Compressive sampling on hologram phase maps

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Abstract

Compressive sampling using a random Gaussian measurement matrix is used to reconstruct hologram phase maps. From these phase maps, hologram images are reconstructed. The text in the resulting hologram images could only be seen from reconstructions from phase maps measured at 90% and above of the Nyquist sampling rate. The errors of the hologram image reconstructions of phase maps sampled below 90% were relatively the same despite decreasing error of the phase map reconstructions.

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1. Introduction

The Shannon/Nyquist theorem requires that when sampling signals, one must sample at a rate at least twice as fast as the signal's bandwidth. However, for certain signals, the Nyquist rate may be too high such that you may need too many samples [1]. Different sampling methods could be used to sample below the Nyquist rate. An example is through non-uniform sampling which is achieved through interpolation [2]. In addition, after data acquisition, transform coding is usually utilized. Transform coding exploits the fact that signals have a K-sparse representation in a fixed basis. An N×1 vector x can be expanded under an orthonormal basis ψ as follows:

$$x = \sum_{i=0}^{N-1} s_i \psi_i \text{ or } x = \psi s \tag{1}$$

where s is an N×1 long coefficient vector of x [1]. The key idea of transform coding is transforming a signal x into an appropriate basis, computing for the complete set of transform coefficients s, getting the K largest coefficients, and discarding the rest [3]. One can store or transmit only these K transform coefficients instead of the entire signal without much loss of information [4]. This process is inherently inefficient because the initial number of samples N may be large despite K being small and that the all transform coefficients are computed despite discarding all but K of them [1] .

Compressive sampling (CS) is a non-adaptive method that can address these inefficiencies. It can be used to bypass the sampling process to directly acquire a compressed signal. It is based on the idea that signals such as real-world images have sparse representations in some basis and that if an image is sampled by a largely incoherent measurement basis, then the signal can be reconstructed from fewer measurements via optimization procedures that search for the sparsest solution [5].

The sampling process is usually done by introducing a measurement matrix Φ to the signal x such that:

$$y = \Phi x \tag{2}$$

Different measurement matrices could be used, however, a standard choice is a Gaussian random matrix.

In order to recover the image x from the random measurements y, l1 optimization is implemented:

$$x = \arg\min \|x'\|_1 \text{ such that } \Phi x' = y \tag{3}$$

The l1 norm of a vector x is the sum of the absolute values of its elements:

$$\|x'\|_1 = \sum_{i=0}^{N-1} |x| \tag{4}$$

Optimization based on the l1 norm is used because it was found to recover K-sparse signals with high probability [1].

The most common applications of compressive sampling are for digital cameras, medical imaging and seismic imaging [6]. In [7], they applied compressive sensing techniques using a pseudo-inverse matrix and total variation minimization algorithms for hologram images.

In this paper, instead of directly using compressive sampling on the hologram images [7], we applied compressive sampling techniques to the hologram phase maps. From the phase maps (ϕ), the hologram image I could be reconstructed by:

$$I = \mathbf{F}[\exp(i\phi)] \tag{5}$$

where **F** is the Fourier transform.

The behavior of errors from the sampled phase maps to the reconstructed hologram images is also determined. Sampling from the phase maps, instead of directly from the images could have further applications in adaptive optics.

2. Methodology

Compressive sampling was done using an M×N random Gaussian measurement matrix, where N is the number of pixels of the image being sampled. A 96×96 pixel image of a hologram phase map was converted into a 9216×1 signal. For the same signal, different numbers of measurements M were taken. The values used were M = 1843, 4608, 5530, 6451, 7373, and 8294, 9216. Reconstruction was done by subjecting the measurements to l1 optimization subject to the constraints in eq. 3.

For each number of measurement, three trials were performed and their averages were taken. Their Normalized Root Mean Square Error (NRMSE) with respect to the original image was then computed. The NRMSE could be computed by:

$$NRMSE = \frac{RMSE}{X_{obs,max} - X_{obs,min}} \tag{6}$$

where

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(X_{obs,i} - X_{model,i})^2}{n}} \tag{7}$$

Here, X_{model} are values from the original image while X_{obs} are from the images reconstructed via CS.

Using eq. 4 and applying a circular aperture mask to the phase maps, reconstructions of the hologram images were also made for the different reconstructed phase maps. The NRMSE of these hologram image reconstructions with respect to the original hologram image was also computed.

3. Results and Discussion

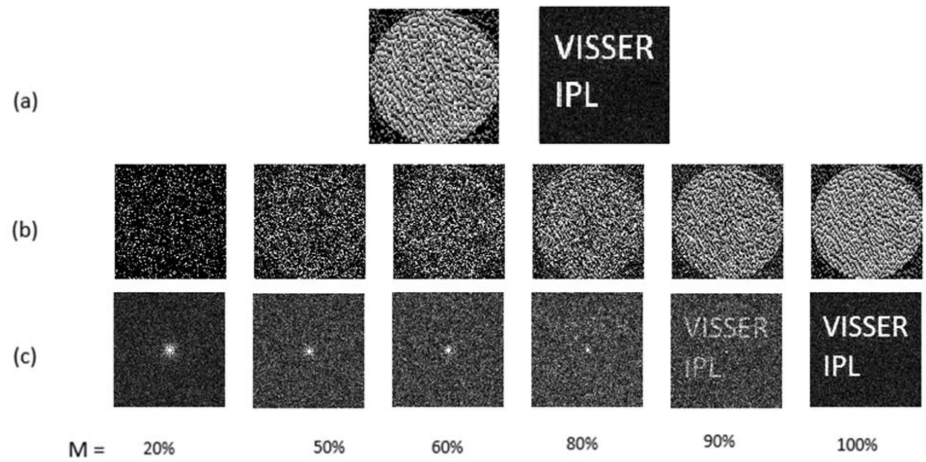

Figure 1. (a) Original 96 x 96 phase map and the reconstruction of its hologram image. (b) Reconstructions using CS with M = 1843 random measurements, M = 4608, M = 5530, M =7373, M = 8294 and M = 9216, respectively, from left to right. (c) Corresponding hologram reconstructions of the phase maps in (b).

Figure 1a shows the original phase map and the reconstructed hologram image when eq. 5 is applied to it. Figure 1b shows the reconstructions of the phase map using CS with varying number of measurements and Figure 1c shows their corresponding hologram reconstructions. For the hologram images, only phase maps sampled at M = 90% and 100% were able to completely show the text in the image, despite the accompanying noise.

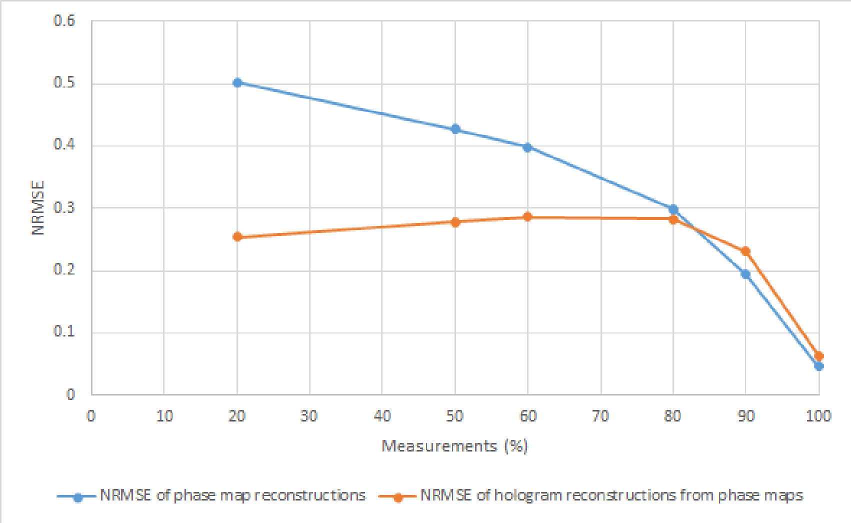

Figure 2. Error propagation of phase map reconstructions and their corresponding hologram reconstructions.

As the number of measurements taken increases, the NRMSE of the phase maps with respect to the original phase map decreases as shown in Figure 2. For the phase map reconstructions at M = 20, 50, 60, 80, 90 and 100%, the computed NRMSEs were 0.50, 0.427, 0.398, 0.300, 0.193, 0.046, respectively. When the hologram images were reconstructed from these phase maps, the NRMSE for each with respect to the hologram reconstruction shown in Figure 1a were 0.251, 0.270, 0.279, 0.229 and 0.063, respectively.

Despite having continuously decreasing NRMSE values for the reconstructed phase maps with measurements at 20, 50, 60, and 80%, their hologram reconstructions had relatively the same NRMSE with their standard deviations at 0.013. The noticeable drop-offs are at 90 and 100% where the text could be read in the hologram reconstructions.

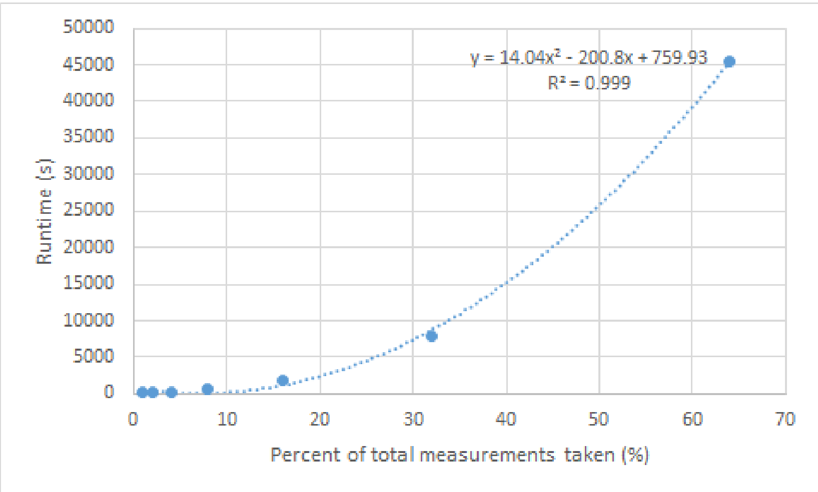

Figure 3. Computational complexity of applying l1 minimization to measurements of the signal.

The running time for minimizing the l1 norm for increasing number of measurements taken was found to be a polynomial of order 2 such that $y = 14.04\,x^2 - 200.8x + 759.93$. Here, y is the running time in seconds and x is the percentage of total measurements taken.

Getting the dot product for eq. 2 was found to be linear and the other parts of the code used had relatively constant run-times. Only seven values for number of measurements taken were tested due to the length of the running time. In addition, at measurements less than M = 20%, there are times when no image could be reconstructed.

The running time for M = 50% was approximately 1/6 that of M = 100%. We took the average of 6 different reconstructed images where M = 50% and found them to have an NRMSE of 0.356. The average of these six images were taken to minimize the image noise. This average was found to have an NRMSE of 0.253. For comparison, one trial for M = 100% had an NRMSE of 0.029.

3. Conclusion

This paper showed that sub-Nyquist sampling of hologram phase maps could be implemented using compressive sensing with a Gaussian random matrix and l1 minimization. For these reconstructions, increasing the number of measurements lowered the NRMSE of the reconstructed phase maps, however, the NRMSE of the corresponding hologram images were relatively the same from M = 20-80%. Only for phase maps where M = 90% and 100% were the text in the reconstructed hologram images visible.

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