
4	4.13	4.25
5	4.03	4.15
6	4.00	4.08
7	3.88	4.12
8	3.99	4.14
9	4.00	4.15
10	4.05	4.21
	Total (cm) : 82.15	
	Average Terminal Velocity (cm.s ⁻¹) : 4.11	

Uncertainties

The uncertainties of taking measurements in this experiment were:

Reading uncertainty of the micrometer gauge = ± 0.005mm

Calibration uncertainty of the micrometer gauge = ± 0.002mm

Reading uncertainty of the velocity image = ± 3 mm

Reading uncertainty of electronic thermometer = ± 0.1°C

Calibration uncertainty of electronic thermometer = ± 0.5°C

Temperature of water = 25 ± 0.5°C

Calculations:

Stokes' Equation:

$$\eta = \frac{gD^2(\rho_p - \rho_f)}{18V_T}$$

Acceleration due to Gravity (g)	= 9.81	m.s ⁻²
Particle Diameter (D)	= 3.47 mm = 0.00347 m	
Density of Particle (ρ_p)	= 7.85 g.cm ³ = 7850000	g.m ³
Density of Fluid (ρ_f)	= 1.262 g.cm ³ = 1262000	g.m ³
Terminal Velocity (V_T)	= 4.11 cm.s ⁻¹ = 0.0411	m.s ⁻¹

$$\eta = \frac{9.81 \times 0.00347^2 (7850000 - 1262000)}{18 \times 0.0411}$$

$$\eta = \frac{9.81 \times (1.20409 \times 10^{-5}) \times 6588000}{0.7398}$$

$$\eta = \frac{9.81 \times (1.20409 \times 10^{-5}) \times 6588000}{0.7398}$$

$$\eta = \frac{778.1826567}{0.7398}$$

$$\eta = 1051.882477233$$

Solution:

$$\text{Viscosity of Fluid } (\eta) = 1051.88 \text{ g.m.s}^{-1}$$

Other Units:

$$\text{Viscosity of Fluid } (\eta) = 3786.78 \text{ Kg.m.h}^{-1}$$

$$\text{Viscosity of Fluid } (\eta) = 1.05 \text{ Kg.m.s}^{-1}$$

$$\text{Viscosity of Fluid } (\eta) = 1.05 \text{ Pa.s}$$

Calculation of Uncertainties

$$\Delta D_T = \pm \sqrt{[(\Delta D_r)^2 + (\Delta D_c)^2]}$$

$$= \pm \sqrt{[(0.005)^2 + (0.002)^2]}$$

$$= \pm 0.005 \text{ mm}$$

Where ΔD_T = Total uncertainty in particle diameter value
 ΔD_r = Reading uncertainty of the micrometer gauge
 ΔD_c = Calibration uncertainty of the micrometer gauge

Therefore the percentage error:

$$D = 3.47 \times 10^{-3} \pm 5 \times 10^{-6} m$$

$$D = 3.47 \times 10^{-3} m \pm 0.14\%$$

$$\Delta V_T = \pm 3 mm$$

Where ΔV_T = Total uncertainty in terminal velocity value

Therefore the percentage error:

$$V = 4.11 \times 10^{-2} \pm 3 \times 10^{-3} m$$

$$V = 4.11 \times 10^{-2} m \pm 7.30\%$$

Total Uncertainty & Resulting Value

$$\Delta \eta = \pm \sqrt{\left[\left(\frac{0.14}{100} \right)^2 + \left(\frac{7.30}{100} \right)^2 \right]}$$

$$= \pm \sqrt{5.33 \times 10^{-3}}$$

$$= \pm 0.0730$$

$$= \pm 7.30\%$$

Therefore

$$\Delta \eta = 0.077 Pa.s$$

The final value for viscosity of water using this method was:

$$\eta = 1.05 \pm 7.7 \times 10^{-2} Pa.s$$

Method 1 - Experiment 3 (30°C)

Ball Bearing	Distance Ball Bearing has traveled during Frame 0 -> 30 (cm)
1	6.59
2	6.92
3	6.53
4	6.85
5	6.69
6	6.70
7	6.74
8	6.77
9	6.84
10	6.71
Total (cm) :	67.34
Average Terminal Velocity (cm.s ⁻¹) :	6.73

Uncertainties

The uncertainties of taking measurements in this experiment were:

Reading uncertainty of the micrometer gauge = $\pm 0.005\text{mm}$

Calibration uncertainty of the micrometer gauge = $\pm 0.002\text{mm}$

Reading uncertainty of the velocity image = $\pm 3\text{ mm}$

Reading uncertainty of electronic thermometer = $\pm 0.1^\circ\text{C}$

Calibration uncertainty of electronic thermometer = $\pm 0.5^\circ\text{C}$

Temperature of water = $30 \pm 0.5^\circ\text{C}$

Calculations:

Stokes' Equation:

$$\eta = \frac{gD^2(\rho_p - \rho_f)}{18V_T}$$

Acceleration due to Gravity (g)	= 9.81	m.s ⁻²
Particle Diameter (D)	= 3.47 mm = 0.00347 m	
Density of Particle (ρ_p)	= 7.85 g.cm ³ = 7850000 g.m ³	
Density of Fluid (ρ_f)	= 1.262 g.cm ³ = 1262000 g.m ³	
Terminal Velocity (V_T)	= 6.73 cm.s ⁻¹ = 0.0673 m.s ⁻¹	

$$\eta = \frac{9.81 \times 0.00347^2 (7850000 - 1262000)}{18 \times 0.0673}$$

$$\eta = \frac{9.81 \times (1.20409 \times 10^{-5}) \times 6588000}{1.2114}$$

$$\eta = \frac{9.81 \times (1.20409 \times 10^{-5}) \times 6588000}{1.2114}$$

$$\eta = \frac{778.1826567}{1.2114}$$

$$\eta = 642.382909569$$

Solution:

$$\text{Viscosity of Fluid } \eta = 642.38 \text{ g.m.s}^{-1}$$

Other Units:

$$\text{Viscosity of Fluid } (\eta) = 2312.58 \text{ Kg.m.h}^{-1}$$

$$\text{Viscosity of Fluid } (\eta) = 0.64 \text{ Kg.m.s}^{-1}$$

$$\text{Viscosity of Fluid } (\eta) = 0.64 \text{ Pa.s}$$

Calculation of Uncertainties

$$\Delta D_T = \pm \sqrt{[(\Delta D_r)^2 + (\Delta D_c)^2]}$$

$$= \pm \sqrt{[(0.005)^2 + (0.002)^2]}$$

$$= \pm 0.005 \text{ mm}$$

Where ΔD_T = Total uncertainty in particle diameter value
 ΔD_r = Reading uncertainty of the micrometer gauge
 ΔD_c = Calibration uncertainty of the micrometer gauge

Therefore the percentage error:

$$D = 3.47 \times 10^{-3} \pm 5 \times 10^{-6} m$$

$$D = 3.47 \times 10^{-3} m \pm 0.14\%$$

$$\Delta V_T = \pm 3 mm$$

Where ΔV_T = Total uncertainty in terminal velocity value

Therefore the percentage error:

$$V = 6.73 \times 10^{-2} \pm 3 \times 10^{-3} m$$

$$V = 4.11 \times 10^{-2} m \pm 4.46\%$$

Total Uncertainty & Resulting Value

$$\Delta \eta = \pm \sqrt{\left[\left(\frac{0.14}{100} \right)^2 + \left(\frac{4.46}{100} \right)^2 \right]}$$

$$= \pm \sqrt{1.99 \times 10^{-3}}$$

$$= \pm 0.0446$$

$$= \pm 4.46\%$$

Therefore

$$\Delta \eta = 0.029 Pa.s$$

The final value for viscosity of water using this method was:

$$\eta = 6.4 \times 10^{-1} \pm 2.9 \times 10^{-2} Pa.s$$

Discussion & Evaluation

The above calculation assumes that the temperature stayed constant throughout the experiment; however this was an impossible task to complete. One method of keeping the glycerol's temperature constant would have been to do the experiment in a warm bath. However this was not possible as the container had to be clearly visible to the camcorder. Therefore the only option I was left with was to heat the glycerol before each run.

I originally completed the experiment using a digital camera to record the experiment; however the resolution & shutter speed was insufficient to give me accurate data. I therefore repeated the experiment using a camcorder to improve the data. Even though the camcorder was able to capture 30 frames per second, it did not give accurate velocities. This is because each frame was slightly blurred giving blurred i.e. enlarged image of the ball. A solution to this would be to either use a camcorder which can record high definition video. Another way how accuracy could be increased is to use a "high speed" camcorder, allowing quicker frames capturing ideally up to 1000 frames per second. The error could be also minimised by using a series of light gates. Above mentioned technical limitations were impossible to overcome in my experimental set up. In order to minimise error I took at least 10 readings per experiment and used average values.

The values obtained from the experiment are as follows:

Temperature of Glycerol	Value Calculated for Viscosity (Pa.s)	Known Value for Viscosity (Pa.s)
$20 \pm 0.5^{\circ}\text{C}$	1.56 ± 0.17	1.495
$25 \pm 0.5^{\circ}\text{C}$	$1.05 \pm 7.7 \times 10^{-2}$	0.942
$30 \pm 0.5^{\circ}\text{C}$	$0.64 \pm 2.9 \times 10^{-2}$	0.622

The main reason behind the difference between experiment and know values in my opinion, is the lack of clarity in the camcorder video. However the temperature changing slightly during the experiment would have had a significant effect on the terminal velocity of the steel ball.

Method 2:

Measuring Viscosity of Water Using a Capillary Tube and Poiseuille's Equation

Aim:

To calculate the viscosity of water through using the capillary method and Poiseuille's Law and then compare results with known values for viscosity.

Method:

Apparatus was set up as shown below in the image below (Fig. 4).

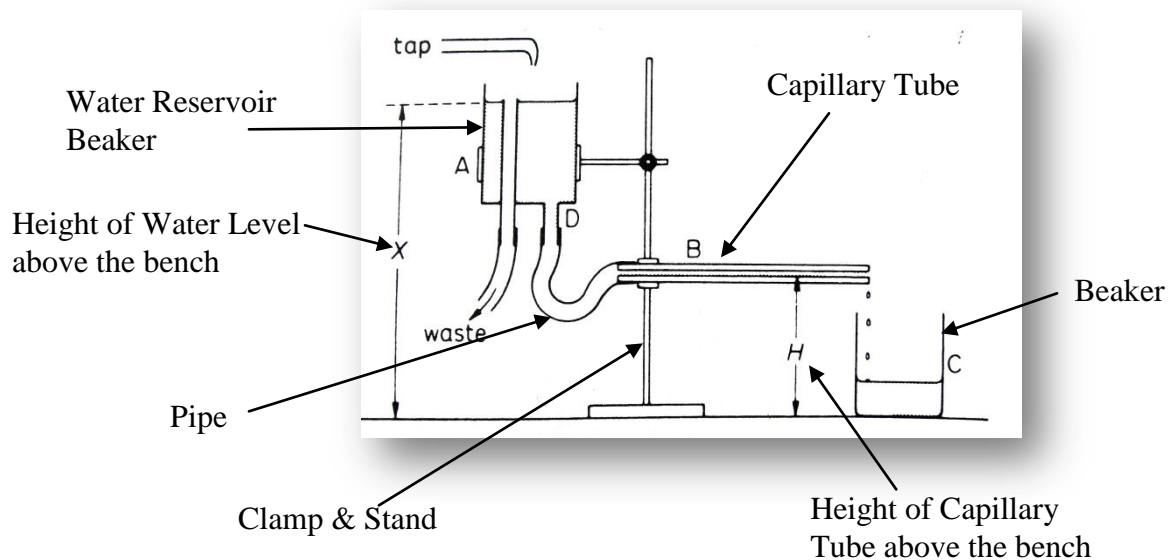


Fig4. Experiment Apparatus

I used the apparatus to get data for calculating viscosity of water. The result was compared to the figure **1.307 milliPas** (at 10°C) (Ref 8.) found in the official tables. The comparison of both allowed me to estimate, the accuracy of my calculation of water viscosity.

Beaker A was placed below the tap and a waste outlet was connected in, to allow for the waste water to flow to the sink. The height of the water level above the work bench was measured by a metal ruler repeated 3 times and average value taken to the calculation. A tube was used to connect the capillary tube (B) to the beaker. A spirit level

was used to make sure that the capillary tube was levelled. The tap was then turned on to allow for a steady stream of water to come out of the waste tube. The height of the capillary tube was similarly measured 3 times using a metal ruler and average taken. The beaker used to collect the water from the capillary was washed out and dried. Dry beaker was then weighed on a digital scale to get a “before” mass of the beaker. Drops from the capillary tube were allowed to drip into the beaker for 5 minutes. The beaker was then weighed again to find the volume of water that had flowed through the capillary tube during the 5 minutes. The temperature of the water was monitored using a digital thermometer throughout the experiment to make sure it was constant; it stayed at 10.1°C. At the end of the experiment the length of capillary tube was measured 5 times using a metal ruler also the diameter of capillary tube was measured 3 times using a travelling microscope to minimise error.

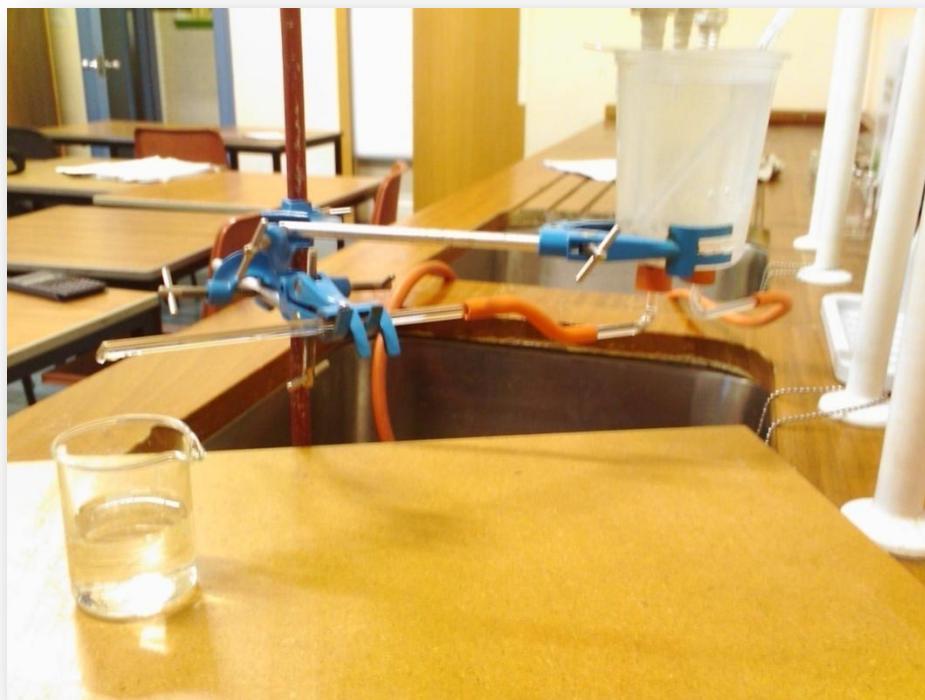


Fig5. Experiment Setup

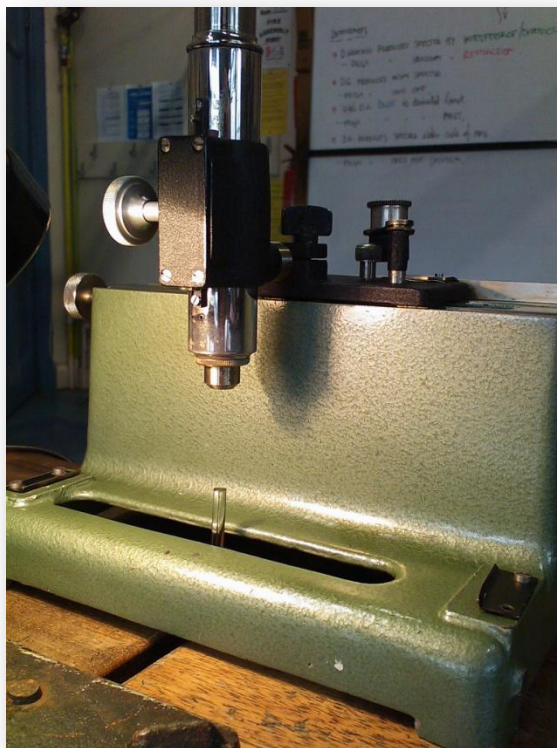


Fig6.Measuring the Diameter of the Capillary Tube

Measurements

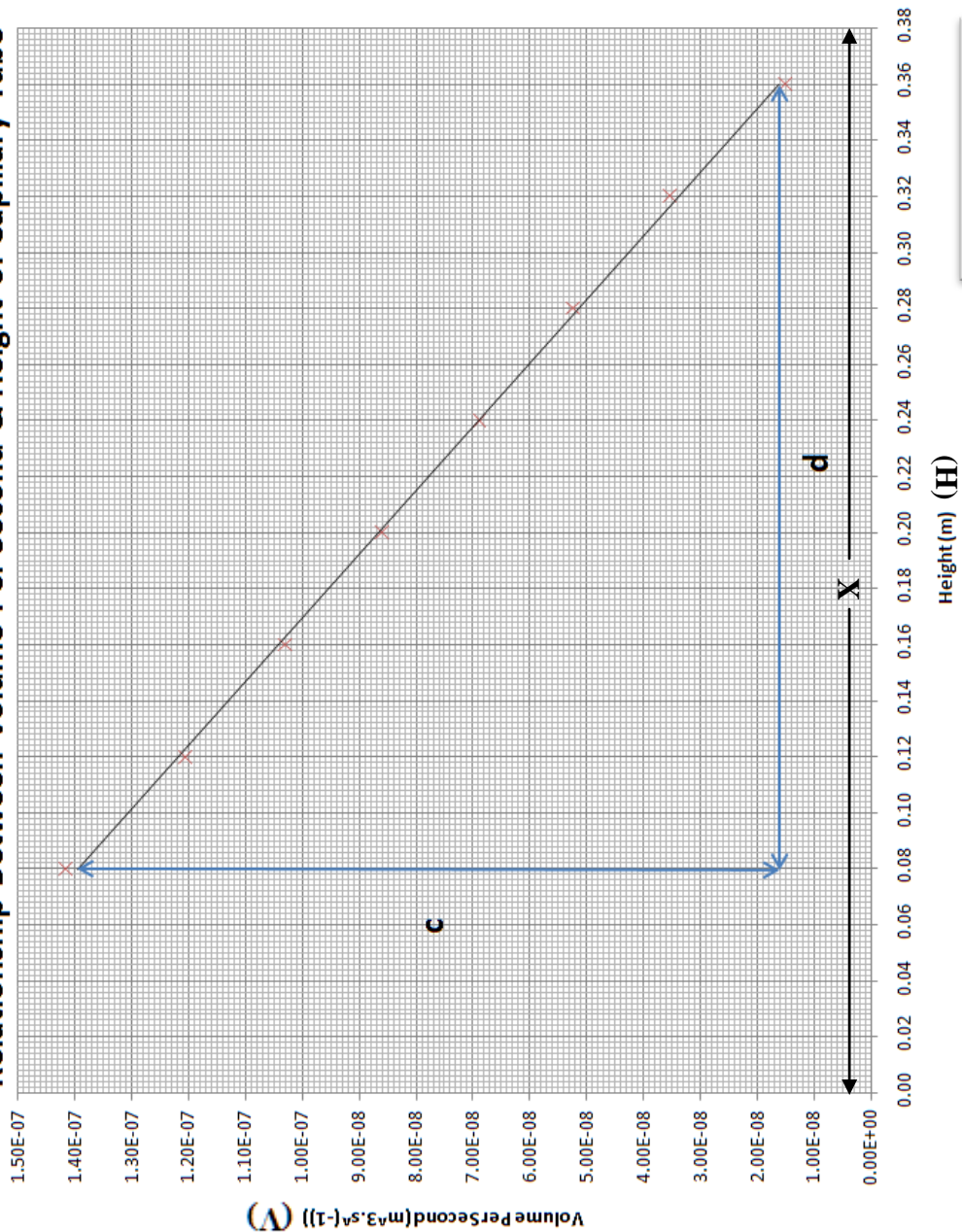
Temperature of water: 10.1°C
 Height of water level (X): 37.8 cm 0.378m
 Length of Capillary tube (l): 18.2cm 0.182m
 Diameter of Capillary tube (O):

Measurement Attempt	Length on Travelling Microscope at point A (cm)	Length on Travelling Microscope at point B (cm)	Diameter (mm)
1	5.451	5.367	0.84
2	5.486	5.403	0.83
3	5.793	5.708	0.85
		Average	0.84

Results

Height (cm)	Height (m)	Time (s)	Mass Before (g)	Mass After (g)	Volume of Water (cm ³)	Volume Per Second (cm ³ .s ⁻¹)	Volume Per Second (m ³ .s ⁻¹)
8	0.08	300	55.747	98.220	42.473	0.1416	1.416E-07
12	0.12	300	50.067	86.214	36.147	0.1205	1.205E-07
16	0.16	300	51.608	82.488	30.880	0.1029	1.029E-07
20	0.20	300	62.118	87.907	25.789	0.08596	8.596E-08
24	0.24	300	47.347	67.990	20.643	0.06881	6.881E-08
28	0.28	300	50.672	66.414	15.742	0.05247	5.247E-08
32	0.32	300	52.774	63.370	10.596	0.03532	3.532E-08

Relationship Between Volume Per Second & Height of Capillary Tube



(Fig5.)

$$\text{Slope} = \frac{c}{d}$$

$$\text{Slope} = \frac{(1.395 \times 10^{-7} - 1.60 \times 10^{-8})}{(0.36 - 0.08)}$$

$$\text{Slope Constant} = 4.411 \times 10^{-7}$$

Uncertainties

The uncertainties of taking measurements in this experiment were:

Reading uncertainty of top pan balance	= ± 0.001g
Calibration uncertainty of top pan balance	= ± 0.005g
Reading uncertainty of the metal ruler	= ± 0.5mm
Calibration uncertainty of the metal ruler	= ± 0.5mm
Reading uncertainty of travelling microscope	= ± 0.005mm
Reading uncertainty of electronic thermometer	= ± 0.1°C
Calibration uncertainty of electronic thermometer	= ± 0.5°C

Temperature of water = $10.1 \pm 0.5^\circ\text{C}$

Calculation

The water level is at a constant of constant X m (0.38m) above the bench the difference between capillary and the water level is X-H m. The graph above, where volume of water per second (V) is plotted against H, has a straight line due to Poiseuille's formula:

$$V = \frac{\pi \rho a^4}{8\eta l} = \pi \frac{(X-H)\rho_w g a^4}{8\eta l}$$

(Ref 9.)

Therefore:

$$\text{Slope} = \frac{c}{d} = \frac{\pi \rho a^4}{8\eta l}$$

$$\eta = \frac{\pi \rho_w g a^4}{8l(\frac{c}{d})}$$

Values:

Acceleration due to Gravity (g)	= 9.81	m.s ⁻²
Radius of Capillary Tube (a)	= 0.42 mm	= 0.00042 m
Density of Water (ρ_w) (Ref 10.)	= 999.7	kg.m ³
Length of Capillary Tube (l)	= 18.2 cm	= 0.182 m
Slope Constant	= 4.411 x 10 ⁻⁷	

$$\eta = \frac{\pi \times 999.7 \times 9.81 \times 0.00042^4}{8 \times 0.182 \times (4.411 \times 10^{-7})}$$

$$\eta = \frac{9.58707 \times 10^{-10}}{6.422 \times 10^{-7}}$$

Solution:

$$\eta = 1.49 \times 10^{-3} \text{ N s m}^{-2}$$

Other Units:

$$\text{Viscosity of Fluid } (\eta) = 1.49 \times 10^{-3} \text{ Pa.s}$$

Calculation of Uncertainties

$$\begin{aligned} \Delta m_T &= \pm \sqrt{[(\Delta m_r)^2 + (\Delta m_c)^2]} \\ &= \pm \sqrt{[(0.01)^2 + (0.005)^2]} \\ &= \pm 0.011g \end{aligned}$$

Where Δm_T = Total uncertainty in mass of water value

Δm_r = Reading uncertainty of balance

Δm_c = Calibration uncertainty of balance

Therefore the maximum percentage error:

$$m = 10.596 \pm 0.011g$$

$$m = 10.596g \pm 0.10\%$$

$$\begin{aligned} \Delta l_T &= \pm \sqrt{[(\Delta l_r)^2 + (\Delta l_c)^2]} \\ &= \pm \sqrt{[(0.5)^2 + (0.5)^2]} \\ &= \pm 0.7mm \end{aligned}$$

Where Δl_T = Total uncertainty in length of the capillary tube value

Δl_r = Reading uncertainty of the metal ruler

Δl_c = Calibration uncertainty of the metal ruler

Therefore the percentage error:

$$l = 0.182 \pm 7 \times 10^{-4} m$$

$$l = 0.182 m \pm 0.38\%$$

$$\Delta o_T = \pm 0.005 \text{ mm}$$

Where Δo_T = Total uncertainty in radius of the capillary tube value

Therefore the percentage error:

$$o = 8.4 \times 10^{-4} \pm 5 \times 10^{-6} m$$

$$o = 8.4 \times 10^{-4} m \pm 0.60\%$$

Total Uncertainty & Resulting Value

$$\Delta \eta = \pm \sqrt{\left[\left(\frac{0.10}{100} \right)^2 + \left(\frac{0.38}{100} \right)^2 + \left(\frac{0.60}{100} \right)^2 \right]}$$

$$= \pm \sqrt{5.14 \times 10^{-5}}$$

$$= \pm 0.0072$$

$$= \pm 0.72\%$$

Therefore

$$\Delta \eta = 0.011 \times 10^{-3} Pa.s$$

The final value for viscosity of water using this method was:

$$\eta = 1.49 \times 10^{-3} \pm 0.01 \times 10^{-3} Pa.s$$

Discussion & Evaluation

During the experiment above I assumed the full length of the capillary tube had exactly the same radius. This was impossible to verify as it could not be checked from an end and it was impossible to measure the radius, accurately from the side due to light diffraction in the glass.

The value, of $1.49 \times 10^{-3} \pm 0.01 \times 10^{-3} \text{ Pa.s}$, obtained from the experiment is very close to the accepted value of $1.307 \times 10^{-3} \text{ Pa.s}$ at 10°C .

The main reason for the error is when measurement of the radius of the capillary tube. This error can be minimised by the use of another method to measure the radius. An alternate method would be to weigh the capillary tube, then fill it with mercury and weigh it again to find the weight of the mercury in the capillary tube. The length of the mercury string would also need to be measured. Because we know the density of mercury we could use the equation below to get a more accurate radius.

$$r = \sqrt{\frac{m}{\pi \cdot L \times 13600}}$$

Where m is the mass of the mercury in the capillary tube, L is the length of the mercury inside the capillary tube & 13600 is the density of mercury.

Investigation Conclusion & Evaluation

I have conducted this investigation in order to find out which of two methods of liquid viscosity measurement would produce most accurate results. The first method of measurement used terminal velocity and Stokes law has an advantage of very easy set up however, it is very instrumentally demanding. On the other hand the second capillary method requires more complicated experimental set up and calibration of instruments. The capillary method was more accurate as the viscosity, calculated on the basis of data from the methods, was very close to the theoretical values and values found in tables.

Despite its drawbacks first method used a very simple principle and does not require lengthy preparation. This could be an advantage when viscosity of an unknown liquid need be roughly established. The technical limitations of the camera, the temperature fluctuation and graphic software used to measure distance will improve over the time as equipment will improve in the future. During collating the results of the study I did, I found that Image Analysis programs might be more appropriate to measure distance on images rather than graphical software. Of course it is possible to think about other means how to measure speed of object freely falling in the liquid. Just by extending the distance of fall or the use more precise distance measurement, i.e. using laser would improve the accuracy of data.

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Ref 10. Constant taken from [www.simetric.co.uk](http://www.simetric.co.uk/si_water.htm)
http://www.simetric.co.uk/si_water.htm

Figures

Fig 1. Diagram taken from www.wikipedia.org

http://upload.wikimedia.org/wikipedia/commons/a/ae/Stokes_sphere.svg

Fig 2. Diagram drawn By Jakub Srsen

Fig 3. Photo of Experiment by Jakub Srsen

Fig 4. Diagram taken from “Advanced Level Practical Physics *Fourth Edition*” By *M. Nelkon and J. M. Ogborn*

Fig 5. Graph by Jakub Srsen

Fig 6. Graph by Jakub Srsen