

The Bifilar Pendulum

Preparatory Task:

Read the entire script; familiarise yourself with the objectives of this experiment and the skills required; and write a brief plan in your lab book of what you will be doing, task by task. Then answer the following question in your lab book:

Pressure is a force (F) per unit area (A). Given this, and the fact, from Newton's second law, that a force has the physical dimensions of a mass times an acceleration, use dimensional analysis to work out the units of pressure. [HINT: an acceleration is a change in velocity (m s^{-1}) per unit time.]

1. What it's about

This practical explores the use of dimensional analysis in the formulation of an equation. You will set up an equation giving the period of oscillation of a complex pendulum, using dimensional analysis to ensure that the equation is dimensionally correct. You will then determine experimentally the relevant exponents in the proposed equation, taking multiple, repeated measurements and doing a statistical analysis of the data. In the process, you will gain experience in plotting and fitting linearised data.

2. Preparation

The international system of units is known as SI units, and is based on the seven units listed in Table 1. Apart from the radian, all other units in Physics are derived from these basic building blocks. Derived units can look rather complicated when expressed in terms of the base units; for convenience, they are often given special names and symbols. (For example, the unit for resistance is the ohm, Ω , which is equivalent to $1 \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$.)

Quantity	Unit name	Symbol
Mass	kilogram	kg
Length	meter	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of a substance	mole	Mol
Luminosity	candela	cd

Table 1: The seven base units of the SI system.

Dimensional analysis is a technique used both to check the validity of equations and to find relationship between physical quantities. The basic principle of dimensional analysis is “for two quantities to be equal, they must have the same dimensions.” That is the object on the left hand side of an equation must have the same physical dimensions as that on the right. For instance, since the force acting on an object is equal the mass of the object multiplied by its acceleration, the physical dimensions of a force must be that of a mass multiplied by an acceleration. Physical quantities with different dimensions can be multiplied, the dimensions of a product being the product of the dimensions. However, physical quantities with different dimensions cannot be added to one another (for instance, it does not make any sense to add a temperature to a length.)

3. Ready to Start

A bifilar suspension pendulum is one in which two (bi) filaments (filar) support a rod. A schematic of this arrangement is shown in Figure 1. **It is important to note that in most of this practical we will consider the situation where the strings hang vertically, parallel to each other, rather than the situation shown in the diagram where the strings hang in oblique.**

If this bar is suspended **symmetrically** in the horizontal plane by two strings of equal length and set to swing **about a vertical axis through its centre**, the period of the swing may depend upon some, or all of the following quantities that define the system:

1. the difference in height between the bar and the support, z ;
2. the distance apart of the upper ends of the strings, x ;
3. the distance apart of the lower ends of the strings, y ;
4. the mass of the suspended bar, m ; and
5. the length of the suspended bar, l .

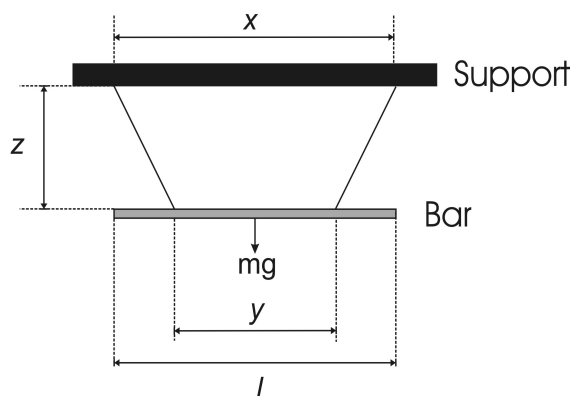


Figure 1: Schematic of a bifilar pendulum. In most of this practical, the lengths x and y are to be equal, contrary to what is indicated in the figure.

As noted above, **for most of this investigation we will consider the simplified situation where x and y are equal and the suspension filaments are vertical.** We thus assume that $x = y$ in the following. Finding how the period of oscillation of this pendulum depends on z , x , m and l by starting from Newton's second law and solving the resulting equations of motion would require complicated calculations. However, one can say something about this dependence from dimensional analysis. To start, let us guess that the period can be written as

$$T = A z^a x^b m^d l^e, \quad (1)$$

where A is a dimensionless constant and a , b , d and e are unknown exponents.

Task 1:

What are the physical dimensions of the left hand side of Equation 1?

What are the physical dimensions of the right hand side of Equation 1?

Task 2:

What other physical quantity would it be reasonable to introduce on the right hand side so that the dimension of time is involved. To what power does this quantity have to be raised? [HINT: Consider the equation that describes a simple pendulum.]

Task 3:

What must the value of the power of m be - i.e., the exponent d in Equation (1) - for this equation to balance dimensionally?

Task 4:

What is the relation that now exists between the powers a , b , and e to balance dimensions after introducing the dimension of time to the correct power?

4. Experiment

The objective here is to find the value of the dimensionless constant A and the individual values of the powers of z , x and l , which the above analysis does not yield. The bar must be suspended symmetrically in a horizontal position by two threads of equal length. **Make sure that the two threads are vertical, so that $x = y$.**

To make your measurements you will set the bar into oscillations about a VERTICAL AXIS THROUGH ITS CENTRE and record the period of oscillation using the stop watch. **(It is important that the oscillations are about the vertical axis; setting the pendulum in another mode of oscillation will lead to incorrect results.)** For each set of parameters (x , z , l , m) you should take a minimum of four measurements of the period and calculate the mean, the standard deviation and the standard error. Do not start taking these measurements yet – you first need to sort out several details.

Task 5:

- *How will you check that the bar is suspended horizontally?*
- *To what magnitude of angle should you limit the swings?*
- *How many swings will you time in order to obtain the period?*
- *How many repeat readings will you take for each set-up? (You must take enough readings to reach a definite conclusion about the values of the unknown powers, which you can assume to be simple fractions.)*
- *How will you extract the unknown powers experimentally? [HINT: Starting from Equation (1), express the logarithm of T in terms of the logarithms of A , z , x , m and l .]*

Remember that all measurements **must** be recorded directly into your lab book with the errors clearly indicated. To this effect, it would be useful to draw up suitable tables before you start collecting data.

We'll begin by measuring the period for different lengths of the bar, in order to find the exponent e . Of course, the mass of the bar will vary, but we know that this should not affect the period of oscillation.

Task 6:

*Find the period of oscillation as a function of the length of the bar, keeping z and x constant. [HINT: x and y should be symmetric about the centre, and smaller than the shortest bar!]. After recording your data, use an Excel spreadsheet to calculate the mean, the standard deviation and the standard error on the mean for the period of oscillation, for each length of the bar. **(Don't do this by hand.)** Copy the values of the period and its error into your lab book.*

Then, plot the logarithm of the period against the logarithm of the length using Excel. Remember to include error bars in this plot. Add a trend line and stick the graph in your lab book. Find the slope of the resulting graph, along with the error in the slope, using the LINEST function of Excel. What is the value of the exponent e in Equation (1)?

Task 7:

Now determine the powers of z and x , i.e., the exponents a and b in Equation (1), by varying one quantity at a time and keeping all the other variables constant. As before, use Excel to plot straight-line graphs and determine slopes. Once again, remember to show the errors bars on your graph. Check your conclusions by dimensional analysis.

Although we have now found all the exponents, we cannot yet accurately describe the period of the pendulum since we still do not know the dimensionless constant A .

Task 8:

When you have found a consistent set of values for a , b , d and e , determine the value of the constant A by measuring the period accurately for a particular set of values of z , x and l .

Estimate the error on your determination of A . In the equation for the period of oscillation in a simple pendulum, $A = 2\pi$. For your bifilar pendulum, is A a simple multiple of π ?

Task 9:

In Task 3 you showed that the period of oscillation should be independent of the mass of the bar. You now should check this experimentally. Measure the period of oscillation of the two long rods made of different materials (do NOT change any other parameters).

Do your measurements support your deduction that the period of oscillation is independent of mass?

5. Exploration

If you have reached this point before the end of the lab session then you could usefully consider the case when the suspension filaments are no longer vertical.

6. To conclude

You will cover simple harmonic motion in the Foundations of Physics module. This topic, and many others related to pendulums can be found in most undergraduate text books. It can be difficult to find out specifically about bifilar pendulums, but there is information available on the worldwide web.