

Topic 12 | Lens Aberrations

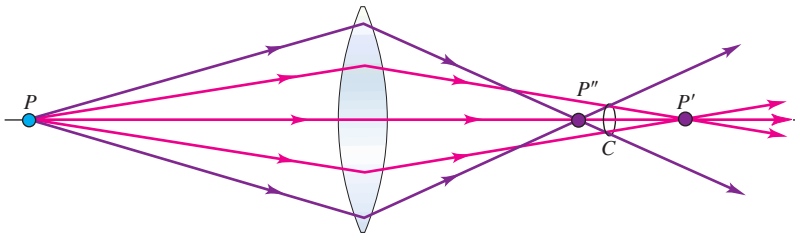
An **aberration** is any failure of a mirror or lens to behave precisely according to the simple formulas we have derived. Aberrations can be classified as **chromatic aberrations**, which involve wavelength-dependent imaging behavior, or **monochromatic aberrations**, which occur even with monochromatic (single-wavelength) light. Lens aberrations are not caused by faulty construction of the lens, such as irregularities in its surfaces, but are inevitable consequences of the laws of refraction at spherical surfaces.

Monochromatic aberrations are all related to the *paraxial approximation*. Our derivations of equations for object and image distances, focal lengths, and magnification have all been based on this approximation. We have assumed that all rays are *paraxial*, that is, that they are very close to the optic axis and make very small angles with it. This condition is never obeyed exactly.

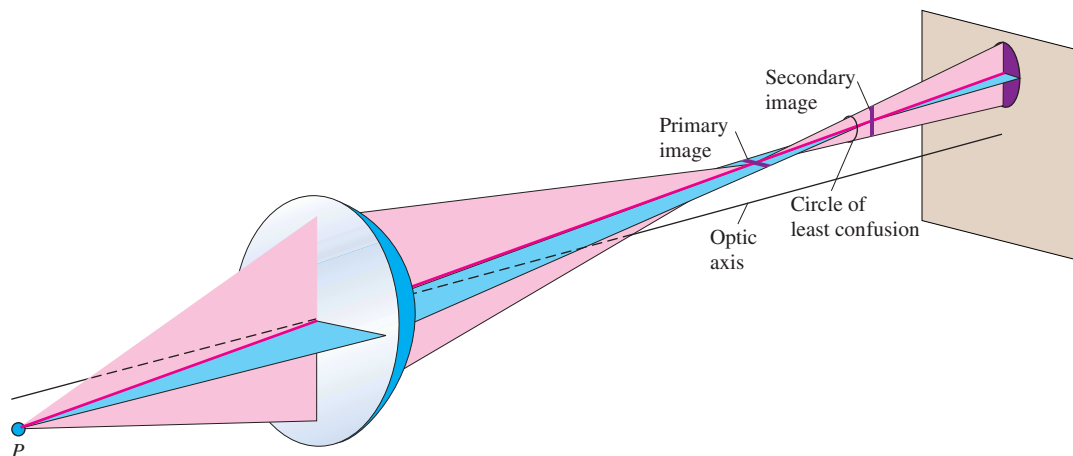
For any lens that has an aperture of finite size, the cone of rays that forms the image of any point has a finite size. In general, nonparaxial rays that proceed from a given object point *do not* all intersect at precisely the same point after they are refracted by a lens. For this reason, the image that is formed by these rays is never perfectly sharp. *Spherical aberration* is the failure of rays from a point object on the optic axis to converge to a point image. Instead, the rays converge within a circle of minimum radius, called *the circle of least confusion*, and then diverge again, as shown in Fig. T12.1. The corresponding effect for points off the optic axis produces images that are comet-shaped figures rather than circles; this is called *coma*. Note that decreasing the size of the lens aperture cuts off the larger-angle rays, thus decreasing spherical aberration.

Spherical aberration is also present in spherical mirrors. Mirrors that are used in astronomical reflecting telescopes are often parabolic rather than spherical; this shape completely eliminates spherical aberration for points on the axis. Parabolic shapes are much more difficult to fabricate precisely than are spherical shapes. The disappointing results from the Hubble Space Telescope when it was first placed in orbit in 1990 were associated with spherical aberration, the result of errors in measurement during the shaping process.

Astigmatism is the imaging of a point off the axis as two perpendicular *lines* in different planes. In this aberration the rays from a point object converge at a certain distance from the lens to a line called the *primary image*, which is perpendicular to the plane defined by the optic axis and the object point. At a somewhat different distance from the lens, they converge to a second line, called the *secondary image*, which is *parallel* to this plane. This effect is shown in Fig. T12.2. The circle of least



T12.1 Spherical aberration for a lens. The circle of least confusion is shown by C .



T12.2 Astigmatism of a lens for a point below the optic axis. The lens forms two images of the point, in planes perpendicular to each other.

confusion (greatest convergence) appears between these two positions.

The location of the circle of least confusion depends on the object point's *transverse* distance from the axis as well as its *longitudinal* distance from the lens. As a result, object points lying in a plane are in general imaged not in a plane but in some curved surface. This effect is called *curvature of field*.

Finally, the image of a straight line that does not pass through the optic axis may be curved. As a result, the image of a square with the axis through its center may resemble a barrel (sides bent outward) or a pincushion (sides bent inward). This effect, called *distortion*, is not related to lack of sharpness of the image but results from a change in lateral magnification with distance from the axis.

Chromatic aberrations are a result of *dispersion*, the variation of index of refraction with wavelength. Dispersion causes the focal length of a lens to be somewhat different for different wavelengths, so different wavelengths are imaged at different points. The magnification of a lens also varies with wavelength; this effect is responsible for the rainbow-fringed images seen with inexpensive binoculars or telescopes. Mirrors are inherently free of chromatic aberrations, which is one of the reasons for their usefulness in large astronomical telescopes.

Example T12.1

Chromatic aberration

A glass plano-convex lens has its flat side toward the object. The other side has a radius of curvature of 30.0 cm. The index of refraction of the glass for violet light (wavelength 400 nm) is 1.537, and that for red light (700 nm) is 1.517. The color purple is a mixture of red and violet. If a purple object is placed 80.0 cm from this lens, where are the red and violet images formed?

SOLUTION

We use the thin-lens equation in the form given by

$$\frac{1}{s} + \frac{1}{s'} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

In this case, using general sign rules, we have $R_1 = \infty$ and $R_2 = -30.0$ cm. For violet light ($n = 1.537$),

$$\frac{1}{80.0 \text{ cm}} + \frac{1}{s'} = (1.537 - 1) \left(\frac{1}{\infty} - \frac{1}{-30.0 \text{ cm}} \right)$$

$$s' = 185 \text{ cm}$$

For red light ($n = 1.517$) we find $s' = 211$ cm. The violet light is refracted more than the red, and its image is formed closer to the lens. We see that a rather small variation in index of refraction causes a substantial displacement of the image.