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Principal component analysis algorithm in video compressed sensing

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ABSTRACT

An algorithm of principal component analysis in video compressed sensing is proposed in the paper. Aiming at the compressed sensing problems of video sequences, the inter-frame correlation among the images is analyzed and the transform coefficients with lower value are removed according to the energy concentration characteristics of principal component analysis. Therefore, the sparse realization of video signals in the form of principal component analysis is accomplished and the possibility of the transformation being used in compressed sensing algorithm is verified. Finally, simulation results show that, with the comparison of the traditional algorithm based on wavelet transform, the proposed algorithm can not only improve the reconstructed quality and the visual effects of the video sequence, but also save the sampling resources. Moreover, it is more suitable for stream transmission of multimedia.

1. Introduction

With the advent of information age, people has increasing demand on the amount of information, especially image, video and other multimedia information, which results in a high cost of sampling. On the other hand, after the sampling process, the signal will be compressed in order to reduce the cost of transmission and storage, and the process will cause the waste of sampling resources again. Thus, the codec technology of video signals in the field of sampling is an important issue to be solved in the encoding and transmission study of streaming media.

Recently, scholars in relevant area come up a new theory of compressed sensing (also referred to as CS for short) [1,2]. In traditional sampling theory – Shannon Theorem, the sampling frequency must be higher than twice the highest frequency in order not to make the signal distortion. Yet compressed sensing theory shows, when the signal is compressible or sparse, its compressed representation can be accessed to directly, which can omit the sampling of abundant useless information. For CS, assuming that signal can be exactly recovered from incomplete information through a random measurement process, the representation of signal in a form of small set of data can be acquired. Each measured value of original signal passed through measurement matrix is considered important or unimportant equivalently and loss of a few can still perfectly reconstructs the original signal, therefore, the method can effectively [3].

CS sampling is a statistical technique of data acquisition and estimation, mainly used in the sampling and compression field of sparse data. Because of the compressibility of video images in some transform domain and their fairly superior sparsity of residual, CS theory has important applications in video encoding [4]. At present, most of the papers for CS sampling studied in image compression field, while seldom pay much attention to operating against autologous characteristic of video sequence. Paper [5,6] proposed a video codec scheme based on compressed sensing, but they both used traditional wavelet basis to get the images sparse in the processing of the key frames. In this paper, principal component analysis algorithm for video compressed sensing is presented based on the inter-frame correlation among the video images. The algorithm can remove inter redundancy to a large extent to get better compression ratio and reconstruction quality.

2. Compressed sensing model

Compressed sensing theory was first proposed by Candès et al. in 2004. The main idea of the theory is that the signal can be observed in a lower frequency as long as it is sparse after some orthogonal transform. In this case, we can get the compression form of the original signal with the minimum number of measurements. Then the measurements help to reconstruct the original signal. At this point, the number of measurements is only determined by the characteristics of the signal, and not restricted by Nyquist frequency. Consequently, compressed sensing is suitable for sampling of the signal with high bandwidth.

Consider a discrete signal $x(n)$ $(n = 0,1,\ldots,N-1)$ of length N as an $N\times 1$ dimensional column vector in $\mathbb{R}^N$ space, denoted by X, which can be represented by a linear combination of a set of orthogonal basis $\left\{\psi_i\right\}_{i=1}^{N}$. Then $\Psi = \left[\psi_1, \psi_2 \cdots, \psi_N\right]$ is an $N \times N$ basis matrix consisted of column vector ψ_i. If X is sparse on this set of the basis,

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the transform coefficients $\Theta = \left[\theta_1, \theta_2, \cdots \theta_N\right]^T$ are equivalently or similarly K-sparse expressions under the basis Ψ through $\Theta = \Psi^T X$. At this point of view, there are K nonzero coefficients and $K \ll N$. We can write the signal X as a linear combination of basis Ψ:

$$X = \Psi\Theta = \sum_{i=1}^{N} \theta_i \psi_i \tag{1}$$

M measurements of $Y = [y_1, y_2, \ldots, y_M]^T$ can be observed. Since $M < N$, this is a dimension reduction process, expressed by linear projection as:

$$Y = \Phi X = \Phi\Psi\Theta \tag{2}$$

Φ is an $M \times N$ measurement matrix. The equation is underdetermined because the dimension M of the measurement Y is lower than that of the original signal with N. That is, we cannot get exact X when Y is known. However, Θ is K-sparse makes it possible to solve the problem with optimal ℓ_0 norm, so as to make sure we have the minimum number of nonzeros in Θ [7], which considers the problem

$$\min_{\Theta} \left\|\Theta\right\|_0 \quad \text{s.t.} \quad Y = \Phi\Psi\Theta \tag{3}$$

On purpose to reconstruct the signal accurately, Candès and Tao proposed and proved that the measurement matrix must satisfy the Restricted Isometry Property (RIP) [8]. That means, for any K-sparse signal x, we have

$$(1 - \delta_K)\|x\|_2^2 \le \left\|\Phi_T x\right\|_2^2 \le (1 + \delta_K)\|x\|_2^2 \tag{4}$$

where $0 < \delta_K < 1$, $\left|T\right| \le K, T \subset \left\{1, 2, \ldots N\right\}$. where $0 < \delta_K < 1 |T| \le K, T \subset \{1, 2, \ldots, N\}$. Φ_T is an $M \times |T|$ submatrix consisting of the related columns of Φ indexed by T.

Paper [9] points out that the equivalent condition for Restricted Isometry Property (RIP) is the observation operator and the basis matrix are irrelated. Since Gaussian random matrix is not related to most orthogonal basis matrices, it is chosen to be the measurement operator. Then irrelation condition can be satisfied well [10]. Otherwise, Rademacher matrix whose elements are all plus or minus 1 and local Fourier matrix can also meet the condition.

The reconstruction of the signal is to find the solution to the optimization problem of ℓ_0 norm which is an NP-hard problem. In such circumstances, we usually change to the consideration of optimization problem of ℓ_1 norm [11]. Then we have

$$\min_{\Theta} \left\|\Theta\right\|_1 \quad \text{s.t.} \quad Y = \Phi\Psi\Theta \tag{5}$$

At present, the main reconstruction algorithm includes basis pursuit (BP), matching pursuit (MP), orthogonal matching pursuit (OMP) and so on.

3. Principal component analysis of video sequences

Compression degree of the data depends on how the redundancy can be removed. And the redundancy is measured by means of correlation. Inter-frame image of the video sequence has a large temporal redundancy, also has a large correlation. Principal component analysis is built on the basis of statistical properties, also known as eigenvectors transform, K-L transform and Hotelling transform. It has outstanding advantage of good decorrelation. The transform determines its transformation matrix according to the statistical characteristics of the image (covariance matrix of the image). Therefore the signal correlation in transform domain can be removed entirely to make the image has the best matching effect. Stated thus, principal component analysis transform, known as the best transform based on minimum mean square error (MSE), occupies an important position in digital image compression technology.

For a single image in the video sequence, there is correlation between adjacent pixels because of intra-frame redundancy. Then principal component analysis can be adopted to get the transformation and compression of the whole image. In the meantime, adjacent frames in the video sequence also have correlation by reason of inter redundancy between natural images of the sequence. With the principle above, we can utilize principal component analysis in the transformation process of the entire video sequence.

Consider an I-frame video sequence composed of images of size $W \times L$ and the sequence can be represented as

$$squ = f_i(x, y) \tag{6}$$

$1 \le x \le L, 1 \le y \le W, 1 \le i \le I$.

Divide the images into blocks of size $n \times n$, and assume W, L are both integral multiples of n. So each image is divided into LW/n^2 blocks. The vector $\mathbf{X}_i^t$ can be generated by row stacking or column stacking of block image $f_i^t(x, y)$, which is the image of the tth block in the ith frame. Here we have

$$\mathbf{X}_i^t = (f_i^t(1,1), f_i^t(1,2) \cdots f_i^t(1,n), f_i^t(2,1), \cdots f_i^t(2,n) \cdots f_i^t(n,1) \cdots f_i^t(n,n))^T \tag{7}$$

where $1 \le t \le LW/n^2$, $1 \le i \le I$.

If the mean vector is defined as

$$\mathbf{m}_f = \mathrm{E}\left\{\mathbf{X}\right\} \tag{8}$$

and the covariance matrix of $\mathbf{X}$ is

$$\mathbf{C}_f = E\{(\mathbf{X} - \mathbf{m_f})(\mathbf{X} - \mathbf{m_f})^T\} \tag{9}$$

For the tth block image in the image sequence of I frames, its mean vector and covariance matrix can be written

$$\mathbf{m}_f^t = E\left\{\mathbf{X}\right\} = \frac{1}{I}\sum_{i=1}^{I} \mathbf{X}_i^t \tag{10}$$

$$\begin{aligned}\mathbf{C}_f^t = E\left\{(\mathbf{X} - \mathbf{m_f})(\mathbf{X} - \mathbf{m_f})^T\right\} &\approx \frac{1}{I}\sum_{i=1}^{I}(\mathbf{X}_i^t - \mathbf{m}_f^t)(\mathbf{X}_i^t - \mathbf{m}_f^t)^T \\ &\approx \frac{1}{I}\sum_{i=1}^{I}\mathbf{X}_i^t\mathbf{X}_i^{tT} - \mathbf{m}_f^t\mathbf{m}_f^{tT}\end{aligned} \tag{11}$$

where $\mathbf{m}_f^t$ is a vector of n^2 elements, $\mathbf{C}_f^t$ is a square matrix of n^2 order.

If $\lambda_i^t (i = 1, 2, \ldots n^2)$ are the eigenvalues of covariance matrix $\mathbf{C}_f^t$ in descending order, $\mathbf{e}_i^t = \left[\mathbf{e}_{i1}^t, \mathbf{e}_{i2}^t, \ldots, \mathbf{e}_{in^2}^t\right]^T (i = 1, 2, \ldots, n^2)$ are the corresponding eigenvectors of $\mathbf{C}_f^t$, then transformation matrix $\mathbf{A}^t$ of the image block is

$$\mathbf{A}^t = \begin{pmatrix} e_{11}^t & e_{12}^t & \cdots & e_{1n^2}^t \\ e_{21}^t & e_{22}^t & \cdots & e_{2n^2}^t \\ \vdots & \vdots & \ddots & \vdots \\ e_{n^2 1}^t & e_{n^2 2}^t & \cdots & e_{n^2 n^2}^t \end{pmatrix} \tag{12}$$

Centralize the image vector, which means calculation of the difference between the original image vector $\mathbf{X}^t$ and the mean vector $\mathbf{m}_f^t$, such that

$$\hat{\mathbf{X}}^t = \mathbf{X}^t - \mathbf{m}_f^t \tag{13}$$

Accordingly, the principal component analysis is

$$\mathbf{S} = (\mathbf{A}^t)^T \hat{\mathbf{X}}^t \tag{14}$$

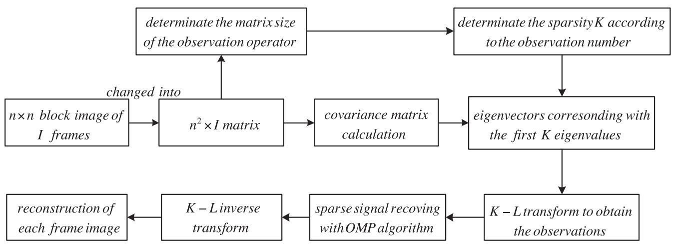

Fig. 1. Flow chart of the algorithm.

4. Design and implementation of principal component analysis algorithm in video compressed sensing

4.1. Sparsity determination of principal component analysis in video compressed sensing

Our goal is to apply principal component analysis to compressed sensing theory, and use less sampling points than traditional sampling methods to restore the original signal. In this case, we must firstly meet two principles, sparsity and irrelevance.

Since matrix $\mathbf{A}^t$ in (12) is the transformation kernel matrix consists of eigenvectors corresponding with the eigenvalues λ^t in descending order. The image energy is mainly concentrated on the coefficients whose corresponding eigenvalue is larger, thus $\mathbf{S}$ can be approximately represented by the first $K(K<n^2)$ principal components

$$\mathbf{S}_K = (\mathbf{A}_K^t)^T \hat{\mathbf{X}}^t \tag{15}$$

Take $\mathbf{S}_K$ as the sparse representation of the input signal of compressed sensing algorithm, and K as the sparsity. We adopt Gaussian random matrix Φ as the measurement matrix with measurement number M. If the elements of Gaussian matrix are independent identically distributed Gaussian random variables, its expectation is 0 and variance is $1/M$.

The estimation of RIP constants is proved below [12].

Theorem 1. *Assume $\Phi \in \mathbb{R}^{M\times N}$ is a random matrix matching centralization property [13], so there exists a constant only dependent on c_0, as long as*

$$M \geq C\delta^{-2}\left(K\log\left(\frac{N}{M}\right) + \log(\varepsilon^{-1})\right) \tag{16}$$

where $C>1$. The probability that the RIP constant of Φ satisfies $\delta_K \leq \delta$ exceeds $1-\varepsilon$, where ε is infinite. In consideration of this RIP estimation, (16) can be approximated as

$$M \geq CK\log\left(\frac{N}{M}\right) \tag{17}$$

According to formula (17), the relationship between the sparsity K and the measurement number M of the proposed algorithm in this paper is:

$$K \leq \frac{M}{C\log(n^2/M)} < \frac{M}{\log(n^2/M)} \tag{18}$$

where $C>1$, $n^2>M$.

4.2. Algorithm implementation

Given the analysis procedure above, Fig. 1 illustrates the overall diagram of principal component analysis algorithm in video compressed sensing developed in this paper. The reason we use principal component analysis to process the video signal is that principal component analysis has the best decorrelation of the corresponding pixels between adjacent frames. Energy concentration characteristic is utilized to remove the eigenvectors who contribute less to the variance, so as to implement the sparse representation of video data in compressed sensing algorithm.

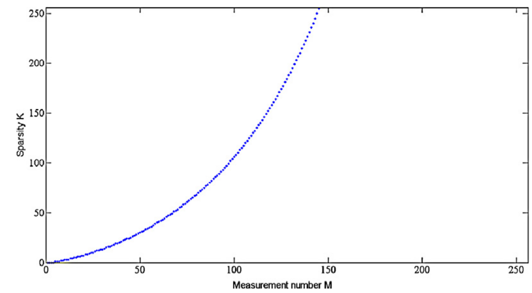

Fig. 2. Constraint relation between the sparsity and the measurement number.

Our proposed algorithm processes each frame image with block partition in order to prevent the covariance matrix being too large. On the other hand, it will facilitate the storage and real-time transmission of the data processed. With the correlation of block pixels at the same location being considered, the covariance matrix $\mathbf{C}_f^t$ in (11) is a real symmetric matrix. As a result, we can find an orthonormal eigenvector matrix that makes $(\mathbf{A}_K^t)^T = (\mathbf{A}_K^t)^{-1}$. If the measurement matrix $\Phi \in \mathbb{R}^{M\times n^2}$, $\hat{\mathbf{X}}^t \in \mathbb{R}^{n^2\times I}$, the observation result is:

$$y = \Phi\hat{\mathbf{X}}^t = \Phi\mathbf{A}_K^t\mathbf{S}_K \tag{19}$$

where $y \in \mathbb{R}^{M\times I}$.

Now the crux of video signal reconstruction is how to recover the sparse matrix $\mathbf{S}_K$,

$$\mathop{arg\,min}_{\mathbf{S}_K} \left\|\mathbf{S}_K\right\|_0 \quad \text{s.t.} \quad y = \Phi\mathbf{A}_K^t\mathbf{S}_K \tag{20}$$

OMP algorithm is applied to reconstruct the video signal, which means we determine which column in dictionary Φ participates in the measurement y, so as to determine the support of $\mathbf{S}_K$. At each iteration, we choose the column of Φ that is most strongly correlated with the remaining part of y. Then we subtract off its contribution to y and iterate on the residual. After each iteration, the atoms that have already been chosen are updated in order that residual is orthogonal to the subspace generated by the chosen atoms.

Assume the reconstructed data of OMP is $\tilde{\mathbf{S}}$. Then the block image recovered after the inverse transform is

$$\tilde{\mathbf{X}}^t = \mathbf{A}_K^t\tilde{\mathbf{S}} + \mathbf{m}_f^t. \tag{21}$$

5. Simulation results

For simple calculation, in the paper, the standard video sequence of size 352 × 288 is cropped into 256 × 256 images and only luminance component is concerned. If the block size is 16 × 16, the vector length of the image is 256 after row stacking or column stacking. From (18), we obtain the relationship between sparsity K and measurement number M

$$K < floor\left(\frac{M}{\log(256/M)}\right) \tag{22}$$

And the relationship curve is shown in Fig. 2. In the subsequent simulation, the sparsity will be identified by the measurement number for the sake of reconstruction requirement.

Select 5 frames of images with intense motion from different sequences and take peak signal noise ratio (PSNR) as the objective evaluation standard. The effects of reconstruction under different measurement numbers are presented in Table 1. We can see

Table 1
Reconstruction quality comparison of the video sequence.

Sequence name	Frames selected	Value of M	Average PSNR	DWT PSNR
CIF_foreman.yuv	11–15	50	27.3814	4.6396
		100	30.0379	25.0236
		190	34.2039	32.8838
CIF_paris.yuv	15–19	50	28.3967	7.2342
		100	31.1989	19.1932
		190	35.0091	24.9004
CIF_mother_daughter.yuv	58–62	50	32.7972	4.6594
		100	34.8478	26.8008
		190	38.1559	33.1162

Fig. 3. Comparison of subjective quality (a) original sequence, (b) reconstructed sequence when measurement number $M = 50$, (c) reconstructed sequence when measurement number $M = 100$, (d) reconstructed sequence when measurement number $M = 190$, (e) reconstructed sequence when measurement number $M = 190$ with DWT.

that, the reconstruction quality of the proposed algorithm outperforms that of the discrete wavelet transform (DWT) method. Especially when the number of measurements is small, we can still get better PSNR improvement. Consequently, the algorithm adoption can dramatically reduce sampling rates. The reconstruction quality remains at a good level and have small changes along with the measurement number varying.

Take sequence "CIF_foreman.yuv" as an example and compare the subjective quality of 12–14 frames of the reconstructed images in Fig. 3.

From Fig. 3 we can see that, compared with DWT, the proposed algorithm is still able to get better reconstruction results in the case of saving almost half of the measurement numbers. When M is small, though reconstruction quality of the foreground image is not so clear for its intense motion, the changing trend is reflected at a certain extent. At this time the reconstruction condition is not satisfied with DWT method to the contrary.

Still background image keeps good reconstruction quality in the process of the measurement number changing. Meanwhile, the image integrity is guaranteed with DWT method which means the whole image has the same but not good definition. At the same time, compression and reconstruction for the whole image result in a waste of calculation resources.

6. Conclusion

The paper puts forward a principal component analysis algorithm in video compressed sensing. After the simulation analysis of multiple video sequences under different measurement numbers, simulation results show that the proposed algorithm has the following advantages compared with the traditional algorithm based on DWT:

(1) The block algorithm is introduced in this paper. As a whole image, the covariance matrix of principal component analysis is huge, and extremely difficult to be achieved in practical simulation. Procession in blocks, on the one hand reduces the requirements of computing environment; on the other hand, can speed up processing, which is convenient for the storage and transmission of the output signal.
(2) In the consideration of the large inter redundancy of adjacent frames in the video sequence, the proposed algorithm fully analyzes the fundamental principle and the main properties of principal component analysis. We obtain the sparse representation of the video signal in principal component analysis, and combine it with compressed sensing. Under the same measurement conditions, better video reconstruction is implemented.
(3) Performance advantages of the algorithm are more obvious for smaller measurement numbers, even at some situation that the traditional DWT method can't recover the image satisfying the requirements of visual effects.
(4) The proposed algorithm is more suitable for streaming transmission scheme of mobile streaming media. There exists a mean architecture of multi-frame images in the video sequence during the principal component analysis. And for each frame, its difference to the mean value will be transformed. So that in practical media transmission, the mean value can be transmitted first to construct the approximation image at decoding end. Then construct the details of image through the received difference information. In this case, we can get higher efficiency and save more resources of transmission or storage.

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