Determination of fluid viscosity using a modified Farneback optical flow algorithm

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Abstract

We propose an image processing-based approach to determine the dynamic viscosity of a fluid using a modified Farneback optical flow algorithm. From Stokes' law, the dynamic viscosity is a function of the terminal velocity of an object free-falling through that fluid. By recording the motion of a steel ball falling through the fluid, its displacement through time can be extracted from the change in pixel intensities using optical equations. The proposed method yield high-error results, due in part to the impurity of the fluids used, uncorrected lens distortions, and the imperfection of the background subtraction process in the algorithm.

Keywords: fluid dynamics, viscosity, optical flow, corner detection.

1 Introduction

Stokes' law states the force due to viscosity or due to a viscous fluid, F_d is given by

$$F_d = 6\pi a \nu v, \tag{1}$$

where a is the cross-sectional area of the fluid, ν is the viscosity and v is the speed of the object. [1] This force is opposite the objects motion and along the buoyant force. The buoyant force is given by

$$F_b = \rho g V, \tag{2}$$

where ρ is the density of the fluid and V is the volume of the object. From the second law of motion, the equation governing the motion of an object in a viscous fluid is given by:

$$m\frac{d^2x}{dt^2} = mg - \rho g V - 6\pi a \nu \frac{dx}{dt}. \tag{3}$$

The solution to this differential equation, $x(t)$ is given by:

$$x(t) = x_0 + \frac{mg(\rho V - m)}{(6\pi a \nu)} - \frac{g(\rho V - m)}{6\pi a \nu}\left[t + \frac{m}{6\pi a \nu}e^{-\frac{6\pi a \nu}{m}t}\right] \tag{4}$$

thus, the speed of the object in a viscous fluid is given by the time derivative of (4):

$$x'(t) = v(t) = \frac{g(m - \rho V)}{6\pi a \nu}\left(1 - e^{-\frac{6\pi a \nu}{m}t}\right) \tag{5}$$

where the initial conditions are $x(0) = x_0$ and $x'(0) = 0$. Getting the limit (5) as $t \to \infty$ yields the terminal velocity given by

$$\lim_{t\to\infty} v(t) = \lim_{t\to\infty} \frac{g(m-\rho V)}{6\pi a\nu}\left(1 - e^{-\frac{6\pi a\nu}{m}t}\right) = \frac{g(m-\rho V)}{6\pi a\nu}. \tag{6}$$

The objective of this paper is to determine the experimental value of the terminal velocity and compare it to the theoretical value.

2 Experimental Setup and Materials

The experiment used two fluids, motor oil (Valvoline All-Climate motor oil SAE 20W-50) and castor oil. The setup, as shown in Figure 1, contains the fluid in a 500 mL graduated cylinder. To capture the motion of the balls, a NIKON D3400 digital single-lens reflex camera was used with a SIGMA 24-70mm lens attachment, recording at 60 fps and 1080p resolution. Six different masses of balls for the castor oil, while five masses were used for the motor oil. Also, a magnet was used to remove the metallic ball from the cylinder.

3 Methodology

First, the diameter and mass of the steel balls were measured using a digital weighing scale and a micrometer. Next, we followed the set up as shown in Figure 1. The metallic ball was placed in the cylinder, just above the meniscus of the fluid in order to minimize surface impact effects. The free-fall of the ball through the fluid was captured. A magnet was used to retrieve the metal ball from the bottom of the cylinder and was washed for the next user.

For the pre-processing stage, the saved videos were imported into Adobe Premiere Pro, slowed down to 15 fps, trimmed to include only the parts where the ball was falling, and rescaled to 640p resolution.

For the next stage, the pre-processed videos were imported into Python. The Shi-Tomasi corner detection algorithm was used to define boundaries in the video and prevent continuous erroneous detection of an object that has stopped moving or moved out of the frame. A modified Farneback optical flow algorithm was then used to detect moving objects and track them throughout the video. The same algorithm was used to subtract the background and display only pixels which change significantly per frame. The mean pixel location of the maximum intensity per frame was obtained in order to generate the position vs time graphs, and consequently, the velocity vs time graphs. The specifications of the camera and lens equation was used in order to convert pixel values to real, SI units. From the position vs time graphs, linear regression was performed to obtain the slope, i.e. the terminal velocity, which was inserted into Stokes' law and averaged per mass to obtain the dynamic viscosity.

4 Results and Discussion

Figure 2 shows representative frames of the output of the modified Farneback optical flow algorithm. Since the camera was placed on a tripod, there is minimal movement in the frames and therefore, a falling ball would be detected as the most significant pixel changes. The camera was also positioned such that the captured video would be in portrait orientation relative to it. Thus, the direction of the ground is actually to the left side of the frame. This also allows the proper and standard definition of positive and negative vectors in one dimension. Figure 3 shows the position and velocity graphs obtained by taking the mean location of the maximum intensity values shown in Figure 2, i.e. it is expected that this value is the geometrical center of the ball.

Applying the algorithm described in Section 3 to all the videos, a list of terminal velocities corresponding to each mass in each fluid was obtained. This was plugged into the derived equation [2]

$$\eta = \frac{2}{9} \frac{r^2}{v_T} \left(\rho_s - \rho_f\right) g \tag{7}$$

where η is the dynamic viscosity, r is the radius of the ball, v_T is its terminal velocity, ρ_s is its density, ρ_f is the fluid density, and g is the acceleration due to Earth's gravity. The list of masses with their corresponding dimensions are shown in Table 1. From this, the viscosity of castor oil was experimentally determined to be 2.86 Pa·s, which deviates by 339% from the literature value of 0.650 Pa·s [3].

Similarly, the motor oil viscosity was determined to be 1.19 Pa·s. The literature value was unclear, since references state that its viscosity is "$<$ 9.50 Pa·s" [4]. Using this value as an upper bound on the literature value, the experimental value deviates from it by 87.50%.

Major errors could stem from the following: (a) the used fluids may have been contaminated with other fluids, (b) uncorrected lens distortions which could shift the displacement values significantly (since a wide lens was used, implying possible barrel distortion), and (c) the ball does not appear perfectly circular after subtracting the background, and noise could also shift the mean location of the maximum intensity.

5 Conclusions

An image processing-based approach to determine the viscosity of fluids based on Shi-Tomasi corner detection algorithm and a modified Farneback optical flow algorithm was proposed. A digital SLR recording at 1080p at 60fps was used to capture the motion of a steel ball free-falling through a fluid. Using optical laws, the displacement of the pixel location of the geometric center of the ball could be converted to real units, and the terminal velocity could be subsequently calculated. Inserting the relevant quantities into Stokes' law allows experimental determination of the fluids' dynamic viscosity.

The dynamic viscosities for castor oil and motor oil are equal to 2.86 Pa·s, 1.19 Pa·s, respectively, with the percent deviation for each type of oil being 339% and 87.50%, respectively.

Such a large error could be attributed to impurity of the used fluids, optical distortions, and distortions caused by the object tracking algorithm itself.

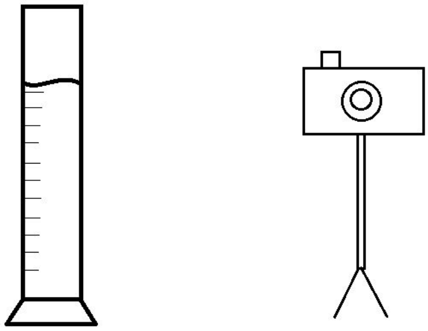
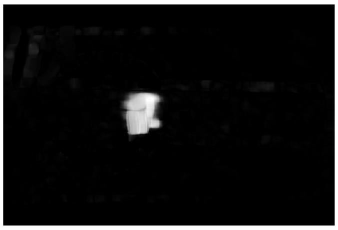


Figure 1: The setup of the experiment

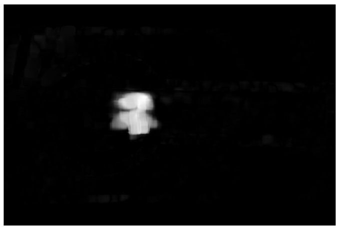
Table 1: Masses with their corresponding characteristics.

Fluid	mass [g]	diameter [mm]
Castor oil		
1	28	14.87
2	8	8.83
3	8	8.80
4	7	8.77
5	4	6.12
6	3	5.12
Motor oil		
1	29	14.84
2	8	12.62
3	8	12.60
4	8	12.61
5	3	9.93

(a) frame 18

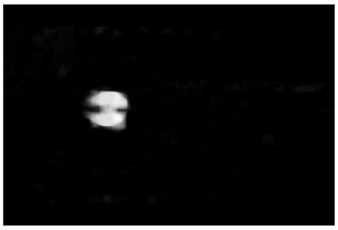


(b) frame 19

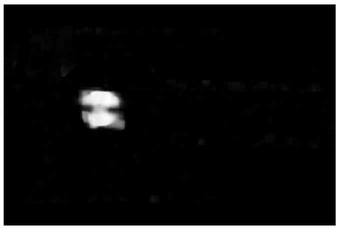


(c) frame 20

(d) frame 21



(e) frame 22



(f) frame 23

(g) frame 24

(h) frame 25

(i) frame 26

Figure 2: Representative frames with subtracted background from the modified Farneback optical flow algorithm.

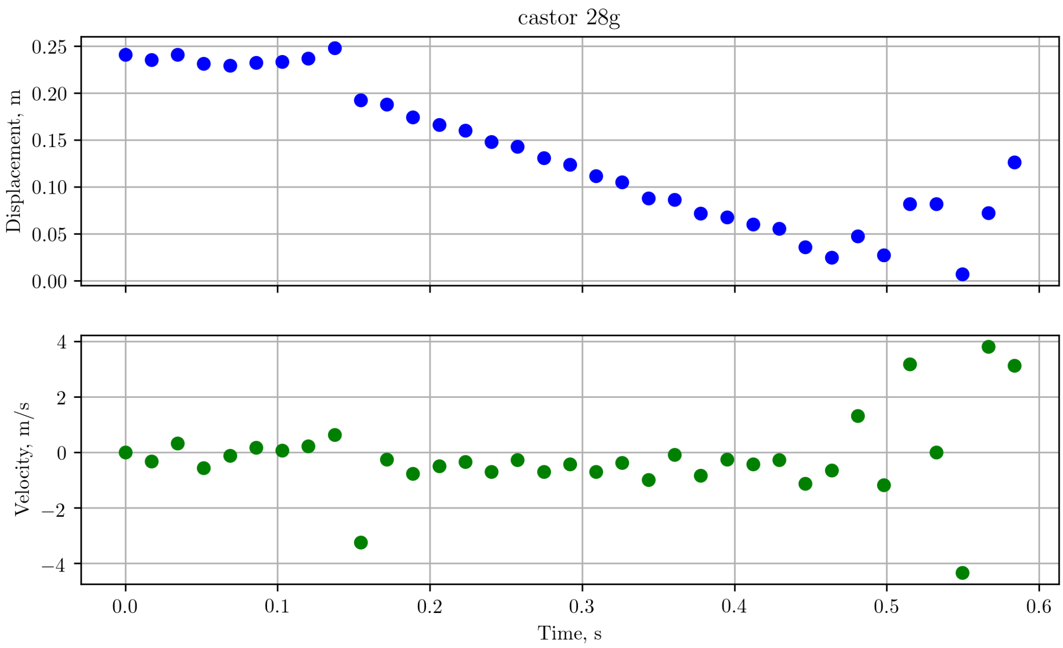


Figure 3: Representative data of the obtained position vs time and velocity vs time graphs from the optical flow algorithm

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