Analyzing thin lens equation and magnification using chi-squared test

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Abstract

Thin lens equation neglects the thickness of a spherical lens. In line with this, this experiment wants to test this equation statistically. A setup consisting of an incandescent bulb, transparent grid, diaphragm, lens, and a viewing screen was prepared. Then sets of values of image and object distances were obtained by varying the position of the lens. Results showed a close agreement between the theoretical and experimental focal lengths as well as between the magnification obtained by getting the ratio of the object and image distances and the magnification computed from the ratio of the object and image heights. Keywords: thin lens equation, chi-squared test

1 Introduction

Lens is an optical element used to refract light in able to form an image from a given object. Lenses are usually made with glass and normally has a spherical surface. A special type of lens where the thickness of the lens appears to be negligible in comparison to the radius of curvature of the lens is called the thin lens. The relationship between the distance of the object from the lens (d_o) and the distance of the image formed from the lens (d_i) is described by:

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i} \tag{1}$$

where f is the focal length of the lens used, and the positive distance from the lens is at the direction of the object being imaged. The magnification of the lens is the negative ratio of the image distance and object distance. The ratio of the image height (y_i) and the original height of the object (y_o) could also be used in the calculation of the magnification [1].

$$M = \frac{-d_i}{d_0} = \frac{y_i}{y_0} \tag{2}$$

However, this system is an over-generalization from a point object which is paraxial with the center of the lens. In observing object with significant height or width, deviations could be seen on its image formed which are called aberrations. The significant area of the object being image results to an array of different rays that may converge that results to error in focusing the image.

Despite the aberrations that might have been observed, the generalized thin lens equation described in Eq. 1 could be tested using statistical methods such as chi-squared test.

The Chi-Squared test compares the experimental value (E_n) and the theoretical value (T_n) with the equation

$$\chi^2 = \sum_{n=1}^{N} \frac{(E_n - T_n)^2}{T_n} \tag{3}$$

where n is the trial number, N is the total number of trials, and d is the degrees of freedom which is given by ($d = N - c$) where c is the number of parameters. The hypothesis is accepted if the result is $\chi^2 \leq n$ [2].

This experiment aimed to test the theoretical relationship of the image distance and object distance in the thin lens equation using quantitative method such as using the chi-squared test.

2 Methodology

The experiment was done using an incandescent bulb as light source, a diaphragm as an aperture, lenses with different focal lengths, a transparent grid as an object.

To get accurate measurements, the diaphragm was aligned first with the light source. The image staying centered as the viewing screen was moved away from the aperture served as the indication that they were aligned. After this, a lens was placed between the diaphragm and the viewing screen while a transparent grid was put between the bulb and the aperture. Then, we determined the location of the lens at which the produced image was the sharpest and this was recorded as its experimental focal length. We also got the distances between the lens and the object, and between the lens and the image. This was repeated several times to account for the measurement's uncertainty. The position of the lens was varied for several times and we did the same as before. The set of data that we obtained was employed into the thin lens equation and was also subjected to statistical analysis.

3 Results and Discussion

We applied the χ^2 test for a lens of focal length $f = 200$mm. We examined the system for three different object distances (and, thus, different image distances and magnification). The calculated value of chi-squared for the calculated focal length, magnitude of magnification using $M = -q/p$, where q and p are the image and object distance, respectively, and magnitude of magnification using $M = d'/d$, where d' and d are the image and object height, respectively, are less than the number of trials $n = 5$ ($\chi^2 < 5$. Moreover, these values are of order of magnitude 10^{-2} or less. This means that the agreement between the expected (taken to be the mean of the calculated values) and the observed values are of close agreement (Figures 1-3). This agreement also holds when we compared the calculated focal length to the theoretical focal length $f = 200$mm, with χ^2 of order of magnitude 10^{-3} or less ($\chi^2 = 7.9 \times 10^{-3}, 2.4 \times 10^{-3}$, and 9.2×10^{-4} for the three analyzed cases).

We also used the χ^2 test to test the agreement of the magnitude of magnification using the ratio of the image and object distances to the magnification using the ratio of the image and object heights, where we used the former to be the expected values. The χ^2 is of magnitude 10^{-2} ($\chi^2 = 0.088, 8.8 \times 10^{-3}$, and 5.0×10^{-3}), which implies that the two ways to compute for the magnification agrees with each other. These values are small enough that we can practically consider that the two equations will result to the same value. The precision of the materials used to measure distances and the possibly imperfect alignment of the apparatus used may have caused errors to propagate throughout our calculations and analysis.

4 Conclusions

Comparison of theoretical and experimental values can be done statistically. In this case, focal lengths computed using the thin lens equation was compared to the measured value from the experiment. Magnification determined from ratios of different quantities was also a topic of interest.

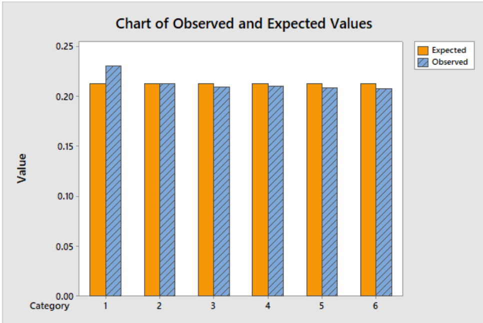
The chi-squared test that was performed on the focal lengths expressed a close agreement between the theoretical and experimental measurements. Measurements of the magnitude of magnification from the ratio of object and image distance, and ratio of object and image height also showed no significant difference

References

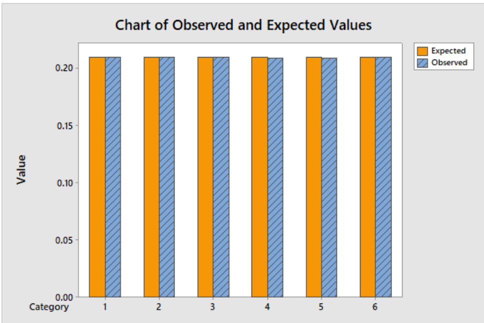
[1] P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers* (W.H. Freeman and Company, New York, 2008).

[2] J. Taylor, *An Introduction to Error Analysis: The study of Uncertainties in Physical Measurements* (University Science Books, Sausalito, California, 1997), 2nd ed.

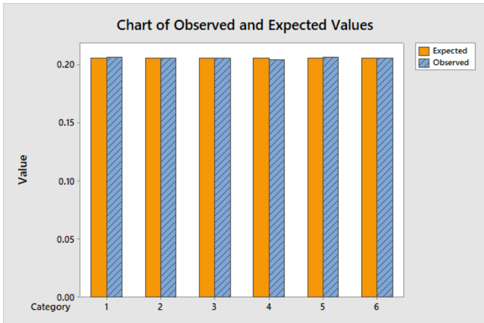
Figures and Diagrams



(a) $\chi^2 = 1.7 \times 10^{-3}$

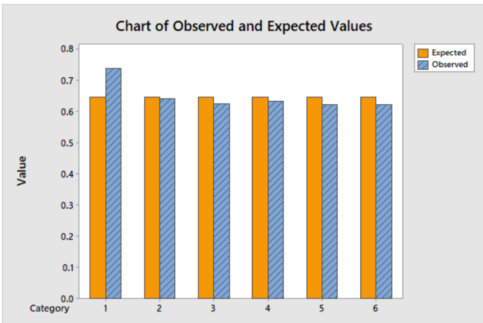


(b) $\chi^2 = 4.7 \times 10^{-6}$

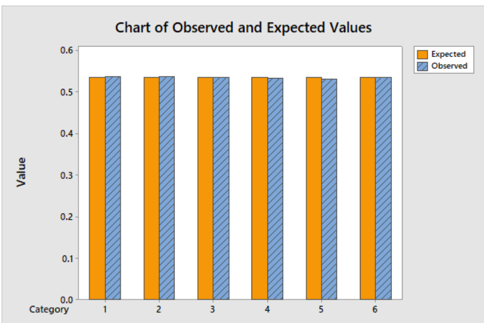


(c) $\chi^2 = 1.7 \times 10^{-5}$

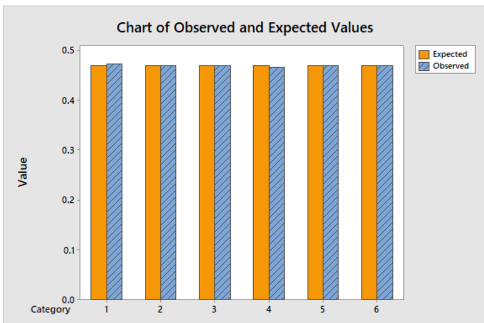
Figure 1: χ^2 analysis for the focal length in three different distances.



(a) $\chi^2 = 0.015$

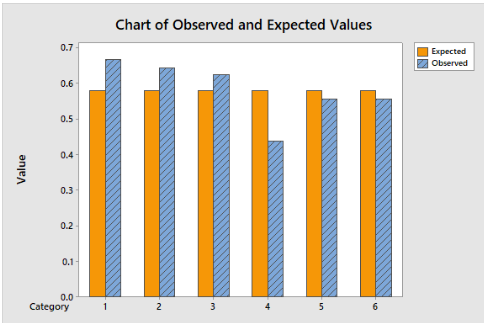


(b) $\chi^2 = 5.5 \times 10^{-5}$

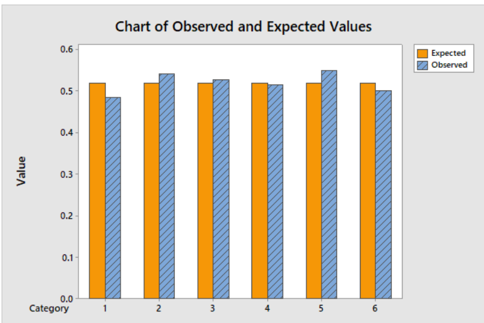


(c) $\chi^2 = 5.5 \times 10^{-5}$

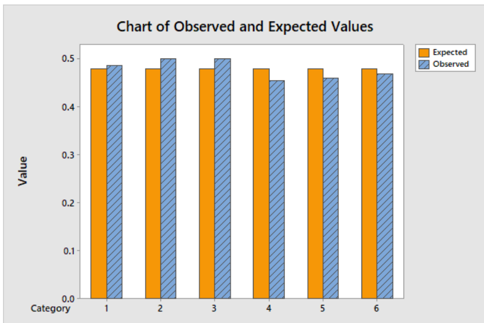
Figure 2: χ^2 analysis for the magnification in three different distances using the ratio of the image and object distances.



(a) $\chi^2 = 0.060$



(b) $\chi^2 = 6.0 \times 10^{-3}$



(c) $\chi^2 = 4.4 \times 10^{-3}$

Figure 3: χ^2 analysis for the magnification in three different distances using the ratio of the image and object heights.