LABORATORY INSTRUMENTATION

Transmission of Radiation

1. COAXIAL TRANSMISSION LINE. VELOCITY OF PROPAGATION, IMPEDANCE MATCHING

APPARATUS

Oscilloscope
Coaxial line (about 20 m in length)
Coaxial line (about 1 m in length)
Various line terminations
Pulse-generating circuit (see Figure 1.7*a*)

OBJECTIVES

To study the amplitude of a reflected pulse as a function of load resistance.

To measure the velocity of a pulse propagating on a coaxial transmission line.

To determine the capacitance of a 1-m coaxial line and use the result to calculate a theoretical velocity of a pulse propagating on a coaxial line.

To observe the decrease in amplitude of a multi-reflected pulse and determine the attenuation constant of the coaxial line.

To observe the time dependence of a reflected pulse for loads of various impedance.

To gain an understanding of impedance, impedance matching, and reflection coefficients.

KEY CONCEPTS

TEM mode
Inductance per unit length
Resistance per unit length
Phase constant
Characteristic impedance
Reflection coefficient

Capacitance per unit length
Siemens per unit length
Propagation constant
Attenuation constant
Load impedance
Source impedance
Impedance matching

REFERENCES

1. W. T. Scott, *The Physics of Electricity and Magnetism*, 2d ed., Wiley, New York, 1966. Coaxial lines are discussed on pages 530–542. Admittance, conductance, and impedance are discussed on various pages. See the index.
2. R. K. Wangsness, *Electromagnetic Fields*, Wiley, New York, 1979. Coaxial lines and TEM modes are a part of Chapter 26.
3. R. B. Adler, L. J. Chu, and R. M. Fano, *Electromagnetic Energy Transmission and Radiation*, Wiley, New York, 1966. Coaxial lines are discussed on pages 524–530. Admittance, conductance, and impedance are discussed on various pages. See the index.
4. A. M. Portis, *Electromagnetic Fields: Sources and Media*, Wiley, New York, 1978. Transmission lines are discussed on various pages. See the index.
5. D. Halliday and R. Resnick, *Physics*, Part 2, 3d ed., Wiley, New York, 1978. Coaxial lines are discussed on pages 899–902. Capacitance and inductance per unit length of coaxial line are discussed on pages 654 and 804.
6. *American Institute of Physics Handbook*, McGraw-Hill, New York, 1957. Characteristics of standard radio-frequency cables are listed. Included are the nominal capacitance per foot and approximate impedance.

INTRODUCTION

A coaxial line is a two-conductor transmission line consisting of a center conductor, a dielectric spacer, and a concentric outer conductor. The electric and magnetic field configurations, or modes, are most commonly transverse electromagnetic (TEM) field modes. In a TEM mode both the electric and magnetic fields are entirely transverse to the direction of propagation. A longitudinal section of a coaxial line is shown in Figure 1.1, along with the (radial) electric field **E** and the (concentric circular) magnetic field **B** of a TEM mode.

A coaxial line is often used to transmit energy from a generator to a load. We ask, "How does the line effect the transmission of energy?"

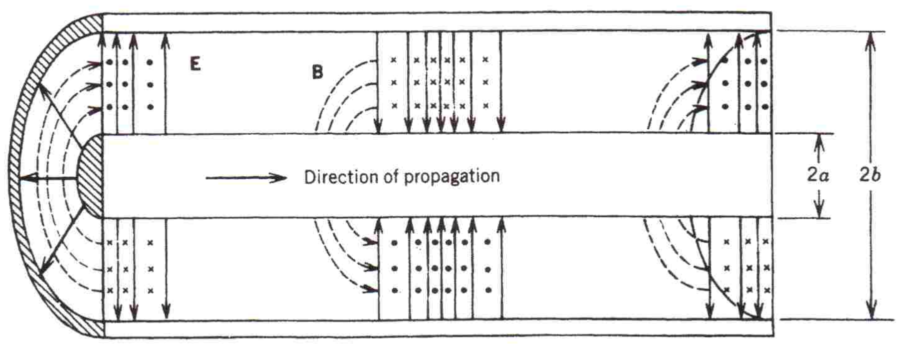


FIGURE 1.1 A TEM mode propagating on a longitudinal section of a coaxial transmission line.

1. A current in the center conductor sets up a magnetic field encircling the conductor. Hence, the line has inductance L'.
2. The line has resistance R', since the conductors making up the line have resistance. R' depends on the "skin depth" and will thus be frequency dependent.
3. There will be a voltage between and charges on the conductors. Hence, the line has capacitance C'.
4. The dielectric between the conductors is not perfect; therefore, it is necessary to associate a conductivity with the dielectric. Hence, the line has a conductance G'.

We shall see that these four parameters determine the impedance of the line, and the velocity, phase, and attenuation of waves propagating on the line. In this experiment, the propagation of voltage and current waves, rather than electric and magnetic field waves, are examined.

The value per unit length of line can be calculated for each parameter. For a coaxial line with an inner conductor of radius a and an outer conductor of inside radius b, the calculated values per unit length of line are

$$L = \frac{\mu}{2\pi} \ln \frac{b}{a} \qquad \text{(H/m)} \tag{1}$$

$$R = \frac{1}{2}\sqrt{\frac{\mu_c \nu}{\pi \sigma_c}}\left(\frac{1}{b} + \frac{1}{a}\right) \qquad (\Omega/\text{m}) \tag{2}$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \qquad \text{(F/m)} \tag{3}$$

$$G = \frac{\sigma}{\varepsilon} C = \frac{2\pi\sigma}{\ln(b/a)} \qquad (\text{S*/m}) \tag{4}$$

where

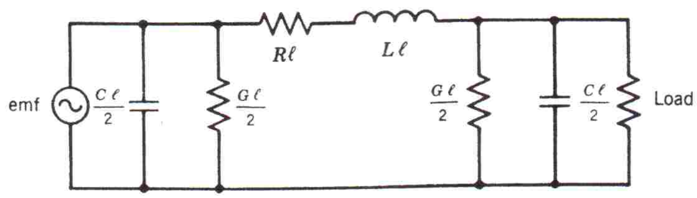
μ is the magnetic permeability of the dielectric (H/m),
ε is the electric permittivity of the dielectric (F/m),
σ is the conductivity of the dielectric (S/m),
μ_c is the magnetic permeability of the conductors (H/m),
σ_c is the conductivity of the conductors (S/m), and
ν is the frequency of the waves.

EXERCISE 1

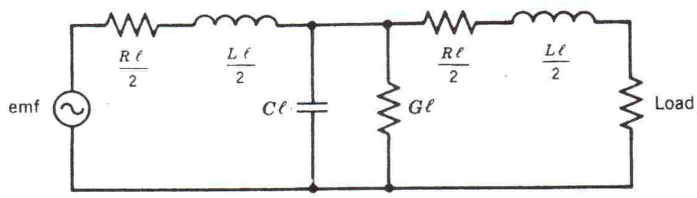
Derive the expressions for the inductance and capacitance per unit length, equations 1 and 3, for a coaxial line.

For the resistance and inductance, all sections of the line are in series; therefore, for a line of length ℓ the total resistance and inductance are $R\ell$ and $L\ell$. For the capacitance and conductance, all sections are in parallel; therefore, for a line of length ℓ the total capacitance and conductance are $C\ell$ and $G\ell$. At very low frequencies the effect of the inductance, capacitance, resistance, and conductance of the line can be taken into account by either

*S stands for siemens after Ernst Werner von Siemens (1816–1892). The siemens was formerly called the mho. One siemens is equal to one ampere/volt.



(a)



(b)

FIGURE 1.2 Both circuits are equivalent to a coaxial line when the propagating fields have very low frequencies.

equivalent circuit shown in Figure 1.2. At very high frequencies the impedance presented to the generator is

(i) for the circuit in Figure 1.2*a*

$$\frac{1}{2\pi\nu C\ell/2} \xrightarrow[\nu\to\infty]{} 0$$

(ii) for the circuit in Figure 1.2*b*

$$\frac{2\pi\nu L\ell}{2} \xrightarrow[\nu\to\infty]{} \infty$$

A way of making the impedances the same and obtaining a circuit that is the exact equivalent of the coaxial line is to divide the total inductance, capacitance, resistance, and conductance into an infinite number of infinitesimal elements. The exact equivalent circuit of a coaxial line is shown in Figure 1.3.

The general equations for the current in each conductor and the voltage between the conductors may be obtained by considering a line element of length Δz (Figure 1.4*a*). Figure 1.4*b* shows the corresponding length of the coaxial line. If we denote by q the charge per unit length of the conductor, we can write, using superior bars to denote average values in

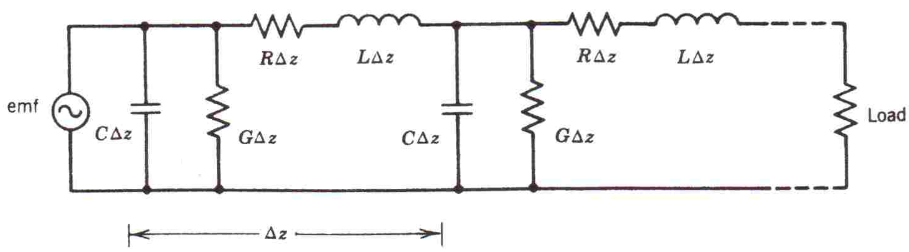


FIGURE 1.3 Equivalent circuit of a coaxial line.

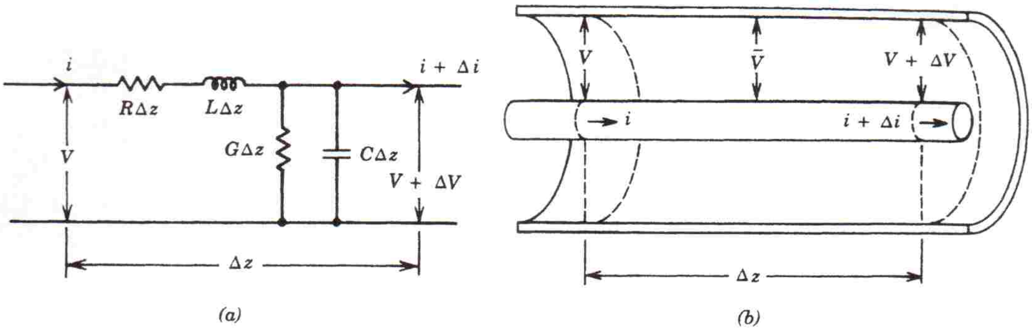


FIGURE 1.4 Current along and voltage across a section of (*a*) an equivalent circuit of a coaxial line and (*b*) a coaxial line.

the length Δz,

$$\bar{V} = \frac{\bar{q}\Delta z}{C\Delta z} = \frac{\bar{q}}{C} \qquad \text{(C/F)} \tag{5}$$

The net rate of decrease of the charge in Δz is given by the net current leaving Δz along the wire plus the leakage current $\bar{V}G\Delta z$, so that

$$-\Delta i = \frac{\partial(\bar{q}\Delta z)}{\partial t} + \bar{V}G\Delta z \qquad \text{(C/s)} \tag{6}$$

Substituting from equation 5 into 6, dividing by Δz, and taking limits, we obtain

$$\frac{\partial i}{\partial z} = -C\frac{\partial V}{\partial t} - GV \qquad (\text{C/s} \cdot \text{m}) \tag{7}$$

with $\bar{V}$ replaced by V.

The total voltage drop in Δz is given by

$$-\Delta V = L\Delta z\frac{\partial i}{\partial t} + iR\Delta z \qquad \text{(V)} \tag{8}$$

Dividing by Δz and taking limits, we obtain

$$\frac{\partial V}{\partial z} = -L\frac{\partial i}{\partial t} - iR \qquad \text{(V/m)} \tag{9}$$

Differentiating equation 9 with respect to z yields

$$\begin{aligned} \frac{\partial^2 V}{\partial z^2} &= -L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right) - R\frac{\partial i}{\partial z} \\ &= -L\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right) - R\frac{\partial i}{\partial z} \qquad (\text{V/m}^2) \end{aligned} \tag{10}$$

If equation 7 is used to replace $\partial i/\partial z$, then equation 10 becomes

$$\frac{\partial^2 V}{\partial z^2} = -L\frac{\partial}{\partial t}\left(-C\frac{\partial V}{\partial t} - GV\right) - R\left(-C\frac{\partial V}{\partial t} - GV\right) \qquad (\text{V/m}^2) \tag{11}$$

Equation 11 may be written

$$\frac{\partial^2 V}{\partial z^2} = RGV + (RC + LG)\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2} \qquad (\mathrm{V/m^2}) \tag{12}$$

Equation 12 is the general equation for the transmission of electric signals along a wire. It is called the *telegraph equation* because it describes the transmission of telegraph signals in conductors.

The equation for the current is of the same form as equation 12, and can be obtained by differentiating equation 7 with respect to z.

We can obtain solutions of equation 12 for the case of a harmonically varying wave with a time dependence given by

$$V(z, t) = V(z)\, e^{j\omega t} \qquad (\mathrm{V}) \tag{13}$$

where $j^2 = -1$. For this case, equation 12 becomes

$$\frac{d^2 V(z)}{dz^2} = RGV(z) + (RC + LG)j\omega V(z) - LC\omega^2 V(z) \qquad (\mathrm{V/m^2}) \tag{14}$$

where the time dependence cancels.

The net effect of the series resistance and inductance can be expressed by the **series impedance** Z per unit length:

$$Z = R + j\omega L \qquad (\Omega/\mathrm{m}) \tag{15}$$

The net effect of the shunt conductance and capacitance can be expressed by the **shunt admittance** Y per unit length:

$$Y = G + j\omega C \qquad (\text{siemens/m}) \tag{16}$$

Writing equation 14 in terms of Y and Z we have

$$\frac{d^2 V(z)}{dz^2} = ZYV(z) \qquad (\mathrm{V/m^2}) \tag{17}$$

The equation for $i(z)$ is obtained in a similar manner:

$$\frac{d^2 i(z)}{dz^2} = ZYi(z) \qquad (\mathrm{A/m^2}) \tag{18}$$

If we try a solution for equation 17 of the form $e^{\gamma z}$, we find

$$V(z) = V_1\, e^{\gamma z} + V_2\, e^{-\gamma z} \qquad (\mathrm{V}) \tag{19}$$

where V_1 and V_2 are arbitrary constants that can be found from initial or boundary conditions, and γ, called the **propagation constant**, is given by

$$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (\mathrm{m^{-1}}) \tag{20}$$

The propagation constant is complex with a real part α called the **attenuation constant** and an imaginary part β called the **phase constant**:

$$\alpha = \mathrm{Re}(\gamma) \qquad \beta = \mathrm{Im}(\gamma) \qquad (\mathrm{m^{-1}}) \tag{21}$$

The solution $V(z, t)$ is obtained by multiplying equation 19 by the harmonic time dependence $e^{j\omega t}$:

$$V(z, t) = V_1 e^{\alpha z} e^{j(\omega t + \beta z)} + V_2 e^{-\alpha z} e^{j(\omega t - \beta z)}$$
$$\equiv V_r(z, t) + V_i(z, t) \qquad \text{(V)} \qquad (22)$$

The term involving $\omega t + \beta z$ represents a reflected wave $V_r(z, t)$, reflected from the load, traveling in the negative z direction along the transmission line. The factor $e^{\alpha z}$ indicates that this wave decreases in magnitude as it travels in the negative z direction. The term involving $\omega t - \beta z$ represents an incident wave $V_i(z, t)$, incident on the load, traveling in the positive z direction, and the factor $e^{-\alpha z}$ indicates that this wave decreases in magnitude as it travels in the positive z direction. The total voltage $V(z, t)$ at any point z along the line is the superposition of the two traveling waves.

The total current $i(z, t)$ may be obtained by substituting equation 22 into equation 7 and then integrating over z:

$$i(z, t) = -\frac{V_1}{(Z/Y)^{1/2}} e^{\alpha z} e^{j(\omega t + \beta z)} + \frac{V_2}{(Z/Y)^{1/2}} e^{-\alpha z} e^{j(\omega t - \beta z)}$$
$$\equiv i_r(z, t) + i_i(z, t) \qquad \text{(A)} \qquad (23)$$

where $i_r(z, t)$ and $i_i(z, t)$ represent reflected and incident current waves that are traveling in the negative and positive directions, respectively. The total current $i(z, t)$ is a superposition of these two traveling waves.

The incident wave, reflected wave, and total wave are shown in Figure 1.5 at three separate points along a section of coaxial line. The load that reflects the reflected wave is to the right and the source of emf that produces the waves is to the left. The outer conductor is shown grounded, which is usually the case, and, hence, current is not shown in the outer conductor. As you will see, V_1, the constant term in the amplitude of the reflected wave, is less than or equal to V_2, the constant term in the amplitude of the incident wave. It is usually less, and letters of different size are used to indicate this in Figure 1.5.

For the waves specified by equations 22 and 23, β equals $2\pi/\lambda$, where λ is the wavelength, and ω/β is the wave velocity.

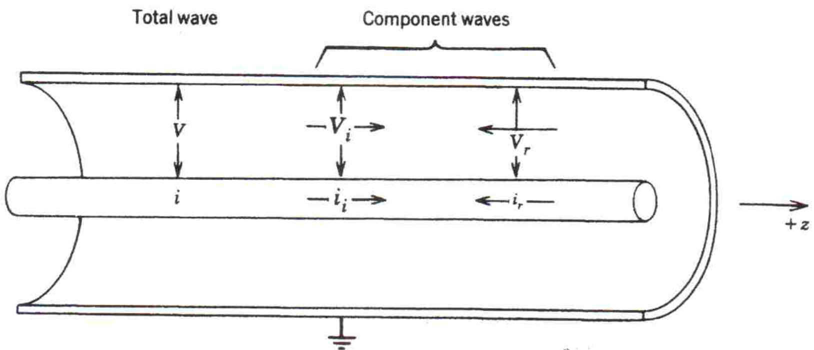


FIGURE 1.5 The incident, reflected, and total waves are shown at three separate points along a coaxial line.

EXERCISE 2

If R and G are small or if the frequency is large so that $\omega L \gg R$ and $\omega C \gg G$, then factor $\omega\sqrt{LC}$ from the right-hand side of equation 20, followed by a binomial expansion, and ignoring small terms in the expansion raised to the second and higher powers, show that

$$\alpha = \mathrm{Re}(\gamma) \simeq \frac{\sqrt{LC}}{2}\left(\frac{G}{C} + \frac{R}{L}\right) \qquad (\mathrm{m}^{-1})$$

$$\beta = \mathrm{Im}(\gamma) \simeq \omega\sqrt{LC} \qquad (\mathrm{m}^{-1})$$

Hence, show that the wave velocity v is given by

$$v = \frac{\omega}{\beta} \simeq \frac{1}{(LC)^{1/2}} = \frac{1}{(\mu\varepsilon)^{1/2}} \qquad (\mathrm{m/s})$$

where the last equality was obtained by using equations 1 and 3. If there is no material between the conductors, then $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, and $v = c$, the velocity of light.

When we confine our attention to a single wave traveling in the positive z direction with voltage $V_i(z, t)$ and current $i_i(z, t)$, the **characteristic impedance** Z_c of the line is defined to be the ratio of the voltage across the line to the current through the line for such a wave:

$$Z_c = \frac{V_i(z, t)}{i_i(z, t)} = \sqrt{\frac{Z}{Y}} \qquad (\Omega) \qquad (24)$$

Using equations 15 and 16,

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad (\Omega) \qquad (25)$$

Note that if R and G are negligibly small (an ideal, lossless line), then

$$Z_c = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}} \ln\frac{b}{a} \qquad (\Omega) \qquad (26)$$

where equations 1 and 3 were used in the last equality.

EXERCISE 3

If R and G are small or if the frequency is large so that $\omega L \gg R$ and $\omega C \gg G$, then expand equation 25, similar to the expansion carried out in Exercise 2, to show that

$$Z_c \simeq \sqrt{\frac{L}{C}}\left[1 + j\left(\frac{G}{2\omega C} - \frac{R}{2\omega L}\right)\right] \qquad (\Omega) \qquad (27)$$

Reflection Coefficient at the Output

Consider a transmission line of length ℓ and characteristic impedance Z_c, which is fed by an emf ϵ with a (source) impedance Z_0 and terminated with a load of impedance Z_L (Figure 1.6). At the output end where $z = \ell$, the load impedance Z_L is defined to be

$$Z_L = \frac{V(\ell, t)}{i(\ell, t)} \qquad (\Omega) \qquad (28)$$

If equations 23 and 22 are substituted, then equation 28 becomes

$$Z_L = Z_c\left(\frac{V_1 e^{\alpha\ell + j\beta\ell} + V_2 e^{-\alpha\ell - j\beta\ell}}{-V_1 e^{\alpha\ell + j\beta\ell} + V_2 e^{-\alpha\ell - j\beta\ell}}\right) \qquad (\Omega) \qquad (29)$$

Solving equation 29 for V_2, we have

$$V_2 = e^{2\alpha\ell + j2\beta\ell} V_1 \frac{Z_L + Z_c}{Z_L - Z_c} \quad \text{(V)} \tag{30}$$

The ratio of the reflected wave to the incident wave at $z = \ell$ is given by

$$\frac{V_r(\ell, t)}{V_i(\ell, t)} = \frac{V_1 e^{\alpha\ell} e^{j(\omega t + \beta\ell)}}{V_2 e^{-\alpha\ell} e^{j(\omega t - \beta\ell)}} = \frac{Z_L - Z_c}{Z_L + Z_c} \tag{31}$$

where equation 30 was used. The ratio of the reflected to the incident current at $z = \ell$ is given by

$$\begin{aligned} \frac{i_r(\ell, t)}{i_i(\ell, t)} &= -\frac{(V_1/Z_c) e^{\alpha\ell} e^{j(\omega t + \beta\ell)}}{(V_2/Z_c) e^{-\alpha\ell} e^{j(\omega t - \beta\ell)}} \\ &= -\frac{Z_L - Z_c}{Z_L + Z_c} \end{aligned} \tag{32}$$

The **output reflection coefficient** Γ_L is defined to be

$$\Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} \tag{33}$$

Note that if Γ_L is positive the current changes sign on reflection and the voltage does not, and if Γ_L is negative then the voltage changes sign and the current does not. For an open-circuited line $\Gamma_L = +1$, and for a short-circuited line $\Gamma_L = -1$, and therefore $-1 \leqq \Gamma_L \leqq +1$.

Impedance matching of the load to the line occurs when $Z_L = Z_c$; then $\Gamma_L = 0$ and there are no reflected waves.

EXERCISE 4

(a) If the line is open-circuited, then show that the current through the load is zero and the voltage across the load is twice the incident voltage. (b) If the line is short-circuited, then show that the voltage across the load is zero and the current through the load is twice the incident current.

Reflection Coefficient at the Input

Let τ be the time for a wave to propagate from one end of the line to the other; that is, $\tau = \ell/v$. If we turn on the emf at $t = 0$, then for $t < 2\tau$ only the incident wave (V_i, i_i) from the emf exists at the input end of the line. For such a case, we apply Kirchhoff's loop theorem to the input of the coaxial line shown in Figure 1.6 to obtain

$$\epsilon(t) - Z_0 i_i(0, t) = V_i(0, t) \quad \text{(V)} \tag{34}$$

where

$$V_i(0, t) = Z_c i_i(0, t) \quad \text{(V)} \tag{35}$$

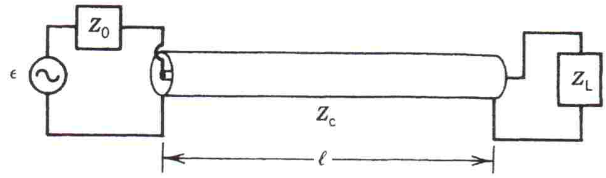


FIGURE 1.6 The source of emf with impedance Z_0 is shown connected to a coaxial line of characteristic impedance Z_c and the line is terminated with a load of impedance Z_L.

Solving for V_i and i_i, we find that

$$V_i(0, t) = \frac{Z_c}{Z_0 + Z_c}\epsilon(t) \quad \text{(V)}$$
$$i_i(0, t) = \frac{\epsilon(t)}{Z_0 + Z_c} \quad \text{(A)} \tag{36}$$

For $t \geqq 2\tau$, a second incident wave (V_i', i_i'), which has reflected off of the load, arrives at the input. There are now three waves at the input of the coaxial line: (1) (V_i, i_i), a wave incident from the generator; (2) (V_i', i_i'), a wave reflected from the load; (3) (V_r', i_r'), the reflection of (V_i', i_i') at the input. Applying Kirchhoff's loop theorem to the input of the coaxial line shown in Figure 1.6, we obtain

$$\epsilon(t) - Z_0[i_i(0, t) + i_i'(0, t) + i_r'(0, t)] = V_i(0, t) + V_i'(0, t) + V_r'(0, t) \quad \text{(V)} \tag{37}$$

Also

$$i_i = \frac{V_i}{Z_c} \qquad i_i' = -\frac{V_i'}{Z_c} \qquad i_r' = \frac{V_r'}{Z_c} \quad \text{(A)} \tag{38}$$

where i_i' and V_i' are out of phase by π following the reflection at $z = \ell$. Substituting equation 38 into 37, and using equaton 36 to eliminate V_i and i_i, we find

$$V_r' = \frac{Z_0 - Z_c}{Z_0 + Z_c} V_i' \quad \text{(V)} \qquad i_r' = -\frac{Z_0 - Z_c}{Z_0 + Z_c} i_i' \quad \text{(A)} \tag{39}$$

If we define

$$\Gamma_0 = \frac{Z_0 - Z_c}{Z_0 + Z_c} \tag{40}$$

to be the **reflection coefficient** at the **input** to the line, then equations 39 become

$$V_r' = \Gamma_0 V_i' \quad \text{(V)} \qquad i_r' = -\Gamma_0 i_i' \quad \text{(A)} \tag{41}$$

where $-1 \leqq \Gamma_0 \leqq 1$.

Impedance matching of the emf to the line occurs when $Z_0 = Z_c$ and $V_r' = 0$.

EXERCISE 5

If the emf is matched to the line, that is, if $Z_0 = Z_c$, but the load is not matched to the line, that is, if $Z_L \neq Z_c$, then show that the voltage across the load and the current through the load are given by

$$i = \frac{2V_i}{Z_L + Z_c} \quad (A) \qquad V = \frac{2Z_L}{Z_L + Z_c} V_i \quad (V)$$

where V_i is the voltage incident on the load.

EXPERIMENT

Connect the circuit shown in Figure 1.7*a* with $R = Z_c$. Set the square wave generator to 10^4 Hz and observe the voltage pulses on the oscilloscope. Connect the circuit shown in Figure 1.7*b* with $Z_L = \infty$. The generator pulse period is much greater than τ, the time for the pulse to travel the length of the coaxial line. The oscilloscope will display the pulse incident on the transmission line from the generator, followed by a pulse that has reflected from the load and returned to the input of the line.

Undesirable reflected signals at the input can be reduced by making the leads connecting the resistor R, the oscilloscope, and the coaxial line as short as possible. One way to do this is by using a male BNC and a female banana connector. The male BNC connects to the oscilloscope and the resistor R and the coax connects directly to the banana receptacles.

Perhaps it is worth comparing the circuits shown in Figures 1.6 and 1.7. In Figure 1.6 the impedance Z_0 is the impedance as seen from the transmission line side of the input. In Figure 1.7*b* the impedance seen from the transmission line side of the input is the total impedance of three parallel impedances: (1) the oscilloscope impedance, (2) the resistance R, and (3) the impedance of the 510 Ω, the 5 pF, and the generator in series. The typical impedance of a generator is 600 Ω; hence, the impedance of the series combination is given by

$$Z_s = \sqrt{R_s^2 + (\omega C')^{-2}} = \sqrt{1110^2 + (\omega 5 \times 10^{-12})^{-2}} \quad (\Omega) \qquad (42)$$

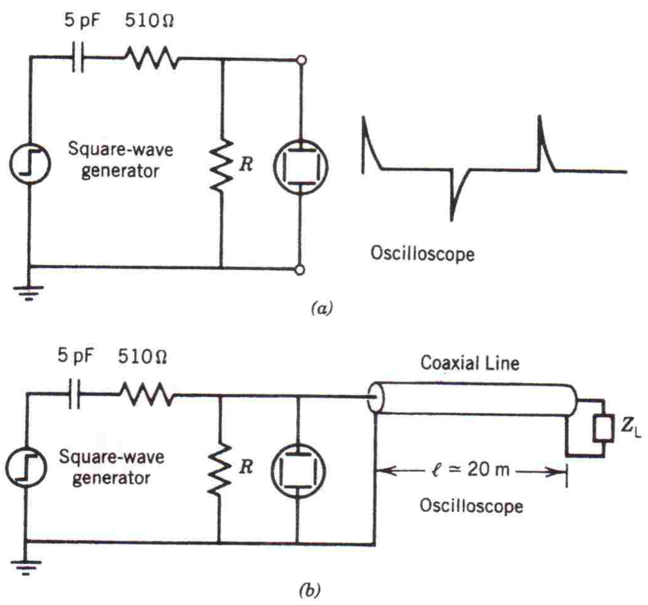


FIGURE 1.7 (*a*) Circuit to generate pulses. The pulses are sketched to the right. (*b*) Circuit to study pulses propagating on a coaxial line.

The oscilloscope impedance is about 1 MΩ, and since the three impedances are in parallel the reciprocal of Z_0 is given by

$$\frac{1}{Z_0} = \frac{1}{R} + \frac{1}{Z_s} + \frac{1}{1\ \mathrm{M}\Omega} \qquad (1/\Omega) \tag{43}$$

Note that when $R = Z_c$, which is much less than Z_s and 1 MΩ, then $Z_0 \simeq R$ and the input is impedance matched to the transmission line.

With $R = Z_c$, observe and compare the pulse incident on the load with the pulse reflected from the load when the load impedance Z_L is (a) open circuit, (b) short circuit, and (c) a resistor that matches the line impedance.

EXERCISE 6

In cases a and b, do you expect the reflected and incident pulses to have the same or opposite signs? Explain.

With $Z_L = \infty$ and $R = \infty$, observe the multireflected pulse.

EXERCISE 7

From your observation of the multireflected pulse determine the attenuation constant α.

With $R = Z_c$, use a 1-kΩ potentiometer as a load and measure the voltage of the reflected pulse as a function of the load resistance. Plot a graph of the reflected voltage versus load resistance. Compare your graph with theory by plotting a theoretical curve of the reflected voltage versus load resistance. See equation 31.

With $R = Z_c$ and $Z_L = \infty$ observe the pulses on the oscilloscope and determine an experimental value for the velocity of a pulse. The theoretical value of the velocity is $1/\sqrt{\mu\varepsilon}$, where $\mu \simeq \mu_0$. To determine ε, and, hence, a theoretical value for the velocity, connect the circuit shown in Figure 1.8. Measure the RC' time constant, use your measured value to determine the capacitance C', and then calculate the capacitance per unit length C, which is C'/ℓ. When C is known, use equation 3 to solve for ε and, hence, the theoretical velocity of the pulse.

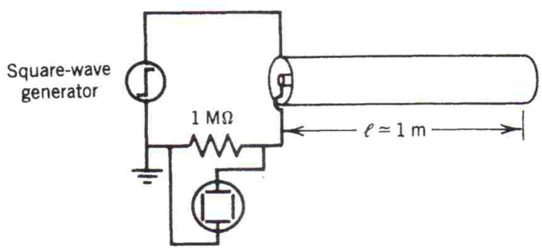


FIGURE 1.8 Circuit to measure the RC' time constant.

EXERCISE 8

Compare the experimental and theoretical values of the velocity of the pulse.

The 1-MΩ resistance shown in Figure 1.8 is of the order of the oscilloscope impedance, and therefore the resistance in the RC time constant will be an appropriate combination of the oscilloscope impedance and the 1-MΩ resistance. The scope impedance may be specified on the scope; if not, look it up in the manufacturer's operating manual.

Reconnect the circuit in Figure 1.7*b* with $R = Z_c$. Terminate the 20-m line with an inductor and observe the reflected pulse. Replace the inductor with a capacitor and observe the reflected pulse.

EXERCISE 9

Explain the shape (time dependence) of the reflected pulse for each termination.