TM16
Universal Vibration

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Educational
PRODUCTS

CONTENTS

SECTION 1: INTRODUCTION

The **TQ Universal Vibration Apparatus** enables students to perform a comprehensive range of vibration experiments with the minimum amount of assembly time and the maximum adaptability.

The experiments lead the student through the basics of vibration theory by, initially, very simple experiments which make way for those of a more extensive nature as experimental aptitude increases.

Although the policy of the experiments is to give the student a general insight into experimental methods, there has been some attempt to evoke further study and critical appreciation by questions posed at the end of some of the tasks.

This manual primarily give details of the apparatus required and the experimental techniques involved for each experiment in turn. Each experiment starts with an 'Introduction' dealing with the purpose and basic theory involved. Further sections detail the apparatus and experimental method with reference to diagrams included in the text.

Finally, the form of calculations and results is given, followed by any 'Further Considerations' which may be significant.

SECTION 2: GENERAL DESCRIPTION OF THE APPARATUS

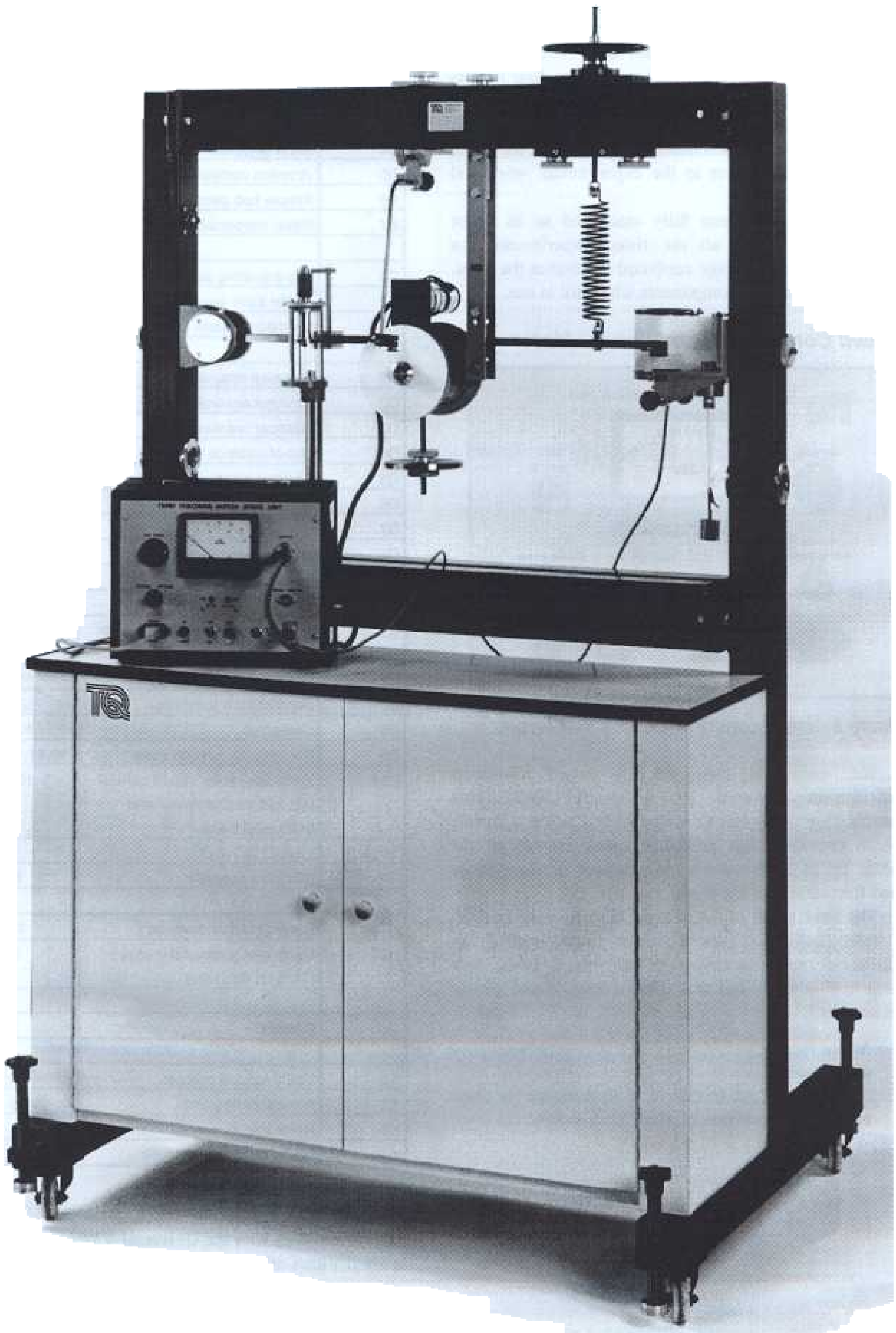


Figure 1 TM16 universal vibration apparatus

Portal Frame

The apparatus shown in Figure 1, consists of a basic portal frame, robustly constructed from square, rolled hollow section, vertical uprights and double channel horizontal members. The frame mounts on four castors for ease of mobility.

Screw jacks allow the weight of the frame to transfer to the floor during experiments, which enables the entire rig to be levelled prior to the experimental work and guarantees rigidity.

The frame has been fully machined so as to be adaptable to accept all the listed experiments. An attractive wooden storage cupboard is fitted at the front, which houses all the components when not in use.

Speed Control Unit and Exciter Motor

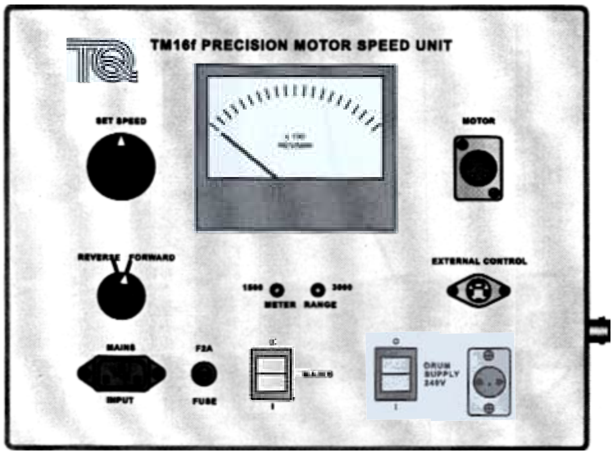


Figure 2 Speed control unit front panel layout

A d.c. motor is used for all forced vibrations experiments powered by a control unit. This combination comprises of a control box and d.c. motor, which provides high precision speed control of the motor up to 3000 rev/min, irrespective of the normal load fluctuations of the motor.

The front panel of the unit contains a speed control, a fully calibrated speed meter incorporating an automatic range switching device (there being two ranges: 0 – 1500 and 0 – 3000 rev/min), and power sockets for:

Mains input;
d.c. motor;
Auxiliary output (either to a stroboscope or chart recorder), sometimes marked drum supply.

List of Components

Part No.	Description	Experiment
B1	Pendulum sub-frame (cross beam)	1,2,3,4,5
B2	Simple pendulum - Wood ball	1
B3	Simple pendulum - Steel ball	1
B4	Kater (adjustable) pendulum	4
B5	Wooden compound pendulum	3
B6	Simple bob pendulum	2,3
B7	Bifilar suspension	5
C1	Top adjusting assembly (Spring)	6,12,13,14
C2	Guide bush assembly	6
C3	Loading platform	6
D1	Trunnion mounting	10,11,12,13,14
D2	Dashpot assembly	10,11,12,13,14
D3	Dashpot bracket	12,13,14
D4	Out-of-balance discs	12,13,14
D5	Beam support	12,13,14
D6	Stylus and support	12,13,14
D7	Chart recorder	12,13,14
D8	Pivot support for stylus	14
D9	Beam clamp	12,13,14
E1	Trunnion mounting with lateral movement	10,11
E2	Support for dashpot	10,11
E3	Support for micrometer	10,11
E5	Contactor	10,11
E6	Rectangular section base	10,11,12,13,14
E11	precision motor speed control unit with exciter motor and graduated discs	10,11
G1	Vibration absorber	11
H1	Rotor (254mm diameter)	7,9
H2	Rotor and additional masses (168mm diameter)	9
I1	Bracket	7
K1	Shaft support bracket	8
K2	Pen-recording unit	8
K3	Rotor and recording drum	8
K4	Transparent oil reservoir	8

Table 1

The oil supplied with the Universal Vibration Apparatus is Shell Vitrea oil.

SECTION 3: EXPERIMENTS

Experiment 1: Simple Pendulum

Introduction

One of the simplest examples of free vibration with negligible damping is the simple pendulum. The motion is simple harmonic, and is characterised by the equation:

$$\frac{d^2x}{dt^2} = \frac{-g}{l}x$$

The periodic time is:

$$t = 2\pi\sqrt{\frac{l}{g}}$$

In this experiment, the object is to analyse the above equation for the periodic time by varying the length of the pendulum, l, and timing the oscillations. The independence of the size of the mass of the particle is demonstrated.

Apparatus

Sub-frame (cross beam)	**B1**
Small wooden ball	**B2**
Small steel ball	**B3**
Inextensible flexible cord (not supplied)	
Stopwatch or clock (not supplied)	
Metre rule (not supplied)	

Both the steel and wooden balls attach to lengths of cord approximately one metre long, each of the two cords suspending from the small chucks at either end of the sub-frame. You can vary the length by pulling the thread through the chuck and the hole above the sub-frame.

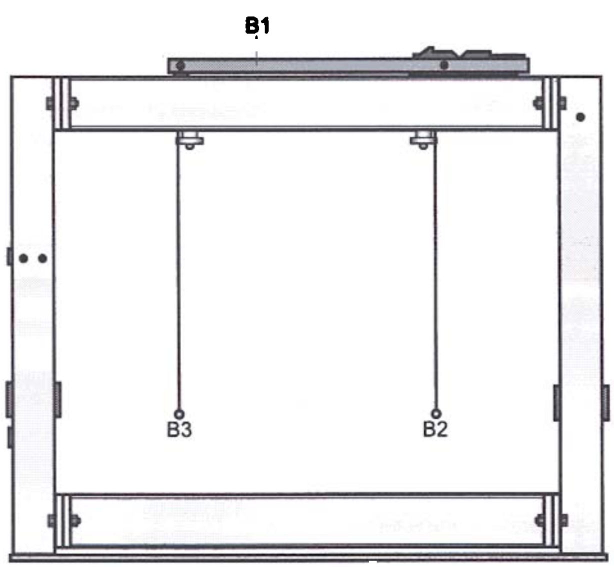


Figure 3

Procedure

Measure and note the length l, the distance from the bottom of the chuck to the centre of the ball. Displace the pendulum through a small angle θ, and allow to swing freely. Once settled, measure the time taken for 50 oscillations and record the periodic time, τ.

Repeat the procedure for various values of l for both the wooden ball and the steel ball. Enter the results in Table 2. Plot a graph for values of τ^2 against values of length l.

Results

Length l (m)	Time for 50 complete oscillations		Period τ	τ^2
	Steel	Wood	Steel	Steel
0.10				
0.15				
0.20				
0.25				
0.30				
0.35				
0.40				
0.45				
0.50				

Table 2 Experiment 1 results

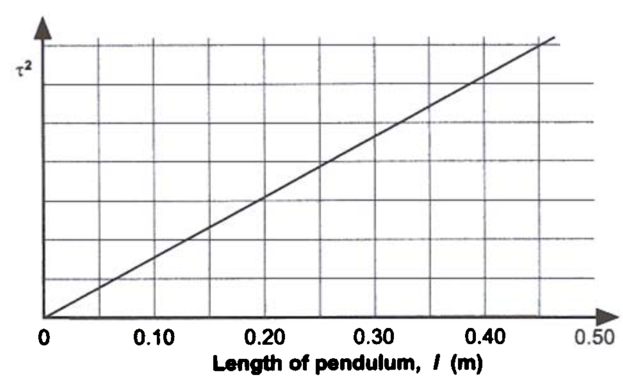


Figure 3 Graph of τ^2 against l for a simple pendulum

The graph results in a straight line, giving a relationship between τ^2 and l of the form:

$$\tau^2 = Kl$$

where K is a constant equal to $\frac{4\pi^2}{g}$

Hence the value of g, the acceleration due to gravity, can be determined.

Further Considerations

1. What inaccuracies exist in this method for calculating a satisfactory value for g?
2. How can you overcome these inaccuracies?

Experiment 2: Compound Pendulum

Introduction

A rigid body that swings about a fixed horizontal axis, shown in Figure 5, displaces through an angle θ and is subject to a restoring couple $mgh \sin\theta$.

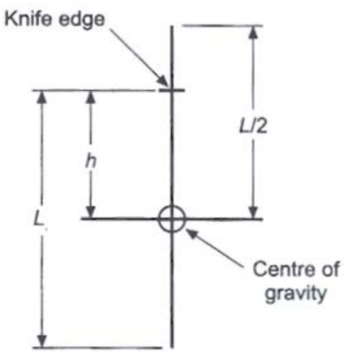


Figure 5 Compound pendulum

If angle θ is sensibly small, the equation of motion becomes:

$$\frac{d^2\theta}{dt^2} + \left[\frac{mgh}{I_A}\right]\theta = 0$$

- m = Mass of the body;
- h = Distance of the mass centre from the swing axis;
- θ = Angular displacement;
- I_A = Moment of inertia of the body about the swing axis.

This is simple harmonic motion so the constant:

$$\frac{mgh}{I_A} = \omega^2$$

and the periodic time $\tau = \frac{2\pi}{\omega}$. This gives:

$$\tau = 2\pi\sqrt{\frac{I_A}{mgh}}$$

$I_A = I_G + mh^2$ (by the parallel axis theorem)

and

$I_G = mk^2$

where k is the radius of gyration of the body about axis through the mass centre parallel to the swing axis. Therefore,

$$\tau = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

Apparatus

The compound pendulum consists of a steel rod of length 762 mm and diameter 12.7 mm. The rod is supported on the cross member B1 by an adjustable knife-edge which, when moved along the rod, effectively alters the value of h discussed above.

Procedure

Determine the location of the centre of gravity of the rod (midway along the rod).

For a given value of L_1 from one end, tighten the knife-edge and then suspend the rod by placing the knife-edge on the cross beam so that it swings freely through a small angle without any rotation of the support.

Once the system is swinging freely measure and note the time taken for 20 complete oscillations and record the periodic time, τ.

Repeat the procedure for differing values of L_1 and enter the values in Table 3. In order to perform further tests, slackened off the knife-edge be and move along the rod to a new position. It is found that removing the pendulum from the cross-beam to carry out any adjustments is the easiest method.

Results

The expression for the periodic time transforms to

$$\tau^2 h = \frac{4\pi^2}{g}h^2 + \frac{4\pi^2}{g}k^2$$

Plot a graph of $\tau^2 h$ to a base of h^2. Determine the slope of the line g, and from the intercept determine k.

L_1 (m)	h (m)	Time for 20 oscillations	Period τ (s)	h^2	$\tau^2 h$
0.45					
0.50					
0.55					
0.60					
0.65					
0.70					
0.75					

Table 3

Theoretical value of k can be found using Routh's Rule which for a rod of small cross-section gives:

$$k^2 = \frac{L^2}{\quad}$$

Further Considerations

1. Calculate the length of the simple equivalent pendulum for the above case where

   $\tau = 2\pi\sqrt{\dfrac{l}{g}}$ (simple pendulum) is equal to

   $2\pi\sqrt{\dfrac{k^2 + h^2}{gh}}$ for a compound pendulum.

2. Find the two values of h which satisfy the resulting quadratic equation giving equal periodic times.

Experiment 3: Centre of Percussion

Introduction

If you subject a compound pendulum supported on a horizontal pivot to an impact force at an arbitrary point, there will be a horizontal reaction at the pivot. We can liken this to a cricket bat striking a ball - there is one particular point at which the strike occurs, for which there is no horizontal reaction at the pivot of the compound pendulum. Such a point is the centre of percussion. The location of such a point is the object of this particular experiment.

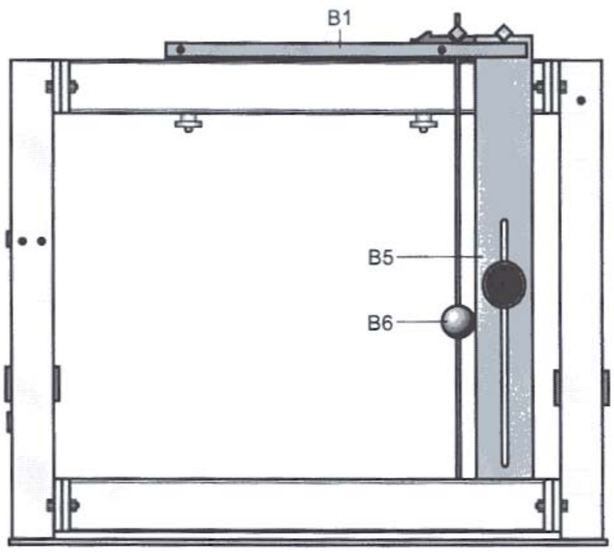


Figure 6

Figure 6 illustrates the apparatus, and consists of a steel ball as part of a simple pendulum (B6) and the rectangular shaped wooden compound pendulum (B5) having an adjustable steel weight slidable in a central slot. Both are suspended on steel knife-edges from the horizontal cross-beam (B1) at the top of the portal frame. The simple pendulum is located in a V-groove whilst the knife-edge of the compound pendulum rests on the flat surface of the beam.

Part A: Determining the Centre of Percussion

Procedure

To find the centre of percussion of the compound pendulum, first determine the periodic time. From this the radius of gyration about the pivot axis, k_A, can be found using formula:

$$\tau = 2\pi\sqrt{\frac{k_A^2}{gh}} \qquad (1)$$

where

h = Distance from the point of suspension to the centre of gravity and

$k_A^2 = k^2 + h^2$ (parallel axis theorem).

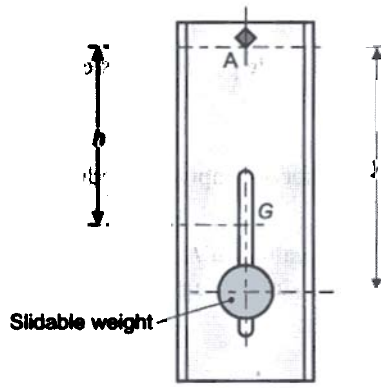


Figure 7

Determine the centre of gravity of the pendulum by resting the board, with the steel weight at distance y, from a knife-edge support. The distance h from the knife-edge of the pendulum to the balancing knife-edge can then be determined (see Figure 7).

For each position of the steel weight measure the time taken for 20 complete oscillations and record the results in Table 4.

From the values of τ and h in Equation (1) calculate the value of k_A and compare with the theoretical values.

Results

Table 4 will indicate the variation of periodic time as the radius of gyration about the point of suspension varies. Calculate a theoretical value for k, the radius of gyration about the centre of gravity, from the dimensions of the pendulum.

Test Number	Time for 20 oscillations	Period τ (s)	y (m)	h (m)	k_A (m)	k (m)
1						
2						
3						
4						
5						

Table 4

Part B: The Centre of Percussion in Relation to the Point of Suspension

Using the results of Part A, show that the centre of percussion can be at a distance from the point of suspension equal to its equivalent length:

$$l = \frac{k^2 + h^2}{h}$$

where

l = Length of the equivalent pendulum;

k = Radius of gyration about the centre of gravity;

h = Distance of the point of suspension from the centre of gravity.

Procedure

Adjust the length of the simple pendulum (B6) so that the length of the bob from the knife-edge is equal to the length of the compound pendulum. Allow the simple pendulum to swing, so that the spherical bob strikes the edge of the compound pendulum at its perigee (lowest point of its path) and causes the latter to swing.

By constraining horizontal movement of the simple pendulum in its V-groove, the only horizontal movement possible is that of the compound pendulum resting on its flat support. It may be observed that no horizontal movement is produced with the simple pendulum = 1 and that for any other values, horizontal movement is produced. A pencil mark on the cross-beam under the initial knife-edge position may be used as a reference mark.

Experiment 4: Determination of the Acceleration due to Gravity by means of a Kater (reversible) Pendulum

Introduction

The Kater pendulum is a device for accurately determining acceleration due to gravity. It consists of two adjustable knife-edges and an adjustable cylindrical bob. Arranging their relative positions to give equal periodic times when suspended from either knife-edge produces two simultaneous equations:

$$\tau_1 = 2\pi\sqrt{\frac{h_1^2 + k^2}{gh_1}}$$

$$\tau_2 = 2\pi\sqrt{\frac{h_2^2 + k^2}{gh_2}}$$

$$\frac{gh_1^2\tau_1^2}{4\pi^2} = h_1^2 + k^2$$

and

$$\frac{gh_2^2\tau_2^2}{4\pi^2} = h_2^2 + k^2$$

By arrangement

$$\frac{4\pi^2}{g} = \frac{\tau_1^2 + \tau_2^2}{2(h_1 + h_2)} + \frac{\tau_1^2 - \tau_2^2}{2(h_1 - h_2)}$$

Apparatus

The apparatus required for this experiment consists of a pendulum having two adjustable knife-edges and an adjustable cylindrical bob (B4) suspended from the hardened steel cross-beam (B1). See Figure 8.

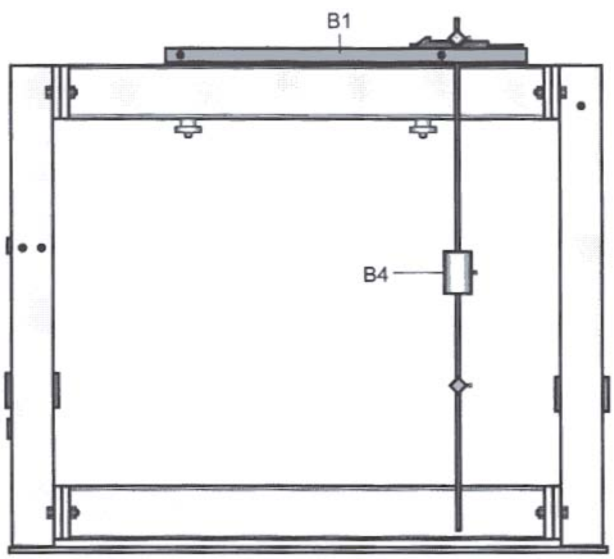


Figure 8

Procedure

Position the knife-edges a set distance apart, and then suspend the pendulum from one of the knife-edges. Allow the pendulum to swing freely and measure the time taken for 50 oscillations, and from this find the periodic time τ_1.

Reverse the pendulum and suspend it from the other knife-edge. By suitable positioning of the cylindrical bob, obtain the periodic time τ_2 to be approximately equal to τ_1.

Recheck τ_1 and carry out any further adjustments to obtain an equal time of swing.

Once τ_2 has been altered to be approximately equal to τ_1, allow the pendulum to swing for 200 oscillations and note the times for both τ_1 and τ_2.

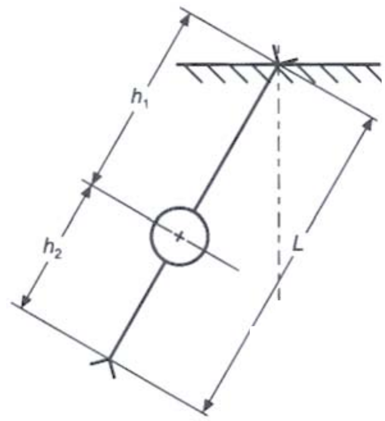


Figure 9

Find the centre of gravity of the pendulum by balancing it on a knife-edge and measuring h_1 and h_2, the respective distances of the knife-edges from the centre of gravity. The distance between the two edges is the length of the simple equivalent pendulum, L.

Results

h_1 = 0.20 m	h_2 = 0.30 m
τ_1 =	τ_2 =

Table 5

$$\frac{4\pi^2}{g} = \frac{(\tau_1^2 + \tau_2^2)}{2(h_1 + h_2)} + \frac{(\tau_1^2 + \tau_2^2)}{2(h_1 - h_2)}$$

From which the value of g is determined

Experiment 5: Bifilar Suspension

Introduction

The bifilar suspension can determine the moment of inertia about an axis by suspending two parallel cords of equal length through the mass centre of bodies, as shown in Figure 10. Angular displacement of the body about the vertical axis through the mass centre G is by angle θ, which is sensibly small.

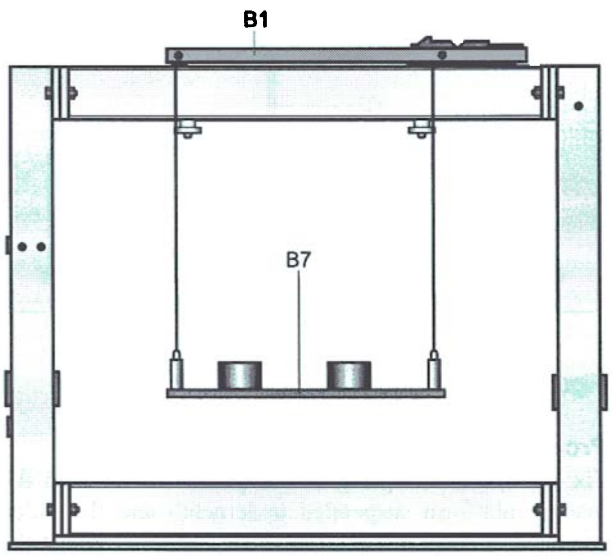


Figure 10

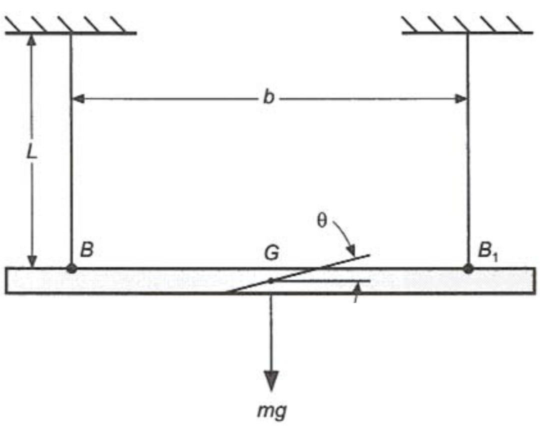


Figure 11

This equation of the angular motion is:

$$\frac{Id^2\theta}{dt^2} = -\frac{mgb^2}{4L}\theta$$

which may be written as:

$$\ddot{\theta} + \frac{gb^2}{4k^2L}\theta = 0$$

The motion is clearly simple harmonic and the period is:

$$\tau = 4\pi\sqrt{\frac{k^2L}{gb^2}}$$

where I is moment of inertia about swing axis through $G(I = mk^2)$.

Knowing the periodic time and the magnitudes of the various parameters, the radius of gyration k and therefore the value of I can be determined.

Apparatus

Figure 10 shows the apparatus and consists of a uniform rectangular bar B7 suspended by fine wires from the small chucks as used in Experiment 1. Drawing the two wires through the chucks and tightening alters the lengths of the suspension. The bar is drilled at regular intervals along its length so that two 1.85 kg masses may be pegged at varying points along it.

Procedure

With the bar is suspended by the wires, adjust length L to a convenient size, and measure the distance between the wires, b. Displace the bar through a small angle and measure the time taken for 20 complete oscillations. From this, calculate the periodic time.

Adjust the length of the wires, L, and measure the time taken for a further 20 swings. Increase the inertia of the body by placing two masses symmetrically on either side of the centreline distance x apart, and repeating the procedure for various values of L and the distance between the masses. Calculate the radius of gyration k of the system as previously outlined. Tabulate the results in Table 6.

Results

Test number	L (m)	x (m)	τ (s)	k (m)	k^2 (m^2)	m (kg)	$I = mk^2$ (kgm^2)
1							
2							
3							
4							

Table 6 Results for bifilar suspension

It is instructive to compare the value of I obtained in a particular test with the value of I determined analytically, using $\sum\delta mx^2$.

Further Considerations

1. Some noteworthy points will have arisen as a result of performing this experiment. Write out your conclusions.
2. How would the radius of gyration, and hence moment of inertia, of a body using the bifilar suspension be determined?

Experiment 6: Mass - Spring Systems

Introduction

A helical spring, deflecting as a result of applied force, conforms to Hooke's Law (deflection proportional to deflecting force).

The graph of force against deflection is a straight line as shown in Figure 12.

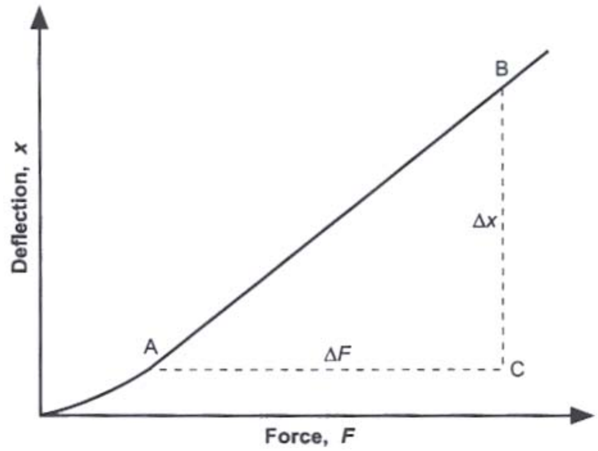


Figure 12

Slope of the line $\frac{\Delta x}{\Delta F}$ is the 'deflection coefficient' in metres per newton.

The reciprocal of this is the stiffness of the spring and is the force required to produce unit deflection. A rigid body of mass M under elastic restraint, supported by spring(s), forms the basis of all analysis of vibrations in mechanical systems. The basic equation is of the form:

$$M\ddot{x} = -kx$$

where k = stiffness of the spring

This clearly simple harmonic motion of periodic time τ:

$$\tau = 2\pi\sqrt{\frac{M}{k}}$$

Apparatus

Figure 13 shows the required set-up for the experiment. Suspend any one of the three helical springs supplied from the upper adjustable assembly (C1) and clamp to the top member of the portal frame.

To the lower end of the spring is bolted a rod and integral platform (C3) onto which 0.4 kg masses may be added. The rod passes through a brass guide bush, fixed to an adjustable plate (C2), which attaches to the lower member. A depth gauge is supplied which, when fitted to the upper assembly with its movable stem resting on the top plate of the guide rod, can be used to measure deflection, and thereby the stiffness, of a given spring.

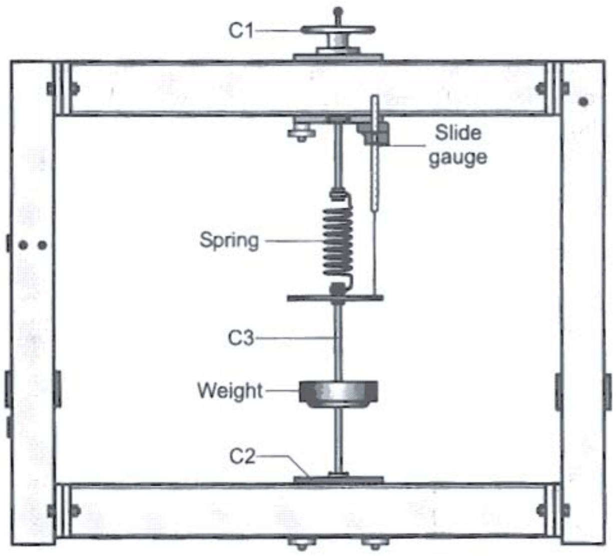


Figure 13

Procedure: Part A

Fix the specimen spring to the portal frame, with the loading platform suspended underneath and the guide rod passing through the guide bush. Carefully adjust the system to ensure that the guide bush is directly below the top anchorage point, since any misalignment will produce experimental errors due to friction. Friction can be minimised by using grease or oil around the bush.

Using the gauge measure the length of the spring with the platform unloaded. Add weights in increments, taking note of the extension in Table 7, until reaching a suitable maximum load. Remove the weights, again noting the length at each increment, as the system is unloaded. From these values determine the mean value of extension for the spring.

M	**Deflection x**		**Mean x**
(kg)	**Loading**	**Unloading**	**(mm)**
0			
0.4			
0.8			
1.2			
1.6			
2.0			
2.4			
2.8			
3.2			
3.6			
4.0			

Table 7

Plot a graph for the extension against load, and from this determine the spring stiffness, k.

Procedure: Part B

Add masses to the platform in varying increments, pull down on the platform and release to produce vertical vibrations in the system. For each increment of weight note the time taken for 20 complete oscillations in Table 8, and from this calculate the periodic time, τ.

Mean coil diameter:			
Mean wire diameter:			
Number of coils:			
M (kg)	Time for 20 oscillations	Period τ (s)	τ^2 (s^2)
1..47			
3.47			
4.27			
4.67			
5.07			
5.47			

Table 8

Note that

$$\tau^2 = \left(\frac{4\pi^2}{k}\right)M$$

The mass of the rod and platform are included in M above. From Table 8 plot a graph of τ^2 against M and find the slope of the graph, g, and the M-axis intercept, m.

Results

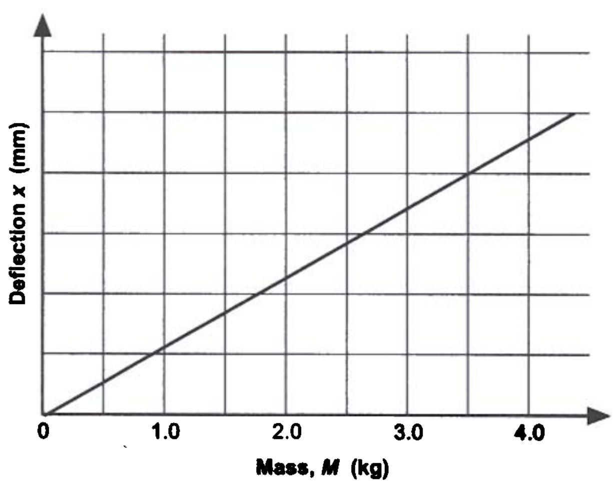


Figure 14 Part A graph

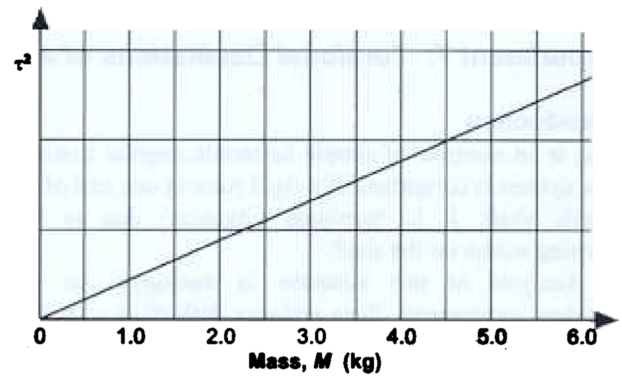


Figure 15 Part B graph

From the intercept of the line on the M-axis, the effective mass of the spring can be found (m). Compare the value of m obtained with the generally accepted value, that is, $\frac{1}{3}$ mass of spring. Repeat the procedure with the other springs provided.

Further Considerations

1. State your conclusions in the light of the results obtained. Has the basic theory been verified?
2. From the experiments so far performed, discuss the relative merits of each in calculating an accurate value for g. Criteria for your comments should be:
   a. Ease of experimentation;
   b. Inherent inaccuracies;
   c. Ease of computation.
3. Choosing some typical results, what error is introduced in calculating g by neglecting the effective mass of the spring?

Experiment 7: Torsional Oscillations of a Single Rotor

Introduction

This is an example of simple harmonic angular motion. The system is comprised of a rigid rotor at one end of an elastic shaft. It is 'torsional vibration' due to the twisting action on the shaft.

Analysis of this situation is analogous to the previous experiment. Zero replaces deflection x, and k, which was stiffness, is now torsional stiffness of the shaft. The polar moment of inertia of the rotor I, replaces mass M.

The equation of motion is $I\ddot{\theta} = -k\theta$, which is simple harmonic motion. It can be shown that the time period, τ, is:

$$\tau = 2\pi\sqrt{\frac{IL}{GJ}}$$

where:

L = Effective length of the shaft;
G = Modulus of rigidity of the material of the shaft
J = Polar moment of area of the shaft section.

Apparatus

For experiments on undamped torsional vibrations, the inertia is provided by two heavy rotors, cylindrical in shape, one 168 mm diameter the other 254 mm diameter. Figure 16 shows the smaller diameter rotor, H2. The rotor mounts on a short axle, which fits in either of the vertical members of the portal frame, and secures by a knurled knob.

The rotor is fitted with a chuck designed to accept shafts of different diameter. An identical chuck rigidly clamps the shaft, which is an integral part of a bracket (I1). This is at the same height as the flywheel chuck and adjustable, relative to the base of the portal frame. Three steel test shafts are supplied - 3.18, 4.76 and 6.35 mm in diameter, each 965 mm long.

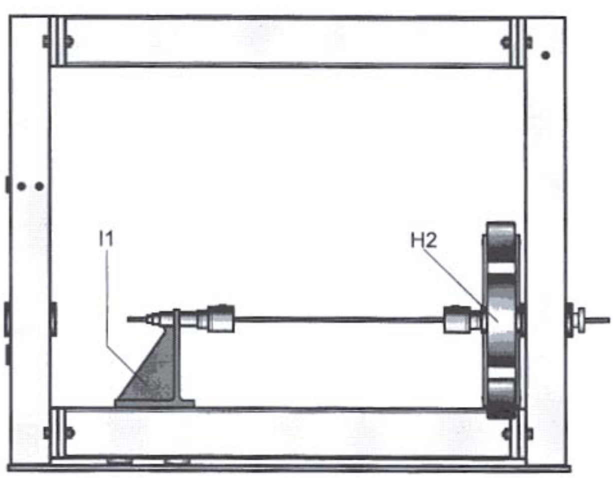


Figure 16

By bolting two pairs of steel arms to each side and attaching heavy masses at each end, we can increase the inertia of the smaller rotor. Two pairs of masses are available of approximately 1800 g and 3200 g.

Part A: To Determine the Moment of Inertia of a Flywheel

Procedure

The moment of inertia of a flywheel (one of the rotors would be most suitable), can be found experimentally by the falling weight method. The flywheel mounts as described above so that it can rotate freely on an axle fitted to one of the vertical members of the frame.

In the case of the smaller rotor with added masses, it is necessary to clamp the rotor in the reversed position, since a complete revolution of the whole assembly is impossible with the rotor clamped inside the portal frame.

Attach a body of mass m to a length of string. Wind the string around the circumference of the rotor, ensuring that it loops around a steel peg projecting from the rim. Allowed the body to fall through a measured height h to the ground and record the time of descent, t_1, by a stopwatch (not supplied). Note the number of revolutions, n_1, during the acceleration period.

Find the number of revolutions, n_2, and the corresponding time, t_2, from the instant the body strikes the ground to the instant the rotor comes to rest. Adjust the length of string so that the string detaches itself as the body strikes the ground. More than one test should be performed to obtain average values for n_1, n_2 and the times t_1 and t_2.

Theory

Apply the basic energy equation: $W = \Delta E$ to the two phases of the motion of the system.

Acceleration period

$$-T_f n_1(2\pi) = \frac{m}{2}\left[v^2 - 0\right] + mg\left[0 - h\right] + \frac{I}{2}\left[\omega^2 - 0\right]$$

Deceleration period

$$-T_f n_2(2\pi) = \frac{I}{2}\left[0 - \omega^2\right]$$

Eliminating T_f from the above two equations gives:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\left[\frac{n_1 + n_2}{n_2}\right]$$

from which I can be calculated

Notation

m = mass of the falling body (kg).
h = height of fall (m).
v = maximum velocity of the body striking ground (m/s).
ω = corresponding maximum angular velocity of the wheel (rad/s).

T_f = Frictional torque at the bearing of the wheel (Nm).

n_1 = Number of revolutions of the flywheel to wind up the string from the ground to the starting point.

n_2 = Number of revolutions the wheel makes after the falling body strikes the ground (deceleration period).

r = Effective radius (m).

t_1 = Time of fall of the body (s).

t_2 = Time for wheel to come to rest after the falling body has reached the ground.

$$n_2 = \frac{\omega t_2}{4\pi} \text{ and } h = \frac{0+v}{2}t_1$$

$$\therefore v = \frac{2h}{t_1} \text{ and } \omega = \frac{v}{r}$$

When performing a practical test the value of *m* should not be too large, otherwise the duration of the second phase of the motion runs into many minutes. A value of *m* equal to about 0.05 kg is suggested. *I* comes to approximately 0.18 kgm^2

Part B: Frequency of Torsional Oscillations (Single Rotor System)

Having determined a value for *I* for a particular rotor by the method described in Part A using one of the three shafts, the frequency of torsional oscillations of a single rotor system can be found experimentally. Compare the result with theoretical prediction.

Procedure

Pass the shaft through the bracket centre hole, so that it enters the chuck on the flywheel and then tighten. Move the bracket along the slotted base until the distance between the jaws of the chuck corresponds to the required length *L*. Tighten the chuck on the bracket.

Ensure that the jaws securely grip the shaft. Displace the rotor (flywheel) angularly and record the time for 20 oscillations.

Vary the distance between the chucks in suitable increments by sliding the bracket, and record and tabulate the values of periodic time for the various shaft lengths. Plot a graph of τ^2 against *L*.

Results

L (mm)	Time for 20 oscillations	Period τ (s)	τ^2
100			
150			
200			
250			
300			
375			
450			

Table 9

From the slope of the graph $\frac{4\pi^2 I}{GJ}$ determine the value of G and compare with the generally accepted value.

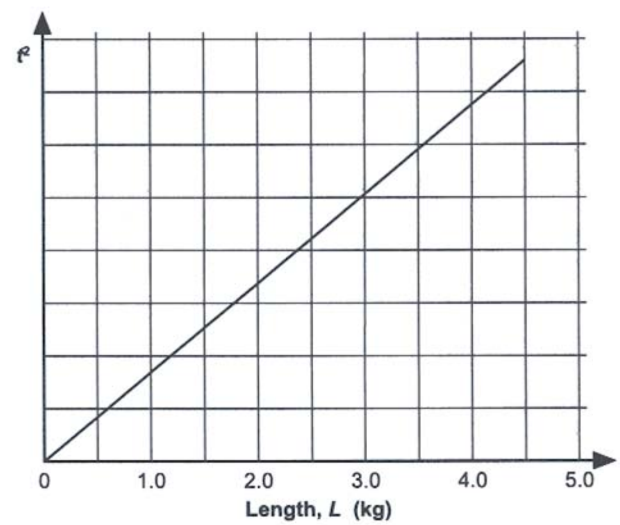


Figure 17 Graph of τ^2 versus L

Further Considerations

1. Using the falling weight method, if the string wrapped around the axle of the wheel instead of around its rim how would this affect the results?
2. What change(s) in procedure would be necessary if you used a stepped shaft instead of one of uniform section throughout its length?

Experiment 8: Torsional Oscillations of a Single Rotor with Viscous Damping

Introduction

In this experiment, the effect of including a damper in a system undergoing torsional oscillations is investigated. The amount of damping in the system depends on the extent to which the conical portion of a rotor is exposed to the viscous effects of a given oil.

Theory

The equation of the angular motion is:

$$\frac{Id^2\theta}{dt^2} = -C\frac{d\theta}{dt} - k\theta$$

which may be written

$$\frac{d^2\theta}{dt^2} + a\frac{d\theta}{dt} + b\theta = 0$$

where $a = \frac{C}{I}$ and $b = \frac{k}{I}$

The angular displacement is:

$$\theta = Ce^{(-a/2t)}\cos(pt + \psi)$$

where C and ψ are constants.

The periodic time is

$$\tau = \frac{2\pi}{p}$$

Measuring amplitudes on the same side of the near position, the nth oscillation is:

$$\frac{x_0}{x_n} = e^{\left[\left(\frac{a}{2}\right)n\tau\right]}$$

where n is a positive integer corresponding to the number of complete oscillations starting at a convenient datum ($t = 0$).

Putting n = 1 gives the logarithmic decrement $\log_e\left[\frac{x_n}{x_{n+1}}\right] = \frac{a}{2}\tau$. This is all that is required by way of basic theory.

Apparatus

Figure 18 shows the apparatus, and consists of a vertical shaft gripped at its upper end by a chuck attached to a bracket (K1) and by a similar chuck attached to a heavy rotor (K3) at its lower end.

The rotor K3 suspends over a transparent cylindrical container, K4, containing damping oil. The oil container can be raised or lowered by means of knurled knobs on its underside, allowing the contact area between the oil in the container and the conical portion of the rotor to vary. This effectively varies the damping torque on the rotor when the latter oscillates. Record damped oscillation traces on paper wrapped round the drum mounted above the flywheel. Unit K2 consists of a pen-holder and pen, which adjust to make proper contact with the paper; the unit undergoes a controlled descent over the length of the drum by means of an oil dashpot clamped to the mainframe.

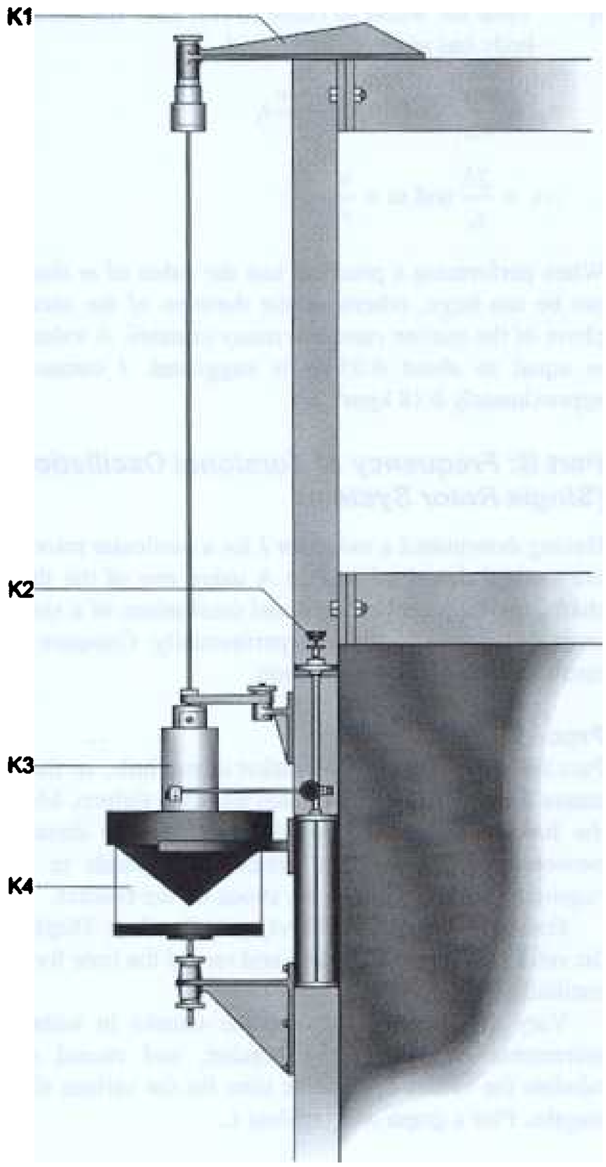


Figure 18

You can use various diameter shafts, but due to the location and necessary fine adjustment of the oil container the length is restricted to approximately 0.75 m. Measure the angular displacement of the flywheel by means of a graduated scale on the upper rim of the rotor. An etched marking on the frame serves as a datum for the measurement of angular displacement.

Part A: Determination of Damping Coefficient

Procedure

Fill the cylindrical container K4 with oil to within 10 mm of the top. Adjust the knobs underneath to level the oil surface with one of the upper graduations on the conical portion of the rotor, K3. A depth, *d* of 175 mm is suggested for maximum damping. Details of the graduations on the rotor are in Figure 19.

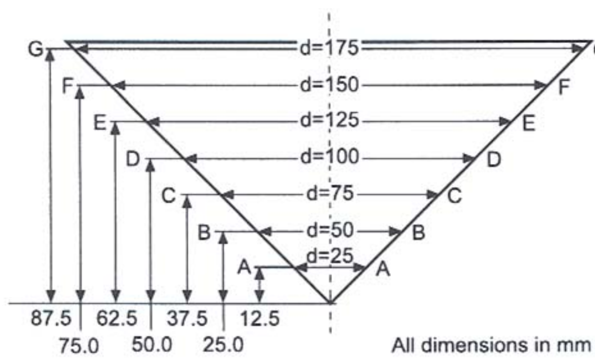


Figure 19 Conical graduations

Select and fit a suitable shaft, noting the length of the shaft between the two inside faces of the chuck, together with the diameter of the shaft. Allow the pen to fall, and measure the rate of descent of the pen (in mm/second) by timing the descent of the pen over a fixed length of paper, using a stopwatch.

The system is now ready for recording torsional oscillations. Raise the pen to the top of the paper on the drum and rotate the rotor to an angle of approximately 40° and then release. A trace of the oscillations can be obtained by bringing the pen into contact with the paper using the thumbnut on the support and allowing the pen to descend.

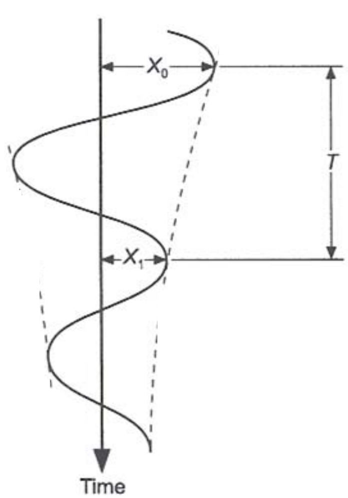


Figure 20

Record a trace of the amplitude of oscillation showing decay of the vibration due to the damping. The rate of descent of the pen previously carried out will provide a suitable time scale.

From the trace given in Figure 20, measure five successive amplitudes starting with the initial one (n = 0) and tabulate the results in Table 10 below.

n	x_n (mm)	$\log_e \frac{x_o}{x_n}$
0	x_0 =	0
1	x_1 =	
2	x_2 =	
3	x_3 =	
4	x_4 =	
5	x_5 =	

Table 10

Results

Plot a graph of $\log_e (x_o/x_n)$ to a base of *n*. Confirm that the damping is viscous, and that the slope of the line is equal to $(a\tau/2)$ (the logarithmic decrement).

The period can be found by timing a convenient number of oscillations using a stopwatch, whereupon the constant, *a*, is determined and hence the value of the damping coefficient (the torque per unit angular velocity) in Nm/rad/s^{-1}. The polar moment of inertia of the rotor is determined as in Experiment 7.

Part B: Investigation of how the Damping Coefficient depends on the Depth of Immersion of the Rotor in the Oil

Repeat Part A for each oil level as defined by the seven graduations on the conical portion of the rotor.

The damping coefficient depends on the area *A* of the curved surface of the conical portion of the rotor exposed to viscous damping. This area is equal to πrl, where *r* is the radius of base of core and *l* is the slant height equal to $\sqrt{r^2 + h^2}$.

Plot a graph of damping coefficient to a base of *A* times mean radius.

Results

Tabulate these as in Table 11

r (mm)	Mean radius r_m (mm)	Area A (mm^2)	$A.r_m$ (mm^3)	Period τ (s)	Constant a	Damping coefficient
12.5	6.25					
25.0	12.50					
37.5	18.75					
50.0	25.00					
62.5	31.25					
75.0	37.50					
87.5	43.75					

Table 11 Results of torsional oscillation with viscous damping

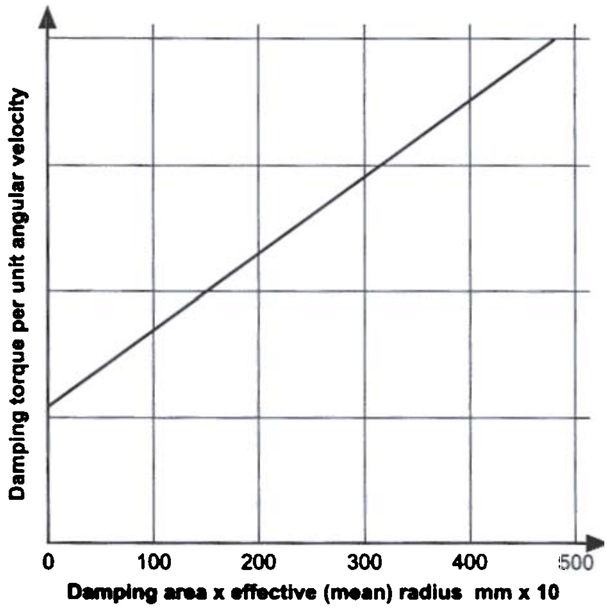


Figure 21

State the probable relationship between the two parameters.

Experiment 9: Torsional Oscillations of Two Rotor System

Introduction

With the addition of a second rotor, the apparatus described in Experiment 7B can be used to investigate the oscillation of a two rotor system. For such a system the periodic time is:

$$\tau = \sqrt{\frac{I_1 I_2 L}{GJ(I_1 + I_2}}$$

where

I_1 = Moment of inertia of the rotor 1;
I_2 = Moment of inertia of rotor 2;
L = Length of the shaft between the rotors;
G = Modulus of rigidity of the material of the shaft;
J = Polar second moment of area of the shaft section.

Apparatus

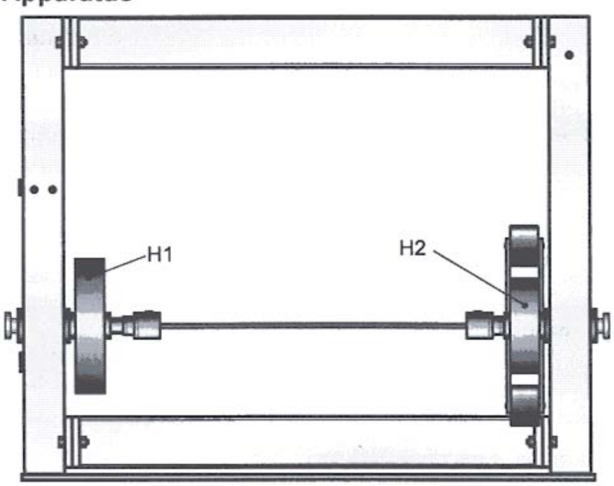


Figure 22

The apparatus, as in Figure 22, is that of Experiment 7, with the bracket (I1) replaced by a second rotor (H1) which is free to rotate on a axle fixed to the left-hand vertical member of portal frame. Both rotors have chucks fitted for use with shafts of various diameters. Since both rotors axles are fixed to their respective vertical members, the length of the shaft may not be varied but three shafts of different diameter are supplied and three combinations of different inertias are possible.

Procedure

One of the shafts clamps between the two rotors H_1 and H_2 of predetermined inertia. Record the effective length of the shaft measured between the jaws of the chucks. Carefully tighten the chucks to ensure that neither rotor can slip relative to the shaft.

Rotate each rotor through a small angle in opposite directions and then release. Torsional oscillations of the system are thereby set up and the time for 20 oscillations recorded.

The periodic time of the system may be determined and compared with the theoretical value given by the formula quoted in the introduction. Determine the moments of inertia of the rotors the method described in Experiment 7.

Results

Polar second moment of area $J = \frac{\pi}{32} d^4$

The generally accepted value of G for steel is 82 GPa and for g 9.81 m/s^2.

Further Considerations

When oscillating torsionally, the two rotors oscillate back-to-back about a non-moving section of the shaft, called the node. It is instructive to locate the position of the node for a given pair of inertias and their shaft. This can be done by introducing a third (dummy) rotor in the form of a cardboard disc (of negligible inertia) and moving it along the shaft to a position where it becomes fixed in space.

Shaft diameter mm	I_1 kgm^2	I_2 kgm^2	Time for 20 oscillations	Period τ	Theoretical value of period
3.17					
3.17					
4.76					
6.35					

Table 12

Experiment 10: Transverse Vibration of a Beam with One or More Bodies Attached

Introduction

The frequency of transverse vibrations of a beam with bodies attached is identical to the critical (whirling) speed of a shaft of the same stiffness as the beam, carrying rotors of masses which correspond to those of the bodies on the beam.

One has to think in terms of small size rotors, otherwise gyroscopic effects are involved. In the case of a beam with just one body attached, the basic theory is the same as that in Experiment 6. For a beam with two or more bodies attached, other methods can determine the frequency of free transverse vibrations. Examples are as follows:

Rayleigh or energy method (gives good results);
Dunkerley equation (only approximate, but quite adequate);
Rigorous (accurate) analysis (arduous);
Experimental analysis, using the equipment described below, (fairly simple and quick).

Apparatus

The basic apparatus for this experiment is in Figure 23. A bar of steel of rectangular cross-section (E6) is supported at each end by trunnion blocks. The left-hand support (D1) pivots in two ball bearings in a housing located on the inside face of the vertical frame member.

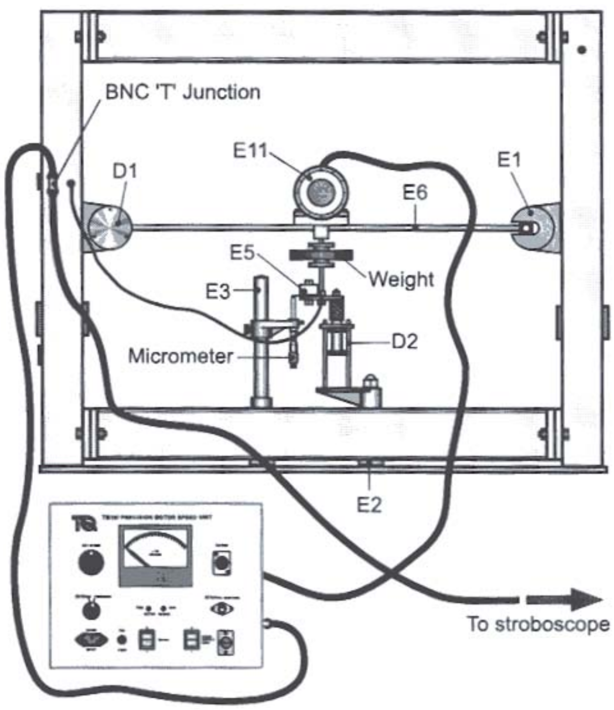


Figure 23

The right-hand support consists of two roller bearings, which are free to move in a guide block located on the inside face. At the centre of the beam bolt a small motor carrying two 'out-of-balance' discs (part of Excitor Motor and Speed Control unit). Connect the motor via leads to the precision speed control unit, which applies a wide range of exciting frequencies to the beam.

Clockwise rotation of the control knob on the speed control unit will increase the speed of the motor - thus increasing the out-of-balance rotating force produced by the unbalanced discs. As the speed increases as indicated by the speed meter on the control unit, the beam begins to vibrate transversely. Over a discrete band of frequencies increasingly larger amplitudes of vibration are produced which reach a peak at a frequency corresponding to the frequency of free natural transverse vibration of the system, i.e. beam plus added components.

Part A: Transverse Vibration of a Beam

Procedure

Suspend bodies of different size mass, m, below the motor. For each mass *m*, adjust the speed control until the beam vibrates at its natural frequency.

In order to determine accurately the exact value on the speed meter, it is expedient to take the beam through the range of excessive amplitudes several times, noting the limits of the range. From these, we can locate the frequency at which the amplitude and resultant noise appears greatest. Record your observations in Table 13.

Mass m kg	Frequency f (Hz)	f^2	$\frac{1}{f^2} \times 10^3$
4.8			
5.2			
6.8			
8.0			
9.2			
10.8			
11.6			
13.2			
14.4			

Table 13 Table of results for Experiment 10A

Results

A graph of $(1/f$ to a base of *m* gives a straight line, as in Figure 23.

The intercept on the vertical axis is equal to (

Natural frequency of the system, i.e. beam added components.
Natural frequency of the beam by itself

Dunkerley's equatior applicable to this situatior and is given by:

+

Here f_1 = natural frequency of a corresponding light beam with mass *m* attached. Clearly when

$m = 0$, $f_1 = \infty$ and $f = f_b$

Evaluate and compare with the theoretical value obtained from:

$$f_b = \frac{\pi^2}{2}\sqrt{\frac{EI}{m_o L^3}}$$

where

- L = Length of the beam (m);
- F = Modulus of elasticity of material of the beam (N/m^2);
- I = Second moment of area of the beam section;
- m_o = Mass of the beam by itself (kg); no masses attached.

Also, from the graph, when the system is not vibrating (period $\tau = 0$) f = ∞ and $1/f^2 = 0$. The corresponding value of mass m is then equal to m_e, the equivalent mass of the beam. $m_e = \lambda m_o$, where λ is a constant.

Determine the value of λ. How does it compare with the generally accepted value of 0.5?

Further Considerations

We can test the validity of the Dunkerley equation in its more familiar form by moving the motor with out-of-balance discs away from the centre of the beam and attaching a heavy body of known mass at some other point on the beam. The Dunkerley equation then becomes:

$$\frac{1}{f^2} = \frac{1}{f_\tau^2} + \frac{1}{f_2^2} + \frac{1}{f_b^2}$$

The f_2 in this equation could be the variable parameter and a graph plotted similar to the one described above. A special block for attaching extra masses to the beam and a suitable vibration generator of variable frequency (not supplied with the standard equipment) would be required to perform this additional test.

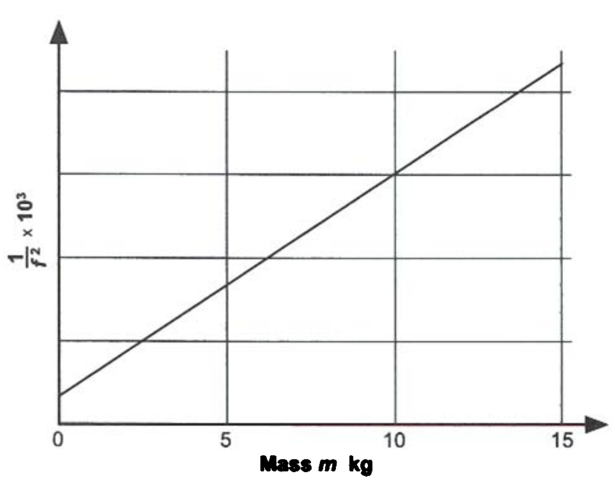


Figure 24 Graph of $1/f^2$ against m *for the system*

Part B: Damped Transverse Vibration of a Beam

Introduction

Damping forces are counteracting forces in a vibration system, which gradually reduce the motion. Damping occurs in all natural vibrations and may be caused by Coulomb friction (rubbing between one solid and another), or viscous resistance of a fluid as in this experiment on damped transverse vibration of a beam where a dashpot is use.

Apparatus

This is shown in Figure 23 (the same set up as for Experiment 10A, but with certain additions). In this experiment you will require the amplitude of vibration and phase angle. Fit a dashpot (D2) and its support (E2) to the beam to create damping. Use the contactor (E5) with its vertically mounted micrometer to determine the amplitude and phase angle very accurately at any exciting frequency. The electric circuit, of which a stroboscope is a part, completes when the contact element (E5) touches the plunger of the micrometer.

Procedure

Allow the speed control unit time to warm-up, then adjust the micrometer plunger so that it just touches the contactor. When the stroboscope switches to **external** stimulus, a discharge occurs on contact. Take the micrometer reading. Use this value as a datum position from which values of amplitude may be determined.

Energise the motor to produce a definite amplitude at a predetermined frequency. To determine the amplitude, lower the micrometer head and then bring up again to produce contact. It is important that the stroboscope discharges at a uniform frequency, so careful adjustment must be made to ensure steady conditions. At this point, find the amplitude of the vibration by comparing the new micrometer reading with that of the original datum position.

You may also find the phase angle by focusing the stroboscope on the graduated disc on the motor shaft. Since the stroboscopic discharge should be at a frequency corresponding to the rotational speed of the motor, the disc may be effectively stopped and the phase angle corresponding to the datum mark on the motor read off. By following this procedure for a range of frequencies, you can assess the effect of damping by varying the piston area of the dashpot and thus altering the damping characteristics of the system.

Rotate the two orifice plates inside the dashpot relative to one another to vary the effective area. Compare the results obtained with these settings with an undamped condition (the system minus dashpot). Plot graphs of amplitude and phase angle against the frequency ratio, ω/ω_n i.e. (exciting frequency/natural frequency).

Note: At low frequencies, phase angle may **not be** obtainable.

Results

Motor speed (rev/min)	$\frac{w}{w_n}$	Phase angle log (°)	Amplitude x max. (mm)
500			
600			
700			
800			
900			
980			
1000			
1010			
1020			
1040			
1050			
1055			
1060			
1075			
1100			
1200			
1300			
1400			
1500			
1800			
2000			
2500			

Table 14

Motor speed (rev/min)	$\frac{w}{w_n}$	Phase angle log (°)	Amplitude x max. (mm)
500			
600			
700			
800			
900			
980			
1000			
1010			
1020			
1040			
1050			
1055			
1060			
1075			
1100			
1200			
1300			
1400			
1500			
1800			
2000			
2500			

Table 15

Motor speed (rev/min)	$\frac{w}{w_n}$	Phase angle log (°)	Amplitude x max. (mm)
500			
600			
700			
800			
900			
980			
1000			
1010			
1020			
1040			
1050			
1055			
1060			
1075			
1100			
1200			
1300			
1400			
1500			
1800			
2000			
2500			

Table 16

The results, in Tables 14 to 16, show the effect of increasing damping on amplitude and phase angle. For each damping condition a graph of amplitude against frequency can be plotted, from which a value for the natural frequency for each damping condition can be found. Typical values obtained in this way are as follows:

No damping	17.36 Hz
Light damping	17.50 Hz
Heavy damping	17.58 Hz

and from these values the frequency ratio can be found being the exciting frequency/natural frequency. Figures 25 and 26 are typical graphs of amplitude and phase angle plotted against frequency ratio.

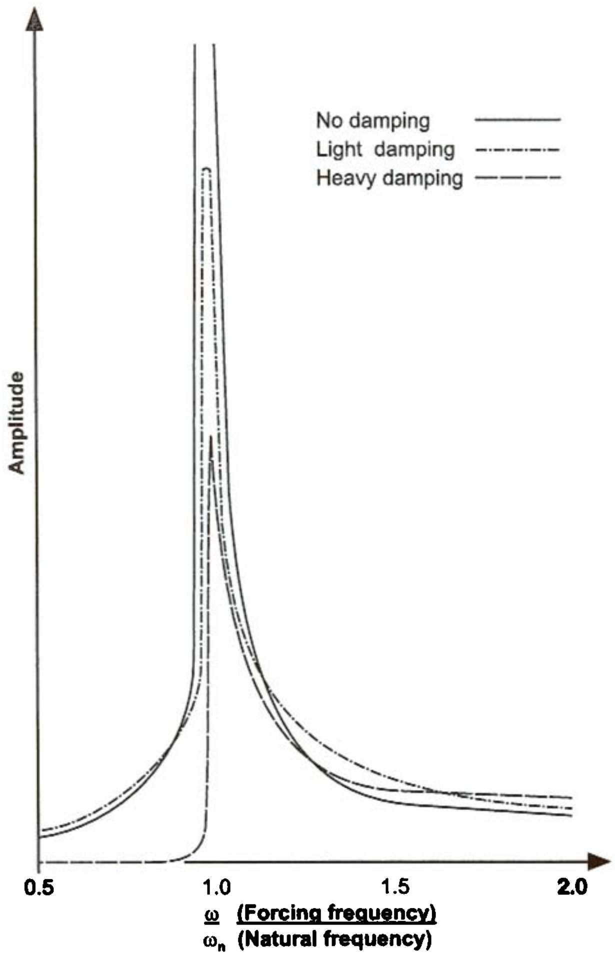


Figure 25

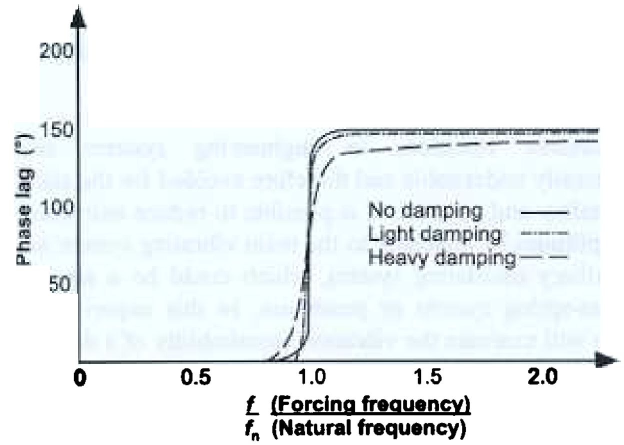


Figure 26

Experiment 11: Undamped Vibration Absorber

Introduction

Excessive vibrations in engineering systems are generally undesirable and therefore avoided for the sake of safety and comfort. It is possible to reduce untoward amplitudes by attaching to the main vibrating system an auxiliary oscillating system, which could be a simple mass-spring system or pendulum. In this experiment, you will examine the vibration absorbability of a double cantilever system.

Apparatus

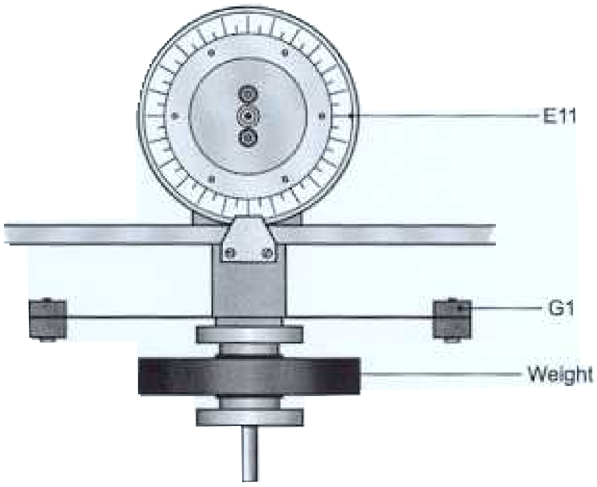


Figure 27

Figure 27 shows the vibration absorber (G1) clamped below the motor. It comprises two bodies of equal mass fixed equidistant from the midpoint of the horizontal cantilever. The distance apart of the bodies varies until the system is 'tuned'.

Procedure

For a given frequency, the masses of the vibration absorber are adjustable along their cantilevered leaf spring so that the energy of vibration transmits to the absorber and the amplitude of the main (primary) system, i.e. the motor and beam, is reduced to zero.

The aim is to determine the length l, the distance of the centre of each of the bodies from the midpoint of the cantilever so that the natural frequency of transverse vibration of this sub-system corresponds to the running speed of the main (primary) system, i.e. the motor and the beam.

The formula for determining l is:

$$f = \frac{l}{2\pi}\sqrt{\frac{3EI}{ml^3}}$$

Here

f = Natural frequency of the sub (auxiliary) system;
m = Mass of each of the bodies;
EI = Flexural rigidity of the double cantilever.

Experiment 12: Forced Vibration of a Rigid Body - Spring System with Negligible Damping

Introduction

When external forces act on a system during its vibratory motion, it is termed forced vibration. Under conditions of forced vibration, the system will tend to vibrate at its own natural frequency superimposed upon the frequency of the excitation force.

Friction and damping effects, though only slight are present in all vibrating systems; that portion of the total amplitude not sustained by the external force will gradually decay. After a short time, the system will vibrate at the frequency of the excitation force, regardless of the initial conditions or natural frequency of the system. In this experiment, observe and compare the natural frequency of the forced vibration of a rectangular section beam with the analytical results.

Theory

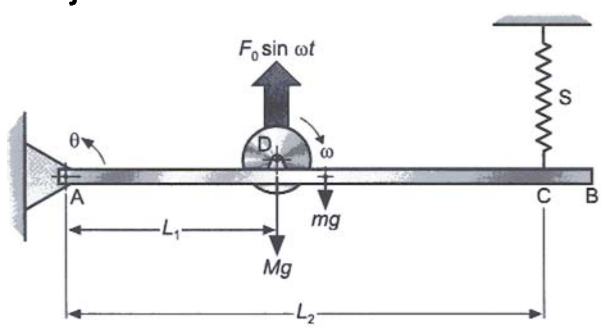


Figure 28

The system is shown in Figure 28 and comprises of:

1. A beam AB, of length *b*, sensibly rigid, of mass *m*, freely pivoted at the left-hand end.
2. A spring of stiffness *S* attached to the beam at the point C.
3. A motor with out-of-balance discs attached to the beam at D.

M = mass of the motor including the two discs.

The equation of the angular motion is:

$$I_A \frac{d^2\theta}{dt^2} = (F_0 \sin \omega t)L_1 - (SL_2\theta)L_2$$

$$I_{,} \approx ML^2 + \frac{mL^2}{3}$$

the moment of inertia of the system about the pivot axis, where:

θ = Angular displacement of the beam;
F_0 = Maximum value of the disturbing force;
ω = Angular velocity of rotation to the discs.

The above equation reduces to the form:

$$\frac{d^2\theta}{dt^2} + b_0\theta = A\sin\omega t$$

The values of the constants *b*, *A* and ω are known. Only the steady-state motion is of interest i.e.

$$\theta = \frac{A\sin\omega t}{b - \omega^2}$$

Amplitude:

$$\theta_{max} = \left|\frac{A}{b - \omega^2}\right|$$

Resonance occurs when $b - \omega^2 = 0$. So the critical angular velocity of the motor is given by $\sqrt{b}$.

Note that in practical circumstances the amplitude, although it may be very large, does not become infinite because of the small amount of damping that is always present.

Apparatus

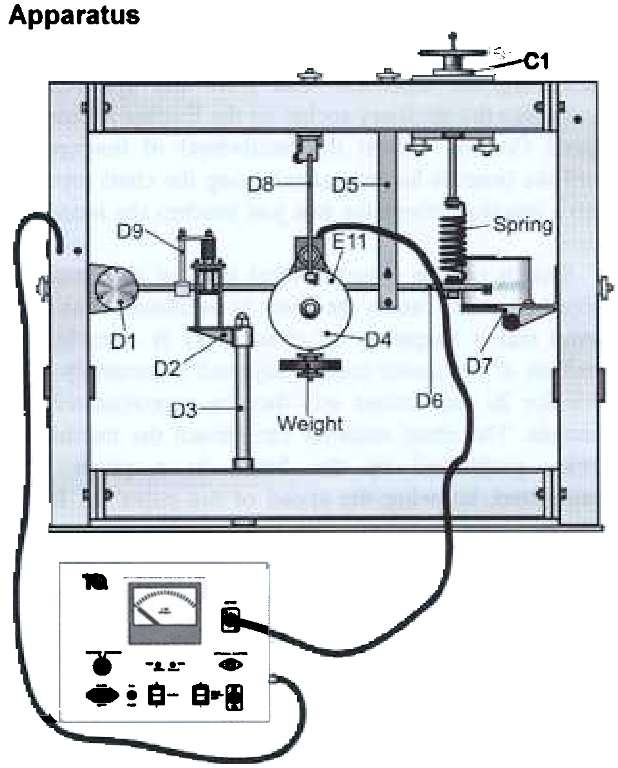


Figure 29

The apparatus shown in Figure 29 consists of a rectangular beam (D6), supported at one end by a trunnion pivoted in ball bearings located in a fixed housing. The outer end of the beam is supported by a helical spring of known stiffness bolted to the bracket C1 fixed to the top member of the frame. This bracket enables fine adjustments of the spring, thus raising and lowering the end of the beam.

The Excitor Motor and Speed Control (E11) rigidly bolts to the beam with additional masses placed on the platform attached. Two out-of-balance discs on the output shaft of the belt driven unit (D4) provide the forcing motion. The forcing frequency adjusts by means of the speed control unit.

The chart recorder (D7) fits to the right-hand vertical member of the frame and provides the means of obtaining a trace of the vibration. The recorder unit consists of a slowly rotating drum driven by a synchronous motor, operated from auxiliary supply on the Excitor Motor and Speed Control unit. A roll of recording paper is adjacent to the drum and is wound round the drum so that the paper is driven at a constant speed. A felt-tipped pen fits to the free end of the beam; means are provided for drum adjustment so that the pen just touches the paper. A small attachable weight guides the paper vertically downwards. By switching on the motor, we can obtain a trace showing the oscillations of the end of the beam.

If the amplitude of vibration near to the resonance condition is too large we can introduce extra damping into the system by fitting the dashpot assembly (part numbers D2, D3 and D9) near to the pivoted end of the beam.

Experimental Procedure

First plug the electrical lead from the synchronous motor into the auxiliary socket on the Excitor Motor and Speed Control. Adjust the handwheel of bracket C1 until the beam is horizontal and bring the chart recorder into a position where the pen just touches the recording paper.

Switch on the speed control unit so the resulting forced vibration causes the beam to oscillate. It has been found that a frequency of about 2 Hz is suitable, the position of the motor can be adjusted accordingly. The time for 20 oscillations will then be approximately 10 seconds. The chart recorder can record the number of cycles performed by the beam in a given time (calculated, knowing the speed of the paper or, better still, by visual counting).

Bring the pen into contact with the paper, then record the number of cycles and calculate the cycles per unit time (i.e. the frequency) of the forced vibration beam.

You need to known the speed of the paper on the chart recorder. To obtain this, record a trace for 20 seconds, for example, then measure the length of the trace, thus calculating the speed in mm/s.

Determine the values of the relevant parameters as described in the theory: lengths L_1, L_2 magnitude of the masses m and M, also the stiffness of the spring.

Results and Calculations

Using a stopwatch, time the linear speed of the drum for 20 vibrations and determine the time for one cycle (period of vibration). Using the two different methods determine the corresponding frequency. Calculate the relevant moment of inertia.

Mass of motor with discs, *M*	kg
Mass of beam, *m*	kg
Lengths, L_1	m
Lengths, L_2	m
Lengths, *L*	m

Table 17

Calculate stiffness of the spring (as in Experiment 6)

$$S = \frac{\text{Deflecting force}}{\text{Deflection}} = \text{N/m}$$

Calculate frequency of the forced vibration. The constant:

$$b = \frac{S}{I_A} = \frac{\mathrm{Nm}^{-1}}{\mathrm{kg\,m}^2} = \mathrm{s}^{-2}$$

$$\therefore f = \frac{\omega}{2\pi} \quad i.e. \quad \frac{b}{2\pi} = (\mathrm{Hz}) \quad \text{cycles}$$

Compare with the values of f found above

Experiment 13: Free Damped Vibrations of a Rigid Body - Spring System

Introduction

During vibrations, energy is dissipated and so a steady amplitude cannot be maintained without continuous replacement. Viscous damping in which force is proportional to velocity affords the simplest mathematical treatment.

A convenient means of measuring the amount of damping present is to measure the rate of decay of oscillation. This is expressed by the term 'logarithmic decrement', which is defined as the natural logarithm of the ratio of successive amplitudes on the same side of the mean position (see Figure 19).

In this experiment, the effect of the position of the dashpot and the corresponding damping coefficient are assessed in terms of the logarithmic decrement, measured by the decay in amplitude of a free vibration of a beam.

Theory

Referring to Figure 29, the disturbing force, $F_0 \sin \omega t$ is replaced by a damping force $cL_1 \frac{d\theta}{dt}$ downward. The equation of the angular motion becomes:

$$I_A\ddot{\theta} = -(cL_1\dot{\theta})L_1 - (SL_2\theta)L_2$$

which can be put in the form:

$$\ddot{\theta} + a\dot{\theta} + b\theta = 0$$

The theory from now on is identical to that set out in Experiment 8 (the same symbols are used).

Apparatus

The apparatus is as shown in Figure 28 and Figure 29 in Experiment 12, except that the exciter motor is not required since only free vibrations are of interest. The Excitor Motor and Speed Control unit is required in order to drive the drum on the recorder unit D7. The system is set vibrating freely by pulling down on the free end of the beam a short distance (15 – 25 mm) and releasing. Use the chart recorder to obtain a trace of just three successive amplitudes on the same side of the mean position. Vary the damping by moving the dashpot (D2) and its clamps along the beam, and also by relative rotation of the two orifice plates in the dashpot to increase or decrease the effective area of the piston as in Experiment 10B.

Procedure

Switch on the speed control unit and connect the lead from the motor of recorder unit D7 to the auxiliary supply socket on the Excitor Motor and Speed Control box. Set the dashpot at distance L_1 (the distance from the trunnion mounting to the centre of the beam clamp D9), and then pull the beam down a short distance, under the point of attachment of the spring, and release.

Bring the recording pen into contact with the paper to produce a trace of the decaying amplitude of vibration and thus produce a trace of the decaying applied amplitude on the chart recorder paper.

For a given piston area, select and obtain traces for various values of L_1. Choose a different piston area and repeat the process.

For each piston area and value L_1, use the trace to evaluate the logarithmic decrement. Find the periodic time of one complete oscillation, τ, in the manner described in Experiment 12. To recap:

$$\text{Ln}\frac{x_0}{x_1} = \frac{a\tau}{2}$$

where:

Constant $a = \frac{cL_1^2}{I_A}$ and

Constant $b = \frac{SL_2^2}{I_A}$ (incidental).

From this the damping coefficient, c, the resisting force per unit relative velocity can be determined.

Results

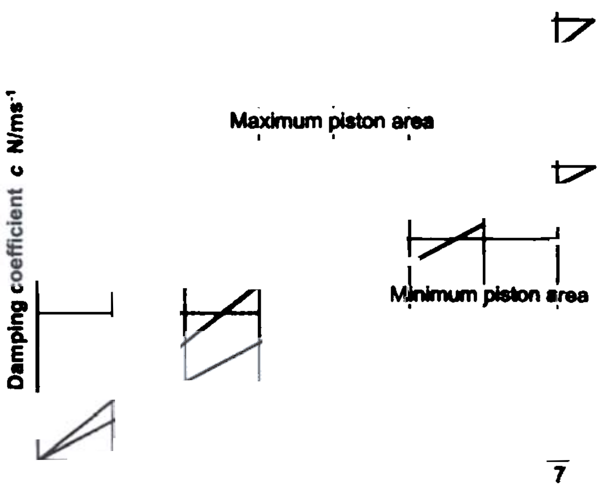
Enter the results in Tables 18 and 19, one relating to maximum damping (orifice plates in the dashpot set to give maximum area) and the other to minimum damping.

Length L_1 (m)	Amplitude ratio $\frac{x_0}{x_1}$	Log dec Log $\frac{x_0}{x_1}$	Period τ (s)	Constant a	Damping coeff c (N/m s^{-1})
0.10					
0.15					
0.20					
0.25					

Table 18 Maximum damping

Length L_1 (m)	Amplitude ratio $\frac{x_0}{x_1}$	Log dec $\log \frac{x_0}{x_1}$	Period τ (s)	Constant a	Damping coeff c (N/m s^{-1})
0.10					
0.15					
0.20					
0.25					

Table 19 Minimum damping



Plot, on the same graph, values of damping coefficient c against L^2. Figure 30 shows typical plots. The logarithmic decrement, hence the damping coefficient varies according to the square of the distance from the dashpot. Adjusting the position of the dashpot on the beam produces any degree of damping by consulting the graph. This information may be used in Experiment 14.

Experiment 14: Forced Damped Vibration of a Rigid Body - Spring System

Introduction

Having established the effect of viscous damping on free vibrations in the previous experiment, the effect on forced vibration is now analysed. To assess the relative magnitude of the forced vibration, use the concept of 'dynamic magnifier'. This is the ratio of the amplitude of the forced vibration to the deflection produced if the maximum value of the disturbing force F is applied **statically**, under the same elastic restraint.

Theory

The out-of-balance force is:

$2mr\omega^2$ (two discs)

where

m = Mass corresponding to hole in each disc (kg);
r = Radius to centre of hole (m);
ω = Angular velocity of discs (rad/s).

Note that the ratio of the rotational speed of the discs to that of the motor is 22:72. Consider this when calculating the angular velocity of the discs, ω rad/s, from the speed indicated on the control unit.

Referring to Figure 29, the equation of the angular motion is:

$$J_A\ddot{\theta} = (F\sin\omega t)L_1 - (cL_1\dot{\theta})L_1 - (SL_2\theta)L_2$$

which reduces to the standard form:

$$\ddot{\theta} + a\dot{\theta} + b\theta = A\sin\omega t$$

Only the steady-state motion is of interest i.e.

$$\theta = \frac{A\sin(\omega t - \phi)}{\sqrt{(b-\omega^2)^2 + \omega^2 a^2}}$$

where ϕ = phase angle lag and $\tan\phi = \frac{\omega \quad a}{b-\omega^2}$.

The other symbols have their usual meaning. Working in terms of angles the Dynamic Magnifier is:

$$D_m = \frac{\theta_{max}}{\theta_0}$$

where θ_0 = angular displacement of the beam due to the force F applied **statically** and:

$$\theta_0 = \frac{FL_1}{k} \tag{2}$$

where

$k = SL_2^2$ (torsional stiffness of the beam).

Deflections measured are those of the end B of the beam and are given by $x = L\theta$ so:

$$D_m = \frac{x_{max}}{L_0\theta_0} \tag{3}$$

It can be shown that:

$$D_m = \frac{}{\sqrt{\left(1-\frac{\omega^2}{b}\right)^2 + \frac{\omega^2 a^2}{b^2}}} \tag{4}$$

and in nearly all practical circumstances, damping is 'light', and therefore a is sensibly small so

$$D_m = \frac{1}{1-\left(\omega^2/\omega_n^2\right)} \tag{5}$$

ω = Circular frequency of the forced vibration (rad/s);
ω_n = Circular frequency of free undamped vibration (rad/s).

Apparatus

Figure 29 shows the apparatus with the dashpot added and the addition of an extra item: a plate clamped to the out-of-balance disc. The plate holds a piece of circular paper.

The recording pen fits to pivot (D8), which clamps to the upper member of the frame and clips above the frame when not in use. The pen makes a trace of the locus of the point at any radius on the rotor. Since the rotor is capable of vertical as well as rotational movement, you can obtain a trace from which the phase lag can be determined.

Procedure

The natural frequency of the system is first found as described in Part 4 of Experiment 13, by analysing the free vibrations of the system, without the dashpot, from a trace produced on the chart recording unit (D7).

Fit the dashpot unit (D2) at a suitable point along the beam to give a definite degree of damping (as determined in Experiment 13). Then rotate the exciter discs at a very low speed and obtain a datum trace on the paper mounted on the plate attached to the nearside disc. Mark the position of the hole in the disc on the trace.

Increase the speed of rotation to develop forced vibration of reasonable amplitude in the beam. Obtain a second 'dynamic' trace on the paper mounted on the plate. Also obtain a trace on the chart recorder at the right-hand end of the beam (as in Experiment 12) in order to determine the amplitude of the vibrations. Repeat the procedure for different speeds below and above the critical speed to show how the value of dynamic magnifier varies with frequency for a give value of the damping coefficient.

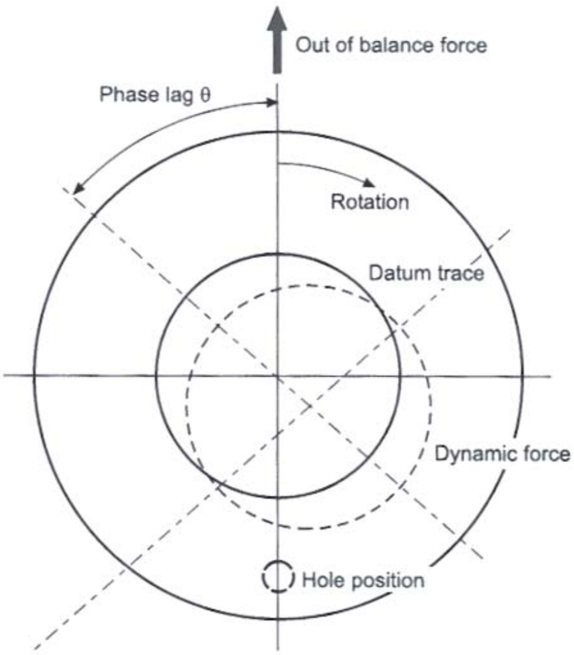


Figure 31

Determine the phase lag from the traces recorded on the paper as shown in Figure 31. Note that the dynamic trace displaces relative to the axis of rotation due to the vibration of the beam. If there is no phase lag between the exciting force and the resulting vibration, the dynamic trace displaces along the datum line corresponding to the out-of-balance force. Join up the points of intersection of the two traces and draw a line through the axis of rotation at right angles to determine the phase lag θ for the various speeds of rotation of the discs.

Results

Tabulate the results as shown in Table 20 and 21, the columns numbering is for the purpose of this explanation only.

There are two tables, one for the case of no damping, the other for a definite degree of damping. Present a specimen of calculations in respect of each table.

Column (ii) $\omega = \left(\frac{n}{60} \times 2\pi\right)\frac{22}{72}$, hence the ratio in column (iii). Obtain the amplitude column from the trace on the drum recorder D7. The corresponding phase angle lag, is obtained in the manner already described. Use Equations (2) and (3) to find the 'Static Deflection'; find D_m using Equation (4).

(i) Excitor motor speed (rev/min)	(ii) Angular velocity of disc ω (rad/s)	(iii) $\frac{w}{w_n}$	(iv) Amplitude x_{max} (mm)	(v) Phase angle lag (°)	(vi) Static deflection (mm)	(vii) Dynamic magnifier (D_m)
500						
550						
600						
625						
650						
660						
675						
700						
800						
900						

Table 20 No damping

(i) Excitor motor speed (rev/min)	(ii) Angular velocity of disc ω (rad/s)	(iii) $\frac{w}{w_n}$	(iv) Amplitude x_{max} (mm)	(v) Phase angle lag (°)	(vi) Static deflection (mm)	(vii) Dynamic magnifier (D_m)
500						
550						
600						
625						
650						
660						
675						
700						
800						
900						

Table 21 Damping

Plot graphs of Dynamic Magnifier D_m and Phase Angle each to a base of the ratio ω/ω_n. Figures 32 and 33 show typical graphs. Obtain similar results for different degrees of damping.

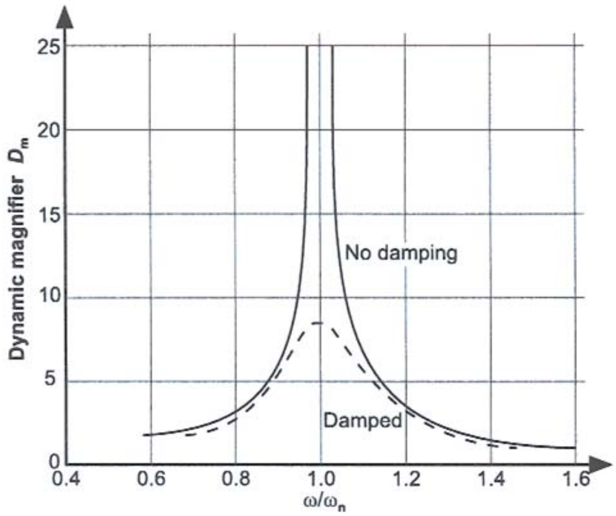


Figure 32 Dynamic magnifier against ratio ω/ω_n

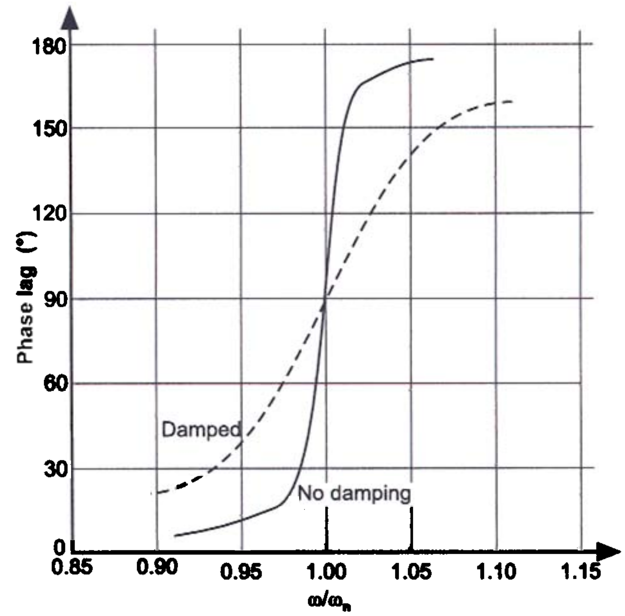


Figure 33 Phase lag against ratio ω/ω_n

SECTION 4: REFERENCES

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APPENDIX: EXCITOR MOTOR AND SPEED CONTROL

Operation and Use

A Speed Control Unit comes as part of the Universal Vibration Apparatus. The unit provides complete bi-directional drive and high precision speed control of the Motor/Tacho-generator under all normal conditions, using a closed-loop control system. An amplifier detects any difference between the motor speed and the input command voltage set by the 'Set Speed' control on the front panel. The amplifier drives the motor until the difference is approximately zero. Provisions for the very high currents for acceleration and deceleration are automatical. The speed control can maintain speeds from 3000 rev/min down to less than 1rev/min when used with the Motor/Tacho-generator.

The front panel of the unit contains a 'set speed' control, a fully calibrated speed meter incorporating automatic range switching, a motor forward/reverse switch, mains input socket and switch, d.c. motor power socket, external control and an auxiliary output socket and switch.

Set Speed Control

A ten turn control giving increasing speed with clockwise rotation.

Speed Meter

The speed meter has two ranges, 0 – 1500 rev/min and 0 – 3000 rev/min. Switching between these occurs automatically when the motor speed increases above or decreases below 1500 rev/min. The lamps below the meter indicates its range.

Motor Forward/Reverse Switch

You can reverse the direction of rotation of the motor at any load or speed without damage. It is, however, recommended that the motor is stopped before reversing.

External Control

A DIN connector provides connection for an external set speed control (1 kΩ) which, when connected, overrides the front panel control. Connections to the DIN connector are:

PIN 1 Potentiometer supply
PIN 2 'earth PIN 3' wiper

Connection and Operation

Motor/Tacho-Generator, TM16f (F1)

Ensure all switches are in the 'off' position before proceeding and the motor and/or recorder has been physically installed as outlined previously.

1. Connect the speed controller to a suitable mains supply.
2. Connect the motor to the socket marked 'd.c. motor'.
3. Switch the unit on and adjust the motor speed using the 'Set Speed' control.

The unit will automatically switch to the correct speed scale.

Drum Recorder, TM16d (D7)

1. Switch the unit off.
2. Connect the drum recorder to the socket marked either 'auxiliary output' or 'drum supply 240 V'.
3. As (3) above.
4. Switch on the drum recorder when required

Connecting the Speed Control Unit in Conjunction with a Stroboscope *(not supplied)*

1. Switch the unit off.
2. Plug the BNC T-piece into either of the sockets on the portal frame. Connect the 'trigger supply' socket on the speed controller to the T-piece and connect the T-piece to a stroboscope using the BNC to jack lead.
3. Connect the leaf contact on the motor to the other BNC socket on the portal frame via its lead.
4. As (3) above. The stroboscope will flash once every revolution of the motor.
5. Connect the stroboscope to a suitable mains supply.