The Bifilar Pendulum

Preparatory Task:

Read the entire script. In particular, familiarise yourself with the theoretical background (Section 2) and identify the main objectives of the measurements you'll make during the session. Then write a 5- to 10-1ine summary of the experiment in your lab book. Answer the following questions, too:

- *What are the physical dimensions of the left-hand side of Equation (1)? What are the physical dimensions of its right-hand side?*

- *What must the exponents γ and δ be to balance Equation (1) dimensionally? What is the relation that must exist between the exponents α, β, and ε?*

1. What it's about

This practical explores the use of dimensional analysis in the formulation of an equation. You will set up an equation giving the period of oscillation of a complex pendulum, using dimensional analysis to ensure that the equation is dimensionally correct. You will then determine experimentally the relevant exponents in the proposed equation, taking multiple, repeated measurements and doing a statistical analysis of the data. In the process, you will gain experience in plotting and fitting linearised data.

2. Preparation

2.1 Physical dimensions

The international system of units, known as the SI, is based on the seven units listed in Table 1. Apart from the radian, all other units in Physics are derived from these basic building blocks. Derived units can look rather complicated when expressed in terms of the base units; for convenience, they are often given special names and symbols. For example, the unit for resistance is the ohm, Ω, which is equivalent to 1 $kg\,m^2\,s^{-3}\,A^{-2}$.

Dimensional analysis is a technique used both to check the validity of equations and to find relationship between physical quantities. The principle of dimensional analysis is very simple: "for two quantities to be equal, they must have the same dimensions". For instance, since the force acting on an object is equal the mass of the object multiplied by its acceleration, the physical dimensions of a force must be that of a mass multiplied by an acceleration.

Table 1: The seven base units of the SI system.

Quantity	Unit name	Symbol
Mass	kilogram	kg
Length	meter	m
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of a substance	mole	Mol
Luminosity	candela	cd

Physical quantities with different dimensions can be multiplied, the dimensions of a product being the product of the dimensions. However, physical quantities with different dimensions cannot be added to one another (for instance, it does not make any sense to add a temperature to a length). Therefore if you obtain an equation in which a term with one physical dimension is added to a term with another physical dimension, you can be sure that this equation is incorrect.

Dimensional analysis also gives you the dimensions of physical quantities. For example, recalling Ohm's law, you will immediately see that a resistance R has the physical dimensions of a voltage divided by a current. Recalling the relation $q=CV$ between the charge stored in a capacitor and the potential difference across the capacitor, you will immediately see that a capacitance C has the physical dimensions of a charge divided by a voltage. From this, and taking into account that a current has the physical dimensions of a charge divided by a time, one sees that the product RC has the physical dimensions of a time.

As a final example, let us take the RC circuit of Figure 3 of the Circuits 2 script. This circuit is characterized by four experimental parameters: the frequency, f, and amplitude of oscillation, V_{G0}, of the voltage imposed by the function generator, the resistance of the resistor, R, and the capacitance of the capacitor, C. As shown in the script of Circuits 2 and demonstrated in that experiment, the voltage across the resistor oscillates with a different phase than the voltage imposed by the function generator. The phase difference, δ, is a pure number. Therefore, δ can depend on f, V_{G0}, R and C only through a combination of these four parameters which has no physical dimensions. Given that a frequency is the inverse of a time, the only combination possible is the product fRC, and indeed, Equation (7) of the Circuit 2 script shows that δ depends on f, V_{G0}, R and C only through this product.

2.2 A bifilar suspension

A bifilar suspension pendulum is one in which two (bi) filaments (filar) support a bar (a thin rod in the experiment). A schematic of this arrangement is shown in Figure 1. **It is important to note that in most of this practical we will consider the situation where the strings hang vertically, parallel to each other, as shown in the diagram, not the more general situation where the strings hang in oblique.** Also, we will only consider oscillations about a vertical axis through the centre of the rod.
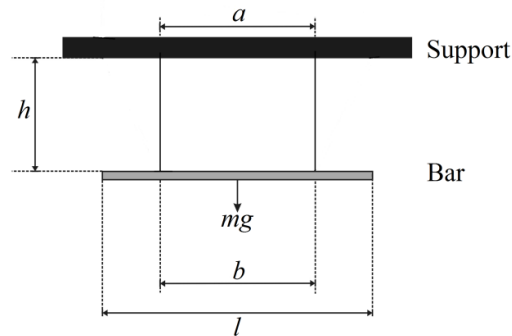
Figure 1: Schematic of a bifilar pendulum. In most of this practical, the lengths *a* and *b* are to be equal, as is indicated in the figure.

The system oscillates because any rotation of the rod away from its equilibrium position moves the strings away from the vertical, which lifts the rod upwards and increases its potential energy.

If the rod is homogeneous and symmetrical (so that its centre of mass coincides with its geometrical centre), if it is suspended **symmetrically** in the horizontal plane by two strings of equal length and if it set to swing **about a vertical axis through its centre**, the period of the oscillation may depend upon some, or all of the following quantities that define the system:

1. the difference in height between the bar and the support, *h;*
2. the distance apart of the upper ends of the strings, *a;*
3. the distance apart of the lower ends of the strings, *b;*
4. the mass of the suspended bar, *m*;
5. its length, *l*;
6. its radius, *r*; and
7. the acceleration due to gravity, *g*.

As noted above, **for most of this investigation we will consider the simplified situation where *a* and *b* are equal and the suspension filaments are vertical.** We thus assume that $a = b$ in the following. Finding how the period of oscillation of this pendulum depends on *h, a, m, l, r* and *g* by starting from Newton's second law and solving the resulting equations of motion is possible but requires rather complicated calculations. However, one can say something about this dependence from dimensional analysis. To start, let us guess that the period can be written as

$$T = A\, h^\alpha a^\beta\, g^\gamma\, m^\delta\, l^\varepsilon, \tag{1}$$

where *A* is a dimensionless constant and *α, β, γ, δ* and *ε* are unknown exponents. In writing this equation we have made the assumption that the period depends on the lengths *h, a,* and *l* in the form of a product of powers, not through other expressions

such as $l + a$ or $l^2 + h^2$ or some other combinations. This is merely a guess at this point. Whether this guess is correct will be ascertained by the measurements: If the variation of the period of oscillation with h, a, and l would be found inconsistent with a product of powers, then Equation (1) would be invalid. We have also assumed that the period T does not significantly depend on the radius of the bar, and have therefore not specified a term in r in Equation (1). The reason is that in the experiment this radius is much smaller than all the other lengths appearing in the problem (h, a, and l) and is therefore unlikely to be of great significance. Again, whether this assumption is valid can be proved or disproved by the measurements, at the level of precision possible in the experiment.

You are invited to find the values of γ and δ and a relation between α, β and ε in advance of the session, as part of the preparatory task. The objective of the measurements is to ascertain whether Equation (1) correctly describes how the period varies with h, a, and l and find the value of the dimensionless constant A and the individual values of the exponents α, β and ε, which a dimensional analysis does not yield.

3. Ready to start

The bar must be suspended symmetrically in a horizontal position by two threads of equal length. **Make sure that the two threads are vertical, so that $a = b$.**

To make your measurements you will set the bar into oscillations about a **vertical axis through its centre** and record the period of oscillation using the stop watch. **(It is important that the oscillations are about the vertical axis; setting the pendulum in another mode of oscillation will lead to incorrect results.)** For each set of parameters (a, h, l, m) you should take a minimum of four measurements of the period and calculate the <u>mean</u>, the <u>standard deviation</u> and the <u>standard error</u>. Do not start taking these measurements yet – you first need to sort out the following details. You may also want to do a few trial runs before beginning, too, to see where you are.

   ?   How will you check that the bar is suspended horizontally?

   ?   To what magnitude of angle should you limit the swings?

   ?   How many swings will you time in order to obtain the period?

   ?   How will you measure the period so as to reduce the experimental error as much as you can? (You may need to try different arrangements before finding the best one.)

   ?   How will you extract the unknown powers experimentally? [Hint: Starting from Equation (1), express the logarithm of T in terms of the logarithms of A, h, a, g, m and l.]

Remember that all measurements **must** be recorded directly into your lab book with the errors clearly indicated. To this effect, it would be useful to draw up suitable tables before you start collecting data.

4. Measurements and analysis

We'll begin by measuring the period for bars of different masses but same lengths and suspended in the same way, in order to verify that the period does not depend on the mass.

Task 1:

Measure the period of oscillation of two of the long rods of same length but made of different materials, keeping h and a constant. **The intervals a and b must be symmetric about the centre and as mentioned above must have the same length. When setting the bars in motion, try and ensure that only the mode of oscillation of interest is excited.**

? Do your measurements support your deduction that the period of oscillation is independent of mass?

? In retrospect, is the method you have used to time the oscillation the best possible with the equipment at your disposal?

We now measure the period for different lengths of the bar, in order to find the exponent ε. Of course, the mass of the bar will vary, but we know that this should not affect the period. We then find the exponents α and β and the overall factor A. As all this will involve many repeated measurements, it is OK to join force with another group and divide the work between you, both for Task 2 and Task 3.

Task 2:

Find the period of oscillation as a function of the length of the bar, keeping h and a constant. As noted before, the suspension should be symmetric about the centre. Clearly, the length a should be **smaller than the shortest bar!**

After recording your data, use an Excel spreadsheet to calculate the mean, the standard deviation and the standard error on the mean for the period of oscillation, for each length of the bar. **Don't do this by hand.** (Information about using Excel for calculating averages and standard errors can be found in the online Physics Laboratory Guide, http://labs.physics.dur.ac.uk/general/index.php, under Skills > General Skills > Data Analysis. You may want to look at this online resource and that mentioned in the next paragraph in advance of the session, to save time.) Keep a record of the values of the period and its error into your lab book. Better: print the spreadsheet and stick the copy in your lab book.

Then, plot the logarithm of the period against the logarithm of the length using Excel. Remember to include error bars in this plot. Add a trend line and stick the graph in your lab book. (See the Excel pages of the Physics Laboratory Guide, in particular http://labs.physics.dur.ac.uk/skills/skills/logscales.php, for all relevant information. Plotting your results on a sheet of log-log graph paper would be faster than using Excel, and you may want to try this; however, make sure that you can also produce the relevant graph with Excel.)

Finally, find the slope of the resulting graph, along with the error in the slope, using Excel. What is the value of the exponent ε in Equation (1)?

?   Have you taken enough readings for each set-up to be confident in the value of ε?

Task 3:

Now determine the powers of h and a, i.e., the exponents α and β in Equation (1), by varying one quantity at a time and keeping all the other variables constant. As before, use Excel to plot straight-line graphs and determine slopes. Once again, remember to show the error bars on your graphs.

?   Are the exponents α, β and ε determined experimentally consistent with the relation obtained in the preparatory task? Can you therefore rule out the possibility that the period of oscillation would depend on the radius of the rod through a product of the form $A\,h^\alpha a^\beta\,g^\gamma\,m^\delta\,l^\varepsilon\,r^\zeta$?

?   Is there any evidence, at the level of precision you have achieved, that the period would depend on h, a and l through a different combination of these variables than the product $h^\alpha a^\beta\,l^\varepsilon$?

Task 4:

When you have found a consistent set of values for α, β and ε, determine the constant A by measuring the period accurately for a particular set of values of h, a and l. Don't forget to estimate the error on your determination of A.

Task 5:

If you have reached this point before the end of the lab session, then you could usefully consider the case when the suspension filaments are no longer vertical (i.e., $a \neq b$).