Physics 233
Experiment 9

The Compound Pendulum

References

Stephenson, *Mechanics and Properties of Matter*, Wiley, 1960,
Ch. 6, (QA 807 S82).

Introduction

In this experiment we shall see how the period of oscillation of a compound, or physical, pendulum depends on the distance between the point of suspension and the center of mass.

The compound pendulum you will use in this experiment is a one metre long bar of steel which may be supported at different points along its length, as shown in Fig. 1.

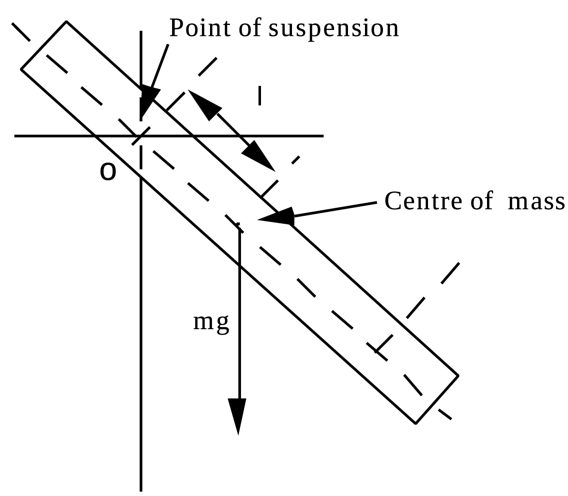

Figure 1 ***A compound pendulum.***

If we denote the distance between the point of suspension, O, and the center of mass, by l, the period of this pendulum is:

$$T = 2\pi\left(\frac{k^2 + l^2}{gl}\right)^{\frac{1}{2}} \tag{1}$$

where k is the radius of gyration of the bar about an axis passing through the centre of mass. You should derive this expression.

The period of a simple pendulum of length l' is:

$$T = 2\pi \left(\frac{l'}{g} \right)^{\frac{1}{2}} \tag{2}$$

By equating Eqs (1) and (2) and solving for l, we may find the values of l such that the compound pendulum has the same period as that of a a simple pendulum of length l':

$$l = \frac{l' \pm \left(l'^2 - 4k^2 \right)^{\frac{1}{2}}}{2} \tag{3}$$

As you can see, there are two values of l , which we will label l_1 and l_2, for which the period of the compound pendulum is the same as that of the given simple pendulum.

There is a value of l for which the compound pendulum has a minimum period. The minimum period may be found from Eq. (1) by setting:

$$\frac{dT}{dl} = 0$$

One finds:

$$T_{\min} = 2\pi \left(\frac{2k}{g} \right)^{\frac{1}{2}} \tag{4}$$

Prelab Questions

1. Derive Eq. (1) and write down an expression for the radius of gyration k in terms of the dimensions of the bar.

2.
   a. Does Eq. (1) apply when:
      i. A large amplitude is used?
      ii. When damping is present, due to friction at the pivot, or to air resistance?
   b. If Eq (1) does not apply, would the value you found for g be too high or too low?

3. Should the presence of holes in the bar be considered when calculating the theoretical value of k?

4. Rewrite Eq (1) in the form $l^2 = f\left(T^2 l\right)$ to give an equation of a straight line, with g related to the slope and k to an intercept.

5. Could there be a systematic error in the stopwatch you will use? How could you check this?

6. Will the increased weights used in the optional experiment alter the damping due to friction?

Apparatus

- pendulum bar and ball bearing mount
- 2 extra masses
- meter stick
- stopwatch

Experiment

1.
   a. Determine the period of the compound pendulum for various values of l. To do this, time about 20 complete swings and repeat each measurement several times. Be careful not to make the amplitude of oscillation too large and explain why this precaution is necessary.
   b. Plot your results and calculate k from the minimum period.
   c. Show that $l_1 l_2 = k^2$ for fixed T.

2. Calculate a theoretical value of k from the bar's dimensions and compare it with your experimental result from item 1.

3. Using the theoretical value of k, plot the theoretically predicted variation of the period with l on the graph on which you displayed your experimental results. Compare and comment.

4. Replot your data exploiting the linear relationship derived in the Prelab Questions and extract k and g from the graph. Compare to calculated or previously measured values.

Optional Experiment

Attach two equal masses symmetrically at each end of the bar. Repeat the experiment and interpret your results.