Experiment #11

The Viscosity of Fluids

References:

1. Your first year physics textbook.
2. D. Tabor, *Gases, Liquids and Solids: and Other States of Matter* (Cambridge Press, 1991).
3. J.R. Van Wazer et al, *Viscosity and Flow Measurement* (Interscience, 1963) pp 272-275.
4. PHYS 233 Reference Manual - section on Errors and Error Propagation.

Introduction:

In this experiment you will investigate how the speed of a sphere falling through a viscous liquid depends on the size of the sphere and measure the viscosity of corn syrup at room temperature.

Definition of Viscosity: The viscosity of a fluid is a measure of the internal friction opposing the deformation or flow of the fluid.

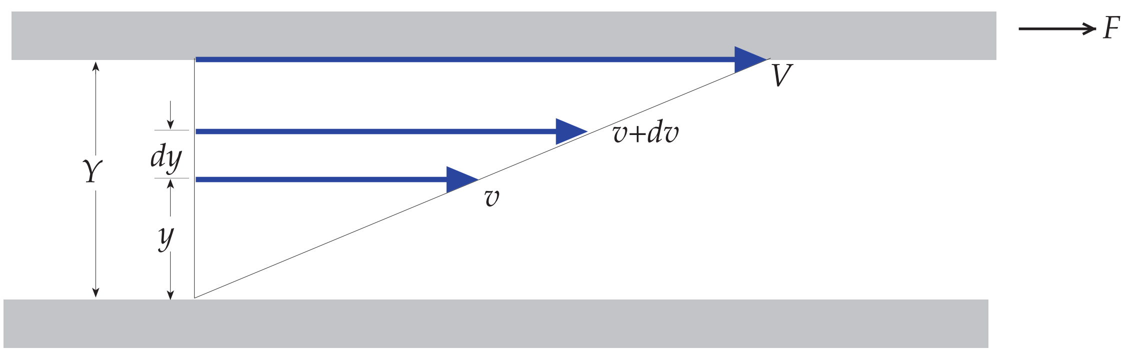


Figure 1: Schematic diagram showing flow of a fluid between two plates.

Consider two parallel plates, as shown in Fig. 1, each of area A and sufficiently large so that edge effects are unimportant, placed a small distance Y apart with the space between the plates being filled with fluid. The upper plate is moved parallel to the lower one with velocity v under the application of a force F. Particles of fluid in contact with each plate will adhere to it and, if Y is not too great nor v too high, the velocity of the fluid will increase linearly with distance from the plate. The action is much as if the fluid were made up of a series of thin sheets, each of which slips a bit relative to the next. Experiments have shown that for a large class of fluids the force required to move the upper plate is proportional to the area of the plates and the velocity gradient v/Y,

$$F \;\propto\; A\,\frac{v}{Y} \;=\; \eta\,A\,\frac{v}{Y} \quad , \tag{1}$$

where the constant of proportionality η (eta) represents the viscosity of the fluid. Equation 1 is commonly referred to as Newton's equation of viscosity and serves to define the coefficient of viscosity η. The shear stress τ (tau) between any two thin sheets of fluid can be expressed as

$$\tau \;\equiv\; \frac{F}{A} \;=\; \eta\,\frac{dv}{dy} \quad . \tag{2}$$

Viscosity Dimensions and Units: The dimensions of viscosity are force per unit area divided by velocity gradient as in Eq. 1. In the MKS system the unit of viscosity is Ns/m^2 whereas in the CGS system the unit of viscosity is the poise (1 poise = 1dyne-sec/cm^2). Handbooks commonly quote viscosity in centipoise (1 cp = 0.01 poise) which is a very convenient unit of viscosity because the viscosity of water at 20 °C is one centipoise.

The motion of spheres in viscous fluids (laminar flow): In principle, the force F required to drag a sphere of radius r at velocity v through a fluid of viscosity η can be calculated. Since the derivation is beyond the scope of this lab, we simply quote the result as :

$$F \;=\; 6\pi r\eta v \quad . \tag{3}$$

Equation 3 is known as Stokes' Law and is valid only for laminar flow, where the flow of the fluid can be treated as consisting of layers, each layer having a well defined velocity. Laminar flow around an airfoil is illustrated in Fig. 2a. Turbulent (non-laminar) flow is illustrated in Fig. 2b.

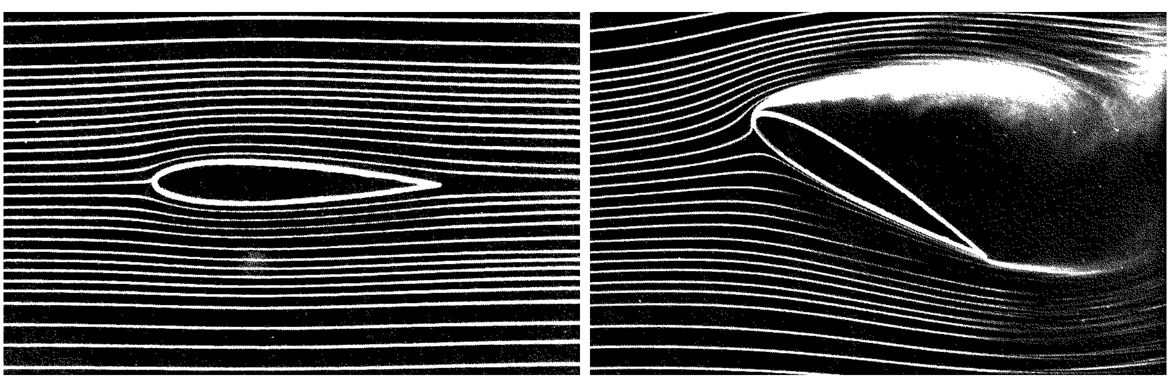


Figure 2: Photographs showing (left) laminar flow and (right) turbulent flow of a fluid around a wing-shaped airfoil. Pictures are from *Shape and Flow* by Ascher Shapiro, Science Study Series # S21.

Sphere falling in a viscous fluid: A sphere of density ρ_s (rho sub s) falling in a stationary fluid of density ρ_l feels a gravitational force given by

$$F = \frac{4}{3}\pi r^3 g\left(\rho_s - \rho_l\right) \quad , \tag{4}$$

where g is the acceleration due to gravity. According to our previous discussion, the sphere also feels a viscous drag force given by Eq. 3. Combining Eqs. 3 and 4, we can obtain an expression for the terminal velocity of the sphere

$$v = \frac{2gr^2}{9\eta}\left(\rho_s - \rho_l\right) \quad . \tag{5}$$

We can now experimentally determine the viscosity of a fluid since all of the remaining quantities in Eq. 5 can be measured easily.

The motion of spheres in viscous fluids (turbulent flow): As the velocity of a sphere through the fluid increases, the flow past the sphere no longer remains perfectly laminar. The formation of eddies is observed and turbulence sets in. Turbulent motion is very difficult to analyze, and one has to resort to empirical methods in order to treat this situation. The degree of turbulence is usually gauged by the magnitude of a dimensionless number R known as the Reynold's Number. The Reynold's Number for a sphere is given by:

$$R = \frac{2rv}{\eta}\rho_l \quad . \tag{6}$$

It is found experimentally that turbulence is present and important whenever $R \geq 1$.

Finite size effects: Eq. 3 is based on the assumption that the sphere is moving through a fluid of infinite extent. This is not the case in the experiment you will be attempting as the presence of the cylinder walls appears to increase the viscosity above its actual value and you will find that the larger spheres fall less rapidly than theory predicts.

Stokes suggested the use of an empirical correction formula to account for this effect (see Ref. 3 for more details). Define an apparent viscosity $\eta\prime$ given by

$$\eta\prime = \frac{\eta}{\left(1 - \frac{r}{R_c}\right)^{\alpha}} \tag{7}$$

where $2.2 < \alpha < 2.4$ and R_c is the cylinder radius. Equation 5 can still be used to interpret the data when it is written in the form

$$v = \frac{2gr^2}{9\eta\prime}\left(\rho_s - \rho_l\right) \quad . \tag{8}$$

Prelab Questions:

1. Consider a sphere released from rest and falling under the influence of gravity. There are two forces on it, the gravitational force and the viscous drag force as given by Eqs. 3 and 4. Write down the force balance equation and derive an expression for the speed $v(t)$ and sketch a graph of it. How should the knowledge gained from this exercise affect your experimental method?

2. From the result of Prelab Question 1, show that the terminal velocity is given by Eq. 5.

3. Calculate Reynold's Number for a representative sphere to check whether the flow might be turbulent.

Apparatus:

- glycerol
- graduated cylinder
- stopwatch
- steel balls of various sizes ranging from 2 to 8 mm
- ruler
- micrometer
- balances

Experiment:

1. Eventually you will need to calculate the viscosity of the glycerol. In order to do this, you will need to determine several other quantities including the densities of the balls and the glycerol and the radii of the balls. Be sure to estimate the error for each quantity. In some cases, the error will just reflect the resolution of the equipment, but in other cases it will be necessary to make several measurements and calculate the mean value and the standard deviation of the mean for the sample population you have obtained. In Appendix A, at the end of this lab script, you will find information on how to use Vernier calipers and micrometers.

2. Determine the terminal velocity for each size ball as it falls through the glycerol. Make sure that you are actually measuring the terminal velocity by measuring the velocity of a single ball of each size at several positions as it falls through the glycerol. To estimate the uncertainty in your measurement of the terminal velocity, you should repeat the measurement for each size multiple times to determine the distribution of terminal velocities. Do as many measurements as possible, focussing on the ones that are least accurate.

3. Plot the terminal velocity for each ball without the finite size correction. Then use the finite size correction with $\alpha = 2.3$ to recalculate η. Arrange your data so that, if it follows Eqs. 5, the plot will be linear. Is there a range of sphere size for which Eq. 5 or Eq. 8 holds?

4. Determine the viscosity of the glycerol and an estimate of the experimental uncertainty in this quantity.

Appendix: The Vernier Caliper and the Micrometer

The Vernier Caliper

The Vernier principle which allows accurate determination to fractions of the divisions on the main scale is used in a multitude of instruments. The Vernier caliper is illustrated in Figure 3. The diagram labelling is self-explanatory, but the way in which the readings are made is as follows (only the metric scale will be explained). The Vernier scale is the small one that moves along the main scale. With the caliper closed, the zero end of the Vernier coincides with the zero mark on the main scale. With the caliper open by 1 mm the zero on the Vernier lines up with the 1 mm mark on the main scale, and so on. The millimeter reading is obtained from the zero end of the Vernier scale. To see the operation of the Vernier, close the calipers and note that nine divisions on the main scale are covered by ten divisions on the Vernier scale as shown in Figure 4A. The first Vernier mark is short of the millimeter mark by 1/10 or 0.1 mm. If the caliper were opened by 0.1 mm, this Vernier mark would line up with the millimeter mark above it. When the instrument is closed, the second Vernier mark is 0.2 mm away from a millimeter line, and if the zero end of the Vernier is 0.2 mm past the millimeter mark, the second Vernier line will line up with the one above it. The tenths of a millimeter are obtained by finding which Vernier line coincides best with a line on the main scale. Figure 4B shows a Vernier reading 5.6 mm. The 5 is read at the zero on the Vernier and the sixth line coincides with a millimeter mark.

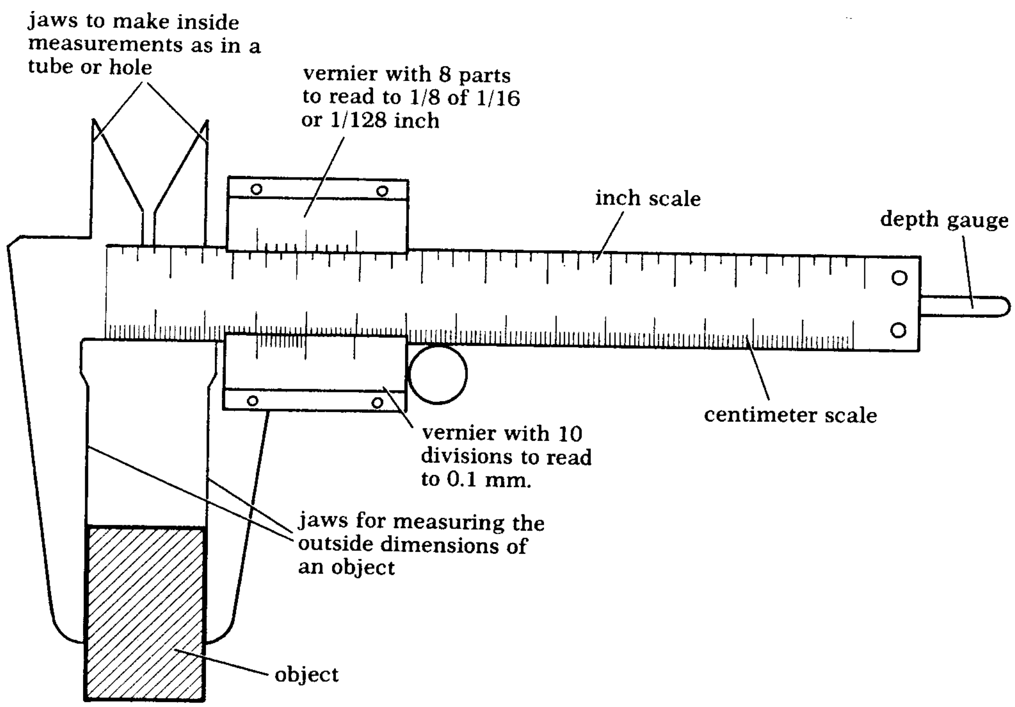


Figure 3: Schematic diagram of a Vernier caliper.

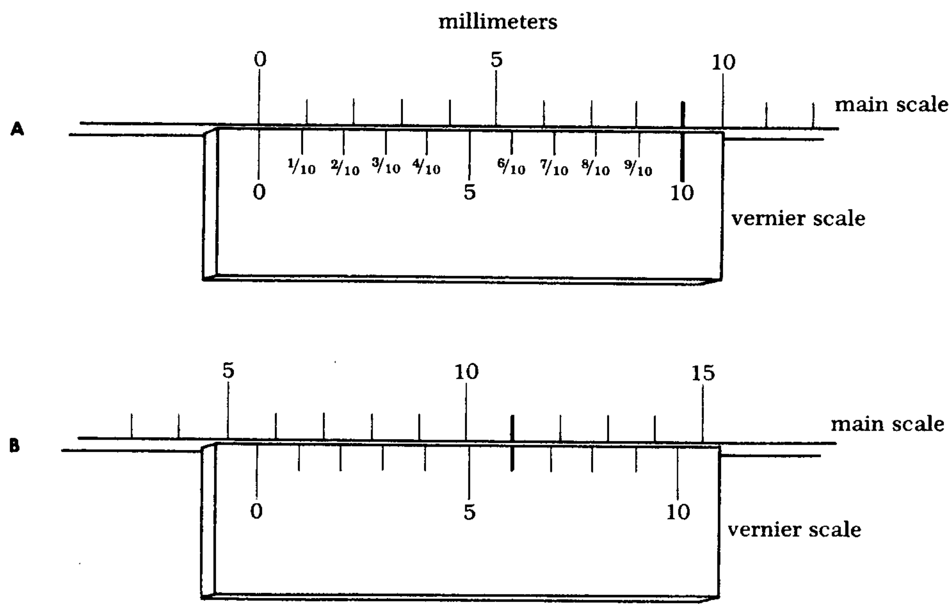


Figure 4: The use of a Vernier scale. In A the jaws of the caliper are closed and it reads 0.0. In B the position of the Vernier scale shows a reading of 5.6.

There are differences on different instruments as to the number of lines on the Vernier. If 50 Vernier division cover 49 main scale divisions, the Vernier divisions read to 1/50 or 0.02 of a scale division. If the scale division corresponds to 0.5 mm, then a Vernier with 50 divisions would read directly to 0.01 mm. In such an instance, the zero end of the Vernier would read to the half millimeter. If the zero end of the Vernier was past a millimeter mark but not as far as the half millimeter mark, the Vernier would read directly. If the zero on the Vernier was beyond a half millimeter mark, 0.5 would have to be added to the Vernier reading.

The Micrometer

Another instrument for measuring small distances is the micrometer or screw caliper which is shown in Figure 5. The principle of the micrometer is that, as a screw is turned by one revolution, it advances a distance equal to the pitch of the screw. A fraction of a rotation advances the screw by a corresponding fraction of the pitch. It is common in a micrometer to have a screw with a pitch of 0.5 mm, or 20 turns/cm, and a thimble to read to fiftieths of revolutions or 0.01 mm. Markings on the sleeve give the readings of the distance to 0.5 mm, and the reading on the thimble gives the number of hundredths to be added to the reading on the sleeve. Figure 6 shows two examples.

Micrometer calipers often do not read zero when the jaws are closed. To correct for this, a zero reading must always be obtained and subtracted from the micrometer reading of an object. If the reading is below the zero mark, it is considered to be negative and is added to

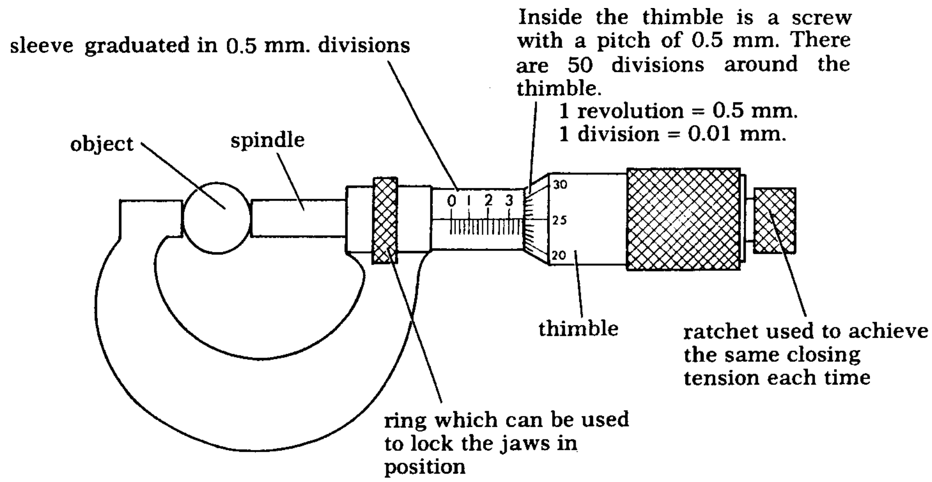


Figure 5: The micrometer caliper

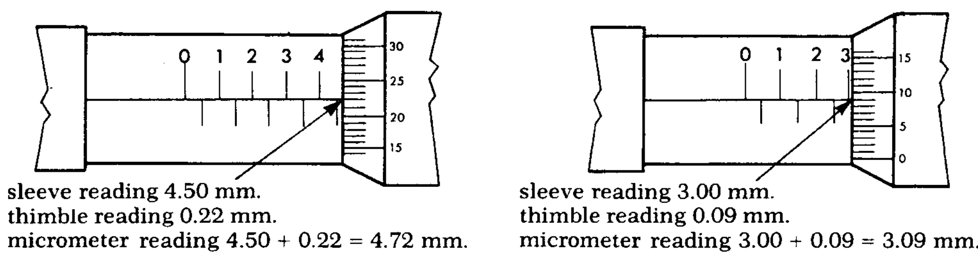
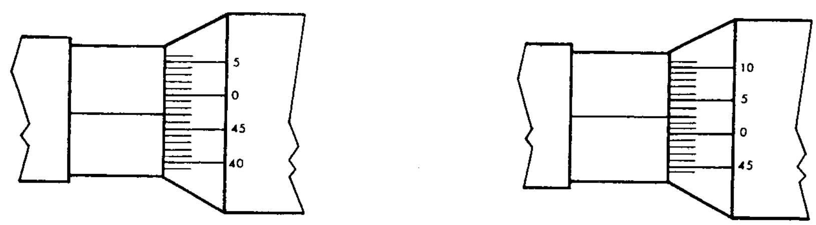


Figure 6: Two micrometer readings illustrated.

the micrometer reading. Examples of zero readings are shown in Figure 7.

A micrometer caliper should always be closed gently, using the ratchet adjustment if there is one. The screw has a high mechanical advantage and if the drum is forced, the object being measured may be squeezed by several hundredths of a millimeter or the jaws may be bent apart by a similar amount. Never force a micrometer.

Always leave the micrometer with the jaws open when you are finished using it.



micrometer reading minus 0.03 mm. to be added to each reading

micrometer reading 0.03 mm. to be subtracted from each reading

Figure 7: Two micrometer zero readings illustrated.