

# Fluid Mechanics: Stokes' Law and Viscosity

## Measurement Laboratory

### Investigation No. 3

*Scott A. Shearer, Professor*

*Jeremy R. Hudson, Graduate Teaching Assistant*

*Biosystems and Agricultural Engineering*

## 1. Introduction

This laboratory investigation involves determining the viscosity and mass density of an unknown fluid using Stokes' Law. Viscosity is a fluid property that provides an indication of the resistance to shear within a fluid. Specifically, you will be using a fluid column as a viscometer. To obtain the viscometer readings you will use a stopwatch to determine the rate of drop of various spheres within the fluid. You will determine both density and viscosity.

## 2. Learning Outcomes

On completion of this laboratory investigation students will:

Appreciate the engineering science of 'fluid mechanics.'

Understand the concept of fluid 'viscosity.'

Understand the concept of dimensionless parameters, and most specifically the determination of Reynold's Number.

Be able to predict the settling time of spheres in a quiescent fluid.

Be able to calculate the viscosity of an unknown fluid using Stokes' Law and the terminal velocity of a sphere in this fluid.

Be able to correct for the diameter effects of fluid container on the determination of fluid viscosity using a 'falling ball' viscometer.

## 3. Definitions

**Fluid** – a substance that deforms continuously when subjected to a shear stress.

**Viscosity** – a fluid property that relates the shear stress in a fluid to the angular rate of deformation.

**Fluid Mechanics** – the study of fluid properties.

**Reynold's Number** – dimensionless parameter that represents the ratio of viscous to inertial forces in a fluid.

#### 4. Stokes' Law



Figure 1: George Gabriel Stokes

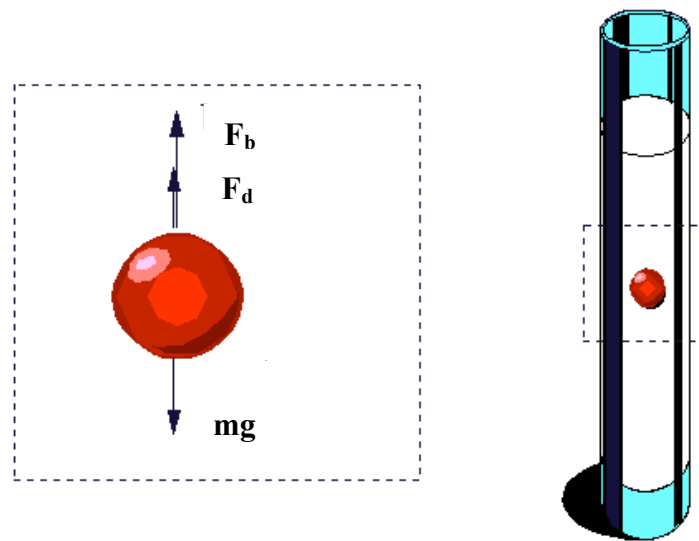
George Gabriel Stokes, an Irish-born mathematician, worked most of his professional life describing fluid properties. Perhaps his most significant accomplishment was the work describing the motion of a sphere in a viscous fluid. This work led to the development of Stokes' Law, a mathematical description of the force required to move a sphere through a quiescent, viscous fluid at specific velocity. This law will form the basis of this laboratory investigation.

Stokes' Law is written as,

$$F_d = 6\pi\mu Vd$$

where  $F_d$  is the drag force of the fluid on a sphere,  $\mu$  is the fluid viscosity,  $V$  is the velocity of the sphere relative to the fluid, and  $d$  is the diameter of the sphere. Using this equation, along with other well-known principle of physics, we can write an expression that describes the rate at which the sphere falls through a quiescent, viscous fluid.

To begin we must draw a free body diagram (FBD) of the sphere. That is we must sketch the sphere and all of the internal and external forces acting on the sphere as it is dropped into the fluid. Figure 2 shows a sketch of the entire system (sphere dropping through a column of liquid). The FBD is the dashed cross-section that has been removed and exploded in the left portion of this figure.



**Figure 2: Free-body diagram of a sphere in a quiescent fluid.**

The FBD in this figure lists three forces acting on the sphere;  $F_b$ ,  $F_d$ , and  $mg$ . The first two forces arise from the buoyancy effect of displacing the fluid in question, and from the viscous drag of the fluid on the sphere, respectively. Both forces act upwards -- buoyancy tending to 'float' the sphere ( $F_b$ ) and the drag force ( $F_d$ ) resisting the acceleration of gravity. The only force acting downwards is the body force resulting from gravitational attraction ( $mg$ ). By summing forces in the vertical direction we can write the following equation,

$$F_b + F_d = mg$$

The buoyancy force is simply the weight of displaced fluid. As you may recall from earlier work in science and math, the volume of a sphere ( $v_{sphere}$ ) is written as,

$$v_{sphere} = \frac{4}{3}\pi r^3$$

Combining this volume with the mass density of the fluid,  $\rho_{fluid}$ , we can now write the buoyancy force as the product ,

$$F_b = m_{df} g = \frac{4}{3}\pi r^3 \rho_{fluid} g$$

where  $g$  is the gravitational acceleration and  $r$  is the radius of the sphere. Combining all of the previous relationships that describe the forces acting on the sphere in a fluid we can write the following expression,

$$\frac{4}{3}\pi r^3 \rho_{fluid} g + 6\pi\mu Vd = mg$$

Rearranging and regrouping the terms from the above equation we arrive at the following relationship,

$$V = \frac{2r^2(\rho_{sphere} - \rho_{fluid})g}{9\mu}$$

While Stokes' Law is straight forward, it is subject to some limitations. Specifically, this relationship is valid only for 'laminar' flow. Laminar flow is defined as a condition where fluid particles move along in smooth paths in lamina (fluid layers gliding over one another). The alternate flow condition is termed 'turbulent' flow. This latter condition is characterized by fluid particles that move in random in irregular paths causing an exchange of momentum between particles.

Engineers utilize a dimensionless parameter known as the Reynold's number to distinguish between these two flow conditions. This number is a ratio between the inertial and viscous forces within the fluid. Engineering students will learn more about the origin of this parameter – the Buckingham Pi Theorem – in the final two years of the curriculum. For now we will define the Reynold's number as,

$$N_R = \frac{\rho Vd}{\mu}$$

where  $N_R$  is Reynold's Number,  $\rho_{fluid}$  is the mass density of the fluid,  $V$  is the velocity of the fluids relative to the sphere, and  $d$  is the diameter of the sphere.

The application of the Reynold's Number to fluids problems is to determine the nature of the fluid flow conditions – laminar or turbulent. For the case where we have a viscous and incompressible fluid flowing around a sphere, Stokes' Law is valid providing the Reynold's Number has a value less than 1.0. When utilizing Stokes' Law, it is appropriate to verify the application of this law is appropriate.

## 5. Falling Ball Viscometers

The falling ball viscometer is based on Stokes' Law, and is what we will use in this laboratory investigation. This type of viscometer consists of a circular cylinder containing the fluid and a smooth ball. The ball is placed in the fluid and the time that it takes to fall the length of the cylinder is recorded. This time is then utilized to back the viscosity out of the velocity relationship that we derived using Stokes' Law and summing forces. As the ball is dropped into the fluid it accelerates as a result of the gravitational field until the ball reaches terminal velocity. Terminal velocity occurs

when the viscous and buoyancy forces equal the weight of the ball. At this point the velocity of the ball is maximum, or terminal. To simplify our approach, we will allow the ball to reach terminal velocity prior to making the time measurements.

## 6. Laboratory Procedures

### Part I: Determine the viscosity of an unknown fluid

- 1) At your lab station you will find several different sizes of spheres of different materials. The materials are brass, Teflon, and glass. For the first procedure you need to use the largest of the Teflon spheres.
- 2) Using the micrometer determine the diameter of the largest Teflon sphere to the nearest 0.001 inch. You must convert this measurement to SI units (Hint: 1.00 in. equals 2.54 cm.) Next using the digital scales, find the mass of the Teflon sphere to the nearest 0.01 g. You can now use these two numbers to determine the density of the Teflon sphere ( $\text{g/cm}^3$ ).
- 3) Next you will need to measure the fall time of the sphere through the fluid in the 2000-mL graduated cylinder (to the nearest 0.01 s). To do this use the stopwatch to measure the amount of time it takes for the sphere to fall from the 1600-mL mark to the 400-mL mark.
- 4) Repeat steps for the remaining two Teflon spheres of that size.
- 5) Measure the distance using the ruler between the 1600-mL graduation line and the 400-mL graduation line.
- 6) Now using the time recorded from the stopwatch for each sphere dropped and the distance measurement between the graduation lines, determine the velocity of each sphere as it passed through the fluid ( $\text{cm/s}$ ). You will need to use the steel scale to determine the distance between the 400 and 1600 ml marks
- 7) Using Stokes' Law provided in the lab manual, determine the viscosity ( $\text{m}$ ) of the fluid using the average velocity of the three spheres. A common unit of viscosity is the Poise, or  $1 \text{ g/cm}\cdot\text{s}$ .
- 8) Calculate the Reynold's Number using the fluid and ball properties determined above.

### Part II: Predict the fall time of similar size spheres of differing materials:

- 1) Using the micrometer, determine the diameter of the largest glass and brass spheres (nearest 0.001 in.). There should be three of each and these diameters should be roughly the same as the diameters of the Teflon spheres used earlier.

- 2) Next use the digital scales to determine the mass of each of the spheres (nearest 0.01 g). Using the mass and diameter measurements, calculate the density of the glass and brass spheres ( $\text{g/cm}^3$ ).
- 3) Now using your calculated viscosity for the unknown fluid, use Stokes Law to determine the velocity for the spheres (cm/s). Using the velocity and the distance between the 1600-mL and the 400-mL graduation lines determine the fall time for the spheres.
- 4) Next confirm your predicted fall times by timing each of the spheres falling through the fluid.

### **Part III: Predict the fall time of differing size spheres of similar material**

- 1) Determine the diameters of the remaining six glass spheres. And measure their respective individual masses in the electronic scale. Use these measurements to confirm the density of the glass spheres.
- 2) Now using your calculated viscosity for the unknown fluid, use Stokes Law to determine the velocity for the spheres. Using the velocity and the distance between the 1600-mL and the 400-mL graduation lines determine the fall time for the spheres.
- 4) Next confirm your predicted fall times by timing each of the spheres falling through the fluid.

### **Part IV: Determine an unknown fluid**

- 1) For this procedure you will need the remaining six Teflon spheres and the smaller fluid filled graduated cylinder.
- 2) As before measure the diameters and masses for each of the Teflon spheres and confirm the density of Teflon.
- 3) Next determine the mass of the fluid in the graduated cylinder and the volume. The mass of the graduated cylinder while empty will be provided in lab. Using the volume and the mass of the fluid, calculate the density.
- 4) Now as before use the stop watch to measure the fall time between two graduation lines on the graduated cylinder. Which lines you use are of your own choosing. Repeat for all six of the spheres. **Note:** For small diameter fluid columns there is an interaction between the fluid and the wall of the cylinder. For this reason you must correct for the diameter interaction using the following relationship,

$$\mu_c = \mu \left[ 1 - 2.104 \frac{d}{D} + 2.09 \left( \frac{d}{D} \right)^3 - 0.95 \left( \frac{d}{D} \right)^5 \right]$$

where  $\mu_c$  is the corrected viscosity, and  $D$  is the internal diameter of the cylinder.

- 5) Using Stokes Law determine the viscosity of the unknown fluid by using the average velocities of each of the two different size spheres.

## 7. Concluding Questions

- 1) In Part I, Step 8, what is the value of the Reynold's Number, and using this value is Stokes' Law valid? Why, or why not?
- 2) In Part II, Step 4, were there any difficulties in measuring the fall times of the brass spheres? Would increasing the diameter of the brass sphere make the problem worse or better?
- 3) In Part III, Step 4, how did your predicted fall times compare to measured fall times? What were the possible sources of error if any that occurred?
- 4) Given your calculated density and viscosity of your unknown fluid in Part IV, confirm your findings with the lab TA to identify your unknown fluid. Who did your results compare to the data from the TA?
- 5) In the lab manual there is a formula listed for Stokes Law that contains a correction factor relating the diameter of the sphere and the diameter of the graduated cylinder. Measure the diameter of the graduated cylinder and determine the corrected fall times for the two different size spheres in Part IV?
- 6) Was there a significant difference between the corrected values for fall times and the non-corrected values? How much did the diameter of the graduated cylinder influence the fall time of the sphere?