Advanced Physics Laboratory

EXPERIMENT 39: LENSES

A Chi-Squared Test of the Thin Lens Equation

Modified by J.H. Thywissen
v2.5: 24 March 2006

Objectives

Primarily, the goal of this lab is to test the thin lens law, using experimental measurements from several lenses. In so doing, you will get a chance to review quantitative methods of testing a hypothesis using Chi Squared.

Section 1. Background

Thin lens relations. The relationship between the image distance q, object distance p, and focal length f is given by the thin lens law

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}, \tag{1}$$

where these quantities are specified with respect to the principle planes of the lens as described in Figure 1. The magnification of the object is given by the ratio

$$M=-\frac{q}{p}, \tag{2}$$

where the minus sign reflects the fact that the image is inverted with respect to the object. The fundamental assumption of equations (1) and (2) is that each lens has a characteristic focal length f, with which you can predict the image location and magnification given any object location p. It should simply be a matter of finding the correct principle planes (see Figure 1)! This is the hypothesis that you should try to test.

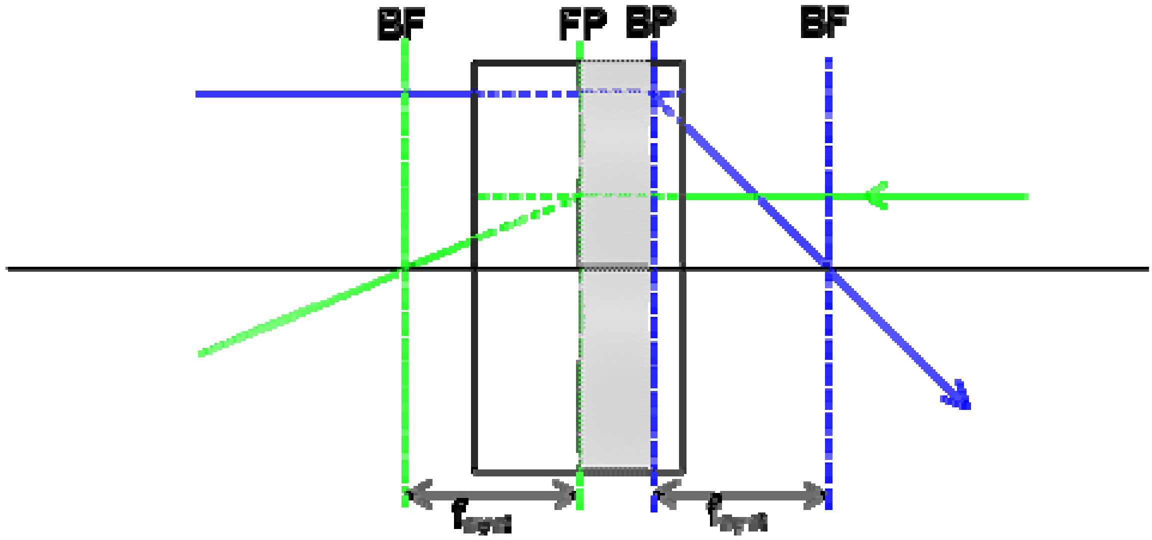

Figure 1. Rays going through a lens or lens system, with front principal plane FP, back principal plane BP, and focal length f marked. As an exercise, show that the front and back focal planes of the spherical lens are at one radius from its center, ie, tangent to the front and back surfaces. What is the relation between f and r, the radius of the spherical lens?

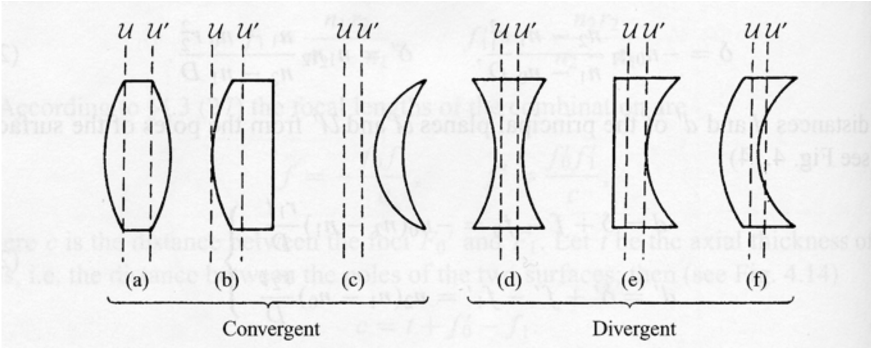

Figure 2: Principle planes of common types of lens: (a) double-convex; (b) plano-convex; (c) convergent meniscus; (d) double-concave; (e) plano-concave; (f) divergent mensicus. U and U' are the principal planes, light being assumed to be incident from the left.

Three important instances of equation (1), depicted in Figure 3, should be kept in mind. First, when p=q, then both the object and image are 2f from the lens. The magnification is M=–1. Note that this configuration is the minimum image-object separation (p+q = 4f) with which an image can be formed. Second, when the object is f away from the lens, then q is infinite. This corresponds to a situation in which rays from a point source are collimated into parallel propagation. Third, when the source is infinitely far away (ie, p is infinite), parallel beams are incident on a lens, and q=f. This can be used to get a rough idea of the focal length of a lens: focus an image of the door at the end of the hallway onto the wall, and see how far the lens is from the wall.

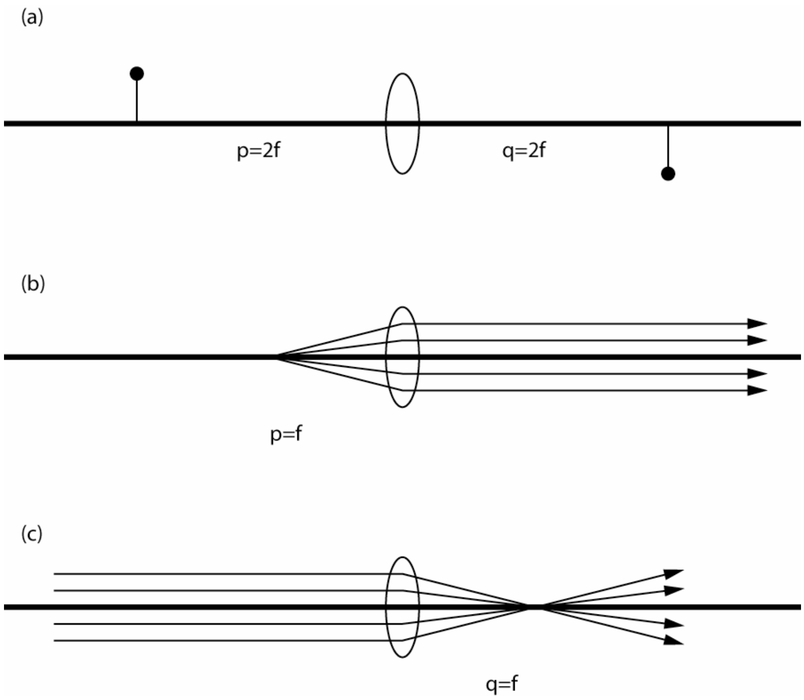

Figure 3. Three important examples of the thin lens equation: (a) Unity magnification (with inversion); (b) collimation of a point source; (c) collimated input to a lens.

Aberrations. Lenses are subject to 6 different aberrations, ie, departures from a perfect mapping of object to image. Here we mention three types of aberrations:

1. Spherical aberration. If parallel beams incident on the edges of the lens are focused closer than beams incident on the inner part of the lens, there will be an imperfect focus. Another way of saying this is that f varies with the distance from the optical axis.
2. Astigmatism. This off-axis effect will cause vertical features to be focused at a different location than horizontal features. This is because the lens “looks” more curved when turned at a slight angle.
3. Distortion. When the appearance of the object is somewhat re-arranged in the image plane, we call this distortion. As discussed in the references, distortion can be either “barrel” or “pin cushion”.
4. Chromatic Aberration. Different frequencies of light see a different index of refraction of glass, and thus refract differently. Thus red and blue light may have a different focal length passing through the same lens.

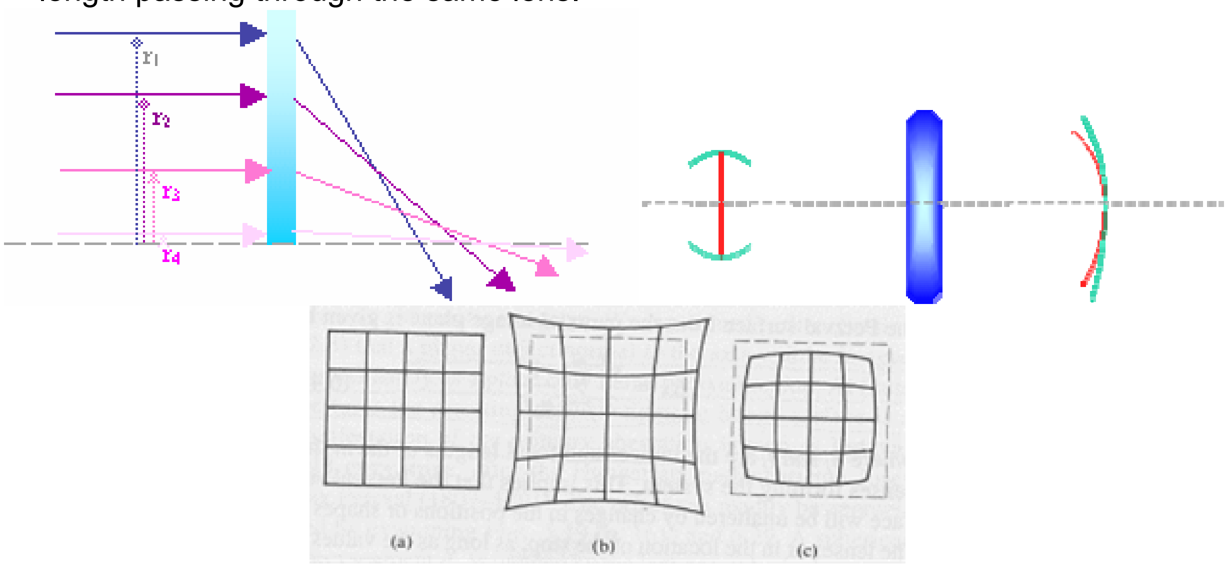

Figure 4. Spherical aberration, Astigmatism, and Distortion.

Section 2. Experiment.

In this experiment, you will set up lenses on a 1.5m optical bench to study lens operation. The apparatus will be in the classic configuration of object, lens, image.

Start by measuring the properties of a thin (perhaps biconvex) lens. Then repeat the measurements for the spherical and hemi-spherical lenses. These three lenses will have qualitatively different behavior.

<u>A. Alignment.</u>

First, the lamp, aperture, and lens must be aligned to the right heights and angles. The goal of this alignment is to have the optical axis parallel to the optical bench, and centered on the lens. If the optical axis is not parallel to the bench, the distances you read of the ruler on the optical bench will not be measuring true distances between the opical elements. If the optical axis does not pass through the center of the lens, aberrations will be induced that will complicate the measurement.

How to align:

1. Plug in the lamp, put an diaphragm in front of it. Close the diaphragm down to a few mm diameter.

First, make sure the center of the diaphragm is at the height of the filament in the lamp:

2. Tape a piece of paper to the screen, and place it close to the diaphragm. Mark the light spot illuminated on the paper.
3. Move the screen away from the diaphragm and source, and see if the expanding light circle stays centered on the mark you made. This would be an indication of light coming out of the source parallel to the optical axis whose height is defined by the aperture.
4. If not, move the diaphragm and lamp around to line up the filament inside the light source and the center of the diaphragm. Repeat steps 2-3.

Now, choose the angle, height, and transverse position of the lens:

5. Move the lens approximately 7 cm away from the diaphragm, to find the back-reflection onto the surface of the diaphragm. Chose the angle and height to retro-reflect back through the center of the diaphragm.
6. Open up the aperture and look at the far-field focus (screen about 1m away) of the lens. Translate the lens transverse to the optical bench until the focused spot aligns with the original mark made on the paper taped to the screen.
7. Look also at the height of this far-field image. It should align with the original mark made.
8. Check the back-reflection to see if still good. If not, iterate.

<u>B. Distance Offsets.</u>

We will measure p and q by reading positions off the ruler built into the optical bench. However, to make sense of these readings, you need to establish the relation between the location of the edge of the mount (where you can read the ruler) and the center of the optical element (lens, screen, etc) in question.

Use a pair of calipers to measure the dimensions of the optical mounts necessary to calculate the centre of the optical element being held from the recorded position of the edge of the mount. Aim for 0.1 mm precision. Note the any uncertainty in this calibration is a *systematic* error, because it would shift all of your data in the same direction. We'll talk more about this later.

An alternate way of measuring these offsets is depicted in Figure 5. You can slide two objects so that a "reference object" (a short bar, for example, with flat ends) touches both of the planes whose position you are trying to measure. Based on the figure, you can see that $o_1 = L_{ref} - o_2 - (x_1 - x_2)$, where the x's are positions measured on the optical bench ruler, the o's are distance offsets, and L_{ref} is the length of your reference object. If you can measure L_{ref} and o_2 well, then you might be able to extract an accurate value for o_1, even if the mount is very complex.

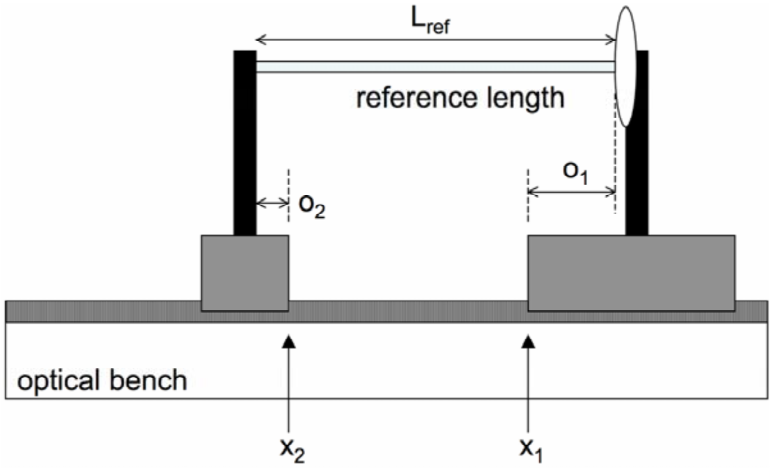

Figure 5: Using a reference length to estimate offsets.

C. Estimating uncertainty in measurements

A good way of estimating the uncertainty of each measurement is to repeat it several times. For instance, when measuring the width of the base of the optical mount, take the calipers, make the measurement, then reset the calipers to fully closed again, and repeat the measurement from zero. Try not to remember your previous measurements, but simply record them and take three to five measurements. The uncertainty can be estimated by the standard deviation of the measurements made. This technique is especially interesting to avoid such subjective judgments such as an estimation of read error. How well can you read a ruler? A set of measurements will tell you.

D. Estimation of focal length

With the lens far from the source, estimate the focal length by finding the image distance. You might also try, in a room with a window, imaging the sun or clouds into a piece paper. This will give you an order of magnitude for f to guide your choices for data in D.

You can also measure the curvature of the lens faces to estimate f. For a thin lens, f=R/[2(n-1)], where R is the radius of curvature and n is the index of glass. Similar relationships can be found for other lenses.

E. Data for p, q, and M.

You are now ready to take a set of data. Set up your optical bench with the source, a diaphragm, the transparent grid "object", a lens, and the screen (without any paper taped to it). Start with the thin lens.

Choose a location for the lens, and find the position of the screen which shows the sharpest focus. Record the positions of all elements. Measure the spacing of the grid lines in the image, from which you can determine the magnification. Repeat these measurements 3 to 5 times for the same lens position, in order to determine the measurement uncertainty.

Repeat these measurements for a variety of lens positions. Try to sample the range between q=f and p=f. (Look back at Figure 3 to see why these are extreme cases)

F. Your first fit, and iterations

After taking a first set of data, try to plot the data and analyze it as soon as possible. The results of your analysis will tell you about what to do next in the lab: did you take data with the right range of p and q? Did you take enough measurements for each configuration? Are some parts of your data seemingly off? Is the Chi Squared reasonable?

Try to "iterate" a few times: take a data set, analyze it, then improve the data. If you are happy with the quality of the fit, then start trying to understand the *results* of the fit: focal length, front plane, and back plane. To see if these results are significant, you will at this point need to include the **systematic uncertainty**, discussed in the analysis section.

G. Thick lenses

Finally, repeat parts D, E, and F for one or two of the thick lenses! Does the thin lens equation work for thick lenses?

Section 3. Analysis.

The goal of the analysis is to measure the properties of each lens (focal length and front and back principal planes) assuming that the lens follows the thin lens equation (1). Out of the analysis will naturally arise a test of the validity of this assumption.

Let's call the true position of the of the lens l, the image screen i, and the object screen o. (Remember that the position of each of these includes some offset measured in section 2.B.) If the principle planes were both located at the center of the lens, then p = |o-l| and q=|i-l|. However, there is some additional offset due to the fact that the principle planes are *not* necessarily at the center of the lens, so p=|o-l|-fp and q=|i-l|-bp. These quantities should follow the ideal lens law, so we can write

$$\frac{1}{f} = \frac{1}{|o-l| - fp} + \frac{1}{|i-l| - bp}, \tag{3}$$

where f, fp, and bp are parameters to be determined, and o, i, and l are measured parameters.

Using your data, and standard function fitting analyses, find the lens parameters for each lens. State the Chi Squared per degree of freedom of your fit, and whether the ideal lens law is a valid assumption for each lens. *When possible, compare to the expected focal length and locations of fp and bp.*

Using the MATLAB suite: datacrunch.m and fitter.m

It is recommended that you use MATLAB to analyze analyze your data. MATLAB is available on the computers in the Nortel computer lab. The MATLAB web site (`http://www.mathworks.com/support/`) has help files, if they are not installed on the computer; there should also be a paper manual floating around. The custom MATLAB files for this experiment are available on the course web site:
`http://www.physics.utoronto.ca/~phy326/lens/`
Updates and new software will also be posted on this site.

How to prepare your data and use in this code:
1. Create a text file with all your data, with four columns, in order
(source_position) (image_position) (lens_position) (grid_image_size)
Skip a line between each set of measurements, for instance

129.100	*40.030*	*115.600*	*39.470*
129.100	*40.102*	*115.600*	*39.390*
129.100	*40.050*	*115.600*	
129.100	*40.260*	*59.490*	*2.510*
129.100	*40.450*	*59.490*	*2.533*
129.100	*40.330*	*59.490*	*2.420*

etc. You can use whatever units you want (mm, cm, inches) so long as you are consistent.
2. If you didn't always measure the image grid size, just leave it out – don't just copy one value several times. The program takes the mean and deviations of the measurements made.
3. Save the file to the working directory you will use. [If you are using excel, save as a MS-DOD .txt file.] Also, include a copy of this file for your lab book.
4. The next step is to determine the uncertainties and real (ie, offset) values of the optical elements and magnification. Run **datacrunch2.m**. for example:
datacrunch2('ThinLensData.txt', 'ThinLensData.out', 9.930, -2.979, -1.945, 1.886)
where *9.930* means an grid size of 9.930 mm, and the other numbers are offsets. A full explanation of inputs and outputs can be seen by typing *help datacrunch2.*
6. Print out the output file for your lab book.

7. Now we use these numbers to fit the thin lens equation to the data. Run **fitter.m**, following the directions from its manual. Both of these are available at
`http://www.physics.utoronto.ca/~phy326/matlabfit.htm`
8. Keep a hard copy of the data file created from the fits, as well as a printout of the curves made.

The goal of the above calculations is to measure the properties of the various lenses, and test the thin lens equation. The conclusion of your lab should address these goals. What is the uncertainty with which you have measured the properties of the lenses? What does the Chi squared tell you? To answer these questions, feel free to modify the analysis programs – they are provided here only to get you going.

Systematic uncertainty

In your final answers, you can quote statistical and systematic uncertainty separately:

f = 15.023 +/- 0.015 +/- 0.250

where the first number is statistical uncertainty and the second is systematic. So far, we have only discussed ways of getting the statistical error: ie, from the fit program and data that gives a good Chi Squared value.

Systematic errors are those shift affect all data points in one direction or another. In this lab, a primary systematic error is the uncertainty on the offsets, estimated in part 2B and 2C above. You can calculate the effect of these by re-analyzing your data with a distribution of offset values. Say the object offset was measured to be 9.9 +/- 0.2. To estimate the resultant systematic error in "bp" (the back plane position), for example, run datacrunch2.m and fitter.m with 9.7, with 9.9, and with 10.1 as the object offset, and keep track of the best fit bp in each case. The *spread* in bp (due to variation of all offsets over their one sigma range) is the systematic error. This should be calculated for f, bp, and fp. In this same way, you could also include systematic error from sources other than the offsets.

References

M. Born and E. Wolf, *Principles of Optics*, (Cambridge University Press,1999).

E. Hecht, *Optics*, 2002.

G. R. Fowles, *Introduction to Modern Optics* (Dover, 1975), pp. 294-299. [Physics, Gerstein: QC395.2 .F68 1975]

F. A. Jenkins and H. E. White, *Fundamentals of Optics*. 1976.

F. W. Sears, *Optics* (Addison-Wesley Press, 1958) [Gerstein: QC355 .S45]

http://www.physics.georgetown.edu/~vankeu/webtext/