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Signal reconstruction of compressed sensing based on recurrent neural networks

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ABSTRACT

In this paper, neural network approach is addressed for signal reconstruction under the frame of compressed sensing. By introducing implicit variables, we convert the basis pursuit denoising model into a quadratic programming problem. Based on a class of generalized Fischer–Burmeister complementarity functions, we establish a neural network method for the signal reconstruction of compressed sensing. A projection neural network is also presented to recover the original signals. These two neural networks can be implemented using integrated circuits and two block diagrams of the neural networks are presented. Based on our proposed method, some potential applications of the compressed sensing are discussed.

1. Introduction

The signal processing ability of physical systems now faces to the challenge of the rapid development of information technology. The traditional signal processing technology is not suitable for such vast amounts of processing data. In recent years, Donoho, Candes, et al., proposed compressed sensing (CS) theory [1,2]. Compared with the usual measure which compresses the data after high rate sampling, the CS theory collects the sample data and compress those data simultaneously, which reduces the collecting period and the level of requirement on hardware. It breaks through the bottleneck of the Shannon sampling theorem, and becomes a hot research direction rapidly. The CS theory has wide applications such as various compression imaging, optical signal processing, hyperspectral image information processing, analog-to-digital conversion, biological computing, remote sensing and other fields [3,4,11].

There are various methods for compressed sensing, such as basis pursuit method [5,6], matching pursuit algorithm [7–9], iterative threshold algorithm [10,11], gradient projection algorithm [12], interior-point method [13] and so on. For 2D images, it is common to use the Gaussian i.i.d. matrix as the measurement matrix, that makes the matrix dimension pretty high and leads to great difficulty in storage and computation. Therefore, compressed sensing is not suitable for the real-time image processing. Gan proposed block compressed sensing, which greatly reduced the storage and the computational complexity [14]. Mun and Fowler developed the block compressed sensing with smooth projecting Landweber [15]. After that they proposed a multiscale algorithm of the block compressed sensing, measuring signals in wavelet domain [16]. Utilizing gray entropy, Wang et al. proposed an algorithm to describe the textural of images for the block compressed sensing [17]. Liu et al. presented an adaptive algorithm with support and signal value detection for compressed sensing [18]. Wang, Yang, Li gave an adaptive sampling method of compressed sensing based on texture feature [19]. The scheme classifies the image blocks in the light of textural, and then combines the statistical characteristic of the coefficient in wavelet domain to allot the measurements. From another way, we establish neural network approaches, which can make the compressed sensing possible for real-time processing.

In this paper, we present two neural networks for the compressed sensing signal reconstruction. Neural networks for optimization were first introduced in the 1980s by Hopfield and Tank [20,21]. Since then, significant research results have been achieved for various optimization problems [22–24]. The main idea of the neural-network approach is to construct a nonnegative energy function and establish a dynamic system that represents an artificial neural network. The dynamic system is usually in the form of first order ordinary differential equations. Furthermore, it is expected that the dynamic system will approach its static state (or an equilibrium point), which corresponds to the solution for the underlying optimization problem, starting from an initial point. In addition, neural networks are hardware-implementable. The neural network method can make the compressed sensing possible for real-time signal processing. So there are some potential applications in forecasting and intelligent information processing.

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The rest of the paper is organized as follows. In Section 2, we give some equivalent compressed sensing signal reconstruction models. In Section 3.1, we establish a neural network method based on a class of generalized Fischer–Burmeister complementarity functions. In Section 3.2, a projection neural network approach is designed to the compressed sensing signal reconstruction. Some potential applications are presented in Section 3.3. Section 4 concludes this paper.

2. Problem formulation under CS framework

Consider a length-n, real-valued signal x. In compressed sensing theory, the signal x to be acquired and subsequently reconstructed is typically assumed to be sparse or compressible in an orthogonal basis Ψ which provides a K-sparse representation of x; that is $x = \Psi\theta$. According to the CS theory, such a signal x can be acquired through the following random linear projection:

$$b = Ax = A\Psi\theta = \Theta\theta, \tag{1}$$

where b is the sampled vector with $m \ll n$ data points, A represents an $m \times n$ sensing matrix and $\Theta = A\Psi$.

To recover the signal x, the approach is to seek a solution of the l_0 minimization problem:

$$\min ||\theta||_0 \quad s.t. \quad b = \Theta\theta. \tag{2}$$

Obviously, the above minimization problem is a NP-hard problem. The solution of (2) is not unique, and we need to enumerate all possible θ that meet the condition. Fortunately, the above problem becomes computationally tractable if the sensing matrix A satisfies a restricted isometry property (RIP) which introduced by Candés and Tao in [25,26,29].

Since l_0-norm minimization problem is hard to solve, there are some other models constructed. Substituting l_1-norm with l_0-norm in (2), we get the following optimization problem:

$$\min ||\theta||_1 \quad s.t. \quad b = \Theta\theta. \tag{3}$$

Donoho et al. showed that problems (2) and (3) are equivalent [5,27,28]. l_1-norm minimization problem is very useful not only in compressed sensing, but also in some other applications.

For a more applicable situation, the measurements are corrupted with noise. We observe

$$b = \Theta\theta + n \tag{4}$$

where n is an unknown term. In this context, we propose two neural network methods reconstructing θ as the solution to the convex optimization problem:

$$\min ||\theta||_1 \quad s.t. \quad ||\Theta\theta - b||_2 \leqslant \lambda, \tag{5}$$

where λ is an upper bound on the size of the noisy contribution. By Lagrange multiplier scheme, the model (5) can be transformed into the following basic pursuit denoising model:

$$\min \frac{1}{2}||\Theta\theta - b||_2^2 + \tau||\theta||_1. \tag{6}$$

By introducing the positive part $a_+ = \max\{0, a\}$ and negative part $a_- = (-a)_+$ of the real number a, the problem is transformed equivalently to the following optimization problem:

$$\min \frac{1}{2}||\Theta(u - v) - b||_2^2 + \tau 1_n^T u + \tau 1_n^T v, \tag{7}$$

where $u = \theta_+, \quad v = \theta_-$. Through appropriate deformation, the above optimization problem can be equivalently converted into the following quadratic programming:

$$\min c^T z + \frac{1}{2} z^T B z$$
$$\text{s.t.} \quad z \geqslant 0$$

where $z = \begin{pmatrix} u \\ v \end{pmatrix}$, $d = \Theta^T b$, $c = \tau 1_{2n} + \begin{pmatrix} -d \\ d \end{pmatrix}$, $B = \begin{pmatrix} \Theta^T\Theta & -\Theta^T\Theta \\ -\Theta^T\Theta & \Theta^T\Theta \end{pmatrix}$. The KKT optimality conditions for the above quadratic programming are given by

$$\begin{cases} c + Bz - w = 0 \\ w^T z = 0 \\ w \geqslant 0, \quad z \geqslant 0 \end{cases} \tag{8}$$

It can also be written as

$$\begin{cases} (c + Bz)^T z = 0 \\ c + Bz \geqslant 0, \quad z \geqslant 0 \end{cases} \tag{9}$$

3. Neural network approach and some potential applications

In this section, we establish two neural network models. The first neural network model is developed based on complementarity functions. And the second one is to use the projection mapping. Using the neural network approach, we will explore some potential applications of the compressed sensing.

3.1. Neural network design with generalized Fischer–Burmeister function

It is known that the complementarity function approach can be used for solving system (8). Motivated by the approach, we propose a neural network model. A function $\phi : \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ is called a complementarity function if it satisfies

$$\phi(a, b) = 0 \quad \text{if and only if} \quad ab = 0, \quad a \geqslant 0, \quad b \geqslant 0.$$

Now we introduce a class of complementarity functions

$$\phi_\tau(a, b) = a + b - \sqrt{a^2 + b^2 + (\tau - 2)ab}, \quad \forall a, b \in \mathcal{R} \tag{10}$$

where $\tau \in [0, 4)$ is an arbitrary but fixed parameter. Then the system (8) is equivalent to the following system

$$G(z, w) = \begin{pmatrix} c + Bz - w \\ \phi_\tau(z_1, w_1) \\ \phi_\tau(z_2, w_2) \\ \dots \\ \phi_\tau(z_{2n}, w_{2n}) \end{pmatrix} = 0. \tag{11}$$

Construct a nonnegative energy function

$$\Psi(z, w) = \frac{1}{2}||G(z, w)||^2 = \frac{1}{2}||c + Bz - w||^2 + \frac{1}{2}\sum_{i=1}^{2n}||\phi_\tau(z_i, w_i)||^2 \tag{12}$$

So the quadratic programming can be equivalently converted into the following unconstrained minimization problem

$$\min \Psi(z, w). \tag{13}$$

Based on the above smooth minimization problem (13), it is natural to propose a neural network:

$$\begin{cases} \dfrac{dX(t)}{dt} = -\rho\nabla\Psi(X(t)) \\ X_0 = X(t_0) \end{cases} \tag{14}$$

where $X = \begin{pmatrix} z \\ w \end{pmatrix}$, $\rho > 0$ is a scaling factor which indicates the convergence rate of the neural network (14), and

$$\nabla\Psi(z,w) = \begin{pmatrix} B^T(c+Bz-w) \\ w \end{pmatrix} + \begin{pmatrix} \phi_\tau(z_1,w_1)\left(1 - \dfrac{2z_1+(\tau-2)w_1}{2\sqrt{z_1^2+w_1^2+(\tau-2)z_1w_1}}\right) \\ \cdots \\ \phi_\tau(z_{2n},w_{2n})\left(1 - \dfrac{2z_{2n}+(\tau-2)w_{2n}}{2\sqrt{z_{2n}^2+w_{2n}^2+(\tau-2)z_{2n}w_{2n}}}\right) \\ \phi_\tau(z_1,w_1)\left(1 - \dfrac{2w_1+(\tau-2)z_1}{2\sqrt{z_1^2+w_1^2+(\tau-2)z_1w_1}}\right) \\ \cdots \\ \phi_\tau(z_{2n},w_{2n})\left(1 - \dfrac{2w_{2n}+(\tau-2)z_{2n}}{2\sqrt{z_{2n}^2+w_{2n}^2+(\tau-2)z_{2n}w_{2n}}}\right) \end{pmatrix}. \quad (15)$$

The dynamic system (4) can be realized by a recurrent neural network as shown in Fig. 1. The circuit for the neural network realization requires $4n$ integrators, $2n$ processors for $\frac{\partial\phi_\tau}{\partial z_i}$, $2n$ processors for $\frac{\partial\phi_\tau}{\partial w_i}$, $2n$ processors for $\phi_\tau(z_i,w_i)$, $4n^2$ connection weights and some summers.

3.2. Projection neural network design

In this subsection, we propose another neural network associated with the projection function to solve system (9) for recovering the original signal. Let $P(\cdot)$ denote the projection function from $\mathcal{R}^{2n}$ to $\mathcal{R}^{2n}_+$. Then the system (9) can be equivalent to the following equation

$$z - P(z - c - Bz) = 0. \quad (16)$$

Based on the equivalent formulation in (16), we can construct the second neural network with the following dynamic equations

$$\begin{cases} \dfrac{dz(t)}{dt} = -\rho\,[z - P(z-c-Bz)] \\ z_0 = z(t_0) \end{cases} \quad (17)$$

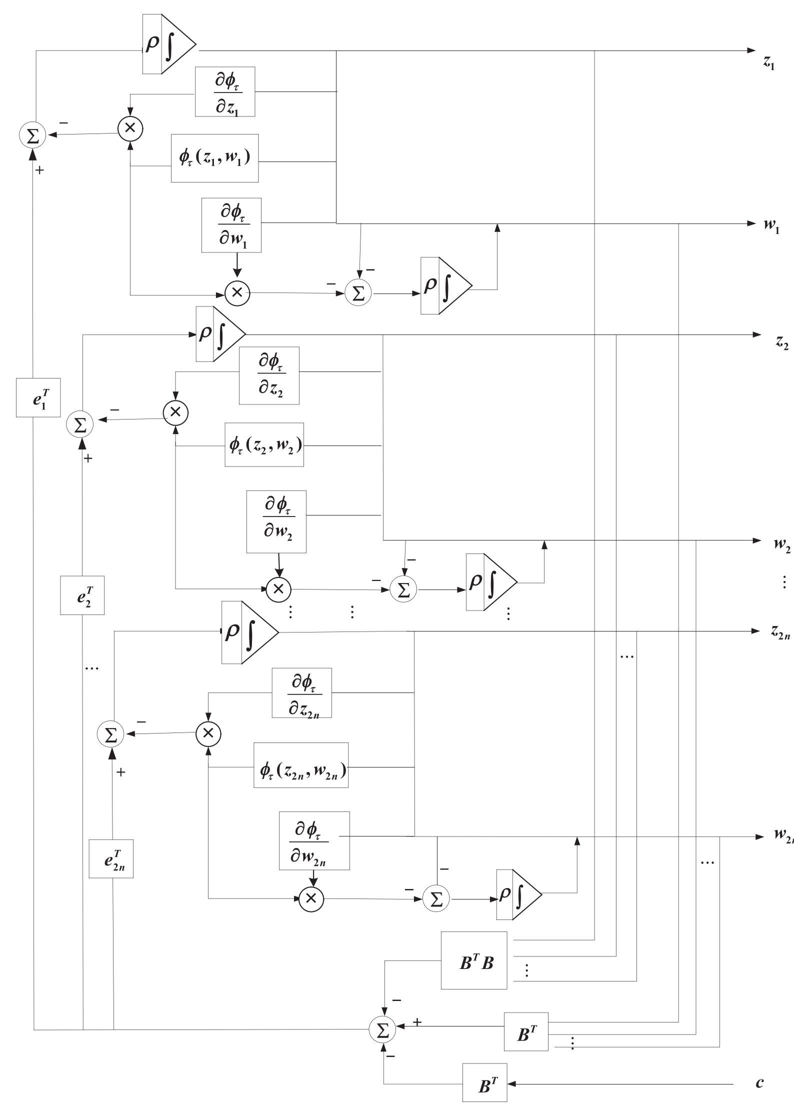

Fig. 1. Block diagram for the neural network with a class of generalized FB function.

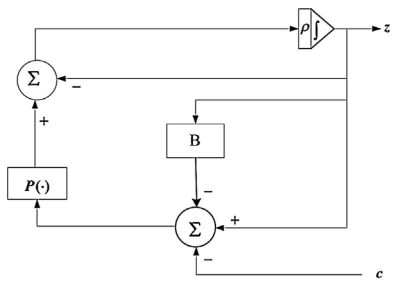

Fig. 2. A simplified block diagram for the projection neural network.

The solution of this first-order ordinary equation above is

$$z(t) = e^{-\rho(t-t_0)}z(t_0) + \rho e^{-\rho t}\int_{t_0}^{t} e^{\rho s}P(z-c-Bz)ds \qquad (18)$$

The dynamic system (17) can be realized by a recurrent neural network as shown in Fig. 2. The circuit for the neural network realization requires $2n$ integrators, $2n$ processors for the projection mapping $p(\cdot)$, $2n$ connection weights and some summers.

3.3. Potential applications

There are many applications of CS, including medical imaging, communications, analog-to-information conversion, geophysical data analysis, compressive radar, genetic screening, and among others. For example, CS can greatly reduce scan times and potentially increase resolution of magnetic resonance imaging [30]. CS microarrays combine group testing and CS principles to accurately identify genetic sequences, e.g., to detect pathogens in a water sample [31]. CS has also been used to image the rupture process of a main shock in an earthquake [32]. Many other applications are being developed including coding and information theory, machine learning, hyperspectral imaging, geophysical data analysis, computational biology, remote sensing, radar analysis, robotics and control, A/D conversion, and many more. All in all, CS is proving to be a powerful and flexible paradigm and will continue to be an exciting field of mathematical research that extends into many other disciplines.

Artificial neural network is a simple modeling and abstraction of a human brain. It has strong data compression capability. In addition, the neural network will also promote the image coding to the intelligence and knowledge transformation. And the strong fault tolerance and associative memory function of the neural network contribute to digital image processing. So the proposed neural network make the compressed sensing suitable for the real-time image processing, which can inspire some other potential applications of the compressed sensing. For example, the compressed sensing can be used for forecasting and intelligent information management: stock forecasting, earthquake prediction, securities management, borrowing and lending risk analysis, IC card management and traffic management, etc. The reconstruction method can be described as follows. First, the neural network is applied to nonlinear time series modeling and prediction, and the theoretical framework of CS is put forward. Second, Fourier transformation is treated for the tested signal, and then Fourier coefficients are obtained. Using orthogonal Gauss observation matrix to compress sampling and getting the observation vector. Then, establish the basis pursuit denoising model, and convert to a quadratic programming. Last, use the proposed neural network approach to recover the original signal.

4. Conclusion

In this paper, two neural network approaches are presented for signal reconstruction of compressed sensing. First, by introducing implicit variables, we convert the original signal reconstruction model into a quadratic programming problem. Second, we construct a nonnegative energy function based on a class of generalized Fischer–Burmeister complementarity functions. Then, the two neural network models for the signal reconstruction of compressed sensing are established. The two neural networks can be implemented using integrated circuits. Last, there are some potential applications of the compressed sensing presented. The neural network approach can make signal reconstruction of compressed sensing in real-time.

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