Physics with a car headlamp and a computer

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An experiment suitable for high school students or undergraduates uses a car headlamp. With the use of a spreadsheet program, extensive data manipulation becomes a simple task, enabling students to answer relevant questions as opposed to just verifying a well-known law. By using the computer in this way, students can be given an awareness of many mathematical techniques used in data analysis not otherwise possible because of time constraints or students lack of knowledge. Templates of varying degrees of complexity can be used to cater for different groups of students.

Experiments investigating the temperature of tungsten filaments have been previously described in the literature by Prasad and Mascarenhas (1978) and Wagner (1991). Such experiments can be extended so that students can answer real-life questions using a common object. Typical questions include: How efficient is the halogen gas-filled lamp with a tungsten filament? What temperature does the filament reach? In what part of the electromagnetic spectrum is most of the radiation emitted from the hot lamp?

The experiment would not be feasible without the use of a computer program such as a spreadsheet because students need to do a large amount of data manipulation and analysis in answering the above questions. Students can use spreadsheets to explore physical phenomena in much more detail by using a variety of data analysis techniques such as extrapolation, goal seeking to find an unknown value, curve fitting, regression and numerical integration.

Theory

When a lamp has reached a steady state, the electrical power dissipated in the filament must be equal to the total rate of heat lost from the filament. The mechanisms of heat transfer can be classified as conduction, convection and radiation. Conduction occurs through the ends of the tungsten filament. Energy is transferred through the gas by both conduction and convection. Electromagnetic radiation is emitted from the hot filament and its glass envelope. When the lamp is glowing we can assume that most of the energy is lost by radiation and so the electrical power dissipated in the filament, $P_{el} = VI$, is effectively equal to the power lost by radiation, P_{rad}.

Measurements of the filament resistance, $R = V/I$, can be used to estimate the absolute temperature of the filament, T. The following empirical relationship (Prasad and Mascarenhas 1978) can be validated in this experiment:

$$T = T_S(R/R_S)^m \qquad (1)$$

where T and T_S are the temperatures for resistance values of R and R_S respectively and m is a constant whose value can be found from the experimental results.

The hot tungsten filament behaves as a greybody and the net power radiated, to a good approximation, is described by the Stefan–Boltzmann law:

$$P_{rad} = \epsilon A\sigma(T^4 - T_S{}^4) \qquad (2)$$

where the filament is at a temperature T and the surrounding room temperature is T_S, ϵ is the emissivity of the filament s surface, A is the area of the surface from which the radiation is emitted and σ is the Stefan–Boltzmann constant. For a blackbody surface, $\epsilon = 1$; for other surfaces, the emissivity is a complicated function of temperature, environment and fabrication (Incropera and De Witt 1990). However, to simplify the analysis the following assumptions are made about the filament: its emissivity is constant and independent of the temperature, it has a uniform surface temperature and all the radiation is emitted from its surface.

For the filament, the constants A and ϵ are unknown. However, these two unknown quantities can be eliminated by taking a ratio. If P_1 is the power when the filament just starts to glow brightly at a temperature T_1 ($T_1^4 >> T_S^4$) and P_i is the power at a higher temperature T_i then

$$P_i / P_1 = (T_i / T_1)^4. \quad (3)$$

Combining equations (1) and (3), it is possible to relate the power radiated to the resistance of the filament:

$$\log(P_i / P_1) = m[4\log(R_i / R_1)]. \quad (4)$$

If the predictions given by equations (1) and (2) are valid, a plot of $\log(P_i / P_1)$ against $4\log(R_i / R_1)$ will yield a straight line with slope m through the origin.

Assuming that the tungsten filament radiates as a greybody, then the spectrum of the emitted radiation will be given by the Planck distribution function (Eisberg and Resnick 1985):

$$p(\lambda)\mathrm{d}\lambda = \{N/[\lambda^5(e^{hc/\lambda kT} - 1)]\}\mathrm{d}\lambda \quad (5)$$

where $p(\lambda)\mathrm{d}\lambda$ is the radiant power emitted in the wavelength interval $\mathrm{d}\lambda$, N is a normalizing factor, λ is the wavelength of the emitted radiation, h is the Planck constant, c is the speed of light and k is the Boltzmann constant. The integral over all wavelengths of the function $p(\lambda)$ given by equation (5) equals the total power radiated by the hot filament at a temperature T:

$$P_{rad} = \int p(\lambda)\mathrm{d}\lambda. \quad (6)$$

With a spreadsheet this integral can be evaluated numerically and the value of N adjusted so that the integral is equal to the input power, P_{el}. The luminous efficiency of the lamp is defined as the ratio of the power radiated in the visible part of the spectrum to the input power. Thus, the efficiency of the lamp can be found by comparing the ratio of areas under the Planck distribution curve. With increasing temperature, the peak in the Planck distribution function shifts to shorter wavelengths. This dependence is given by Wien s displacement law:

$$\lambda_{max}T = 2.898 \times 10^{-3} \text{ m K} \quad (7)$$

where λ_{max} is the wavelength at which the curve peaks.

The experiment

Voltage and current measurements were made using a car headlight assembly (12 V, 60/50 W). These lamps are easy to obtain (often for no charge) from an automobile wrecker or autoelectrician. During the measurements, the potential difference should not be decreased as there may be insufficient time for the filament to cool because at higher current values the filament will be at a higher temperature and have a higher resistance. As the voltage is increased it is necessary to wait before the readings are recorded so that an equilibrium is established. It is necessary to make a few measurements of the voltages across the lamp for current values up to about 200 mA (a 100 Ω resistor may be placed temporarily in series with the filament to ensure low current values to prevent excessive heating of the filament). From these V–I data, the room temperature resistance, R_S, can be found by extrapolation of a R–I graph (figure 1) to find the resistance at zero current. The 100 Ω resistor was removed and then the potential difference across the lamp was increased to 12 V. Normally students take only a few data points. However, using a spreadsheet, it does not take long for students to manually enter 100 measurements of V and I. With a spreadsheet large data sets can be analysed and graphed just as easily as small data sets.

Analysis

Once the V and I data have been entered into a spreadsheet, columns can be created for R, P, $\log(P/P_1)$, $4\log(R/R_1)$, T, ($T^4 - T_S^4$) and λ_{max}. An advantage of using a spreadsheet is that the large amounts of data can be viewed graphically. Students can investigate many relationships between the physical parameters such as the V–I, R–I, P–I, T–P, T–I and λ_{max}–T characteristics.

Figure 1 shows the R–I characteristic for the lamp. Extrapolation can be used to find the resistance at

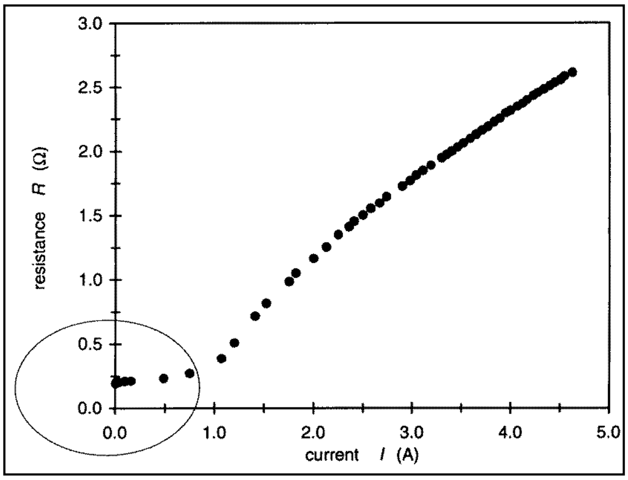

Figure 1. *The R–I characteristic of the filament. By extrapolation of the data the resistance at zero current can be estimated.*

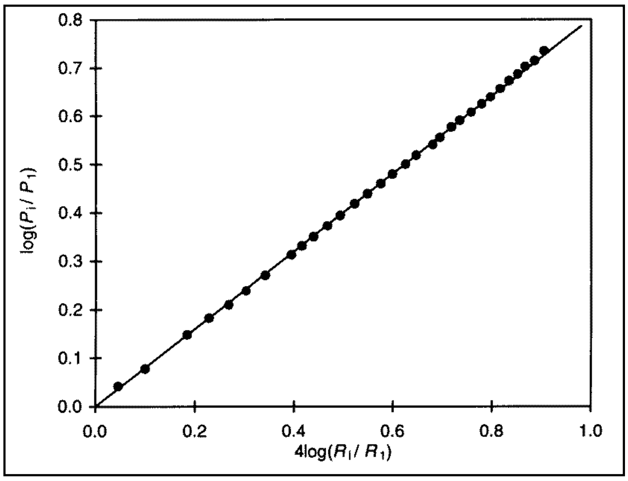

Figure 2. *A straight line through the origin supports the assumptions of the power-law relationship between termperature and resistance and the Stefan–Boltzmann law. A regression analysis gave the slope as 0.802 ± 0.001.*

zero current. This can be made easier by temporarily reducing the maximum current value on the graph. It is a good idea to subtract 0.01 Ω from the resistance value at zero current to account for the resistance of the leads and contacts to the lamp to give the room temperature resistance, R_S.

To ensure that the power lost by radiation is significantly greater than for the other mechanisms, the data for which the lamp is glowing are used to determine the temperature as a function of resistance. Figure 2 shows the graph of $\log(P_i/P_1)$ against $4\log(R_i/R_1)$ for one set of data. Using the curve-fitting and regression analysis tools, a straight line through the origin gave a very good fit. The results were in agreement with the prediction given by equation (4), and the regression analysis gave $m = 0.802 \pm 0.001$. Applying equation (1) to the results gave the relationship between temperature of the filament and its resistance as $T = T_S(R_i/R_S)^{0.802}$ where $T_S = 294$ K and $R_S = 0.19$ Ω. For the lamp used, the maximum temperature reached was 2400 K at a power of 55 W. The peak wavelength of the radiation was 1200 nm. This peak wavelength emitted was not in the visible part of the spectrum but occurs in the infrared. A graph of λ_{max} against P_{rad} indicated a shift towards shorter wavelengths as the power increased. Figure 3 shows the graph of P_{rad} against $(T^4 - T_S^4)$. A straight line through the origin fitted the data and thus supports the use of equation (2).

The luminous efficiency of the lamp was be found by using equation (5). The Planck distribution function was evaluated at the maximum temperature of 2400 K for 501 wavelength values from 200 nm to 9200 nm with $\Delta\lambda = (9200 - 200)/501$ nm. The area under this graph was determined numerically by simply adding the area of 501 rectangles given by multiplying each cell in the column for the Planck distribution function by $\Delta\lambda$. The value of the normalizing constant N was adjusted to make the area under the graph equal in value to the maximum electrical power of 55 W. This was easily accomplished by using a goal-seeking procedure within a spreadsheet; for example, in Microsoft *Excel* (Version 5), select Solver from the Tools Menu. The goal-seeking procedure automates the procedure of adjusting the value of N so that the area under the graph was changed to the desired value of 55 corresponding to the total power. A graph of the Planck distribution function for $T = 2400$ K is shown in figure 4. The vertical lines at 400 nm and 700 nm show the region corresponding to the visible part of the spectrum in which the human eye is sensitive too (Halliday *et al* 1993). The area under this portion of the Planck distribution function gave a power of 1.6 W. This corresponds to a luminous efficiency of only 3%, a very low value. Sixty-one first-year undergraduate physics students were asked to estimate the efficiency of the lamp tested. For this sample, the average predicted efficiency was 34% with a sample standard deviation of 23%. These students had done a traditional experiment on radiation, but they still had little idea that the efficiency of the lamp would be so low.

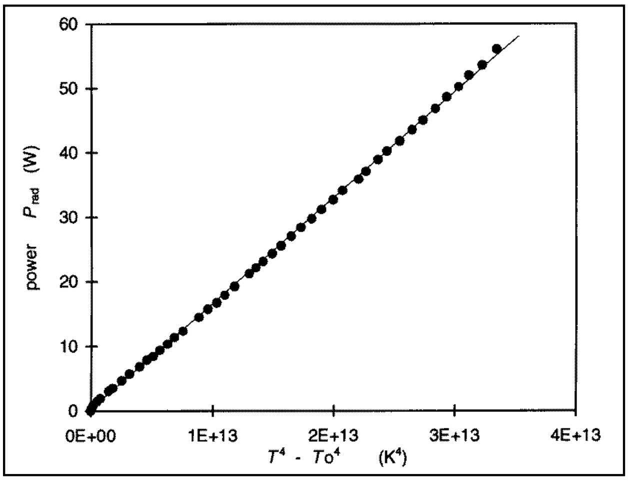

Figure 3. *The straight line through the origin supports the application of the Stefan–Boltzmann law to the hot tungsten filament.*

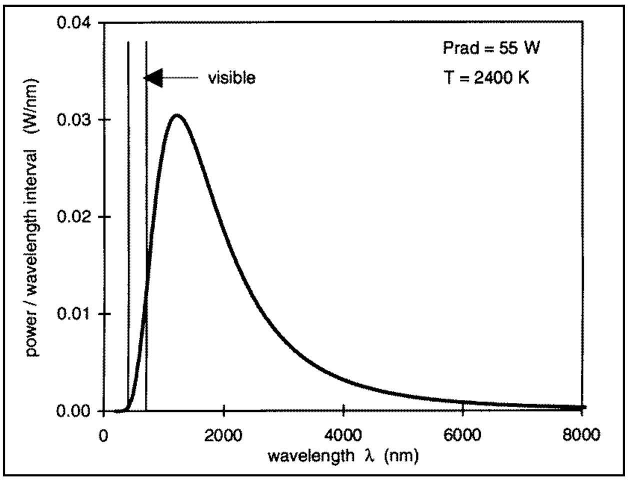

Figure 4. *The Planck distribution function for a temperature of 2400 K. The normalizing constant was adjusted so that the area under the curve was 55 W. The graph indicates that only a small proportion of the radiation is emitted in the visible part of the electromagnetic spectrum.*

Conclusion

The experiment described can indicate to students a way in which physicists can tackle a real-life problem such as finding the luminous efficiency of different types of lamps by applying some of the well established laws of physics with appropriate assumptions. The results of the experiment show that in normal operation the car headlamp tested dissipated a power of about 55 W at a filament temperature of about 2400 K with only about 3% of the radiation emitted in the visible part of the spectrum. Most of the radiation was emitted in the infrared and not in the visible part of the spectrum as desired. The lamp appears to be bright because our eyes are highly sensitive to the very small intensity of the visible radiation emitted by the hot filament (Jain 1996).

The chief reason for using a spreadsheet is that it enables large amounts of data to be easily manipulated, analysed and graphed. Students can gain a better insight into the physical principles involved and develop their skills in data analysis by doing such experiments with spreadsheets. Templates can be designed with different degrees of complexity to suit students at various levels of sophistication. Good undergraduate students could construct the entire spreadsheet, while school students can be given a nearly complete template.

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