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Citation: American Journal of Physics **33**, 545 (1965); doi: 10.1119/1.1971900
View online: http://dx.doi.org/10.1119/1.1971900
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/33/7?ver=pdfcov
Published by the American Association of Physics Teachers

Bifilar Pendulum—An Experimental Study for the Advanced Laboratory

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(Received 16 December 1964)

This study is concerned with various forms of bifilar suspensions and a quantitative comparison of experiment with theory. The various forms of the equations and the surprisingly simple form taken by some are considered. Moments of inertia are obtained experimentally and compared with the calculated moments from the theory.

THE experimental study of bifilar pendulums together with a correlation with the theory should be a valuable adjunct in the development of a technique and the acquisition of a practical tool in the general field of mechanics.

THEORY

The fundamentals of bifilar suspensions are indicated by the derivation of the equations applied to a slender bar of length d suspended horizontally by two vertical strings from a rigid support.[1,2] The strings have length y and the distance between the strings are designated 2b attached equally distant from the center of gravity [see Fig. 1(a)]. Let M be the mass of the bar and k its radius of gyration. If the bar is turned through a *small* angle θ about the vertical through the center of gravity, the lower end of each string describes a small arc which may be taken as practically horizontal and equal to $b\theta$ [see Fig. 1(b)]; and the inclination of each string to the vertical becomes $b\theta/y$, approximately as shown in Fig. 1(c). Since the vertical displacement of the center of gravity in θ is of the second order, it may be experimentally neglected. The tension in each string is approximately constant and equal to $(\frac{1}{2})Mg$. The horizontal component of each tension (the restoring force) is therefore $(\frac{1}{2})Mgb(\theta/y)$, nearly. The restoring couple is therefore $Mgb^2\theta/y$. Hence in free oscillation

$$Mk^2(d^2\theta/dt^2) = -Mg(b^2/y)\theta \tag{1}$$

or

$$(d^2\theta/dt^2)+(gb^2/k^2y)\theta=0. \tag{2}$$

The period is therefore

$$T=2\pi\left(\frac{k^2y}{gb^2}\right)^{\frac{1}{2}}=2\pi\frac{k}{b}\left(\frac{y}{g}\right)^{\frac{1}{2}}. \tag{3}$$

If applied to a slender rod $I=(1/12)Md^2$ and

$$k=\left(\frac{I}{M}\right)^{\frac{1}{2}}=\left(\frac{Md^2}{12M}\right)^{\frac{1}{2}}=\left(\frac{d^2}{12}\right)^{\frac{1}{2}}. \tag{4}$$

If now the strings were at the extremities of the rod then d would equal $2b$ and Eq. (4) would become

$$k=(4b^2/12)^{\frac{1}{2}}=b/\sqrt{3};$$

then Eq. (3) becomes

$$T=2\pi[(y/3g)^{\frac{1}{2}}]. \tag{5}$$

It should be noted that Eq. (5) is a special case referring to a slender rod. Equation (3) is a more general equation with much wider applicability. Jerrard and McNiel[3] in their text

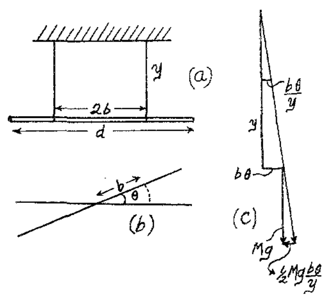

FIG. 1. (a) Bifilar suspension, (b) arc described by lower end of supporting strings, (c) diagram showing restoring force.

[1] H. Lamb, *Dynamics* (Cambridge University Press, Cambridge, England, 1929), p. 164.
[2] W. D. MacMillan, *Dynamics of Rigid Bodies* (McGraw-Hill Book Company, Inc., New York, 1936), p. 130.
[3] H. Jerrard and D. McNeill, *Theoretical and Experimental Physics* (Reinhold Publishing Corporation, New York, 1960), p. 210.
[4] R. Stephenson, *Mechanics and Properties of Matter* (John Wiley & Sons, Inc., New York, 1960), 2nd ed., pp. 175–182.

supply the equation

$$T=\frac{2\pi k}{(b_1b_2)^{\frac{1}{2}}}\left(\frac{y}{g}\right)^{\frac{1}{2}}. \tag{6}$$

Its application is illustrated by Fig. 2(C) and Fig. 2(D). Equation (6) reduces to Eq. (3) when $b_1=b_2$. The equation

$$T=2\pi[(1\cos\beta/3g)^{\frac{1}{2}}] \tag{7}$$

takes into account the case wherein the supporting strings of length 1 are not vertical but are inclined at an angle β with the vertical. Equation (7) applies only to slender rods. Here 1 is the length of the supporting strings and obviously $y=1\cos\beta$ and (7) reduces to (5). Certain geometric forms such as a wire hoop or a metal cylinder supported with their axes vertical, by vertical strings, reduce to the simple pendulum equation

$$T=2\pi[(y/g)^{\frac{1}{2}}]. \tag{8}$$

This is obvious from the general equation (3) since here the radius of gyration is equal to b. (In experiment; the wire hoop must be supported by three vertical strings for stability). The metal cylinder when supported by strings making an angle with the vertical can be analyzed and checked using Eq. (3) only. It is interesting to note that in the case of slender rods if the vertical supporting strings are each attached in from the ends by $\frac{1}{3}$ the length of the rod, the latter period is 3 times that obtained when the strings are at the ends.

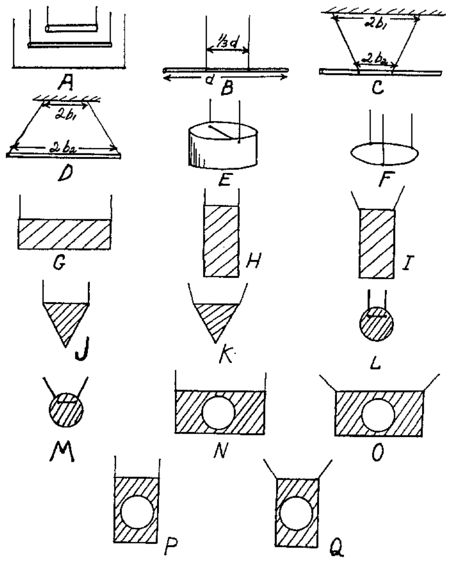

FIG. 2. Diagrams of diverse bifilar pendulums for checking periods and moments of inertia.

EXPERIMENTAL PROCEDURE

Referring to the lettered diagrams in Fig. 2, it is seen in (A) that one can fix the strings satisfactorily near the ends of the rods. Drilled small holes prove satisfactory for the thin walled cylinder (E) and the circular disk (M). A plastic, insulating tape proves very satisfactory to fix the strings at the corners of the rectangular and triangular objects.

Arrange three relatively thin brass rods of varying length and diameter, support with different lengths of strings as indicated in (A) of Fig. 2. Check the experimental value of the period against the calculated value using Eqs. (3) and (5). They should check to roughly 2%. With care some will yield results to 0.1% error.

Take one or more of the rods used in arrangement (A) and set up as indicated in (B) of Fig. 2. Check experimental period with that calculated from Eq. (3). Also compare this period with that obtained in (A) for the same rod or rods.

Check the experimental period indicated in (C) against that calculated by Eq. (6). If $b_1=3b_2$ Eq. (7) becomes

$$T=2\pi[(1\ \cos\beta/g)^{\frac{1}{2}}]. \tag{9}$$

Also check the set up (D) with Eq. (6) also Eq. (9).

Support the thin-walled cylinder, also the heavy wire ring as indicated in (E) and (F) using Eq. (8).

Determine the period experimentally for the plane parallelogram (G) and the same object oriented as in H. Weigh on a balance. Use Eq. (3) and solve for k, the radius of gyration. Determine the moment of inertia (see Chap. 6, pp. 176 to 181, Ref. 4). Compare with the calculated value for the moment of inertia. Do similarly for the setup I using Eq. (6). Compare experimental with calculated moments of inertia for the remainder of the objects shown in (J) to (Q) of Fig. 2.

The following observations and directives are presented by this study: (1) The insignificant cost and simplicity of the apparatus; (2) It would seem that the bifilar method should merit a significant place as an instructional medium; (3) These simple experiments could be supplemented by application to the determination of moments of inertia of three dimensional bodies of regular shape which can be computed and also of irregularly shaped bodies which cannot be computed.

Connection between Macroscopic and Microscopic Transport Phenomena in Solid Conductors

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(Received 6 April 1965)

An expression for the rate of production of entropy in a homogeneous solid conductor is obtained and in the process leads to a natural selection of current densities. It is shown that these current densities when written from the microscopic point of view give a system of equations satisfying Onsager's theorem, thus facilitating a comparison between microscopic and macroscopic transport phenomena.

INTRODUCTION

To obtain a connection between macroscopic and microscopic phenomena it is necessary to obtain a system of equations of the form

$$\mathbf{J}_i = \sum_i L_{ij}\mathbf{X}_j, \qquad (1)$$

where the $\mathbf{J}_i$ are current densities and the $\mathbf{X}_i$ are the conjugate forces. Onsager's theorem states that if the $\mathbf{X}_i$ are computed according to the prescription

$$R(S) = \sum_i \mathbf{J}_i \cdot \mathbf{X}_i, \qquad (2)$$

where $R(S)$ is the rate of production of entropy, then

$$L_{ij} = L_{ji}. \qquad (3)$$

For current densities we have electric, particle, entropy and heat-current densities. With reference to the heat-current densities there are several in vogue at the present time.[1] It is just here that some pedagogical difficulties arise when one comes to write Eqs. (1) from the microscopic point of view and tries to relate the results to those obtained from the macroscopic viewpoint. The difficulty centers around the fact that for macroscopic theory Eqs. (1) are always written with Onsager's theorem in mind and when these same currents are expressed in microscopic terms the off diagonal elements are seldom if ever equal. (For an example, see Ziman.[2])

It is the purpose of this article to show that if a natural selection of current densities is chosen for a homogeneous solid conductor then the Eqs. (1) resulting from microscopic considerations do occur with the off diagonal elements equal.

I. ENTROPY PRODUCTION IN A HOMOGENEOUS CONDUCTOR

Let us consider a homogeneous conductor in which there are electrical and thermal gradients. We assume steady-state conditions and further assume that the conductor is made up of cross-sectional volumes as in Fig. 1, each of which is in local equilibrium. This enables us to apply the usual thermodynamic relations connecting entropy, internal energy, and numbers of particles.

Suppose we fix our attention on two of these

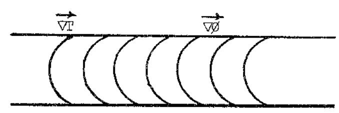

Fig. 1.

[1] S. R. De Groot, *Thermodynamics of Irreversible Processes* (North-Holland Publishing Company, Amsterdam, 1951), p. 27.

[2] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, London, 1960), p. 384, Eqs. 9.9.6.