The ballistic pendulum is a heavy block of wood suspended by four string supports, as in Fig. 11.4. The strings should be about 2 m long and slanted outward in the plane perpendicular to the path of the pendulum, to prevent twisting. The meterstick is mounted just below the pendulum. A small cardboard or aluminum rider serves as an indicator of the position of the pendulum.

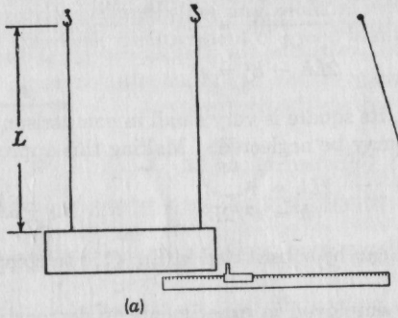

FIG. 11.4 Form of ballistic pendulum

Procedure: 1. Mount the rifle in its clamp with the barrel parallel to the rotating shaft. *Keep the breech open at all times except during the actual firing.* Close the motor switch and allow the system to reach full speed. Fire the rifle, using a .22 short shell.

2. Stop the motor and turn the shaft until the hole in the first disk is visible through the barrel of the rifle. With the disks stationary in this position fire the rifle. Measure the angle θ between the holes in the second disk. Use for this purpose the special protractor which has a semicircular notch at its center to fit the shaft.

3. Repeat the procedure of Steps 1 and 2, using a .22 long-rifle shell.

4. Measure and record the distance between the two disks. With the motor and shaft at full speed attach a revolution counter and allow it to run for 30 sec. Record the initial and final readings of the revolution counter.

5. Mount the rifle in the clamp before the ballistic pendulum with the barrel horizontal and parallel to the plane of the pendulum. Adjust the height of the rifle so that the bullet will strike at about the level of the center of gravity of the wooden block.

6. With the pendulum stationary set the rider on the scale against the end of the block. Read the scale at the end of the rider and record the reading. Insert a .22 short shell and fire the rifle. Read and record the new position of the rider on the scale. Repeat for two other .22 short shells.

7. Repeat Step 6, using .22 long-rifle shells.

8. Remove from its shell a sample of each kind of bullet and measure its mass with a suitable balance. Measure also the mass and length L of the pendulum. (This length should not include one-half the thickness of the block.)

Computations and Analysis: 1. Compute the angular speed ω of the rotating shaft, from the data of Step 4. Compute the time of travel of the bullet from $t = \theta/\omega$. Calculate the speed of each bullet from $v = S/t$, where S is the distance between the two disks.

2. From the data of Step 6, calculate the average distance s that the pendulum moved horizontally. By the use of Eq. (15) find the value of h. By the use of Eq. (11) compute the initial speed V of the pendulum.

3. Calculate the momentum of the .22 short bullet using its mass and the speed determined in Part 1 of the Analysis. Compute the momentum of the pendulum just after the impact. Compare the two values thus obtained.

4. Repeat Parts 2 and 3 using the data for the .22 long-rifle bullet.

5. For each type of bullet, calculate the kinetic energy of the bullet before impact and the kinetic energy of the pendulum after impact. Compare the corresponding energies and explain the discrepancy.

Experiment 11.2

CONSERVATION OF MOMENTUM: BLACKWOOD PENDULUM

Object: To study the elements of projectile motion and the law of conservation of momentum.

Method: A ball is fired horizontally and its range and vertical distance of fall are observed. From these distances and the value of g, the speed is calculated. The ball is fired into a suspended holder, arranged to swing as a ballistic

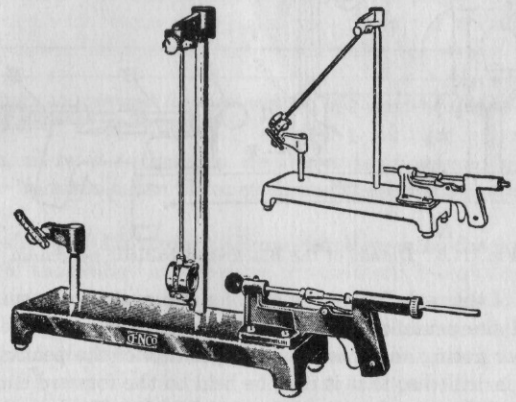

FIG. 11.5 Blackwood ballistic pendulum

pendulum. The speed of the pendulum and ball after impact is computed from the rise of the pendulum. The momentum of the ball before impact and that of the ball and pendulum after impact are calculated and compared.

Apparatus: Blackwood ballistic pendulum, Fig. 11.5; trip scales with masses; metric rule; steel tape measure; level; table clamp; carbon paper; box for catching ball; inclined plane.

The Blackwood pendulum, details of which are shown in Fig. 11.6, is a combination of a ballistic pendulum and a spring gun for propelling the projectile. The ballistic pendulum consists of a massive cylindrical bob C, hollowed out to receive the projectile and suspended by a strong light rod K. The rod is supported at its upper end by steel cone-pivot bearings. The pendulum may be removed from its supporting yoke by unscrewing the shouldered screw L. This screw, when tight against its shoulder, automatically adjusts the bearings so that the pendulum is securely held with little friction.

The projectile is a brass ball B which, when propelled into the pendulum bob, is caught and held by the spring S in such a position that its center of gravity lies in the axis of suspension rod K. The pendulum therefore hangs freely in the same position whether or not it contains the ball. A brass indicator I is attached to the pendulum bob C in such a way that its tip indicates the height of the center of gravity.

When the projectile is caught, the pendulum swings upward and is retained at its highest point by the pawl P, which engages a tooth in the curved rack R. The

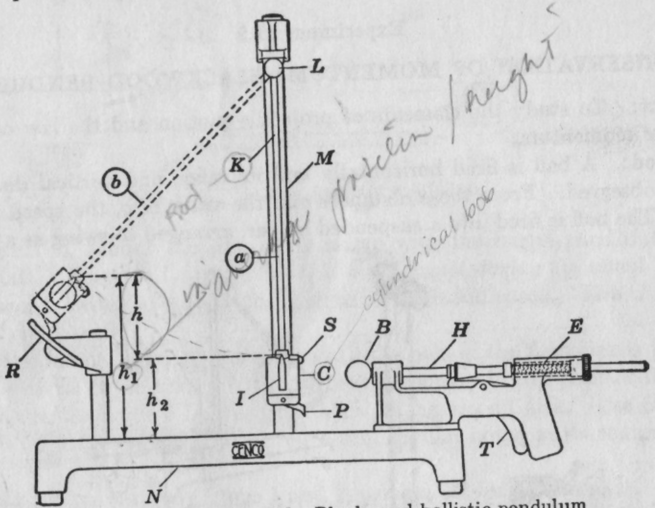

FIG. 11.6 Details of the Blackwood ballistic pendulum

toothed surface of the rack lies on the arc of a circle having its center in the axis of suspension of the pendulum. A scale along the outer edge of the rack provides a means for noting and recording the position of the pendulum after each shot. The ball is drilled so that it may be held on the forward end of the rod H, which is propelled forward by the compressed spring E when the trigger T is pulled.

Procedure: 1. To determine the initial speed of the ball, set up the apparatus as shown in Fig. 11.7. Set the apparatus near one edge of the table. If necessary, wedge it up with cardboard until the base N (Fig. 11.6) is accurately horizontal, as shown by the level. Clamp the frame to the table. Fire the gun and note the approximate place where the ball strikes the floor. To make the gun ready for firing, rest the pendulum on the rack R, put the ball in position on the end of rod H, and holding the base with one hand, pull back on the ball with the other hand until the collar on rod H engages the trigger T. This action compresses the spring E by a definite amount, and the ball is given approximately the same initial speed every time the gun is fired.

2. Place the box at the appropriate location to receive the ball when fired. Lay a piece of paper in the bottom of the box and cover it with carbon paper.

This arrangement automatically marks the spot where the ball strikes. See that paper, box, and gun do not move during the firing. Fire the gun five or six times. Locate the average position of striking and measure the range S_H (Fig. 11.7). Also measure the vertical distance S_V traveled by the ball.

3. To study the collision of the ball with the pendulum, release the pendulum from the rack and allow it to hang freely at rest. Fire the ball into the pendulum and record the position reached by the pendulum. Repeat this procedure five or six times. Record the position each time and calculate the average position. Set the pendulum at this average position and measure the distance h_1 (Fig. 11.6) from the base to the index point for the center of gravity. Release

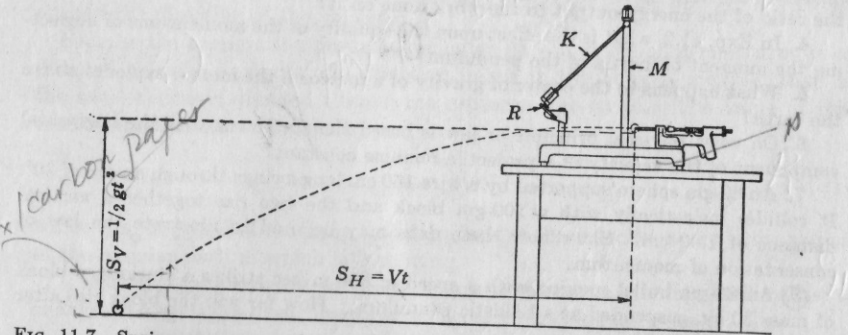

FIG. 11.7 Spring gun arranged for measurement of the initial speed of a projectile from observations of the range and height of fall

the pendulum, allow it to hang in its lowest position, and measure h_2. The difference $h_1 - h_2$ gives h, the distance through which the center of gravity rose after the impact.

4. Remove the pendulum from its support and measure the mass of the pendulum and that of the ball.

5. Elevate the gun so that the ball will be shot at an angle of about 30° above the horizontal. Measure and record the angle. Measure the *vertical* distance between the point of discharge and the level of the receiving surface in the box on the floor.

Assume the initial speed calculated in Part 1 of the Analysis and compute the range of the ball for this position of the gun. Set the box at the calculated position, fire the gun, and compare the calculated and observed ranges.

Computations and Analysis: 1. By use of the data of Step 2 and Eq. (9) calculate the initial speed of the ball.

2. Compute the momentum of the ball just before the collision and that of the ball and pendulum just after the collision. Compare these two quantities. Explain any difference that appears.

3. Compute the kinetic energy of the ball just before the impact and that of the ball and pendulum just after the impact. Compare the two kinetic energies and discuss the result.

Review Questions: 1. Define momentum and write its defining equation. Name the cgs and fps units. 2. Is momentum a vector or a scalar quantity? 3. State the law of conservation of momentum. Show how this law follows as a consequence of

Newton's laws of motion. 4. Derive, in terms of its horizontal range and the vertical distance fallen, an expression for the initial speed of a projectile fired horizontally. 5. What happens to the horizontal component of the velocity of a projectile as time goes on? To the vertical component of the velocity? 6. Derive the expression for the initial speed of a ballistic pendulum. What fundamental assumption is made in this derivation? 7. Describe the essential features of the Blackwood ballistic pendulum and the technique of the observations made with it.

Questions and Problems: 1. Suggest some probable reasons for the difference between observed values of momentum before and after impact.

2. How could one account for an apparent increase in momentum in Exp. 11.2?

3. How might one proceed to determine the efficiency of the spring gun, that is, the ratio of the energy output to the work done on it?

4. In Exp. 11.2, what is the effect upon the equality of the momentums of neglecting the moment of inertia of the pendulum?

5. What happens to the center of gravity of a meteor if the meteor explodes above the earth?

6. On what physical principle or law is based the statement that the horizontal component of the velocity of a projectile remains constant?

7. An 80-gm sphere supported by a wire 150 cm long swings through an arc of 30°. It collides inelastically with a 200-gm block and the two rise together a vertical distance of 1.00 cm. Show how these data may be used to illustrate the law of conservation of momentum.

8. An 80-gm bullet moving with a speed of 300 m/sec strikes a stationary block of mass 30 kg, suspended as a ballistic pendulum. How far will the block rise after the inelastic impact?

9. Show that the fractional loss in kinetic energy before and after impact of a ballistic pendulum of mass m struck inelastically by a moving ball of mass M is given by $m/(M + m)$. Into what form of energy is the lost kinetic energy transformed?

10. In the measurement of v by the free-flight method as in Exp. 11.2, what would be the effect on the accuracy of the measurements if the floor were not truly horizontal? If the base of the apparatus were not accurately horizontal?

11. When a pendulum is set vibrating it ultimately comes to rest. What are the various causes of its coming to rest? Do any such causes affect these momentum experiments?

CHAPTER 12

UNIFORM CIRCULAR MOTION

A body moving with uniform speed in a circle is said to move with *uniform circular motion.* Although the *speed* of the body is constant, its *velocity* is continually changing, for the *direction* of the motion is always changing. Consequently such a body has an acceleration, but this acceleration produces a change only in the direction of the velocity.

Because the acceleration produces a change only in direction it must always be directed at right angles to the direction in which the body is moving; that is, the acceleration is directed toward the center of the circle. The value of this central acceleration a is given by

$$a = v^2/r \tag{1}$$

where v is the speed of the body, or magnitude of the velocity, and r is the radius of the circular path in which it is moving.

A net force is necessary to produce any acceleration whether it involves a change of magnitude or of direction. According to Newton's second law

$$f = ma = m(v^2/r) \tag{2}$$

This net force, directed toward the center of the circle, producing the central acceleration, is called the *centripetal* force. The force is measured in any units suitable for Newton's second law. If the mass is in grams, the speed in centimeters per second, and the radius in centimeters, the force is in dynes.

The name *centrifugal* force is usually given to the equal and reacting force which, by Newton's third law, the *body* exerts away from the center. *Note that the centrifugal force does not act on the moving body.* Both centripetal and centrifugal forces are radial and not tangential.

The equation for centripetal force, Eq. (2), may be written in terms of angular speed by setting $v = \omega r$, whence

$$f = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = m\omega^2 r \tag{3}$$

The angular speed ω, in radians per second, is 2π times the angular speed n, in revolutions per second. Therefore Eq. (3) may be written

$$f = m\omega^2 r = m(2\pi n)^2 r = 4\pi^2 n^2 rm \tag{4}$$

A measure of the central force on a rotating object may be obtained from Eq. (4) by measuring the mass, radius of rotation, and angular speed of the rotating body.