Determination of the natural frequency of a driven spring pendulum

Kenneth Domingo, Mary Chris Go, and Marc Arvie Talavera

National Institute of Physics, University of the Philippines, Diliman, Quezon City

*Corresponding author: mcgo@nip.upd.edu.ph

Abstract

The objective of the experiment is to determine the resonant frequency of a spring-mass system undergoing a driven oscillation, and check if the experimental result is consistent with theory. The spring constant was determined by finding the slope of the force versus displacement plot. A spring constant of 16.36 was obtained. The resonant frequency was then determined by finding at what frequency of the function generator would produce a peak from the displacement versus frequency plot. The theoretical resonant frequency of a 200 g mass were computed to be 1.20 Hz, while the experimental was observed to be 1.29 Hz and this yieled a 7.5% deviation.

Keywords: driven oscillation, resonance, spring constant.

1 Introduction

Consider a massless, rigid rod of length L, whose one end is attached to a driver of fixed position. Suppose that an object of mass m is attached to its other end so that when the system is perturbed, it behaves as a pendulum. A general equation of motion for such a system is given by [1]

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L}\sin\theta = f(t) \tag{1}$$

where we have assumed that there are no dissipative forces, and g is the acceleration due to gravity. Using a function generator, we can set $f(t)$ so that it is of the form $\cos(\omega t)$, with driving frequency ω. (1) becomes

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L}\sin\theta = \cos(\omega t) \tag{2}$$

(2) can be solved via the elliptic integral. For simplicity, we assume the driving amplitude to be small such that the displacement θ is also small. We can then use the small angle approximation $\sin\theta \approx \theta$. (2) becomes

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{L}\theta = \cos(\omega t) \tag{3}$$

whose general solution is [2]

$$\theta(t) \approx \frac{\theta_0}{L}\frac{\omega^2}{\omega_0^2 - \omega^2}[\cos(\omega t) - \cos(\omega_0 t)] \tag{4}$$

where we have set the natural frequency $\omega_0 \equiv (g/L)^{1/2}$, and θ_0 is the initial angular displacement relative to the equilibrium position. If we let the pendulum rod be a spring with intrinsic constant k and also consider small displacements such that

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -kx \tag{5}$$

then we can also express the natural frequency as [3]

$$\omega_0 = \sqrt{\frac{g}{L}} = \sqrt{\frac{k}{m}} \tag{6}$$

From (4), we see that resonance occurs when $\theta(t)$ is maximum, i.e. when $\omega \to \omega_0$.

2 Methodology

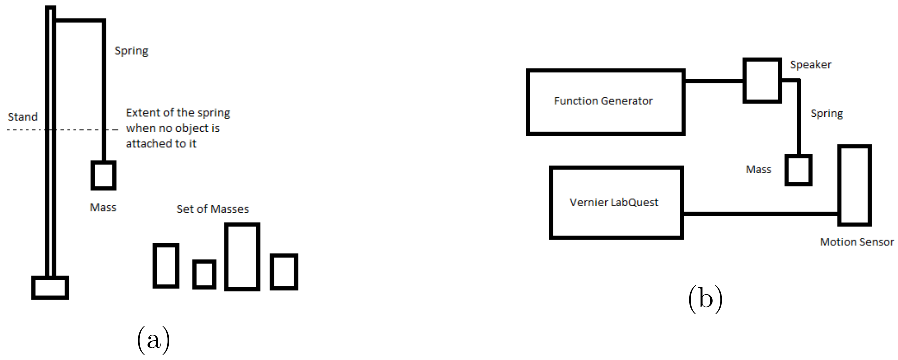

Figure 1: The figures above show the experimental set-up for determining the spring constant (a) and for determining the resonant frequency (b)

The materials used in this experiment are a set of masses which will be used to determine the spring constant of the spring that was used; a spring; a speaker; a function generator whose frequency can be varied, motion sensor for motion detection; and Vernier LabQuest to which the motion sensor was connected to analyze the motion of the spring-mass system.

The experiment was divided into two parts. The first part was determining the spring constant of the spring used, and the second part was determining the plots of position versus time for different driving frequencies as to find at what frequency resonance happens.

For the first set-up, different masses were attached to the spring. The length of the spring at equilibrium was determined because the difference between this and each new length of the stretched spring corresponding to a particular mass was determined. The forces due to gravity done on the given masses were plotted against this set of differences, and the slope was determined because it is equal to the spring constant k.

The second part consisted of a speaker attached to a function generator whose frequency was varied. The function generator served as the driving force to the spring-mass system, and using a motion sensor to detect the motion of the spring with the presence of this driving force, the graph of position versus time was plotted. Resonance occurs at a frequency when the plot of position versus time shows that the amplitude increases as time progresses.

A graph of displacement versus frequency was also plotted to see that a peak occurs at the resonant frequency, or the frequency at which resonance occurs.

3 Results and Discussion

3.1 Natural Frequency

By using the set up shown from Figure (1a), the goal of determining the spring constant was made easy by simply plotting the graph of force vs. the inverse of displacement (Figure 2). The spring constant was found out to be $k = 16.36$. The graph obtained a value of $R^2 = 0.995$ which was close to one and can be considered to be accurate. This value of k was used to identify the natural frequency using equation ********. From the and by using m as 200 g, the theoretical natural frequency was found out to be 1.29.

3.2 Resonance

We know that resonance can be observed from the determination of the natural frequency of the spring. Take note that we used $m = 200\,\text{g}$ also for this part for consistency. We looked for the resonance by obtaining the graphs ranging from 0.5 Hz to 2.0 Hz. From figure (3), the graph peaked at 1.20 which we considered to be the experimental resonant frequency. This deviates from the theoretical resonant frequency by 7.5%.

4 Conclusions

We were able to obtain the spring constant of 16.36 that lead to knowing the theoretical value of the natural frequency for 200 g which is 1.29 Hz. Another experiment was done to calculate the experimental resonant frequency. We obtained a value of 1.20 Hz. Calculating the percent deviation, this yielded to 7.5%. This is a good result but can be improved by using a better condition of the spring. Also, take note of the motion sensor to be sensing the actual mass instead of the surrounding objects.

For further experiments, the experimenters can try to try a different mass and to weigh each mass instead of relying to its label. Also, a better straight spring could be used for more accuracy. They should also consider to align the motion sensor exactly to the moving mass.

References

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[2] Russel, D.A. (2018). The simple pendulum. Retrieved 13 March 2019, from *Acoustics and vibration animations*: https://www.acs.psu.edu/drussell/Demos/Pendulum/Pendula.html.

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Appendix
Figures and Diagrams

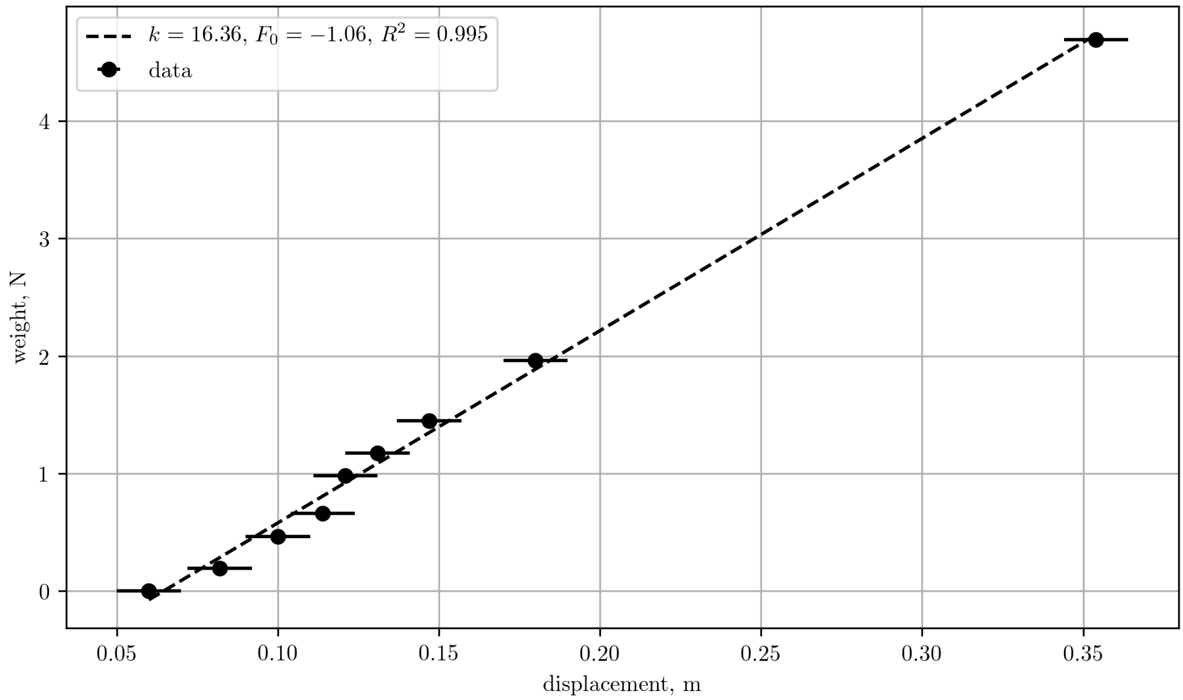

Figure 2: Fit of weight vs displacement to determine the spring constant k.

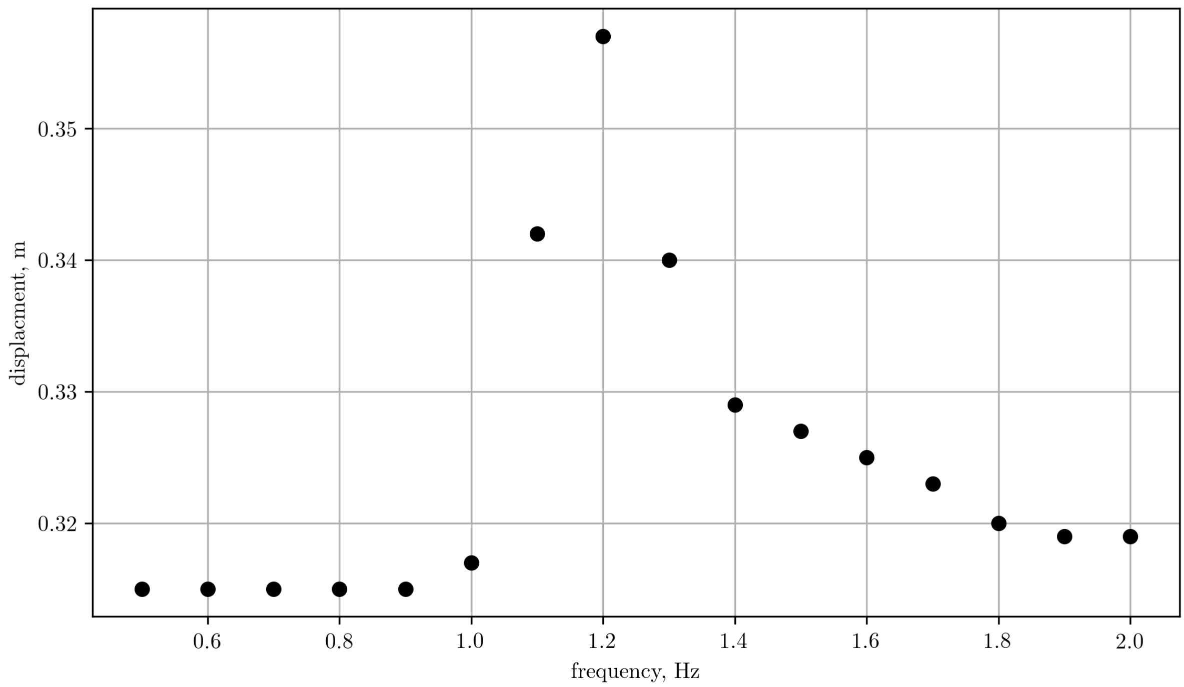

Figure 3: Identification of the experimental natural frequency for $m = 0.2$ kg.

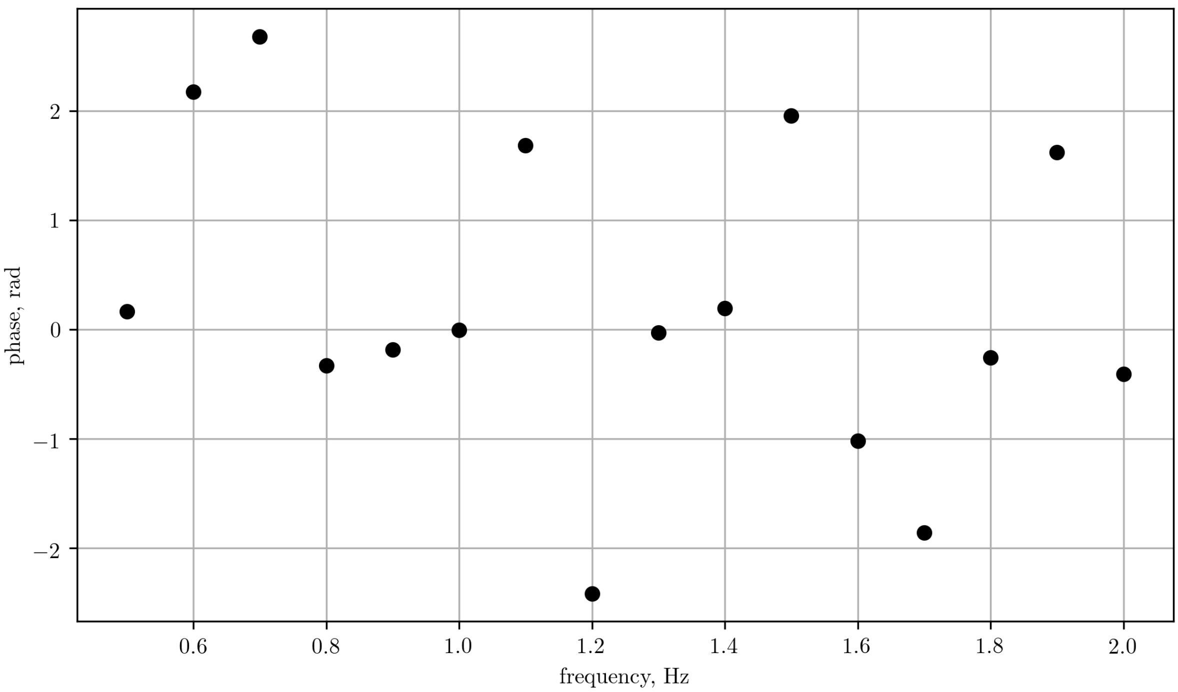

Figure 4: Phase vs frequency plot for $m = 0.2$ kg.