# Inertia Balance

**Items Included:**

1. Inertia Balance Apparatus – 1 No.
2. C-Clamp – 1 No.
3. Nylon Cord
4. Cylindrical Masses - 3 Nos.

**Additional Items Required:**

1. Physical Balance
2. 0-1000g weight box
3. Stop Watch
4. Spring Balance (20N)
5. Adhesive tape

**Theory:**

The concept of inertia originated from Newton's First Law of Motion. According to the law, the property of an object that resists change in its state of rest or of motion is called inertia and mass of the object is said to be the measure of its inertia.

In general practice, different methods are used for determining the mass of an object, such as double pan balance, triple beam balance etc. In all of the above methods, mass is measured by comparing the force of attraction due to gravitation between earth and the object and that between earth and comparing masses. Even in spring balance the mass is measured by the extension caused in the spring due to downward pull by the object under the effect of gravity. The electronic balances also use the phenomenon of change in the physical property of the transducers (producing measurable electrical signal) due to the downward stress exerted on the transducer by the object under the gravitational pull of the earth. As such, these masses are also commonly called gravitational mass. Thus, it is evident that in the absence of gravity, all of the above methods will fail to determine the mass of an object accurately.

The inertia balance is basically a device that is used to measure the mass of an object quantitatively by using the principle of inertia i.e., by comparing the resistance to the change in the motion of the object on the balance. Hence, it is also referred to as inertial mass of the object. This method is used to determine the mass of the objects under weightless conditions such as in spacecrafts.

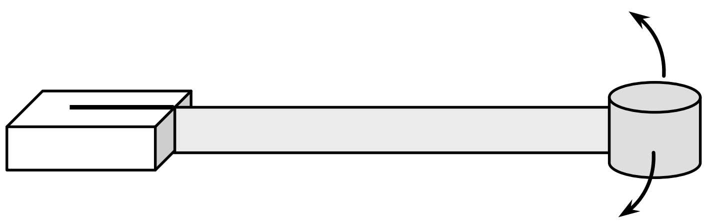

***Figure 1***

In its basic form, the inertia balance consists of a lever arm clamped on one end rigidly to a solid support with the other end free to vibrate in horizontal plane (as shown in figure 1). The free end of the balance is also capable of carrying additional weights or objects. Here, it is evident that inertia balance is like a spring system having its motion similar to simple harmonic oscillator (SHO). In case of Simple pendulum undergoing SHM, time period of its oscillation 'T' is given as,

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{……… (1)}$$

Where, L is the length of oscillation and g is the acceleration due to gravity.

If a spring system undergoes a displacement x under the effect of force F, then from Hooke's law

$$F = -Kx \qquad \text{……… (2)}$$

Where, K is the restoring force constant of the Hooke's law. When the disturbing force is removed abruptly from the spring system, it is set into SHM. The time period of any SHM is given as

$$\begin{aligned} T &= \frac{2\pi}{\sqrt{\text{acceleration per unit displacement}}} \\ &= 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} \end{aligned} \qquad \text{……… (3)}$$

If free end of the inertia balance is displaced to one side in horizontal plane by an angle θ about the vertical axis through the point of clamping of lever arm, the restoring force due to elasticity of the lever arm (from Hooke's Law) is given as

$$F = -K'\theta \qquad \text{……… (4)}$$

Where, K′ is the angular restoring force constant of the lever arm and is expressed in terms of force per radian or simply force (radians being dimensionless quantity). Negative sign in the equation (4) represents that the restoring angular force is directed in the opposite direction as that of displacement. If this angular restoring force produce an angular acceleration α in the lever arm having moment of inertia I, then

$$F = I\alpha \qquad \text{……… (5)}$$

From equations (4) and (5),

$$\frac{\text{acceleration}}{\text{displacement}} = \frac{\alpha}{\theta} = \frac{K}{I} \qquad \text{……… (6)}$$

From equations (3) and (6), the time period of vibration of lever arm can be given as

$$T = 2\pi\sqrt{\frac{I}{K}} \qquad \text{……….(7)}$$

As is evident from the above equation, in case of inertia balance, the term moment of inertia is more relevant than the actual mass since here the force producing angular strain (or the restoring force) is better expressed in terms of moment of inertia.

For finding moment of inertia of lever arm, let L be the length of lever arm (from its free end to the point of clamping) having mass m, as shown in Figure 2.

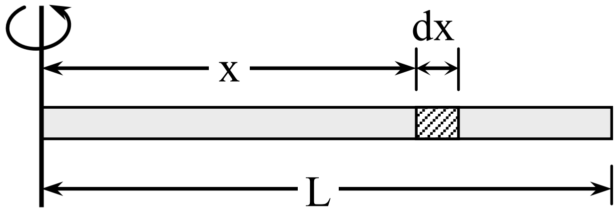


***Figure 2***

Moment of Inertia is the mass of rotating body distributed about the axis of rotation and is constant for a particular rigid body for a particular axis of rotation. The body is made of large number of particles. If $m_i$ be the mass of $i^{th}$ particle and $r_i$ be its distance from the axis of rotation (i.e., the radius of rotation of the particle), then moment of inertia of the body is,

$$MI = \sum m_i r_i^2 \qquad \text{……….(8)}$$

Select an infinitesimally small element of the lever arm having length dx at a distance x from the axis of rotation and having mass dm. The ratio of the mass dm of this element to the total mass m is equal to the ratio of its length dx to the total length L i.e.,

$$\frac{dm}{m} = \frac{dx}{L}$$

$$\Rightarrow dm = \frac{m}{L}dx \qquad \text{……….(9)}$$

From equation (8), it follows that

Moment of Inertia of the element = Mass of the Element × Square of its distance from axis of rotation

$$\therefore \text{Moment of Inertia of Element} = \left(\frac{m}{L}dx\right)x^2 = \frac{m}{L}x^2 dx$$

The complete lever arm can be assumed to be composed of large number of such small elements whose distances from the axis of rotation varies from 0 to L. Hence, the moment of inertia of the lever arm is the sum of moment of inertia of all such small elements and can be given by integral of expressions from x=0 to x=L as

$$I = \int_{x=0}^{x=L} \frac{m}{L} x^2 dx = \frac{m}{L} \left| \frac{x^3}{3} \right|_0^L$$

solving above equation, we get moment of Inertia of lever arm I as

$$I = \frac{1}{3} mL^2 \qquad \text{.......... (10)}$$

If an object having mass M is positioned on the free end of the lever arm, whose overall dimensions are considerably less than its distance from the axis of rotation L, then the moment of Inertia of the system I′ becomes

$$I' = \frac{1}{3} mL^2 + ML^2 \qquad \text{.......... (11)}$$

From equation (7), the time period of oscillation of the lever arm with mass at its end T′ becomes

$$T' = 2\pi L \sqrt{\frac{\frac{1}{3} m + M}{K'}} = \frac{2\pi L}{\sqrt{K'}} \sqrt{\frac{1}{3} m + M} \qquad \text{.......... (12)}$$

Our Inertia balance consists of 2 rectangular platforms connected to each other by two lightweight, non-sagging spring steel strips, as shown in Figure 2.

***Figure 2***

In the above figure, the left platform is clamped to the laboratory table using C-clamp and right platform (having 3 holes) is projected horizontally and is free to oscillate in the horizontal plane. Three cylindrical weights with shoulders are included in the apparatus, which can rest on the holes in the free platform. As is evident, the mass of platform (M) is substantially higher than that of spring steel strips and increases further on loading the platform with cylindrical weights. Therefore, the term m/3 can be ignored when compared to M in equation (5). Hence, it follows from equation (5) that

$$T \propto \sqrt{M}$$

or, if $T_1$ and $T_2$ are the time periods of oscillation of Masses $M_1$ and $M_2$ respectively, then

$$\frac{M_1}{M_2} = \frac{T_1^{\,2}}{T_2^{\,2}} \qquad \text{………. (6)}$$

From the above equation it is obvious that the plot of graph between different masses of vibration and their time periods is a straight-line curve. Graphically, it can be expressed as shown in Figure 3. This curve is also called calibration curve of the inertia balance, since from this curve mass of an unknown object can be determined after knowing its time period of oscillation.

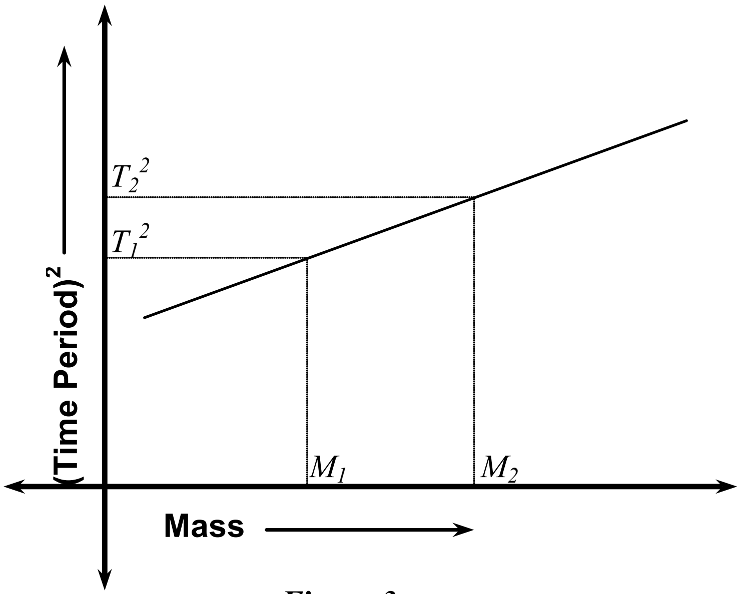


***Figure 3***

If, in place of positioning the cylindrical mass on the tray, it is suspended through the tray from the same hole using nylon cord, the moment of inertia of the oscillating mass along the axis of oscillation remains unchanged (since the distance of the mass from the axis of oscillation does not change and according the theorem of parallel axes, the moment of inertia remains unchanged). As a result, theoretically there is no change in the time period of oscillations.

**EXPERIMENT: Calibration of the Inertia Balance and determination of mass of an unknown object.**

**Procedure:**

1. Fix the platform of the inertia balance (without holes) using C-clamp rigidly to the laboratory table so that the holed platform at the other end of the balance is projecting away from the table horizontally.

2. Pull the free end of the platform to one side and release it. It will set the inertia balance to oscillate. Make sure that the balance remains rigidly fixed at the clamped end without any motion (as this will cause error in the experimental results), else tighten the C-clamp. Also set the stopwatch to zero. Once you are well in tune with the oscillations of the balance, start the stopwatch and start counting the number of oscillations. One oscillation is completed when the oscillating platform starts on one side, swings across from the mean position to the other side and then returns to the starting point. Note the time taken for at least 20-25 oscillations. Repeat this 2-3 times and note the average time for one oscillation. This time period ($T_1$) corresponds to the mass of the inertia balance only (m).

3. Position a 100g weight on the platform and fix it using a tape so that the weight does not move during oscillations of the platform. Set the balance into oscillation and note the time period of the oscillation as in step 2. The time period of oscillations obtained corresponds to mass of inertia balance & weight placed on it i.e., (m+100)g. Repeat the procedure to know the time period of oscillation for weights up to 1000g, increasing 100g weight in each subsequent step.

4. Determine the mass of inertia balance from the first two observations as explained in calculations and plot the graph of time period of oscillations against the respective mass.

5. Similarly measure the time period of oscillations after placing 1, 2 and 3 cylindrical weights included with the apparatus in the holes of the platform.

6. Also determine the time period of oscillation of the balance after suspending 1, 2 and 3 cylindrical weights from the holes of the platform using a nylon thread.

7. Measure the mass of the cylindrical weights included with the apparatus using a physical balance (or any other weighing balance). Also note the mass of cylindrical weights from the graph plotted from their time period of oscillations. Compare the two.

**Observations & Calculations:**

1. Time period of oscillation of inertia balance alone = $T_1$ =

2. Table for Time Period of oscillations with known weights:

| S. No. | Weight in the Pan (in g) | No. of Oscillations (n) | Time Period of Oscillations | | | | Time Period =(Average/n) |
|---|---|---|---|---|---|---|---|
| | | | 1st | 2nd | 3rd | Average | |
| 1. | 100 | | | | | | |
| 2. | 200 | | | | | | |
| 3. | 300 | | | | | | |
| 4. | 400 | | | | | | |
| 5. | 500 | | | | | | |
| 6. | 600 | | | | | | |
| 7. | 700 | | | | | | |
| 8. | 800 | | | | | | |
| 9. | 900 | | | | | | |
| 10. | 1000 | | | | | | |

3. Let T2 be the time period of oscillation of inertia balance with 100g weight. Thus from equation 6, mass of the inertia balance (in grams) is

$$m = \frac{100T_1^{\ 2}}{(T_2^{\ 2} - T_1^{\ 2})} =$$

4. Plot the graph between mass and time period of oscillations. Add the mass of inertia balance in each observation in the above table to the weight place in the platform. This gives the total mass of oscillation which is to be plotted against the time period.

5. Table for time period of oscillations of unknown weight (cylindrical weights).

| S. No. | Cylindrical weights in platform | No. of Oscillations (n) | Time Period of Oscillations | | | | Time Period =(Average/n) |
|---|---|---|---|---|---|---|---|
| | | | 1st | 2nd | 3rd | Average | |
| 1. | 1st | | | | | | |
| 2. | 1st + 2nd | | | | | | |
| 3. | 1st + 2nd + 3rd | | | | | | |

6. From the graph plotted note the masses corresponding to each of the three observations. Mass for 1st observation directly gives the mass of 1st cylindrical weight. Difference between mass for 2nd and 1st observation gives the mass for 2nd cylindrical weight. Similarly, find the mass of 3rd cylindrical weight. Compare the masses obtained here with that of the one from direct measurement using a balance.

7. Table for time period of oscillations of unknown weight (cylindrical weights) when they are suspended using a nylon cord.

| S. No. | Cylindrical weights suspended | No. of Oscillations (n) | Time Period of Oscillations | | | | Time Period =(Average/n) |
|---|---|---|---|---|---|---|---|
| | | | $1^{st}$ | $2^{nd}$ | $3^{rd}$ | Average | |
| 1. | $1^{st}$ | | | | | | |
| 2. | $1^{st} + 2^{nd}$ | | | | | | |
| 3. | $1^{st} + 2^{nd} + 3^{rd}$ | | | | | | |

8. Compare the results obtained in observation 7 with that of observation 5.