Compressive sampling on the Fourier domain as a method for increasing image signal-to-noise ratio

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Abstract
Compressive sampling using measurements taken from a line mask on the Fourier domain is used to reconstruct test images with Gaussian, speckle, 1/f, and uniform noise. Using signal-to-noise ratio (SNR), and Linfoot's criteria as similarity measurements, we are able to show that the images with noise of varying strengths sampled at 5%, 10%, 20% and 30% of Nyquist criteria are generally better compared to completely sampled images.

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1. Introduction
Common signal sampling and imaging techniques such as those used in microscopy are susceptible to different kinds of noises such as speckle noise, Gaussian white noise and 1/f noise. Techniques such as optical coherence microscopy (OCM) are affected by speckle noise [1] which is characterized in images as

$$J = I + nI \tag{1}$$

where I is the pixel value and n is uniformly distributed random noise.

Speckle noise usually occurs due to variations in the gains of individual elements in an image sensor array [2]. Gaussian white noise is noise with a Gaussian probability distribution and a constant power spectral density. This type of noise is commonly found as a result of thermal noise [3]. 1/f noise is noise characterized having its power spectrum inversely proportional to frequency. This type of noise is found due to random variations in the diffusion of charge carriers within devices. It is found in other forms such as current-dependent fluctuations of resistance exhibited by real resistors [3].

Noise affects signals differently and a quantitative method for describing the quality of a signal is in terms of its corruption by noise which is measured using its signal-to-noise ratio, defined as:

$$SNR = 20log\frac{P_{signal}}{P_{noise}} \tag{2}$$

In signal acquisition and imaging, we aim to have samples with high SNRs.

For time-stationary signals, multiple measurements can be averaged to increase the SNR. However, for transient events, other methods such as Fourier decomposition can help isolate signal from noise. For fast signals that run against the resolution limits of detectors and analog-to-digital converters, however, calculating the Fourier transform is difficult and frequency combs need to be used [4].

Compressive sampling (CS) is a non-adaptive sampling method that can be used to bypass the sampling process to directly acquire a compressed signal [5] [6]. Instead of increasing the resolution limits of detectors to satisfy the Nyquist theorem [5], we can use CS. CS is based on the idea that signals such as real-world images have sparse representations in some basis and that if an image is sampled by a largely incoherent measurement basis, then the signal can be reconstructed from fewer measurements via optimization procedures that search for the sparsest solution [7], thus a small number of non-traditional samples in the form of randomized projections can be used to capture the most important information in an image [8].

L1 minimization is the common optimization procedure used that returns a sparse representation and is defined as:

$$x = \arg\min \|x'\|_1 \text{ such that } Ax' = y \tag{3}$$

Note that the l1 norm of a vector x is the sum of the absolute values of its elements where $||x'||_1 = \sum_{i=0}^{N-1}|x|$.

If signal is a 2D image, an alternate recovery model is using a gradient that is sparse and the total variation (TV) of the image is minimized, where the TV is defined as $TV(x) := \sum_{ij}\sqrt{(D_{h;ij}x)^2 + (D_{v;ij}x)^2}$ where, $D_{h;ij}x = x_{i+1,j} - x_{ij}\ if\ i < n$, $D_{h;ij}x = 0\ if\ i = n$, $D_{v;ij}x = x_{i,j+1} - x_{ij}\ if\ j < n$ and $D_{v;ij}x = 0\ x_{ij}\ if\ j = n$. Here, we consider a 2D image where x_{ij} denotes the *ith* and *jth* column of an n x n image [9].

Instead of Equation 3, the constraint we follow is now:

$$(TV_1)\ min\ TV(x)\ subject\ to\ Ax = y \quad (7)$$

An advantage of using CS is that we no longer need to increase the resolution and sampling limits of detectors such as in [4] and that we only need to take a smaller number of sparse samples such as a line mask in Fourier space in order to reconstruct the image [8]. Sampling using CS in the frequency domain is a method that has been used for ultrasound and MRI imaging [10][11]. CS can have applications to advance technologies in digital cameras, medical imaging, seismic imaging and other imaging methods since we can reconstruct images with less samples needed which has been shown to reduce patient exposure times and discomfort for methods such as MRI [12].

In this paper we sample test images with added noise with a line mask in the Fourier domain and show the effect of noise strengths on the quality of the reconstructed images. In the reconstruction procedure, we use TV minimization for our compressive sampling method in order to fill the missing data before transforming it back into the spatial domain.

2. Methodology

The sample images used are the test images Lenna, the USAF resolution target, a Mandelbrot fractal and three different images of neuron samples as shown in (a- f) of Figure 1. Each image is converted into grayscale and is added with Gaussian white noise, speckle noise, 1/f noise, and uniform noise with varying strengths. This method can be applied to single channel (R,G, or B) images without modification. The noise strengths are varied by changing the parameters of the noise added such as the standard deviation for the Gaussian white noise. Pixel values for the images are from 0-255. When noise added sets a pixel value above 255 or below 0 they are set to 255 and 0, respectively.

Sampling is done in the Fourier space by getting the FFT and multiplying it with a line mask as shown in (g) of Figure 1. This line mask has a varying number of equally spaced lines passing through the center. The number of lines used were 30, 54, 110, and 170 lines, corresponding to 5%, 10%, 20%, and 30% sampling in the frequency domain. We then use compressive sampling, minimizing the total variation to reconstruct the image by filling in the missing data. In normal sampling, following the Nyquist criterion, masking or reduction of sampling points generally results to signal loss, thus lowering SNR. This is not observed in compressive sampling where the selection of sampling vectors and the minimization procedure compensates for the loss at the expense of computational time [10].

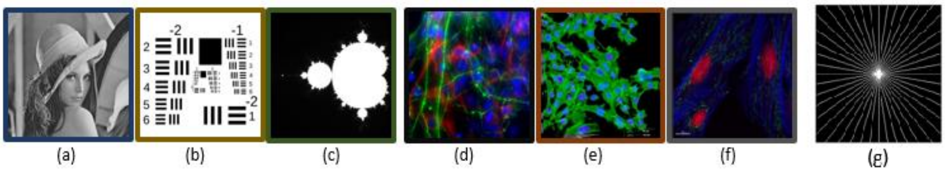

Figure 1. Test images used: (a) Lenna, (b) USAF resolution target, (c) Mandelbrot fractal, (d) Immunostaining of mouse embryonic whole brain culture, (e) network of neurons and (f) Muntjac skin fibroblasts. An example of a line mask with 22 lines used to sample the images in Fourier space is shown in (g).

For the Mandelbrot image and the second neuron image, 70% of the total energies are in the lowest 40% in frequency space while for the other images, 70% of the total energies are located within the lowest 25%.

The SNR and Linfoot's criteria for each reconstructed and sampled image were computed and compared with each other. The Linfoot's criteria used were fidelity (F), relative structural content (C) and correlation quality (Q), defined as $F = 1 - \langle (I_{rec}-I_{orig})^2 \rangle / \langle I_{orig}^2 \rangle$, $C = \langle I_{rec}^2 \rangle / \langle I_{orig}^2 \rangle$, $Q = \langle I_{rec} I_{orig} \rangle / \langle I_{orig}^2 \rangle$, where I_{rec} is the reconstructed image and I_{orig} is the original image. Note that fidelity is one minus the error of the reconstructed image, relative structural content is the sharpness of the image and correlation quality is the alignment of the peaks. For all three criteria, the closer the value is to 1, the better the image.

3. Results and Discussion

The addition of the same kind of noise affects each test image differently so for comparison's sake, we use SNR of the image after addition of noise instead of the parameters varied in shaping the noise.

For all the test images, when sampled without noise, the Linfoot's criteria values approach 1 at around 30% sampling. Thus, for these simulations, with noise, we sample only up to 30%. This is due to the fact that the information needed to reconstruct the images are mostly at the lower frequencies. Despite undersampling in the Fourier domain, CS can be used to reconstruct the images.

Increasing the number of samples results in the spatial frequency content of the reconstructed images becoming more similar to that of the original image. However, decreasing the number of samples shows more differences in the high frequency regions due to the nature of the line mask with sparser sampling in high frequencies.

Figure 2 shows the SNR values of reconstructed images versus the SNR values of the original sampled noisy images for varying noise distributions. The dashed, black line indicates points where the SNR of the sampled and reconstructed images are the same. Each image has a corresponding color and darker shades of the color correspond to higher sampling values.

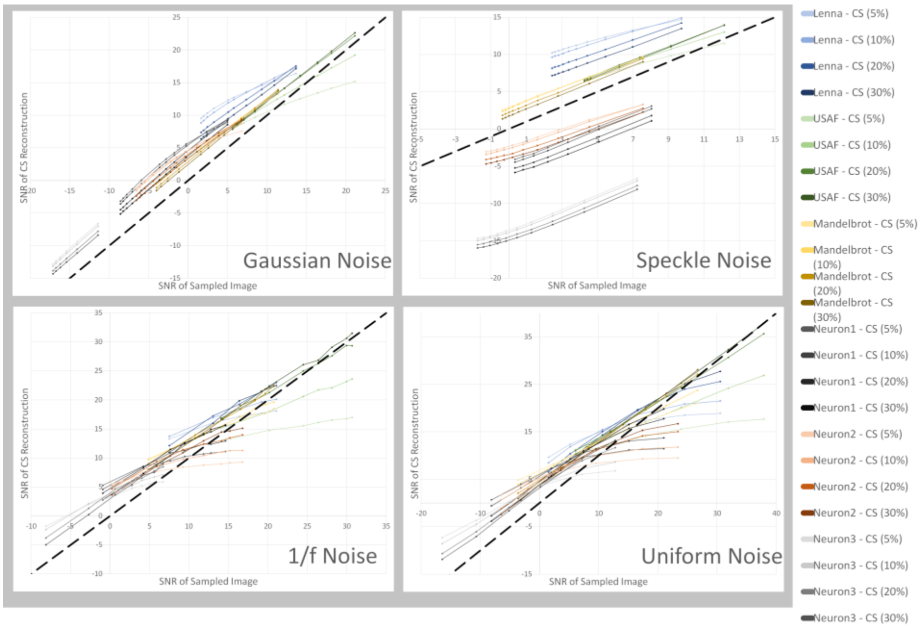

Figure 2. Signal-to-noise ratio (SNR) of CS reconstructions and sampled images for varying types of noises. The dashed line are where the sampled and reconstructed SNR are the same

For Gaussian noise, for test images sampled up to 5% sampling, except for the USAF images at 5 and 10% sampling, the SNR of the reconstructed image was greater than that of the sampled image. This could be explained due to the fact that the sharp edges of the USAF image are from high frequency components that are not sampled at 5% and 10%. At lower SNRs of the sampled image, however, the SNRs of the reconstructed images become better than that of the sampled image.

For all images, the SNR of the reconstructed image also drops off slower than that of the sampled image so at lower SNRs of the sampled image, the improvement in SNR by using CS is greater.

For speckle noise, the Lenna, USAF and Mandelbrot test images were found to have better SNRs using CS reconstruction. However, for all three neuron images, the reconstructed images had worse SNRs compared to the original images.

For both 1/f and uniform noise the SNR for all reconstructed images sampled at 20% and 30% were found to be greater than the sampled images, similar to the results of those with Gaussian noise. The images reconstructed from 5% and 10% sampling have worse SNRs at higher SNRs of the sampled image. However, as the SNR of the sampled image decreases, the SNR of the reconstructed images becomes better than the sampled images. Unlike Gaussian noise, as SNR of the sampled image decreases, the improvement in SNR by using CS also decreases.

CS reconstructions are therefore robust despite added noise since sampling in frequency space is mostly done in low frequencies. The increases in SNR can be attributed to the fact that the noises added range across all frequencies and by using CS to bypass the traditional sampling process, we are able to suppress the destructive effect of the added noise.

Additionally, we also use Linfoot's criteria to evaluate our CS reconstructions. For those with Gaussian noise, in terms of fidelity (F), CS reconstructed images maintain better values compared to those that are equally sampled with noise despite a decrease in the SNR of the sampled image. The CS reconstructions were also found to have better relative structural content (C) values for the Lenna, USAF and neuron images which have most of their energies in the lower frequencies.

The Mandelbrot reconstructions do not have as good C values since most of its energies are in the higher frequencies. We undersample at higher frequencies leading to a loss of detail and sharpness which greatly affects the Mandelbrot image. In terms of correlation quality (Q), there is not much difference between the reconstructed and equally sampled images since the alignment of the peaks are not really affected as much by the added noise.

For speckle noise, in terms of fidelity (F), and relative structural content (C), the Lenna, Mandelbrot and USAF images reconstructed using CS have better values for the two criteria compared to the completely sampled images. In terms of correlation quality (Q), however, the values are comparable to those completely sampled, which is similar to those with added Gaussian noise.

For the neuron images, at higher signal-to-noise ratios, the completely sampled images have better values in terms of Linfoot's criteria. However, as the signal-to-noise ratio decreases, there are crossover values where CS reconstructions now have better values for fidelity and structural content. In terms of correlation quality, however, CS reconstructions perform worse for all cases.

Lastly, the Linfoot's criteria values for those added with 1/f and uniform noise have similar results to those with added Gaussian noise. However, the resulting improvements are not as great.

3. Conclusion

We were able to demonstrate that sampling noisy images in the Fourier domain and reconstructing them using compressive sampling can be used to increase their signal-to-noise ratios. Using SNR and Linfoot's criteria as similarity measurements, we are able to show that the images sampled at 5%, 10%, 20% and 30% of Nyquist criteria are generally better to the original images with Gaussian, 1/f and uniform noise. However, CS reconstructions of the neuron test images were not found to have improvements in terms of SNR, but have improvements in terms of fidelity and relative structural content at lower values of SNR in the sampled image. We take note that CS reconstructions are robust despite added noise since sampling in frequency space is mostly done in low frequencies. For images like the Mandelbrot image who have energies in the higher frequencies, the relative improvements for these reconstructed images are less.

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References

1. Schmitt, "Array detection for speckle reduction in optical coherence microscopy", Physics in Medicine and Biology 42, pp 1427-1439, 1997.
2. Maria Petrou, Costas Petrou - Image Processing: The Fundamentals. John Wiley & Sons, 2010.
3. N. Storey, Electronics—A Systems Approach. Prentice Hall, 2009
4. Sung Chang, Combing for a signal buried in noise, Physics Today 69(3), 21, 2016.
5. Baraniuk, Richard. "Compressive sensing," IEEE Signal Processing Magazine, 24(4), pp. 118-121, July 2007.
6. Romberg, Justin. "Imaging via compressive sampling," IEEE Signal Processing Magazine, 25(2), pp. 14-20, March 2008
7. Magalhães, Filipe, Mehrdad Abolbashari, Francisco M. Araújo, Miguel V. Correia, and Faramarz Farahi. "High-Resolution Hyperspectral Single-Pixel Imaging System Based on Compressive Sensing." Optical Engineering 51, no. 7, 2012
8. Haupt, Jarvis and Nowar Robert. "Compressive Sampling vs. Conventional Imaging.", IEEE, 2006.
9. Romberg, J. l1-Magic. *l1-Magic* Available at: http://users.ece.gatech.edu/justin/l1magic/. (Accessed: 20th March 2016)
10. Candes, E. & Wakin, M. An Introduction To Compressive Sampling. IEEE Signal Process. Mag. IEEE Signal Processing Magazine 25, 21–30 (2008).
11. Céline Quinsac, Adrian Basarab, and Denis Kouamé, "Frequency Domain Compressive Sampling for Ultrasound Imaging," Advances in Acoustics and Vibration, vol. 2012, Article ID 231317, 16 pages, 2012.

12. Qaisar, Saad, Bilal, Rana, Iqbal, Wafa, Muqadas, Naureen and Sungyoung Lee. "Compressive Sensing: From Theory to Applications, A Survey." *Journal of Communications and Networks* 15, no. 5, 2013.