ARTICLE IN PRESS

Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

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Compressive sampling for spectrally sparse signal recovery via one-bit random demodulator

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A R T I C L E I N F O

Article history:
Available online xxxx

Keywords:
Spectrally sparse signals
One-bit compressive sampling
Random demodulator (RD)
Binary iterative hard thresholding (BIHT)

A B S T R A C T

The one-bit compressive sampling (CS) framework aims at alleviating the quantization burden on analog-to-digital converters by quantizing each sample to one bit, i.e., capturing just the signs of samples. Motivated by one-bit CS theory, this paper addresses a new type of data acquisition system to recover spectrally sparse signals. This system is composed of a random demodulator and a one-bit quantizer. The former yields the signal compressed samples while the latter records the sign of each sample. With the observation sign data, the signal is eventually recovered by using the binary iterative hard thresholding algorithm. Through numerical experiments, we demonstrate that our scheme is high-efficient for spectrally sparse signal recovery in the situations of low signal-to-noise ratio, stringent bit budget and weak sparsity.

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1. Introduction

Shannon sampling theorem provides one of the foundations in modern signal processing. For a continuous real-valued time signal $f(t)$ whose highest frequency is not larger than $W/2$ hertz, the sampling theorem suggests that we sample the signal uniformly at a rate equal to W hertz. At a sampling rate of W, the values of the signal at intermediate points are determined completely by

$$f(t) = \sum_{n\in\mathbb{Z}} f(n/W)\mathrm{sinc}(Wt-n), \tag{1}$$

where $\mathbb{Z}$ denotes the set of integer and $\mathrm{sinc}(x) = \sin(x)/x$.

In the absence of extra information, Nyquist-rate sampling can completely recover $f(t)$. However, this well-known approach becomes impractical when the bandwidth W becomes very large because it is challenging for sampling hardware to operate at such a high rate. Even though recent developments in analog-to-digital converter (ADC) technologies have boosted sampling speeds, they cannot meet the requirements in many real-world applications, such as ultra-wideband communications and cognitive radar etc., in which the signal bandwidth might be larger than 10 GHz. On the other hand, the high power consumption is another important restriction, which prohibits the ADC from many wide-band applications [1].

Fortunately, in many implementations, the signals are spectrally sparse, that is, they can be approximated as harmonic tones. Inspired by compressive sampling (CS) theory [2,3], Tropp et al. [4] have designed a new type of sampling system, called random demodulator (RD), which exploits the spectral sparsity. It is shown in [4] that the RD requires just $O(K\log(W/K))$ samples per second to reliably reconstruct the signal, where K is the number of significant frequency tones.

In practice, the samples must be quantized, i.e., each sample is mapped from a continuous value with infinite precision to a discrete value with some finite precision. In [5,6], X. Gu et al. have analyzed the recovery performance of quantized compressed sensing (QCS). The most extreme form of quantization is reducing the signal to one-bit for each sample, which may be accomplished by repeatedly comparing the signal to some reference level, and recording whether the signal is above or below it [7,8]. In [9], Jacques et al. have bridged the one-bit and high-resolution quantized CS theories.

One-bit CS theory extends the CS framework to the extreme quantization, and it shows that some signal parameters can be recovered with high accuracy from samples of which each is quantized to just one bit [10–20]. The one-bit CS theory is of two important advantages in practical implementations. First, simple one-bit hardware quantizers consist of only a comparator and can operate at high speeds. Thus, we can reduce the sampling complexity by reducing the bit depth, rather than decreasing the number of measurements [21]. Second, because the one-bit encoding is

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https://doi.org/10.1016/j.dsp.2018.04.014
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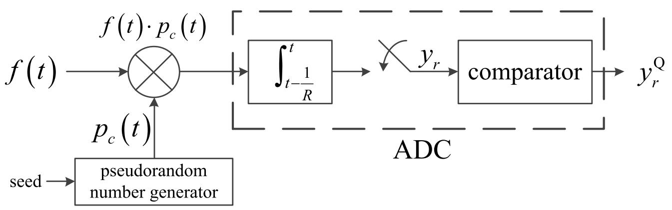

Fig. 1. Block diagram for the compressive sampling via one-bit random demodulator. The components include a random number generator, a mixer, an accumulator, a sampler, and a comparator.

invariant under many kinds of nonlinearities, the one-bit CS techniques is robust against gross sampling nonlinearities.

This work addresses a new type of data acquisition system to recover spectrally sparse signals based on one-bit random demodulator. In particular, the spectrum of spectrally sparse signal is firstly spread uniformly over a greater bandwidth by mixing $f(t)$ with a fast random chipping sequence, i.e., by convolving the sparse spectrum of $f(t)$ with a wide spectrum of the random chipping sequence. This randomization guarantees the excellent performance of the sensing matrix [22]. Secondly, the randomized signals are lowpass filtered and sampled by an integrate-and-dump sampler. Once again, the samples are quantized using the comparator to record the signs. Finally, the sparse bandlimited signals are recovered by using the binary iterative hard thresholding (BIHT) algorithm. It is important to note that the one-bit sampling rate of our scheme could be much faster than W Hertz. Our results show that the proposed scheme outperforms the state-of-the-art RD approaches in the situations of low signal-to-noise ratio (SNR), stringent bit budget and weak sparsity.

The organization of this paper is as follows. Section 2 presents a signal model for the class of spectrally sparse signals and describes the architecture of the random demodulator with one-bit CS. In Section 3, we write the one-bit random demodulator in a matrix form. Section 4 demonstrates the performance of our frame and compares it with previous methods. Conclusions are drawn in Section 5.

2. Problem formulation

2.1. Signal model

Let $f(t)$ be a continuous-time signal with the highest frequency $W/2$. Moreover, let K represent the number of tones in $f(t)$. Then $f(t)$ can be expressed as

$$f(t)=\sum_{\omega\in\Omega} a_\omega e^{j2\pi\omega t}, \quad \text{for } t\in[0,1), \tag{2}$$

where $j=\sqrt{-1}$, Ω is a set of K integer-valued frequencies satisfying $\Omega\subset\{0,\pm1,\cdots,\pm(W/2-1),-W/2\}$ and $\{a_\omega:\omega\in\Omega\}$ is a set of complex-valued amplitudes. In this paper, we consider the situation where the number of tones K is much smaller than W due to the spectral sparsity of the signals.

2.2. One-bit random demodulator

Fig. 1 displays a block diagram for the one-bit random demodulator, where the pseudorandom number generator and mixer implement the demodulation process. The pseudorandom number generator produces a discrete-time sequence $\varepsilon_0,\varepsilon_1,\varepsilon_2,\cdots$, taking values $\{+1,-1\}$ with equal probability. We refer to this random sequence as a chipping sequence, which is used to create a continuous-time demodulation signal $p_c(t)$, given as

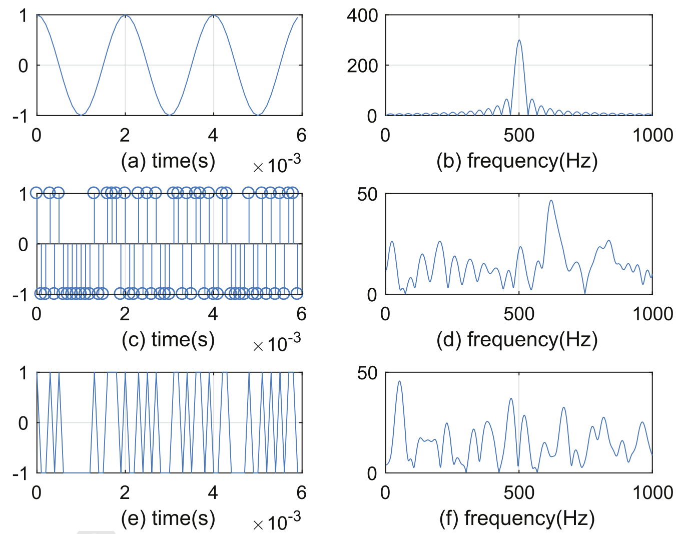

Fig. 2. Action of the demodulator on a pure tone. The demodulation process multiplies the continuous-time input signal by a random chipping sequence. (a) is the continuous-time pure tone signal with frequency 500 Hz, (b) is its spectrum, (c) and (d) are the random chipping sequence and its spectrum, (e) and (f) are the demodulated signal and its spectrum. This figure show that the random demodulator smears the pure tone across the entire spectrum.

$$p_c(t)=\varepsilon_n,\ t\in\Big[\frac{n}{W_s},\frac{n+1}{W_s}\Big),\ n=0,1,2,\cdots,W_s-1. \tag{3}$$

The demodulation signal switches between the levels ±1 randomly at a rate of W_s hertz and $W_s=\alpha W(\alpha\geq1)$.

Next, the mixer multiplies the continuous-time input $f(t)$ by the demodulation signal $p_c(t)$ to obtain a continuous-time demodulated signal $y(t)$, namely

$$y(t)=f(t)\cdot p_c(t), \quad t\in[0,1). \tag{4}$$

This mixing operation will expand the spectrum of the spectrally sparse signal to cover the entire band of width W_s. Fig. 2 describes the intuition behind the design. In summary, we demodulate the pure tone signal by multiplying it with a high-rate random chipping sequence, which smears the tones across the entire spectrum. The random demodulator is a foundation for low-rate sampling because the demodulator expands the tones to cover the entire spectrum.

The next three components within the dash line box behave like a standard one-bit ADC, which performs lowpass filtering to prevent aliasing, samples the signal and quantizes the samples using a comparator. Here, the lowpass filter is simply an accumulator that sums up the demodulated signal $y(t)$ for $1/R$ seconds, where $\beta\in\mathbb{N}$ and $\beta=W_s/R$. The filtered signal is sampled instantaneously every $1/R$ seconds to obtain a sequence of measurements $\{y_r\}$, which is expressed as

$$y_r=R\int_{r/R}^{(r+1)/R} y(t)dt, \quad r=0,1,2,\cdots,R-1. \tag{5}$$

This approach is called the integrate-and-dump sampling.

Finally, the samples are quantized using the comparator to record the sign:

$$y_r^{Q}=\text{sign}(y_r), \quad r=0,1,2,\cdots,R-1. \tag{6}$$

Compared with the random demodulator in [4], the proposed one-bit random demodulator has three major advantages:

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- Power efficiency. In [4], the samples are quantized to infinite precision. However, only the signs of the samples are recorded in our one-bit random demodulation. This reduces the ADC power consumption.
- Spectrum spreading. We expand the bandwidth W of $f(t)$ to $\alpha W (\alpha \geq 1)$ by using the mixer. This allows us to collect sufficient information about the original signal even with one-bit sampling.
- Increased sampling rate but decreased bits per sample. In the devised one-bit RD, the sampling rate R may exceed the bandwidth W of $f(t)$. The total number of bits after sampling can be much smaller than the existing RD method because of the one-bit quantization.

3. Signal recovery via one-bit compressive sampling technique

In this section, the spectrally sparse signal is recovered by using the one-bit CS technique. More specifically, we first express the one-bit random demodulator in terms of a matrix form.

3.1. Discrete-time representation

Recall that, in the proposed one-bit CS scheme depicted in Fig. 1, each $(1/W_s)$-second block of the signal is multiplied by a random sign. Subsequently these blocks are aggregated, summed, sampled and quantized. Therefore, we can average the input signal over blocks of duration $1/W_s$ without affecting the subsequent steps.

Consider a time instant $t_n = n/W_s$ for an integer n. Let x_n denote the average value of the signal $f(t)$ over a time interval of duration $1/W_s$ starting at t_n. Thus, we have

$$x_n = W_s \int_{t_n}^{t_n+1/W_s} f(t)dt = \sum_{\omega\in\Omega} a_\omega \Big[\frac{W_s\big(e^{j2\pi\omega/W_s}-1\big)}{j2\pi\omega}\Big] e^{j2\pi\omega t_n}. \tag{7}$$

Define the attenuation matrix of amplitude coefficients as

$$\mathbf{C} = \mathrm{diag}\left\{\left[\frac{\big(e^{j2\pi\omega/W_s}-1\big)W_s}{j2\pi\omega}\right]_\omega\right\} \in \mathbb{C}^{W\times W} \tag{8}$$

where $\omega = -W/2, -W/2+1, \cdots, -1, 0, 1, \cdots, W/2-1$.

It is important to note that the matrix $\mathbf{C}$ defined in (8) results in a nonlinear attenuation of the amplitude coefficients. However, the nonlinear effects can be alleviated considerably as α increases. This is an additional advantage of one-bit RD.

Define the $W_s \times W$ Fourier matrix as

$$\mathbf{F} = \frac{1}{\sqrt{W_s}}\big[e^{j2\pi n\omega/W_s}\big]_{n,\omega} \tag{9}$$

where $n = 0, 1, 2, \cdots, W_s - 1$. Therefore, it follows from (7) that

$$\mathbf{x} = \mathbf{FCa} \tag{10}$$

where $\mathbf{a} = [a_{-W/2}, \cdots, a_0, \cdots, a_{W/2-1}]^{\mathrm{T}}$ and $\mathbf{x} = [x_0, x_1, \cdots, x_{W_s-1}]^{\mathrm{T}}$.

The random demodulation performs $\mathbf{x} \mapsto \mathbf{Dx}$, where

$$\mathbf{D} = \mathrm{diag}\big([\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_{W_s-1}]\big) \in \mathbb{C}^{W_s\times W_s}. \tag{11}$$

Let us now consider the discrete expression of the accumulator integrate-and-dump sampler. Define the following $R \times W_s$ block-diagonal matrix

$$\mathbf{H} = \mathbf{I}_R \otimes \mathbf{1}_{1\times\beta} \tag{12}$$

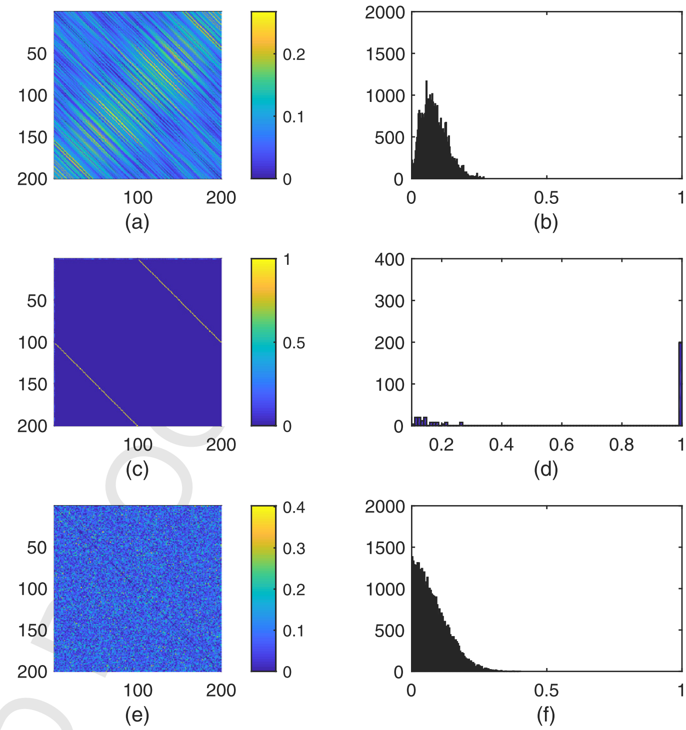

Fig. 3. Mutual Coherence (MC) of the sensing matrix. Each pixel of left figure denotes the normalized correlation coefficient of corresponding atoms, and the right figure denotes the coefficients distribution. The first row ((a) and (b)) is the MC with random demodulator, the middle row ((c) and (d)) is the MC with conventional approach, and the last row ((e) and (f)) is the MC of Gaussian random matrix.

where $\otimes$ denotes the Kronecker product. Then, we obtain the measurements vector as

$$\mathbf{y} = \mathbf{\Phi a} \tag{13}$$

where $\mathbf{y} = [y_0, y_1, \cdots, y_{R-1}]^{\mathrm{T}}$ and

$$\mathbf{\Phi} = \mathbf{HDFC} \tag{14}$$

is the $R \times W$ sensing matrix.

3.2. Performance of sensing matrix

In terms of theory, it is easy to verify the two basic conditions that the sensing matrix $\mathbf{\Phi}$ satisfies the restricted isometry property (RIP). First, the rows of sensing matrix $\mathbf{\Phi}$ are statistically independent due to the randomness of chipping sequence. Second, its Gram matrix $\boldsymbol{G} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}$ averages to the identity matrix. Actually, it has been revealed in [4, Theorem 16] that the sensing matrix has the RIP with high probability for $R \geq \mathcal{O}(K\log^6(W))$. Moreover, L. Jacques et al. [17] have proven that the reconstructed error is $\epsilon = \mathcal{O}\big(\sqrt{\frac{K}{R}\log\frac{RW}{K}}\big)$ when the sensing matrix satisfies the binary ϵ-stable embedding (BϵSE) property, and $\mathbf{\Phi}$ also satisfies this property as R becomes larger.

In term of application practice, the RIP of the sensing matrix is very hard to estimate. So the mutual coherence (MC) is applied to evaluate the performance of sensing matrix in practice [2]. In short, the MC of a sensing matrix is the normalized Gram matrix without diagonal elements. Fig. 3 describes the MC of three sensing matrix including random demodulator, conventional approach and Gaussian random matrix. Obviously, the conventional approach is not suitable for sparse sampling because the under-sampling causes aliasing. Moreover, the random demodulator outperforms the Gauss random matrix in the case of reduce the strong correlation coefficients.

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3.3. One-bit signal recovery algorithms

In practice, hardware systems may be inaccurate when taking measurements, due to the additive noise. In this paper, we take into account additive Gaussian noise on the measurements prior to quantization, i.e.,

$$\mathbf{y}_{1\text{bit}} = \text{sign}(\mathbf{y}_e) = \text{sign}(\boldsymbol{\Phi}\mathbf{a} + \text{e}) \tag{15}$$

where $\mathbf{e} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2 \mathbf{I}_R)$ with σ^2 being the noise variance and $\mathbf{I}_R$ being the $R \times R$ identity matrix, and the sign operator is applied to the real and imagery parts, respectively.

In the one-bit CS framework, the goal is to recover the sparse vector $\mathbf{a}$ from the one-bit measurements $\mathbf{y}_{1\text{bit}}$. The one-bit CS framework was first studied by Boufounos and Baraniuk [10], in which renormalized fixed point iteration (RFPI) algorithm has been devised. Later on, many improved algorithms have been developed, including matching sign pursuit (MSP) [14], restricted-step shrinkage (RSS) [15], and BIHT [17]. It is demonstrated in [17] that the BIHT performs better than the previous algorithms and is robust to sign flips. Therefore, BIHT is used in our scheme.

The BIHT algorithm can be thought of as trying to solve the problem

$$\hat{\mathbf{a}} = \underset{\mathbf{a}}{\text{argmin}} \, \|[\mathbf{y}_{1\text{bit}} \circ (\boldsymbol{\Phi}\mathbf{a})]_-\|_1 \text{s.t.} \|\mathbf{a}\|_0 \le K, \, \|\mathbf{a}\|_2 = 1, \tag{16}$$

where $[\cdot]_-$ denotes the negative function, i.e., $([\mathbf{a}]_-)_i = [a_i]_-$ with $[a_i]_- = a_i$ if $a_i < 0$ and 0 else, and $\circ$ denotes the Hadamard product for two vectors. In this algorithm, we need to know K a prior to computes the best K-term approximation in each iteration, as is similar to the existing works [14].

Observe that minimizing the one-sided ℓ_1 objective enforces a positivity requirement

$$\mathbf{y}_{1\text{bit}} \odot (\boldsymbol{\Phi}\mathbf{a}) \succeq \mathbf{0} \tag{17}$$

which, when satisfied, implies consistency of signs.

4. Numerical results

4.1. Signal recovery

In this section, we will show the proposed scheme is feasible for spectrally sparse signal recovery. Fig. 4 shows the results of spectrally sparse signal recovery. In this experiments, the number of tones is $K = 10$, the length of signal is $W = 100$ and the signal-to-noise ratio (SNR) is 10 dB. Except amplitude error, the proposed scheme can estimate the position and numbers of tones accurately.

Moreover, the proposed scheme is developed based on random demodulator. So we compare our method with traditional random demodulator, which is shown in Fig. 5. This experiment examines the reconstruction performances of the proposed 1-bit scheme and the traditional approach varying with B_T/W, which stands for the bit-depth. The input SNR increased from 4 dB, 10 dB to 16 dB. It is seen that the proposed 1-bit scheme significantly suppresses the RD approach, particularly for low SNR and/or low bit depth (i.e. $B_T/W \le 2$) situations.

4.2. Bits budget reducing

In this section, we will demonstrate the superiority of the proposed one-bit CS scheme over the classical CS scheme [4] and QCS scheme [5] [6] [9] in the bit budget. For the classical CS approach, the ℓ_1-regularized least squares (ℓ_1-LS or Lasso) algorithm [23] is used to recover spectrally sparse signals, and for the QCS method, the quantized iterative hard thresholding (QIHT) algorithm [9] is used.

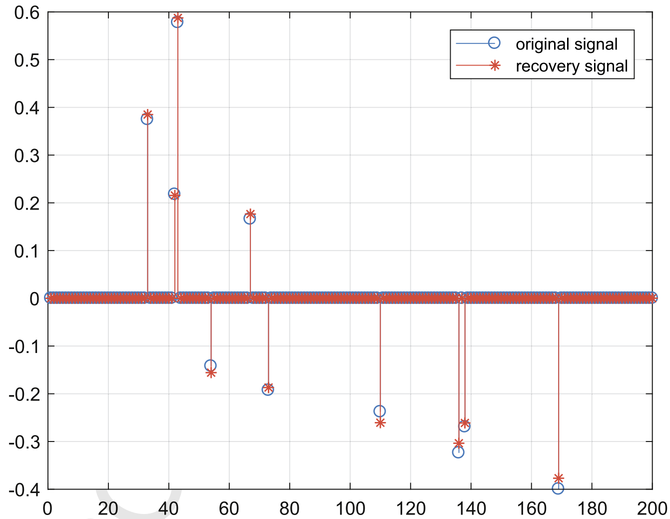

Fig. 4. Spectrally sparse signal recovery, $K = 10$, $W = 100$, SNR = 10 dB. Except amplitude error, the proposed scheme can estimate the position and numbers of tones accurately.

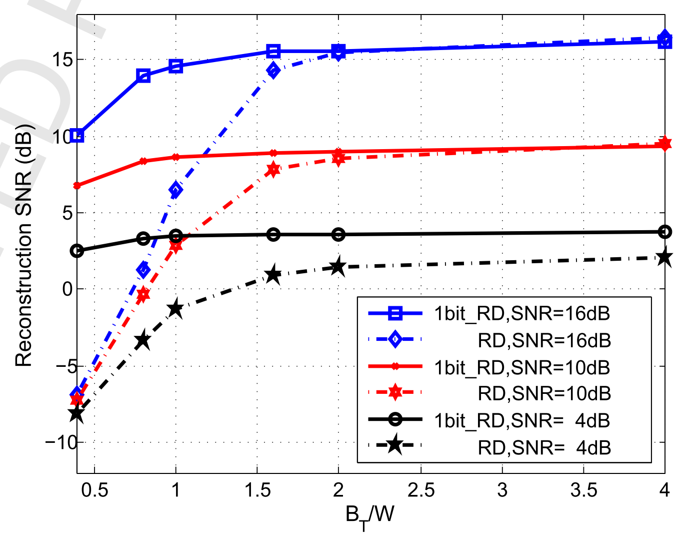

Fig. 5. Performance comparison of 1 bit RD and RD with different bit-depth, here input SNR increased from 4 dB, 10 dB to 16 dB.

For the classical CS, each sample is quantized using 4 bits, which is a medium precision quantization. Let B_{T} denote the bit budget of random demodulator. Then the number of measurements is $R = B_{\text{T}}/4$ for the classical CS scheme.[1] In contrast, the number of measurements for the proposed one-bit CS approach is $R = B_{\text{T}}$. For the QCS scheme, each sample is quantized by 2, 3, and 4 bits. Accordingly, the number of measurements R varies from $B_{\text{T}}/2$, $B_{\text{T}}/3$ to $B_{\text{T}}/4$ respectively.

The experimental setup is as follows: the locations of non-zero frequency components are uniformly distributed, and the amplitudes of coefficients are drawn from a Gaussian distribution with zero mean and unitary variance. Moreover, since the scale of the signal is lost during the quantization process, we can restrict the sparse signals to be on the unit hyper-sphere, i.e., $\|\mathbf{a}\|_2 = 1$. Each reconstruction in this setup is repeated for 1000 trials with $W = 100$ and $K = 20$ unless otherwise noted.

[1] For complex-valued signals, the quantizer is applied to both the real and imagery parts, and have the total number of bits are twice as that of the real-valued signal case.

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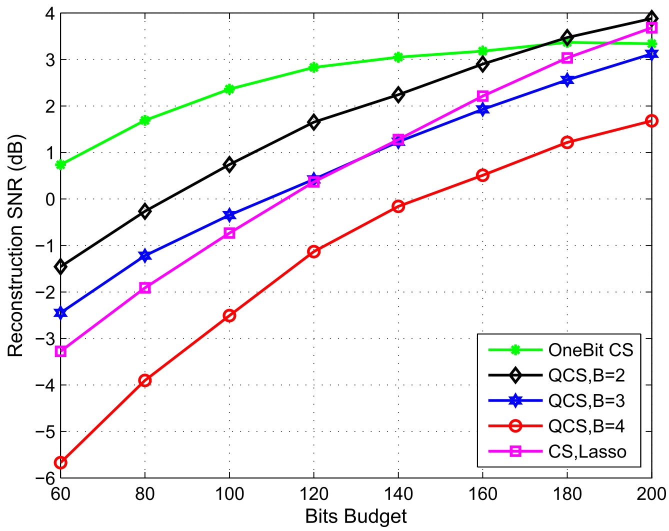

Fig. 6. Performance comparison of one-bit CS, QCS and classical CS schemes with bit budget varies from 20 to 200 bits, here input SNR is 5 dB.

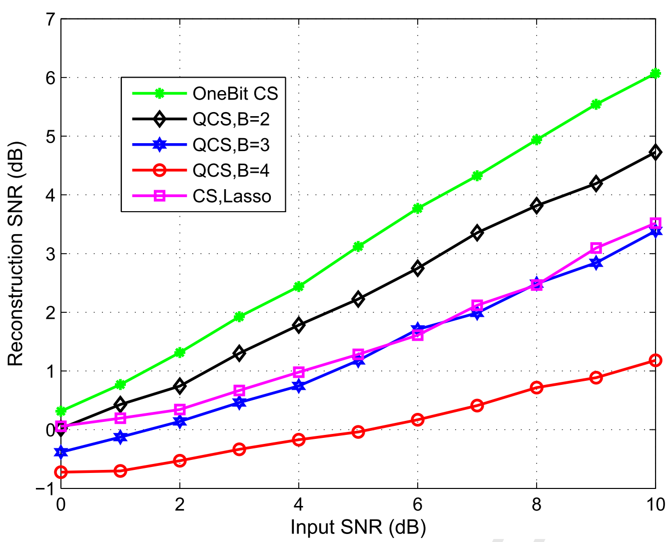

Fig. 7. Performance comparison of one-bit CS, QCS and classical CS on Gaussian noise corrupted data with different noise level, here bit budget $B_{\mathrm{T}} = 1.4W$.

As a result, the input SNR is defined as $\mathrm{SNR_I} := -10\log_{10}(W\sigma^2)$ because of $\|\mathbf{a}\|_2^2 = 1$. The reconstruction SNR is computed as $\mathrm{SNR_R} := 10\log_{10}(\|\hat{\mathbf{a}}\|_2^2/\|\hat{\mathbf{a}} - \mathbf{a}\|_2^2)$.

4.2.1. Bit budget test

This experiment examines the reconstruction performances of the one-bit CS, QCS and classical CS schemes varying with bit budget B_{T}. The input SNR is 5 dB. The empirical reconstruct SNRs versus bit budget are depicted in Fig. 6. It is seen that the reconstruction SNR of one-bit CS method outperforms the classical CS and QCS schemes when the bit budgets is less than 180, i.e., the bit depth B_T/W is less than 1.8, which agrees with Fig. 5. This means that the one-bit CS scheme is high-efficient for stringent bit budget.

4.2.2. Noise level test

In this experiment, the bit budget is set as $B_{\mathrm{T}} = 1.4W$. The input SNR varies from 0 to 10 dB. The empirical results are depicted in Fig. 7. It is seen in Fig. 7 that the proposed one-bit CS approach outperforms the QCS and classical CS schemes, especially in the relatively high SNR region.

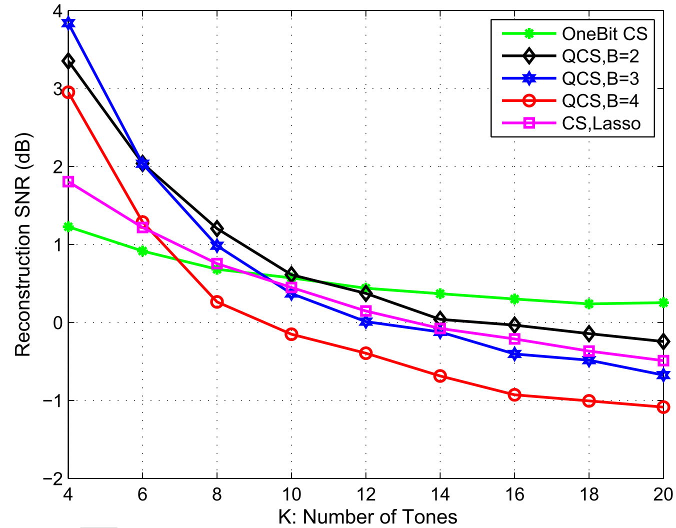

Fig. 8. Performance comparison of one-bit CS, QCS and classical CS scheme with different number of tones, here input SNR is 0 dB and bit budget $B_{\mathrm{T}} = 1.2W$.

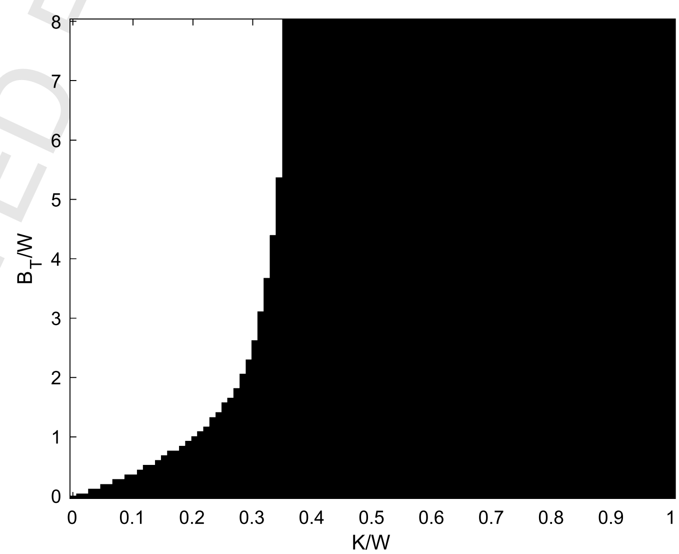

Fig. 9. 'Success' or 'failure' of spectrally sparse signal recovery as a function of sparsity level K/W and bit depth B_T/W. The white pixel indicates the successful recovery for the corresponding combination of parameter values. In this experimental, the SNR is 30 dB.

4.2.3. Sparsity level test

Let us now evaluate the reconstruct performance versus the signal sparsity. We set $B_{\mathrm{T}} = 1.2W$, and the input SNR is fixed at 0 dB. The number of tones varies from 4 to 20, i.e., the sparsity varies from strong to weak. Fig. 8 plots the empirical reconstructed SNRs versus number of tones, which indicates that the one-bit CS method is considerably superior to the QCS and classical CS schemes for the weak sparse signal.

4.2.4. Probability of recovery

In this section, we evaluating the relationship between the bit depth B_T/W and sparsity level K/W required to achieve a successful recovery. In this experimental, the SNR is very low (30 dB). The 'success' of recovery means that 90% trials can recover 95% tones. The 'success' and 'failure' of recovery as a function of sparsity level and bit depth is described in Fig. 9. From this figure, we conclude that the spectrally sparse signal can not be recovered if the sparsity level K/W is larger than 0.35.

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4.2.5. Discussion

From Fig. 6 and Fig. 8 we can see that the reconstruction SNR of one-bit CS scheme is lower than QCS and classical CS methods for larger bit budget and small number of tones. This is mostly because the number of measurements in this situation, with regard to the QCS and classical CS methods, is enough to recovery the spectrally sparse signals accurately. This phenomenon means that the one-bit CS scheme is more suitable for stringent bit budget and weak sparsity situations.

5. Conclusion

We have proposed a new scheme, called one-bit random demodulator, to recover the spectrally sparse signals. In particular, the one-bit compressive sampling scheme consists of three significant components. The first one is the spectrum spreading conducted via the random binary mixer, the second one is the sub-Nyquist sampling by the accumulator, and the third one is the one-bit quantization by the comparator. These three components are expressed in terms of matrix form, and then the spectrally sparse bandlimited signal is recovered by using the BIHT algorithm. Numerical results are presented to demonstrate the superiority of the proposed one-bit CS scheme over the classical CS framework and QCS scheme, especially for the cases of weak sparsity and stringent bit budget.

Acknowledgments

This work was supported in part by the Natural Science Foundation of China (NSFC) under Grants (61501485, U1501253 and 61601300), by the Natural Science Foundation of Guangdong Province under Grant 2015A030311030, by the Foundation of Shenzhen under Grants (ZDSYS20150708, 1625213 and KC2015ZDYF0023A), by the China Postdoctoral Science Foundation under Grant 2015M582413 and by the Science and Technology Innovation Commission of Shenzhen Under Grant JCYJ20150324-140036835.

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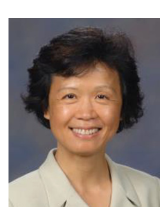

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Please cite this article in press as: H.-F. Zhou et al., Compressive sampling for spectrally sparse signal recovery via one-bit random demodulator, Digit. Signal Process. (2018), https://doi.org/10.1016/j.dsp.2018.04.014