

Radioactive Branching Using Dice

Sarmistha Sahu

Department of Physics, Maharani Lakshmi Ammanni College for Women, Bangalore, India
Email: sarmistha.sahu@gmail.com

Received July 7th, 2012; revised August 12th, 2012; accepted August 22nd, 2012

Dice rolling (Emeric, 1997) is a useful pedagogical tool (Arthur & Ian, 2012; Todd, Clifton, Ingrid, & Zdravko, 2006) to introduce students to the concepts and essential features of radioactivity. It can be extended to explain radioactive branching. In the process, the students learn about half life, decay constant and activity of a radioactive substance. Terms like stochastic processes, probability of decay, statistical fluctuations, and mutually exclusive processes; becomes clear in this process.

Keywords: Radioactivity; Probability; Transition Rate; Half Life; Radioactive Branching

Introduction

Dice rolling is being used as a pedagogical tool in schools as well as undergraduate studies in physics, mathematics, statistics and computer science curriculum. Students learn while they play. Todd W. Neller et al. has used dice rolling in a dice game Pig (Todd, Clifton, Ingrid, & Zdravko, 2006) for undergraduate research in machine learning. Arthur Murray et al. mention that “the ‘radioactive dice’ experiment is a commonly used classroom analogue to model the decay of radioactive nuclei” (Arthur & Ian, 2012). Simple dice rolling can unfold important concepts elegantly.

Theory

Some radionuclides may have several different paths of decay. For example, approximately 36% of Bismuth-212 decays through alpha-emission to thallium-208 while approximately 64% of Bismuth-212 decays through beta-emission to Polonium-212. Both the Thallium-208 and the Polonium-212 are radioactive daughter products of Bismuth-212 and both decay directly to stable Lead-208 (Tayal, 1988).

If a nucleus can decay by several different processes for which the probabilities per unit time are $\lambda_1, \lambda_2, \lambda_3, \dots$ then the total probability λ per unit time for decay is

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3 + \dots$$

and the half lives are related as

$$\frac{1}{T_{1/2}} = \frac{1}{T_{(1/2)_1}} + \frac{1}{T_{(1/2)_2}} + \frac{1}{T_{(1/2)_3}} + \dots$$

where $T_{(1/2)_i}$ is the half-life if only process 1 was available and so on. These are called the **partial half-lives**. If one of them is shorter than the others then it is **dominant** in determining $T_{1/2}$.

Experiment

Dice Used

About 100 cuboctahedron (truncated cubes with 6 square faces and eight triangular faces) have been used for the experi-

ments (**Figure 1**). In this experiment one of the six square faces (suitably marked) and all of the triangular faces represent two unstable states. The unmarked five square faces represent the stable state of radioactive nuclei.

The dice represents a radioactive nucleus having many energy states. *One* of the states is represented by the *yellow-square-face*. *Eight* of the *triangular faces* represent yet another energy state. All the eight states have the same energy (degenerate states). *Five* of the *unpainted square faces* represent another energy state.

Experimental Procedure

By rolling the dice a large number of times quantify the probability per throw of yellow square face “up” (λ_1) and any red triangular face “up” (λ_2). (In this experiment λ_1 is smaller than λ_2 .) Let the yellow square face “up” represent the alpha decay and any red triangular face “up” represent the beta decay.

For each throw t (0, 1, 2, ...), start with N_t number of dice, roll and remove the decayed nuclei (die with the specified face “up”). Continue with the remaining un-decayed dice till about ten percent of the dice is left. The entire process is repeated once for alpha decay (**Figure 2**), once for beta decay (**Figure 3**) and once for both the processes together (**Figure 4**).



Figure 1.
Truncated Cube (Cuboctahedron) with three types of faces made from 3 cm wooden cubes.

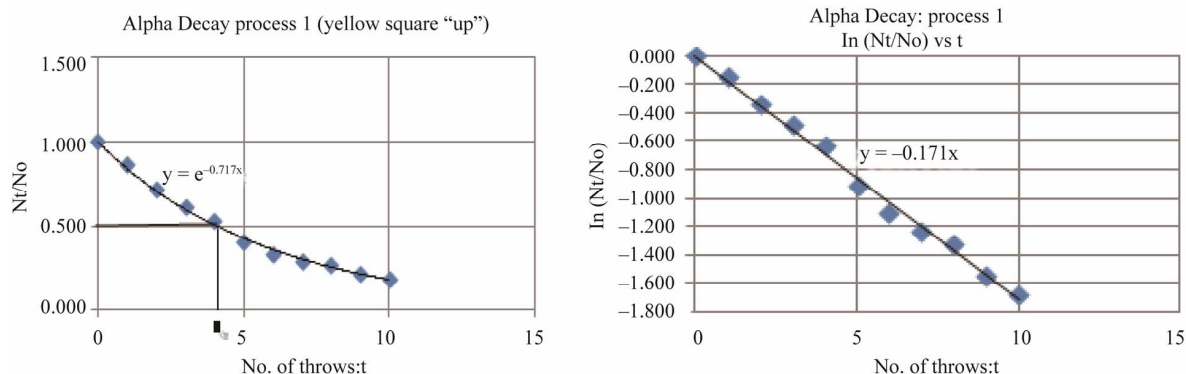


Figure 2.
The partial half life for process 1 is 3.30 throw and the decay constant is .17 per throw.

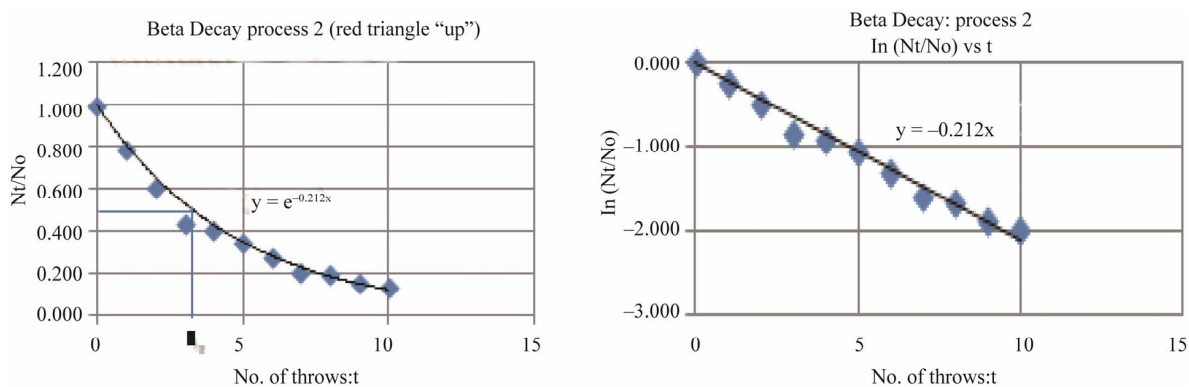


Figure 3.
The partial half life for process 2 is 4.08 throw and the decay constant is .21 per throw.

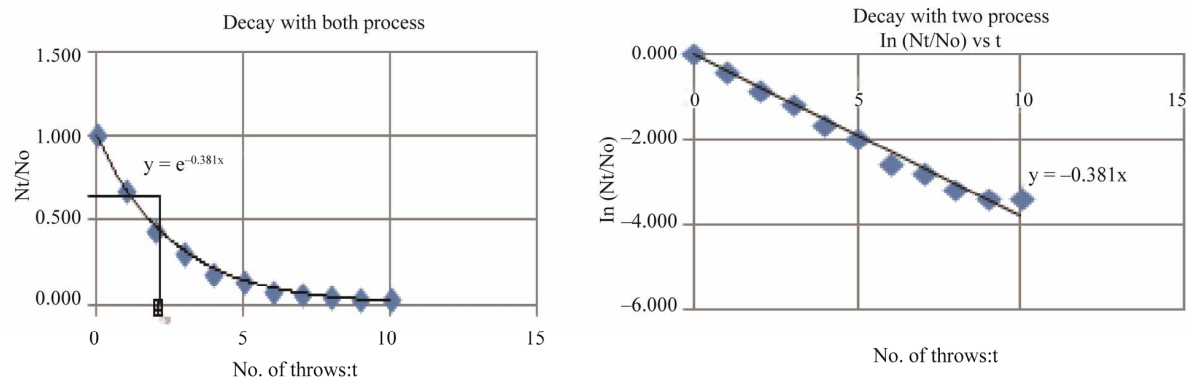


Figure 4.
The half life for both the processes is 1.82 throw and the decay constant is .38 per throw.

Results and Discussion

1. The process with shorter half life dominates. In this experiment, the red triangle decay is dominant.

2. The decay constant (probabilities per unit throw) of all the processes is equal to the sum of the decay constant of the individual processes.

$$\lambda_1 + \lambda_2 = (.17 \pm .04) + (.21 \pm .07) = (.38 \pm .11) \text{ and } \lambda = (.38 \pm .07).$$

Decay constant was determined by plotting

$$\ln \frac{N_t}{N_o} \text{ vs } t \text{ for the three processes.}$$

3. The reciprocal of the half life of the total processes is equal to the sum of the reciprocal of the partial half lives.

$$\frac{1}{T_{1/2}} = .548 \text{ and } \frac{1}{T_{(1/2)_1}} + \frac{1}{T_{(1/2)_2}} = .303 + .245 = .548$$

4. Alpha decay is 45 % $\left(\frac{\lambda_1}{\lambda}\right)$ and beta decay is 55 %

$\left(\frac{\lambda_2}{\lambda}\right)$ with the Cuboctahedron used.

Precaution

1. The probability of **one** triangle up was extremely small, hence **any** triangle up was chosen.
2. When the number of dice is large, throwing all the dice together hinders the motion of the dice as they hit each other. It is advisable to throw in small batches so that the “randomness” is maintained.

Conclusion

This experiment can be done by the students as an *activity based learning*. Many concepts are studied by dice rolling in radioactivity. Simulation using different types of dice has been demonstrated in class room to study decay constant, half life, laws of radioactivity (Sahu, 2011). This technique has been extended to simulate successive radioactive decay (Sahu, 2011) and explain concepts like a) different generations (parent, daughter...), b) change of activity of parent and daughter nuclei with time, and c) behavior of parent and daughter activities in special situations including “radioactive equilibrium”.

Radioactive decay and dice rolling are both stochastic. The statistical fluctuations become obvious when the number of dice rolled becomes smaller.

Acknowledgements

My sincere thanks to my husband Dr. R. P. Sahu for his guidance and support in using the Microsoft Excel for tabulation and graphs.

REFERENCES

- Arthur, M., & Ian, H. (2012). The “radioactive dice” experiment: Why is the “half-life” slightly wrong? *Physics Education*, 47, 197.
[doi:10.1088/0031-9120/47/2/197](https://doi.org/10.1088/0031-9120/47/2/197)
- Emeric, S. (1997). Dice shaking as an analogy for radioactive decay and first order kinematics. *Journal of Chemical Education*, 74, 505.
[doi:10.1021/ed074p505](https://doi.org/10.1021/ed074p505)
- Sahu, S. (2011). *Frontline, Physics, Education*, 46, 255-256.
- Sahu, S. (2011). *Lab Experiments*, 39, 3.
- Tayal, D. C. (1988). *Nuclear physics*. Bombay: Himalaya Publishing House.
- Neller, T. W., Presser, C. G. M., Russell, I., Markov, Z. (2006). Pedagogical possibilities for the dice game pig. *Journal of Computing Sciences in Colleges*, 21, 149-161.