Subject: MDM4UI **Unit:** 1.2
Topic: Exerimental Probability (2) **Bring: Toothpicks and Paper**

Objective:

To learn the terminology and notation applied to the probability of an event

Definition:

Fair Game: all players have an equal chance of winning, or each player can expect to win or lose the same number of times in the long run.

Trial: One repetition of an experiment.

Random Variable: A variable whose value corresponds to the outcome of a random event.

Discrete Random Variable: A variable that assumes a unique value for each outcome.

Expected Value: informally, the value of the average (mean) of the random variable's values tends toward after many repetitions.

Sometimes expressed as E(X)

Experimental Probability: the observed probability of an event A, in an experiment is found by using the formula

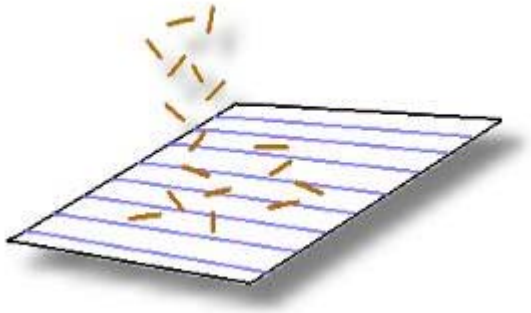
$$P(A) = \frac{\text{number of times the desired event occurred}}{\text{number of trials}}$$

Simulation: an experiment that models an actual event

Calculating Pi with toothpicks

The experiment is performed by randomly dropping needles on a grid of equally spaced parallel lines. The needles are all the same length which is exactly 1/2 the spacing between the lines. After all the needles are dropped a count is made of the number of needles which intersect any line. A given needle may intersect at most one line, but most needles (about 2/3+) do not intersect any line. The total number of needles divided by the number of intersecting needles approximates the number Pi. This experiment is named after the French naturalist Count Buffon (1707-1788) who dropped needles on a floor made of wooden planks. The lines correspond to the cracks between the planks. Buffon was trying to estimate the probability that a needle would fall across or into a crack. The mathematically precise answer is surprisingly related to Pi.

One way to *calculate for yourself* the value of pi is to drop a lot of toothpicks onto a large piece of paper that has lines drawn on it!



Here's how it works. You'll need several boxes of toothpicks. Get a large piece of chart paper, and draw parallel lines on it, from one side to the other. The lines should be separated by a distance just slightly larger than twice the length of a toothpick.

From a height of about one metre, drop a measured number of toothpicks onto the chart paper, so that they all fall randomly somewhere on the paper. **Count how many toothpicks are touching a line (or would be, if they weren't resting on another toothpick).**

Repeat this process as many times as you can. Lots of people can do it at once. All that's important is that, each time you drop some toothpicks, you write down how many you dropped, and how many of those ended up touching a line.

When you're done, find a **total** for each quantity.

You now have all the numbers you need to calculate Pi:

- **c** ... toothpick length (in mm)
- **a** ... line separation (in mm)
- **N** ... total number of toothpicks dropped
- **M** ... total number of intersections

(**c** must be less than **a)**

Here's the formula you need to calculate Pi:

$$\pi = \frac{2cN}{aM}$$