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Sub-Nyquist Sampling of Acoustic Signals Based on Chaotic Compressed Sensing

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Abstract

Compressed sensing (CS) is a new approach to signal sampling that allows signal recovery from an incomplete set of samples. For the faithful reconstruction of the signal from few measurements or samples, the sensing matrix must satisfy the restricted isometry property (RIP). The random matrices obey this property, but the practical implementation is expensive. Chaotic sequences are used to construct the sensing matrix and exact reconstruction is, under specific conditions, possible with high probability. In this paper, a chaotic compressive sampler is employed to sample the acoustic signals at sub-Nyquist rate. In contrast with traditional Nyquist sampling and linear reconstruction, orthogonal matching pursuit (OMP) algorithm is applied to recover the signal from the chaotic measurements. This paper shows that the chaotic compressive sampler outperforms the random demodulator architecture in terms of the reconstruction accuracy.

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Keywords: Compressed Sensing, Random Demodulator, Chaotic Compressed Sensing, Logistic Map

1. Introduction

Nowadays, the storage of acoustic signals, such as musical signals demands large memory requirements. Therefore, the signals are compressed prior to storage or transmission. However, the acoustic signals are directly acquired in compressive manner by applying a new sampling theory, called compressed sensing (CS). CS,

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introduced by Candès, Romberg, Tao [1] and Donoho [2], has gained an overwhelming attention over the recent years. The basic idea behind CS theory is that a very sparse or compressible signal in a known basis can be exactly reconstructed from far few measurements or samples than needed by the conventional Nyquist-Shannon theory. Inspired by the CS theory, S. Kirolos, et al. [3] developed an analog-to-information converter (AIC) as an alternative to conventional ADC. For sparse input signals, AIC samples the signal at a rate proportional to information rate rather than the Nyquist rate. Thus AIC is an effective sampling system for implementing the random measurement procedure.

Tropp, et al. [4] proposed the random demodulator (RD) architecture, the basic unit of complex AIC structures, to sample the bandlimited signals that can be well approximated through a small number of frequency tones. The advantage of RD is that it uses a low-rate ADC and thus it can be constructed from low power, readily available components. But when it is implemented in analog hardware, it suffers some non-idealities like filter imperfections [5], thermal noise, aperture jitter etc. The net effect of these non-idealities introduces noise to the signal, which emphasizes the need of a robust signal reconstruction process. Therefore, the use of random undersampling is expensive in practice.

Chaos based CS overcomes these limitations. The output of the chaos system behaves as random in just a few steps under specific initial and control conditions. Simple chaotic systems behave like a large size of random systems and hence it can be implemented with simple hardware. The chaotic CS is mostly suitable for frequency sparse signals in real time applications. Linh-Trung, et al. [6] introduced a deterministic CS method called chaos filter, utilizing chaotic systems for CS. L.Yu, et al. [7,8] constructed the measurement matrix with chaotic sequence derived from Logistic map and proved that this matrix is Toeplitz structured and satisfies RIP with high probability. Tent map can also be used to construct the sensing matrix and it gives similar performance to random matrix or Logistic chaotic matrix [9].

A tuning fork resonates at a specific constant pitch when set vibrating by hitting it against a surface. It produces a very pure tone with most of the vibrational energy at the fundamental frequency (pitch) and little at the multiples of fundamental frequency called harmonics (overtones). In acoustic musical instruments, the sound is generated by the vibration of strings or air. These vibrations appear on the fundamental frequency and on the harmonics. Hence, the musical signals consist of small number of time varying sinusoidal signals. For that reason, these signals are said to be sparse in the frequency domain and are suitable for the CS problem.

The main contribution of this paper is to present a chaotic compressive sampler based on Gaussian-Logistic map to sample the acoustic signals and to compare its performance with the random demodulator architecture. The paper is organized as follows. In section 2, we present an overview of compressed sensing and random demodulator system. Section 3 describes the chaotic sequences and the construction of chaotic matrix. The performance comparison between chaos based compressive sampler and random demodulator architecture for acoustic signals is given in section 4. At the end, the conclusion is given.

2. Theoretical Background

2.1. Compressed Sensing

Consider a real valued, finite length, discrete time signal $\mathbf{x}$ which can be represented as an *N* X *1* column vector i.e. $\mathbf{x} \in \mathbb{R}^N$. Define $\mathbf{\Psi}$ as an *N* X *N* orthonormal basis with the vectors $\{\psi_i\}_{i=1}^{N}$ as columns. The signal $\mathbf{x}$ can be expressed as

$$\mathbf{x} = \sum_{i=1}^{N} \alpha_i \, \psi_i \tag{1}$$

or $\mathbf{x} = \mathbf{\Psi\alpha}$ where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots., \alpha_N]^{\mathbf{T}}$ is the coefficient sequence. The signal $\mathbf{x}$ is said to be *K*-sparse in $\mathbf{\Psi}$ domain if it contains atmost *K* non-zero coefficients. The compressed vector $\mathbf{y}$ of dimension *M*X*1* where $M < N$ is computed as

$$\mathbf{y} = \mathbf{\Phi x} = \mathbf{\Phi\Psi\alpha} = \mathbf{\Theta\alpha} \tag{2}$$

where $\boldsymbol{\Theta} = \boldsymbol{\Phi\Psi}$ is an *M*X*N* matrix. $\boldsymbol{\Phi}$ is of *M*X*N* dimension and is called the measurement or the sensing matrix. The measurements of the signal are computed using the measurement matrix. The measurement matrix must obey the restricted isometry property (RIP) and thus finding a proper measurement matrix is a challenging problem. A matrix $\boldsymbol{\Phi}$ satisfies the RIP of order *K* if there exists a constant $\delta \in (0, 1)$, such that

$$(1-\delta)\|\mathbf{x}\|_2^2 \leq \|\boldsymbol{\Phi}\mathbf{x}\|_2^2 \leq (1+\delta)\|\mathbf{x}\|_2^2 \tag{3}$$

holds for all $\mathbf{x}$ belonging to the set of all *K*-sparse signals. When this property does not hold, it is possible for a *K*-sparse signal to be in the null space of $\boldsymbol{\Phi}$ and in that case it may be impossible to reconstruct these vectors.

To successfully recover the signal $\mathbf{x}$ from $\mathbf{y}$, the measurement matrix $\boldsymbol{\Phi}$ must be incoherent with the sparsifying matrix $\boldsymbol{\Psi}$ so that the sparsity information is maintained. The random matrices exhibit a very low coherence with any fixed representation $\boldsymbol{\Psi}$. Hence, the measurement matrices are constructed using the i.i.d. samples drawn from random distribution such as Gaussian, uniform, Bernoulli distribution. The CS recovery process search for the sparsest signal $\mathbf{x}$ that yields the measurement vector $\mathbf{y}$. The sparse representation problem thus can be posed as

$$\min\|\boldsymbol{\alpha}\|_1 : \boldsymbol{\Theta\alpha} = \mathbf{y} \tag{4}$$

where $\|.\|_1$ denotes the norm-1 operator. Using various sparse approximation methods, $\mathbf{x}$ (and hence $\boldsymbol{\alpha}$) can be faithfully reconstructed from $\mathbf{y}$. The signal recovery algorithms can be classified into convex optimization techniques (l_1-norm optimization based basis pursuit (BP) [10]), greedy algorithms (matching pursuit (MP), orthogonal matching pursuit (OMP) [11]) and hybrid methods (stage-wise OMP (StOMP)). OMP is an iterative greedy algorithm that solves a portion of the problem at each step. The advantages of OMP are simplicity and fast implementation. For ultra-sparse solution problems, the OMP algorithm is more powerful than convex optimization techniques.

2.2. Random Demodulator

The random demodulator (RD) is a new type of signal acquisition system used to acquire sparse, bandlimited and periodic signals. The block diagram for the system is shown in Fig.1. The random demodulator performs three operations: demodulation, low pass filtering and low rate sampling. The signal is demodulated by multiplying it with a pseudo-noise sequence which alternates between -1 and +1 at the Nyquist rate or higher. The spectrum of PN sequence is similar to that of white noise. When a pure tone is multiplied by this random sequence, the spectrum of the noise gets translated. The tone is smeared across the entire spectrum. It can be detected by a low rate sampler as it leaves a signature. When a frequency sparse signal is multiplied by the random sequence, a superposition of translates of the noise spectrum is obtained in the frequency domain with one translate for each tone. By examining a small part of the spectrum, each tone can be distinguished [4].

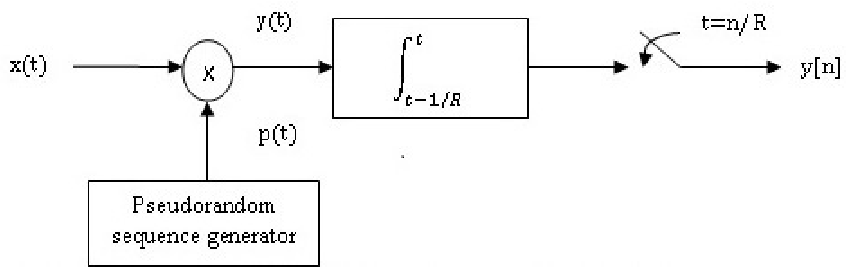


Fig. 1. Block diagram of random demodulator

The signal from the mixer is low pass filtered to prevent aliasing. The low pass filter is an accumulator that sums the demodulated signal for $1/R$ seconds, where R is the sampling rate much lower than the Nyquist rate. The filtered signal is then sampled using a low rate ADC. Hence, a sequence of compressed measurements $\{y_m\}$ is obtained through integrate-and- dump sampling approach [4].

$$\mathrm{y_m} = R\int_{m/R}^{(m+1)/R} \mathrm{y}(t)dt \qquad m = 0,1,\ldots.,R-1 \tag{5}$$

2.3. Non-idealities in Random Demodulator

The important component of random demodulator is the random waveform that changes polarity at a rate above the signal bandwidth. Given the nature of analog electronics, there is a hard bandwidth limit beyond which such waveforms cannot be generated without shape distortion [12]. The most important non-ideality in the pseudo-random number generator is that the clock may arrive before or after its expected time which is referred to as jitter effect. Clock jitter and non-linear distortion can be neglected since a relatively low rate ADC is used. But the component impairments cannot be neglected.

Due to factors such as supply voltage, ambient temperature variations, manufacturing process etc., the analog components in the random demodulator deviate from their ideal behaviour in practice. Thus the hardware devices such as filters do not behave as desired, which lead to measurement matrix perturbations. To measure the actual impulse response of each device and to change the measurement matrix accordingly is an impractical solution. CS reconstruction algorithms are sensitive to such mismatches and hence the measurement matrix needs to be calibrated [13]. Due to the unpredicted nature of these non-linearities, the matrix calibration is difficult. This calibration decreases the impact of these fluctuations only in limited cases.

3. Chaotic Compressed Sensing

Chaotic compressed sensing is a non-linear framework for compressed sensing. The chaotic dynamics have been successfully applied in various engineering fields such as communications, automatic control, signals processing and watermarking. For certain parameter values, the non-linear dynamical systems exhibit chaotic behaviour. Two chaotic trajectories starting from close initial conditions eventually diverge from each other, thus exhibiting exponential sensitivity to small changes in initial conditions. Hence, a large number of uncorrelated, random like, yet deterministic chaotic sequences can be generated by changing the initial values. Only a few parameters and functions are necessary to be memorized even for very long sequences. The hardware implementation of a deterministic system is simpler than that of a random system. Therefore, $\mathbf{\Phi}$ is made chaotic, which is generated by a deterministic system. The chaotic sequences are generated by chaotic maps and the most commonly used is Logistic map.

Consider the quadratic recurrence equation

$$x(n+1) = ax(n)[1 - x(n)] \tag{6}$$

where a is a positive constant known as the bifurcation parameter. Since chaotic sequences exhibit sensitive dependence on initial conditions, the value of a and $x(0)$ ensure the signal to be chaotic. The logistic sequence, generated by (6) with a = 4, $x(0)$ = 0.3, results in chaotic behaviour. To obtain a chaotic sequence that behaves Gaussian-like, transform the Logistic map using the logit transform as follows:

$$x_{GL}(n) = ln\,[x(n)/(1 - x(n))] \tag{7}$$

The resulting sequence is called Gaussian-Logistic map.

3.1. Chaotic Sensing Matrix

Since the hardware design using pure random sequences is difficult in practice, the chaotic sequences which exhibit similar properties to random sequences are employed to construct the measurement matrix, called the chaotic matrix. It generates the "pseudo-random" matrix in deterministic approach and satisfies the RIP with high probability. The chaotic sequence $\mathbf{c} = [c_0, c_1, \ldots, c_{MN-1}]$ sampled from the output sequence produced by a chaotic map, say Logistic map, is generated. An MXN matrix $\mathbf{\Phi}$ is constructed [9] taking M contiguous samples and entering them in a column of $\mathbf{\Phi}$.

$$\mathbf{\Phi} = \frac{1}{\sigma\sqrt{M}} \begin{pmatrix} c_0 & c_M & \cdots & c_{M(N-1)} \\ c_1 & c_{M+1} & \cdots & c_{M(N-1)+1} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M-1} & c_{2M-1} & \cdots & c_{MN-1} \end{pmatrix} \tag{8}$$

where σ^2 is the variance of the sequence. The chaotic matrix $\mathbf{\Phi} \in \mathbb{R}^{MXN}$ constructed following (8) satisfies RIP for constant $\delta > 0$ with overwhelming probability, providing that $K \leq O(M/\log(N/K))$ [7]. A Toeplitz-structured matrix with chaotic sequence can also be constructed which retains RIP with high probability [8].

4. Simulation results

For experiments with the acoustic signals, the tuning fork vibration with a single frequency component at 440 Hz is considered. The Nyquist rate of the signal is 880 Hz. The signal is compressively sampled using the chaotic sensing matrix derived from the Gaussian-Logistic map. The histogram of Gaussian-Logistic map resembles Gaussian probability density function and therefore RIP is satisfied. The reconstruction algorithm used is OMP algorithm. The reconstruction performance is analysed for different number of measurements. The reconstruction accuracy is quantified using the mean square error (MSE) between the original signal and the reconstructed signal. Fig.2. shows the time domain plot of the original signal and the reconstructed signal using 500 samples/sec. The corresponding frequency domain plot of the original signal and the reconstructed is signal shown in Fig.3. The comparison of MSE performance between chaotic compressive sampler and random demodulator for tuning fork vibration is shown in Table 1.

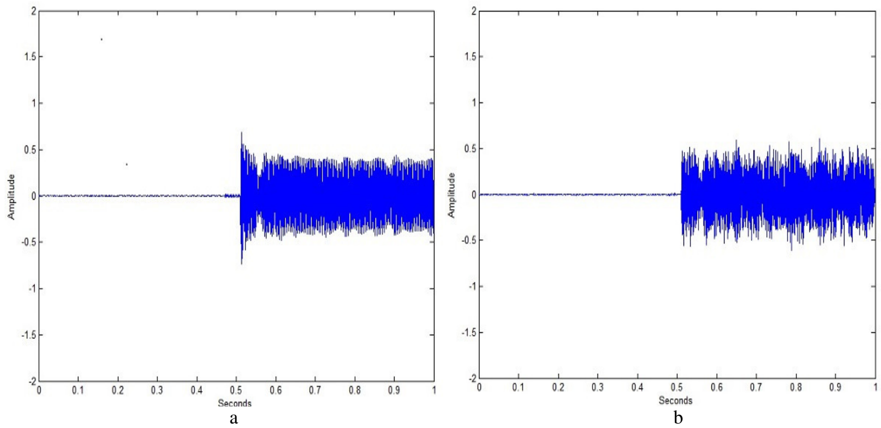
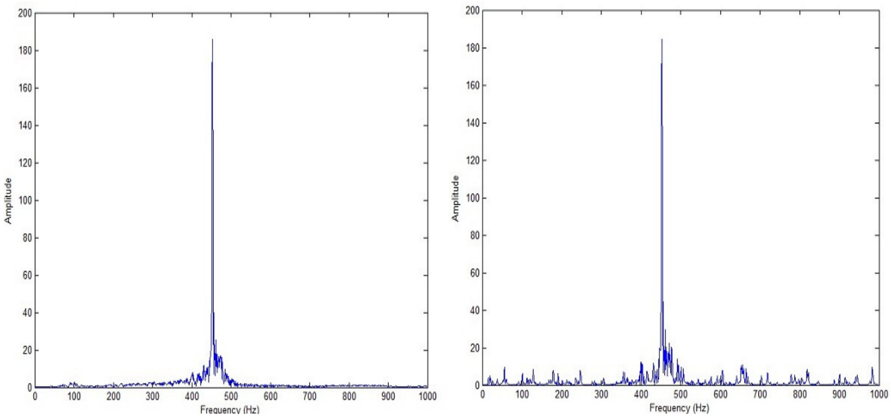


Fig.2. (a) The original tuning fork vibration; (b) The reconstructed tuning fork vibration using 500 samples/sec



a b

Fig. 3. (a) The frequency spectrum of the original signal; (b) The frequency spectrum of the reconstructed signal

Table 1. Comparison of MSE performance for tuning fork vibration

No. of compressive measurements	Chaotic CS	RD architecture
500	0.0031	0.0054
400	0.0044	0.0420
250	0.0052	0.0444
100	0.0058	0.0512

The piano signal with the length of 2000 samples for 1 second duration is chosen as another example of acoustic signal. The discrete cosine transform of the signal consists of a small number of non-zero coefficients, and signal can be considered as sparse in the frequency domain, i.e. the sparse signal domain is DCT domain. The highest significant frequency present in the test piano signal is found to be 330 Hz. According to Nyquist sampling theorem, the signal can be reconstructed using above 660 samples/sec. The signal is reconstructed from far fewer samples by applying chaotic CS theory. The time domain plot of the original piano signal and the reconstructed piano signal using 400 samples/sec is shown in Fig.4. and its frequency domain plot is shown in Fig.5.

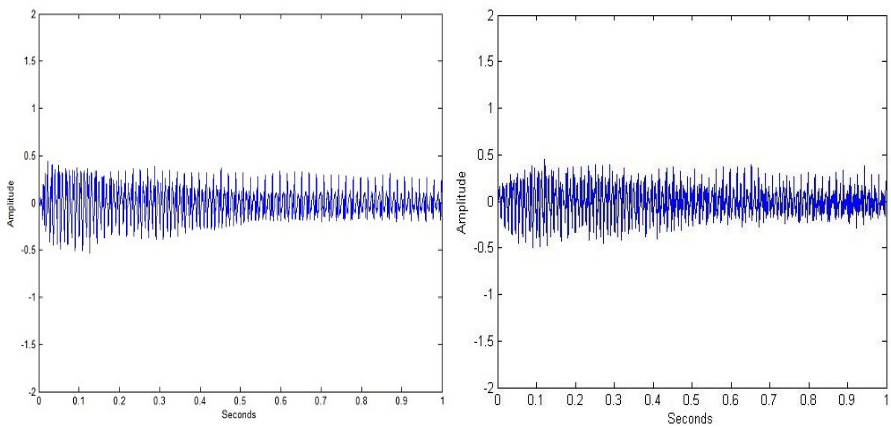


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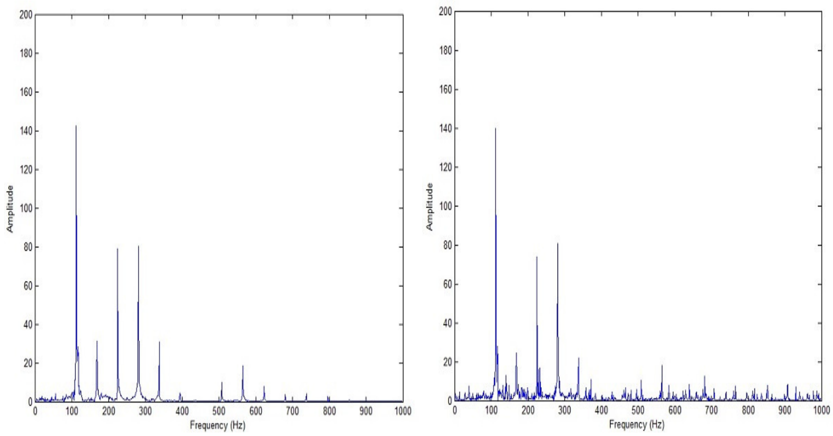
Fig.4. (a) The original piano signal; (b) The reconstructed piano signal using 400 samples/sec



a b

Fig.5. (a) The frequency spectrum of the original signal; (b) The frequency spectrum of the reconstructed signal

The variation of MSE for different number of chaotic compressed measurements for piano signal is observed and its comparison with the random demodulator architecture is shown in Table 2. The errors are shown to be of 10^{-3} order for chaotic compressive sampler, while for the random demodulator architecture it is of the order of 10^{-2}. Therefore, the chaotic compressive sampler shows better performance than the random demodulator architecture for acoustic signals.

Table 2. Comparison of MSE performance for piano signal

No. of compressive measurements	Chaotic CS	RD architecture
500	0.0026	0.0011
400	0.0036	0.0175
200	0.0066	0.0179
100	0.0107	0.0272

5. Conclusion

The acoustic signals which are sparse in the frequency domain are sampled at sub-Nyquist rate based on chaotic compressed sensing and successfully reconstructed from these chaotic measurements using orthogonal matching pursuit algorithm. The difference between the original signal and the reconstructed signal is calculated using mean square error for the chaotic compressive sampler and compared with that of the random demodulator architecture. It is found that the chaotic compressive sampler outperforms the random demodulator architecture for acoustic signals.

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