

# 11. Compound Pendulum

## Background

Compound pendulum  
Simple harmonic Oscillation  
Radius of gyration

## Aim of the experiment

- To determine acceleration due to gravity,  $g$ , using a compound pendulum.
- To determine radius of gyration about an axis through the center of gravity for the compound pendulum.

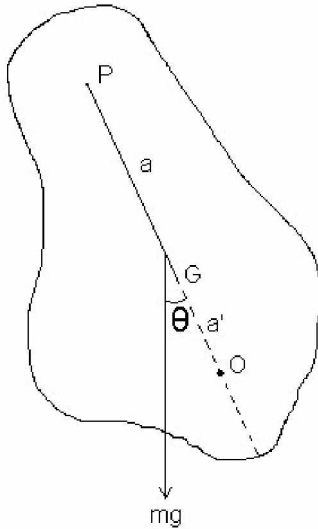
## Apparatus required

Compound pendulum  
Stop watch

## Theory:

A rigid body which can swing in a vertical plane about some axis passing through it is called a compound or physical pendulum.

In Fig.1 a body of irregular shape is pivoted about a horizontal frictionless axis through P and is displaced from its equilibrium position by an angle  $\theta$ . In the equilibrium position the center of gravity G of the body is vertically below P. The distance GP is  $a$  and the mass of the body is  $m$ . The restoring torque for an angular displacement  $\theta$  is



$$\tau = -mg a \sin\theta \quad \dots(1)$$

for small amplitudes,

$$I \frac{d^2\theta}{dt^2} = -mg a \theta, \quad \dots(2)$$

where  $I$  is the moment of inertia of the body through the axis P. Expression (2) represents a simple harmonic motion and hence the time period of oscillation is given by

$$T = 2\pi \sqrt{\frac{I}{mga}} \quad \dots(3)$$

Fig. 1

Now  $I = I_G + ma^2$ , where  $I_G$  is moments of inertia of the body about an axis parallel with axis of oscillation and passing through the center of gravity G.

$$I_G = m k^2 \quad \dots(4)$$

where k is the radius of gyration about the axis passing through G. Thus

$$T = 2\pi \sqrt{\frac{mk^2 + ma^2}{mag}} = 2\pi \sqrt{\frac{\frac{k^2}{a} + a}{g}} \quad \dots(5)$$

Comparing expression (5) with an expression of time period  $\left(T = 2\pi \sqrt{\frac{l}{g}}\right)$  for a simple

pendulum suggests,  $l = \frac{k^2}{a} + a$ . This is the length of “equivalent simple pendulum”. If all the mass of the body were concentrated at a point O, along PG produced such that  $OP = \frac{k^2}{a} + a$ , we would have a simple pendulum with the same time period. The point O is called the ‘Centre of Oscillation’.

Now since

$$l = \frac{k^2}{a} + a$$

or,

$$a^2 - al + k^2 = 0 \quad \dots(6)$$

Equation (6) has two values of GP (or a), which produces the same length l as the length of the equivalent simple pendulum. Since one of the roots for equation (6) is a, the other root a' will satisfy

$$a + a' = l$$

and

$$aa' = k^2 \quad \dots(7)$$

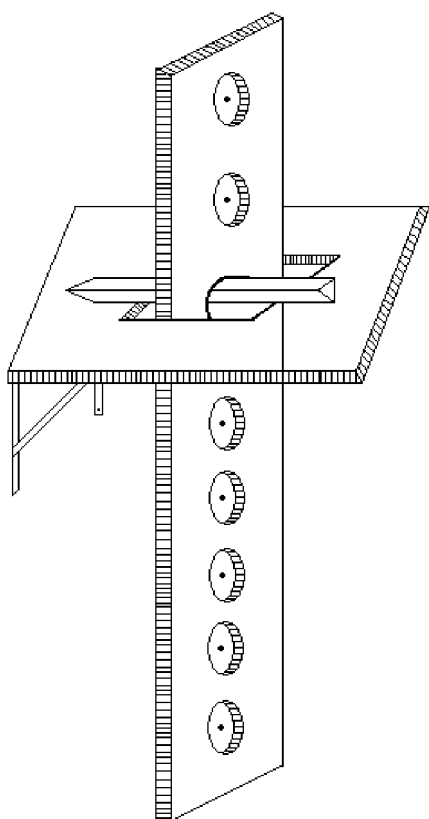
Thus if the body were supported on a parallel axis through the ‘Centre of Oscillation’ point O, it would oscillate with the same time period T as when supported at P.

Now it is evident that there are an infinite number of points distant a and  $\frac{k^2}{a} = a'$  from G in a rigid body. If the body were supported by an axis through G, the time period of oscillation would be infinite. From any other axis in the body the time period is given by expression (5).

The time period has a minimum value when  $a + \frac{k^2}{a}$  is minimum, and that happen when  $a = k$ , and the corresponding time period is

$$T_{\min} = 2\pi \sqrt{\frac{2k}{g}} \quad \dots(8)$$

This experiment can be performed with help of a rectangular metallic rod about 1m long. This may be suspended on a knife-edge at various points along its length through circular holes drilled along the bore at about 2 or 3 cm intervals. (Fig.2)



### Procedure:

- 1) Level the knife-edge and suspend the bar at, say, every other hole in turn, and note time for twenty oscillations several times also note the distance of the hole from the center of the bar.
- 2) Having obtained a set of values for time periods  $T$ , and corresponding distances from the center of gravity, plot a curve with time period vs distance of suspension from the center of gravity. A curve such as shown in (Fig.3) will be obtained.
- 3) It will be found that curve is symmetrical about the line representing center of gravity. Draw any line CAGBD parallel to the axis. This cuts the curve in four points, which have the same time period. The equivalent length  $l$  for this time period is

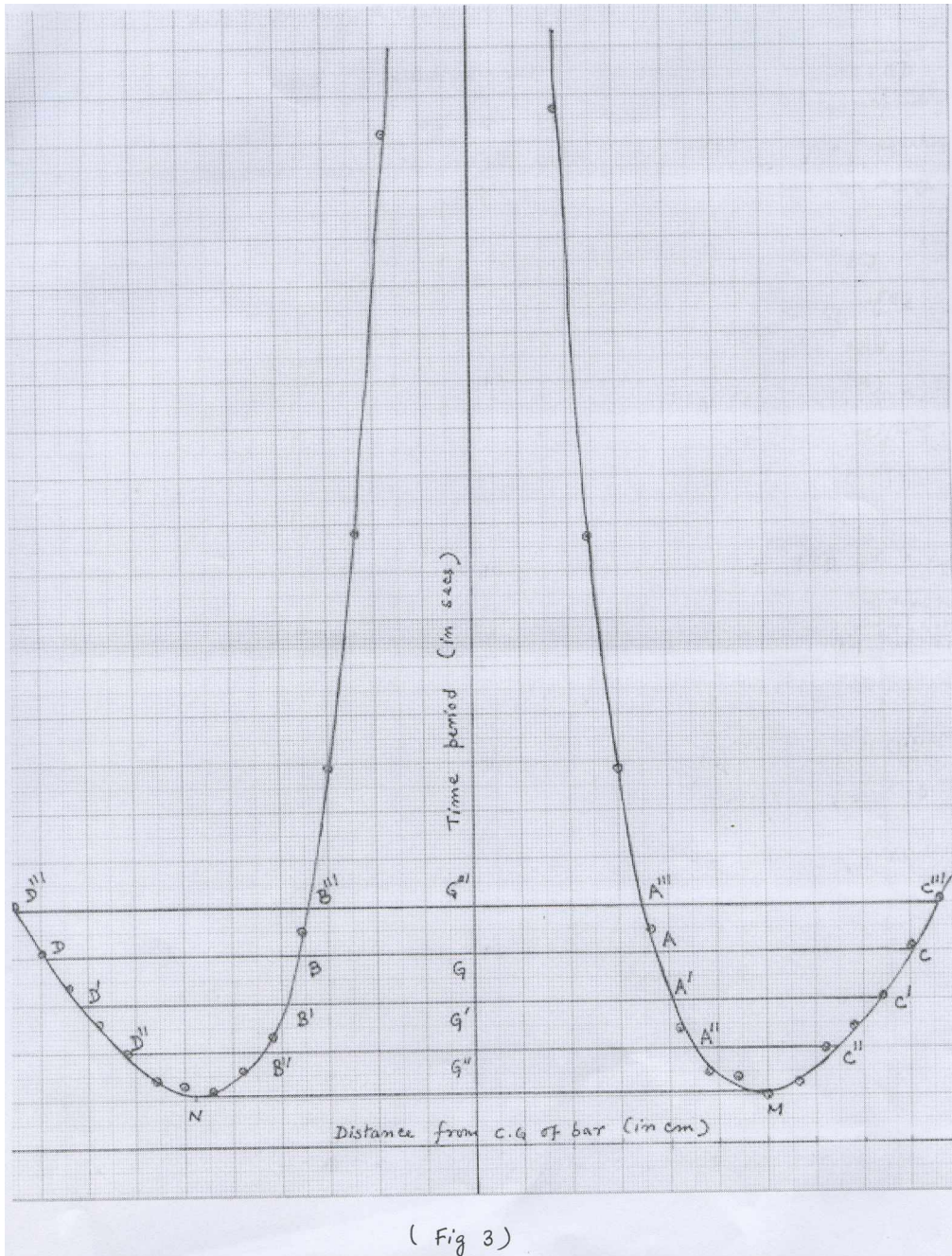
$$l = \frac{AD + BC}{2} \quad \dots(9)$$

**Fig. 2**

The acceleration due to gravity is found using,  $g = \frac{4\pi^2 l}{T^2}$  ...(10)

- 4) Draw several lines parallel to CAGBD, (Fig.3)  $C'A'G'B'D'$ ,  $C''A''G''B''D''$  etc. and obtain the corresponding values of  $l$  and  $T$ . The mean value of  $\frac{4\pi^2 l}{T^2}$  is used to calculate the value of  $g$ .

- 5) If now a tangent is drawn to the curve such as MN, then radius of gyration about an axis through center of gravity is equal to  $MN/2$ .
- 6) Further using equation (7)  $k = \sqrt{a a'} = \sqrt{AG.GD}$ . So a second value of  $k$  may be found. The corresponding time period may be noted from the graph. Using equation (8) of may be evaluated.
- 7) Mean value of  $k$  may be obtained averaging all  $k$ s found from lines CAGBD, C'A'G'B'D'.....etc, and the moment of inertia about a parallel axis through the center of gravity calculated using equation (4), where mass of the rod is obtained by direct weighing.



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***Compound Pendulum***

[illegible]

**Calculations & Results :**

$$T_{\min} =$$

S.N	Line	Time Period (in sec)	$l$ (in cm)	$k$ (in cm)	$k$ (in cm)	$g = \frac{4\pi^2 l}{T^2}$	Ave. $g$	$g = \frac{4\pi^2 (a+b)}{T_{\min}^2}$	Ave. $g$
(i)	<i>CAGBD</i>	<i>T</i>	$\frac{AD+BC}{2}$	$\sqrt{AG \times GD}$ $= (a)$	$\sqrt{BG \times GC}$ $= (b)$	.....	.....	.....	.....
1									
2									
3									
4									
5									
6									

Acceleration due to gravity at Kharagpur is found to be equal to ..... cm/sec<sup>2</sup>.

Radius of gyration about an axis through the center of gravity is found to be equal to ..... MN/2 cm.

### **Error calculation**

$$\frac{\delta g}{g} = \frac{\delta l}{l} + \frac{2\delta T}{T}$$

### **References**

References: Worsenop & Flint : Advanced Practical Physics for Students  
Resnick & Halliday : Physics

**Graph : Compound Pendulum**