

A wave lab inside a coaxial cable

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Abstract

The study of electromagnetic wave propagation in a coaxial cable can be a powerful approach to the study of waves at an undergraduate level. This study can explore different experimental situations, going from those where the finite velocity of propagation must be considered (distributed or transmission line behaviour), to those where this velocity may be considered infinite (lumped behaviour). We believe that the student observation of the existence of these two regimes can be important for the understanding of wave phenomena in general. In this work we show that this can be achieved using low-cost equipment and a set of quite simple experiments, such as the measurement of wave propagation velocity or the study of standing waves and resonance. The results obtained in a coherent set of selected experiments are discussed.

1. Introduction

Experimental activities aimed at the exploration of the distributed or transmission line behaviour of a coaxial cable are usually undertaken using very expensive pulse generators and high speed oscilloscopes [1–4]. Moreover, these experimental activities are not designed to show that in the limit of very large pulse widths (compared with transit time in the transmission line) the physics of the coaxial cable can be well understood in the framework of circuit theory and how both descriptions become equivalent. For these reasons, we have planned a coherent set of simple experiments to explore both limits (propagation time higher than pulse width and pulse width much larger than propagation time) and designed a low-cost kit specially suited for this purpose (this can be easily done if long cables are used).

We shall start by making a quick review of some well-known fundamental properties of a coaxial cable, that can be found in almost any course on electromagnetism [5]. A coaxial cable can be described as a transmission line with cylindrical geometry, composed of a central conductor, with diameter r_i , and an external tube-shaped conductor, with diameter r_e . The space between these two conductors is filled with a dielectric.

Electromagnetic waves will propagate in the direction of cable length and the associated fields will have nonzero values mainly in the dielectric, being shielded from the exterior by the external conductor. Considering this geometry, and assuming that the boundary conditions of the dielectric are imposed by perfectly conducting walls (the conductor surfaces), the field lines are quite simple to predict: the electric field must be normal to these walls and will thus have radial field lines, while the magnetic field must be tangential to the conductor surfaces, which means that magnetic field lines are essentially circular.

Using Gauss' theorem and Ampere's law, the capacitance and the self-inductance of this transmission line, per unit length, are easily calculated to be

$$C_1 = \frac{2\pi\epsilon}{\ln \frac{r_e}{r_i}} \quad L_1 = \frac{\mu}{2\pi} \ln \frac{r_e}{r_i} \quad (1)$$

where ϵ represents the dielectric constant and μ is the magnetic permeability of the dielectric (which will be considered equal to the magnetic permeability of vacuum, μ_0).

Since the electric (E_E) and magnetic energy (E_M) densities are equal we can write

$$E_E = \frac{1}{2}C_1V^2 = E_M = \frac{1}{2}L_1I^2 \quad (2)$$

where V and I represent the potential and electric current at the same point, respectively. The ratio V/I is thus a constant that depends only on the dielectric constant, the magnetic permeability of the medium and the geometry of the cable. This constant is usually called its characteristic impedance, Z_0 . Combining equations (1) and (2) we obtain

$$Z_0 = \frac{V}{I} = \sqrt{\frac{L_1}{C_1}} = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon}} \ln \frac{r_e}{r_i}. \quad (3)$$

The velocity of propagation of the electromagnetic waves is given by

$$c = \frac{1}{\sqrt{\epsilon\mu_0}} = \frac{c_0}{n} \quad (4)$$

where c_0 stands for the light velocity in vacuum and $n = \sqrt{\epsilon_{\text{rel}}}$ represents the refractive index of the dielectric.

If the line is terminated with a purely resistive load of impedance R then waves will be reflected at the end of the line. The reflection coefficient \mathfrak{R} depends on the boundary conditions according to

$$\mathfrak{R} = \frac{R - Z_0}{R + Z_0}. \quad (5)$$

2. Experimental setup

The kit, shown in figure 1, consists of a specially built pulse generator, one (or more) 50 m long RG58 coaxial cable(s) and 'coupling boxes' to adjust the impedance of the cable ends and set the desired length.

The pulse generator was designed to meet the following requirements: low cost and simple to build, adjustable pulse width capability, and repetition rate compatible with the use of a low-cost 20 MHz oscilloscope. Figure 2 shows the electric schematics of the pulse generator developed according to these requirements.

The main specifications of the pulse generator are 15 V output in open circuit conditions, 50 Ω output impedance, pulse width adjustable from 35 ns to 10 μ s with a fall time of 10 ns, and a repetition rate that can be adjusted from 3 to 50 kHz. A pre-pulse signal for synchronizing the oscilloscope is also available at the front panel of the unit. In figure 3 we

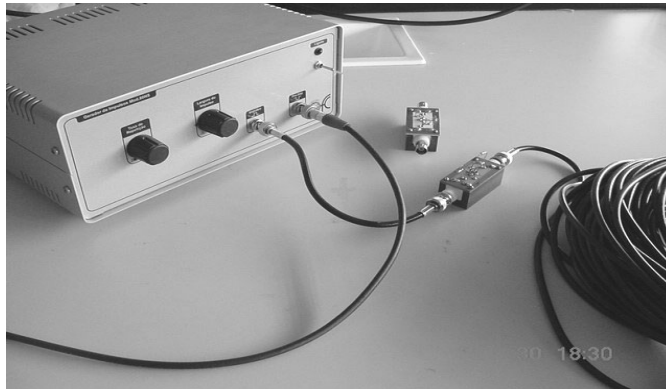


Figure 1. Experimental setup: the coaxial cable wave propagation kit.

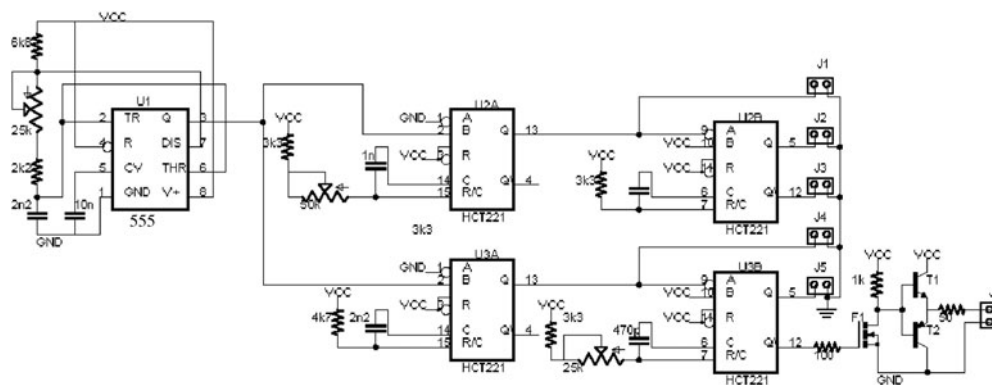


Figure 2. Schematics of the pulse generator.

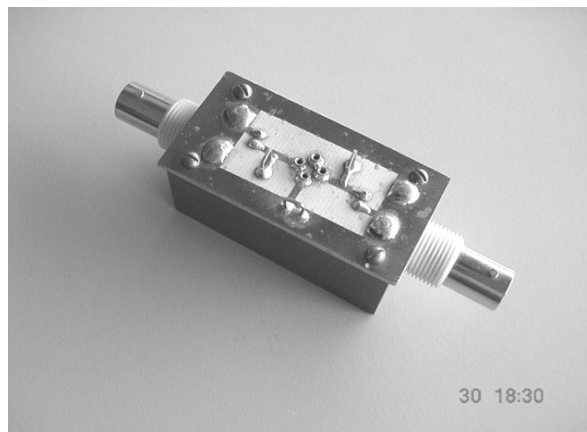
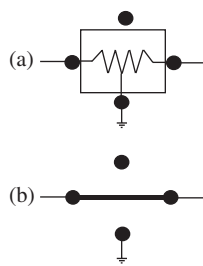


Figure 3. The coupling box can be used to (a) set the terminating impedance of the coaxial cable or (b) as a coupling circuit.

show a picture of one of the coupling boxes that can be used to insert a variable terminating impedance for the coaxial cable (figure 3(a)) or to couple two cables together (figure 3(b)).

Any of these two functions can easily be selected by the user by simply inserting a (variable) resistor or a copper conductor in the appropriate position.

3. Experimental activities

3.1. Wave propagation velocity and reflection coefficient

We shall start with a classic experimental situation that clearly shows the distributed or transmission line behaviour of a coaxial cable. This will happen if the time of propagation of an electrical pulse in the transmission line (ℓ/c) has at least the same order of magnitude as the pulse width. In this limit, the coaxial cable can be described as an extended network where electromagnetic wave propagation must be taken into account.

In figure 4 we show photographs of the signals that can be measured with a 20 MHz oscilloscope when a voltage pulse is introduced in a 100 m long coaxial cable. Each of the three insets corresponds to what is measured at the beginning and end of the cable for different boundary conditions at the end of the cable (open, 50 Ω resistor and short circuit).

Students should be alerted to the fact that, since we are exploring the transmission line behaviour of the coaxial cable, the conductors are not in general equipotentials and for that reason the ground connection must be made in the end of the cable in which measurements are being made.

The discussion of these signals can be interesting for the student since it will show the exact meaning of having a reflection coefficient $\mathfrak{R} = \pm 1$ (open or short circuited end) and $\mathfrak{R} = 0$ when the purely resistive cable termination has an impedance that equals the characteristic cable impedance. Comparing the signals measured at the end of the cable with the echoes received at the origin will help students to understand the mirror wave concept [7] ($\mathfrak{R} = \pm 1$) and the energy absorption that occurs when the wave enters a purely resistive medium with the same characteristic impedance ($\mathfrak{R} = 0$) [7]. Note that this is precisely what happens at the beginning of the cable because the pulse generator output impedance is adjusted to 50 Ω .

If a variable resistor is inserted at the cable end coupling box, students will be able to continuously change the reflection coefficient from -1 to close to $+1$ and observe the evolution of the different signals. When the terminating resistance R is larger than Z_0 the reflected impulse has the same polarity as the incident one. If R exactly matches Z_0 there is no reflected signal. If R is smaller than Z_0 the reflected pulse has the opposite polarity of the incident one. The characteristic impedance Z_0 of the cable can thus be determined by adjusting a variable resistor at the end of the cable in order to eliminate the reflected impulse.

Using the falling edges of two pulses to measure the pulse transit time and knowing the corresponding travelled length, one can calculate the velocity of electromagnetic waves in the transmission line. In particular, from figures 4(a) and (c) the time transit corresponding to a 200 m distance is about 1 μs . The velocity of electromagnetic waves in this transmission line is therefore roughly

$$c = 2\ell / \Delta t \approx \frac{200}{1 \times 10^{-6}} \text{ m s}^{-1} = 2.0 \times 10^8 \text{ m s}^{-1} \approx 0.66c_0.$$

This experimental value is to be compared with the value $1.98 \times 10^8 \text{ m s}^{-1}$ that can be computed using equation (3) and the tabulated properties for the cable RG-58 [6], namely, $\varepsilon = 2.036 \times 10^{-11} \text{ F m}^{-1}$.

If one compares the amplitude and shape of the pulses as they travel longer distances in the transmission line it becomes quite clear (figure 4) that high frequency Fourier components of the pulse are preferentially attenuated, since the echoes look much less sharper than the

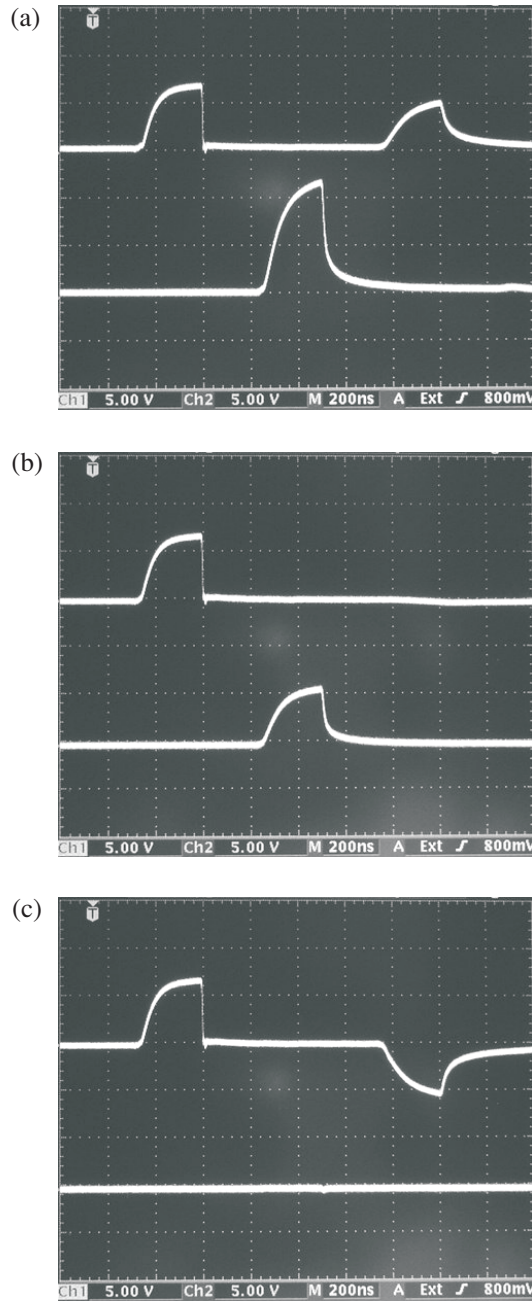


Figure 4. Signals measured at the beginning (top) and end (bottom) of the cable corresponding to different boundary conditions at the end of the cable: (a) open end; (b) $50\ \Omega$ resistor end; (c) short-circuited end. The cable length is 100 m.

original pulses. This is what is to be expected in a dispersive medium where the propagation velocity depends on the frequency.

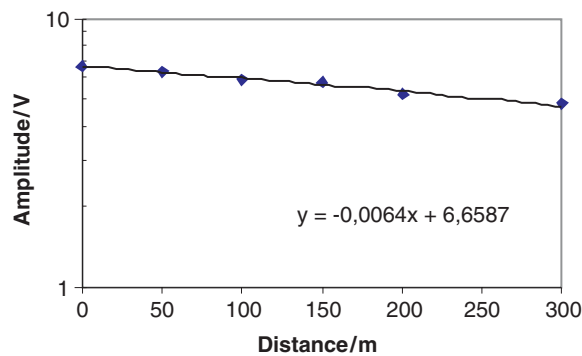


Figure 5. Pulse losses in the transmission line.



Figure 6. Multiple reflections in the ‘coaxial cavity’ configuration corresponding to (a) open end and (b) short circuited end.

In figure 5 we represent the amplitude of the pulse as a function of ‘travelled’ distance. The amplitude decays exponentially with distance and from the best line fit coefficient adjusted to the results we can obtain a pulse loss coefficient (analogous to an absorption coefficient that can be rigorously defined for a single frequency) which we find to be about 0.128 db m^{-1} .

3.2. Multiple reflections

If we introduce a resistor with an impedance greater than the characteristic impedance of the cable (for instance $1 \text{ k}\Omega$) at the input of the transmission line (i.e. between the pulse generator and the cable), we will have a reflected wave at the beginning of the cable, just as we have at the end (since the impedance ‘seen’ by the echo is no longer the output impedance of the pulse generator). This can be done simply by using a small coaxial cable connecting the pulse generator to a coupling box whose other terminal is connected to the transmission line, and inserting the resistor in place of the copper wire that is used to connect cables (figure 3(b)). In this case the cable will act as a cavity for the electromagnetic waves. Successive echoes, with decreasing amplitude due to attenuation along the length of the cable, will travel back and forward in the transmission line. Figure 6 shows a photograph of the signal as measured

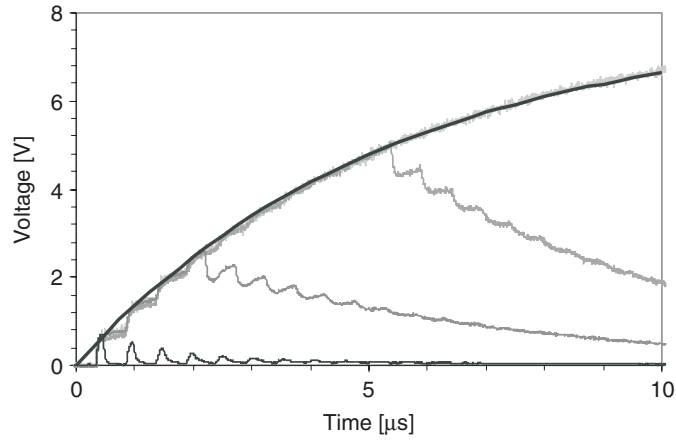


Figure 7. Measured signal at the input of the transmission line for increasing pulse widths. The solid dark line shows the fit to the experimental data using RC circuit theory.

at the input of the coaxial cable. The two lines correspond to different boundary conditions at the cable end: (a) open end; (b) short-circuited end.

3.3. Multiple reflections in the limit of long pulse width

Now, if one increases the pulse width, the successive pulses start adding up, since the reflected pulse arrives before the end of the input pulse. In the limit situation, when the pulses can be well described by Heaviside-type functions (meaning that they last until the amplitude of the reflected echoes is almost zero) the coaxial cable can be described as a capacitor, since all points of the cable are at the same potential.

Figure 7 shows the signal measured with the oscilloscope at the beginning of the cable for increasing pulse widths. The measured signal is clearly approaching the typical capacitance charge curve seen for RC circuits. So, in this limit of ‘slowly’ varying potentials, circuit theory comes in.

Assuming that we use a resistor R to create the electromagnetic cavity and that the output of the pulse generator is a pulse with amplitude V_0 , the resulting pulse on the cable will have an amplitude given by $V_0[Z_0/(R + Z_0)]$. If the cable end is open (reflection coefficient at this end equal to unity) the voltage at the beginning of the coaxial cable after n reflections will then be

$$V_n = V_0 \left[\frac{2Z_0}{(R + Z_0)} \right] [1 + \Re + \Re^2 + \dots + \Re^n] \quad (6)$$

where \Re represents the reflection coefficient at the beginning of the transmission line for waves propagating from the end to the beginning of the cable. In the limit when n goes to infinity the geometric series converges and we will have

$$V_\infty = V_0 \left[\frac{2Z_0}{(R + Z_0)} \right] \left[\frac{1}{(1 - \Re)} \right]. \quad (7)$$

Using the definition of \Re (equation (5)), it can be easily seen that the voltage measured at the beginning of the cable will go to V_0 for large n , as it should be if the cable can be well represented by a capacitor in this limit.

The ratio V_{n+1}/V_n can be written as

$$\frac{V(t_{n+1})}{V(t_n)} = \frac{1 - \Re^{(n+1)}}{1 - \Re^n} \quad \text{where} \quad t_{(n+1)} = t_n + \frac{2\ell}{c}. \quad (8)$$

From circuit theory we know that if we apply a Heaviside pulse of amplitude V_0 in a cable with a capacitance C , connected to the pulse generator through a resistor R , the voltage $V_c(t)$ is given by

$$V_c(t) = V_0(1 - e^{-\frac{t}{\tau}}) \quad (9)$$

where the time constant τ is equal to the RC time constant. From the circuit theory point of view, equation (8) should then be written as

$$\frac{V(t_{n+1})}{V(t_n)} = \frac{1 - e^{-(n+1)\frac{2\ell}{c\tau}}}{1 - e^{-n\frac{2\ell}{c\tau}}}. \quad (10)$$

Comparing equations (8) and (10) we must conclude that

$$e^{-n\frac{2\ell}{c\tau}} = \Re^n \quad \Rightarrow \quad \tau = -\frac{2\ell}{c \ln(\Re)}. \quad (11)$$

For \Re close to unity ($R \gg Z_0$) we can expand $\ln(\Re)$ in series retaining only the first term. Equation (11) will then be approximated by

$$\tau \approx -\frac{2\ell}{c} \left[\frac{1}{(\Re - 1)} \right] = \frac{\ell}{c} \frac{(R + Z_0)}{Z_0}. \quad (12)$$

Once again using the fact that \Re is close to unity, we can make another approximation taking $(R + Z_0) \approx R$. Under these circumstances, and using equation (4), we can rewrite equation (12) in the form

$$\tau \approx \frac{\ell}{c} \frac{R}{Z_0} = \frac{\ell \sqrt{\varepsilon \mu_0}}{Z_0} R. \quad (13)$$

Since from circuit theory $\tau = RC$ we conclude that the quantity $(\ell \sqrt{\varepsilon \mu_0})/Z_0$ must be the capacitance of the transmission line, C_{TL} , that we can use to model our transmission line as a lumped network in the limit of large pulse widths.

Using equations (3) and (13) we can write C_{TL} in the form

$$C_{TL} = \ell \frac{2\pi \varepsilon}{\ln\left(\frac{r_o}{r_i}\right)} \quad (14)$$

which is consistent with the capacitance per unit length expressed by equation (1).

The solid dark line shown in figure 7 yields a time constant of $5.22 \mu s$. Since a $1 \text{ k}\Omega$ resistor was used to create the electromagnetic cavity this means that the measured capacitance of the cable is about $4.96 \times 10^{-9} \text{ F}$, corresponding to a capacitance of about 99 pF m^{-1} (the cable is 50 m long). This value is in good agreement with what is obtained from equation (1): 100.5 pF m^{-1} .

3.4. Resonances in the electromagnetic cavity

If we now introduce a sinusoidal signal in the electromagnetic cavity (using a common signal generator instead of our pulse generator), wave theory tells us that standing waves will appear, since waves travelling in opposite directions will interfere with each other. For a discrete set of frequencies (that depend on the boundary conditions) we will have a resonance in the cavity. If the end of the cable is left open ($\Re = +1$) this set is composed of frequencies such that the cable length is an integer multiple of $\lambda/2$. If the cable end is short circuited ($\Re = 0$) we will have a resonance when the cable length is an odd multiple of $\lambda/4$.

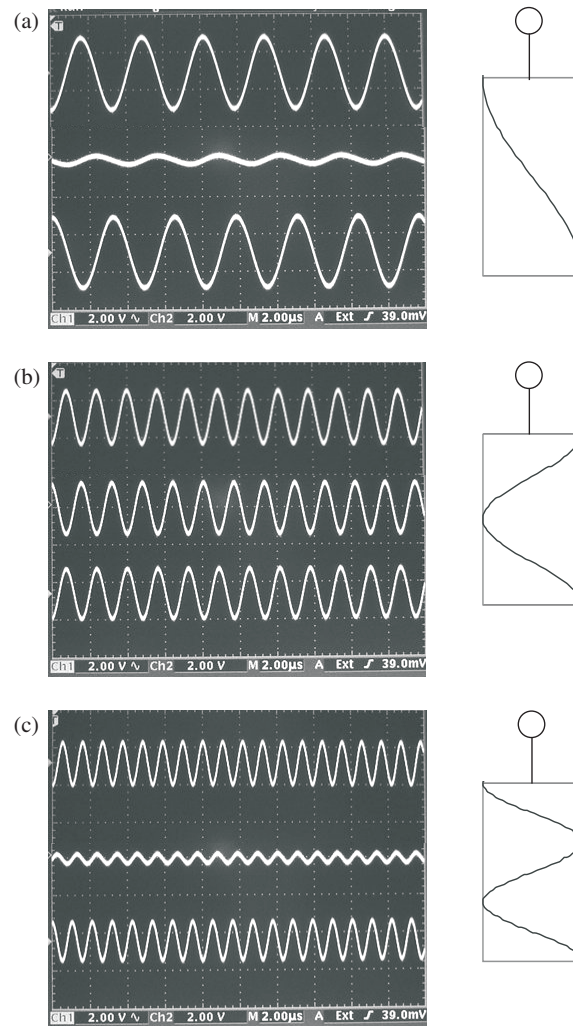


Figure 8. Standing waves inside a coaxial cable with both ends open. The insets show signals measured at the beginning (a), halfway (b) and end of the cable (c) for the first three resonance frequencies.

In figure 8 we show the signals measured at the two ends and in the middle of the coaxial cable for the first three resonance frequencies (304 kHz, 612 kHz and 925 kHz) with the cable end open. In fact the origin of the cable is not entirely open because this cable end is ‘closed’ through a 1 k Ω resistor. For the top inset, the wavelength is twice the cable length, and the node occurs at the middle. Note the 180° phase difference between ends. The next resonance frequency is shown in the middle inset. Here the wavelength is equal to the cable length, and the nodes occur now at $\ell/4$ and $3\ell/4$. As expected the signal in the middle is at 180° with the end signals. When we reach the next resonance frequency (wavelength equal to $2\ell/3$) we again have a node at the middle of the cable. However, the phase relationship of the potential between ends has now changed from 0 to 180°. It should be noted that we never have an exact zero in the middle of the cable as should be expected since waves are attenuated as they

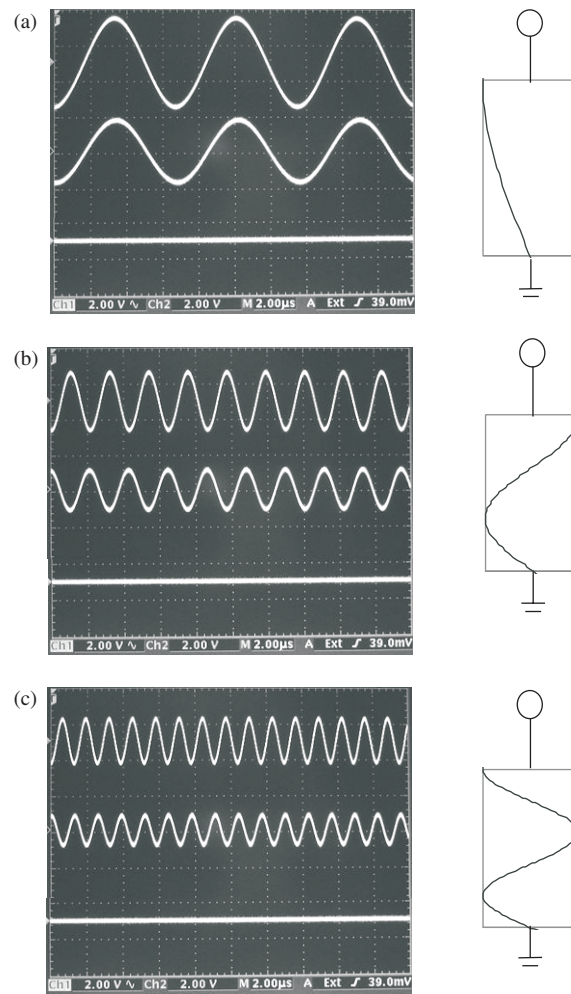


Figure 9. Standing waves inside a coaxial cable with one end open and one end shorted. The insets show signals measured at the beginning (a), halfway (b) and at the end of the cable (c) for the first three resonance frequencies.

propagate in the transmission line and the reflection coefficient near the signal generator is smaller than 1.

In figure 9 we show equivalent signals for the first three resonant frequencies (146 kHz, 460 kHz and 781 kHz) with the cable end shorted.

Once again the phase relationship between the signals is in good agreement with what is to be expected as can be seen by comparison with the schemes close to the images.

Using all the measured frequencies we can compute the propagation velocity c . The mean value obtained by this calculation is $1.8 \times 10^8 \text{ m s}^{-1}$, showing once again the internal consistency of all the results.

When pulse attenuation was discussed it was argued that progressive pulse rounding as they travel in the transmission line could be a consequence of an attenuation increase with frequency. That this is the case, can be seen in figure 10 where the maximum amplitude of the resonance is represented as a function of frequency (results obtained with a 350 m cable

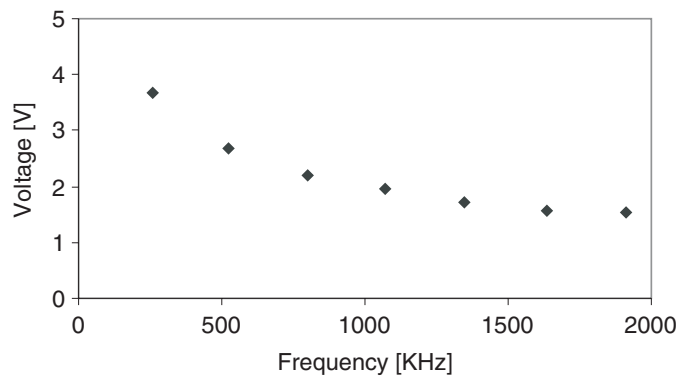


Figure 10. Amplitude of resonance as a function of frequency.

using always the same input amplitude for the sine wave). The resonance amplitude clearly decreases with frequency showing qualitatively that we have in fact an increase in attenuation at higher frequencies.

4. Conclusions

We have presented a low-cost kit for the study of electromagnetic wave propagation using a coaxial cable. Different levels of experiments can be carried out, illustrating several concepts of wave propagation such as boundary conditions, standing waves and dispersive effects. Also, a cross bridge from wave theory to circuit theory is extremely simple to illustrate using this equipment. We must emphasize the good internal coherency of all the results that can be obtained using this equipment.

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