VISUAL PHYSICS
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FLUID FLOW
VISCOSITY
POISEUILLE'S LAW

? Why do cars need different oils in hot and cold countries?

Why does the engine runs more freely as it heats up?

Have you noticed that skin lotions are easier to pour in summer than winter?

Why is honey sticky?

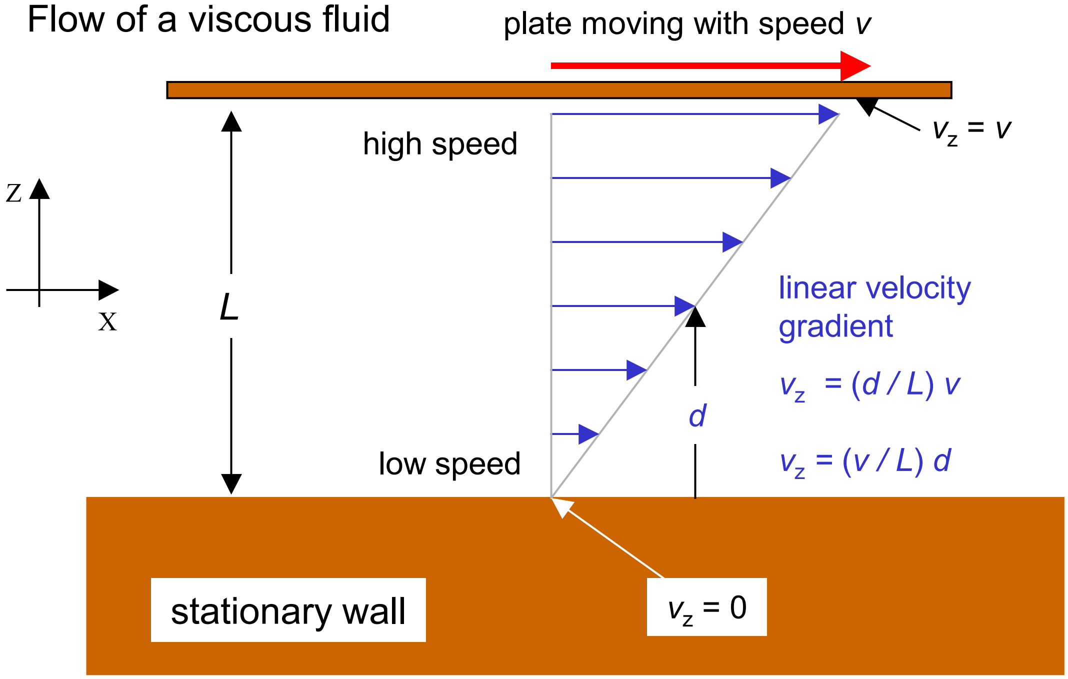
- Viscous fluids tend to cling to a solid surface.
- Syrup and honey are more viscous than water.
- Grease is more viscous than engine oils.
- Liquids are more viscous than gases.
- Lava is an example of a very viscous material.

! When real fluids flow they have a certain amount of internal friction called **viscosity**. It exists in both liquids and gases and is essentially a friction force between different layers of fluid as they move past one another. In liquids the viscosity is due to the cohesive forces between the molecules whilst in gases the viscosity is due to collisions between the molecules.

Coefficient of viscosity

When a fluid (e.g. air) flows past a stationary wall (e.g. table top), the fluid right close to the wall does not move. However, away from the wall the flow speed is not zero. So a velocity gradient exists. This is due to adhesive, cohesive and frictional forces. We find that the magnitude of this gradient (how fast the speed changes with distance) is characteristic of the fluid. This is used to define the coefficient of viscosity η (Greek letter eta).

Flow of a viscous fluid



Fluid in contact with either surface is held to that surface by adhesive forces between the molecules of the fluid and surface. Therefore, the molecules at the surface of the stationary wall are at rest and the molecules at the surface of the moving plate will be moving with velocity *v*. The stationary layer of fluid in contact with the stationary wall will retard the flow of the layer just above it. This layer will retard the layer above and so on. Thus the velocity will vary linearly with distance above the stationary wall. The force required to move the plate at speed v is

$F \propto A$ $\quad\quad$ $A =$ area of either plate

$F \propto (v / L)$ $\quad\quad$ (v / L) = velocity gradient

The constant of proportionality for the fluid is called the coefficient of viscosity η

!

$$F = \eta A v / L$$

The greater the coefficient of viscosity η, the greater the force required to move the plate at a velocity *v*.

This equation does not hold for all fluids. Viscous fluids that obey this equation are called newtonian fluids and η = constant independent of the speed of flow. When η does depend upon the velocity of flow the fluids are called non-newtonian. Blood is an example of a non-newtonian mixture because it contains corpuscles and other suspended particles. The corpuscles can deform and become preferentially oriented so that the viscosity decreases to maintain the flow rate. Corn flour and water mixture is another non-newtonian fluid.

Viscosity

$\eta = (F / A)(L / v)$ $(\mathrm{N.m^{-2}})(\mathrm{m}).(\mathrm{m^{-1}.s}) \equiv \mathrm{Pa.s}$

SI unit for viscosity is Pa.s

A common unit is the poise P where 1 Pa.s = 10 P

1 mPa.s = 10^{-2} P

Fluid	η (mPa.s)
water (0 °C)	1.8
water (20 °C)	**1.0**
water (100 °C)	0.3
white blood (37 °C)	~4
blood plasma (37 °)	~1.5
engine oil (AE10)	~ 200
air	0.018

Viscosity is very temperature dependent

- Viscosity of a liquid decreases with increasing temperature.
- Viscosity of a gas increases with increasing temperature.

Why can't you get all the dust off your car by just squirting water from a hose onto it?

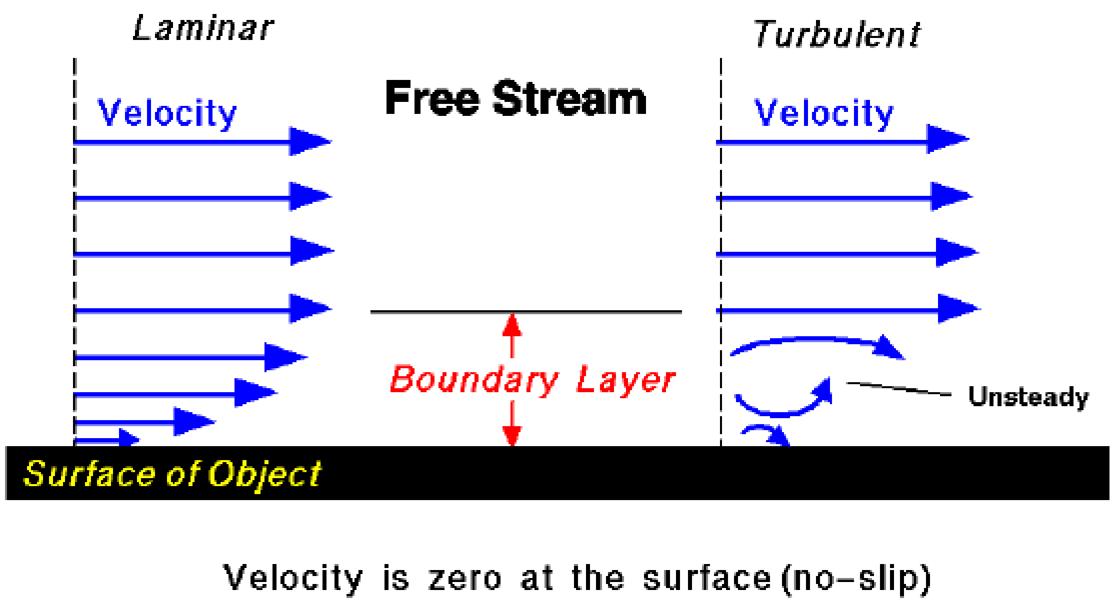
Why can't you simply remove dust just be blowing across the surface?

Why does dust cling to a fast rotating fan?

How can a leaf stay on a car moving at high speed?

Boundary layer

! When a fluid moves over a surface, there is a thin layer of the fluid near the surface which is nearly at rest. This thin layer is called the boundary layer.



Flow of a fluid through a pipe
Poiseuille's Law

In trying to find out what factors control how fast fluids can flow through pipes, the following factors are easy to isolate:

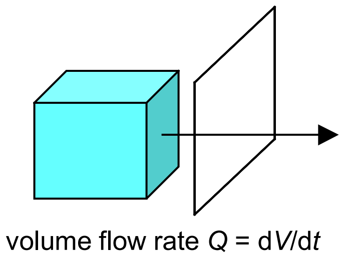
The pressure difference between the ends of the pipe. The bigger the pressure difference, the faster will be the flow.

The length of the pipe. More liquid will flow through a shorter than a longer pipe in the same time.

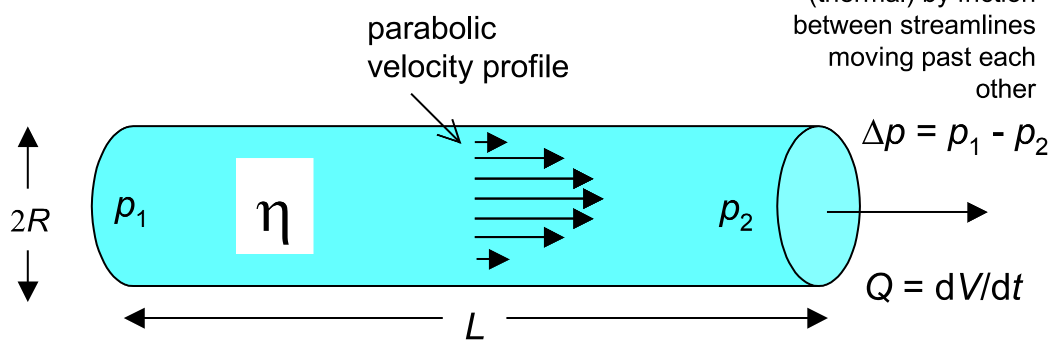
The radius of the pipe. More liquid will flow through a wide than a narrow pipe in the same time. This dependence is very marked.

The coefficient of viscosity of the liquid. Water flows much more easily than glycerine.

Poiseuille's Law: laminar flow of a newtonian fluid through a pipe



$$Q = \frac{dV}{dt} = \frac{\Delta p \, \pi \, R^4}{8 \, \eta \, L}$$



The **volume flow rate**

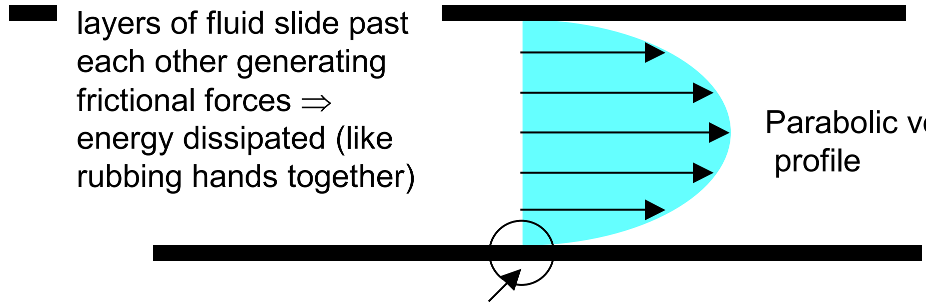
! $\boldsymbol{Q = dV/dt}$

of a fluid of viscosity η, through a pipe of radius R and length L, when driven by a pressure difference Δp is given by

! $\boldsymbol{dV/dt = Q = \Delta p \, \pi \, R^4 / (8 \, \eta \, L)}$

This is known as **Poiseuille's law**. Poiseuille's law only applies to newtonian fluids. Non-newtonian liquids do not obey Poiseuille's law because their viscosities are velocity dependent. The assumption of streamlined (laminar) flow is built in to Poiseuille's law. If turbulence occurs than you must be very careful about using Poiseuille's law to calculate flow rates. If turbulence does occur in the flow then the volume flow rate is dramatically reduced.

Flow of a viscous newtonain fluid through a pipe
Velocity Profile



Alternative view of Poiseuille's Law

Consider an electrical circuit in which a potential V between the ends of a resistance R results in a current I. Then the flow is determined by the ratio of potential to resistance.

flow (current) = potential / resistance

$I = V / R$

Poiseuille's Law can be arranged in this form

flow = potential / resistance

$Q \;=\; \Delta p \,/\, (8\,\eta\, L \,/\, \pi\, R^4)$

flow $\Rightarrow$ Q potential $\Rightarrow$ Δp

resistance $\Rightarrow$ $(8\,\eta\, L \,/\, \pi\, R^4)$

resistance $\propto L$ resistance $\propto \eta$ resistance $\propto (1 / R^4)$

Just as electrical energy is dissipated when an electrical current flows, energy is dissipated when a fluid flows through a pipe. In the electrical circuit this is manifest by the drop in potential around the circuit whereas for the flow in the pipe there is a drop in pressure along the pipe.

Why should turbulence mean that the volume rate of flow is less than in streamlined flow?

? When a builder designs the drainage system for the roof of a house, what factors should influence the choice of the size of the downpipe? Would he be correct in basing his calculations on Poiseuille's Law?

In an experiment water rose by capillary attraction through two columns of soils, one with coarse grains and the other with fine gains. It was observed that the water rose faster in the column with the coarser grains. Can you say why this is so?

The volume flow rate Q is very sensitive to changes in radius of the pipe R since $Q \propto R^4$ this fact and leads to many situations in which Poiseuille's law has important effects.

Irrigation pipes It is uneconomical to use spray irrigation too far from a river since the resistance of a pipe increases with its length, and you need too big a pump.

Pipes from Warragamba Dam Here Δp and L are fixed (by geography), and the volume rate of flow is fixed by the requirements of the population of Sydney. When Sydney doubles in size, the Water Board will have to use twice as many pipes or replace the present pipes by ones of $(2)^{1/4}$ times the radius. (This is an oversimplification.)

Respiratory system The flow of gas here is also Poiseuillean. The resistance to flow is determined primarily by the narrow tubes leading to the alveoli. Any general constriction of the pipes, as occurs in bronchospasm for instance, increases the resistance to flow and makes breathing much more difficult. Asthma.

Circulatory system
(a) There is a decrease in pressure across each section of the tubes. Blood pressure is highest when it leaves the heart (through the aorta) and lowest when it returns (through the inferior vena cava). Most pressure loss occurs over the capillaries. Why?

(b) Any constriction of the tubes - for example a build up of cholesterol on the walls of the arteries (arteriosclerosis)- increases the resistance and hence the pressure drop (it goes as R^4 remember). So the heart has to work harder to compensate. And at times of stress, when an increased flow rate is required, there can be a breakdown.

Soils Water will rise quicker in large grain soils ($Q \propto R^4$) but it will rise to greater height by capillary attraction on fine grain soils ($h \propto 1/R$)

Home activity

Make a sloppy mixture of corn flour and water. Slowly move your hands through the mixture. Also, hit the mixture with your finger quickly.

Why the difference?