Frequency domain reconstruction of stochastically sampled signals based on compressive sensing

Kenneth V. Domingo*

Adviser: **Maricor N. Soriano, Ph.D.**

*Corresponding author: kdomingo@nip.upd.edu.ph

Video and Image Processing Group, Instrumentation Physics Laboratory

National Institute of Physics, University of the Philippines Diliman

The Problem

The Nyquist-Shannon sampling theorem (NSST) states that in order to digitally reconstruct an analog signal faithfully, it has to be sampled at a rate twice the signal's highest frequency. However, sampling signals with periods on the order of 10^{-9} s become impractical to implement on hardware. Compressed sensing (CS) circumvents this limitation by purposely sampling below the Nyquist rate. Currently, several CS algorithms exist. **In this paper, we aim to quantify the performance of three algorithms: least absolute shrinkage and selection operator (LASSO) [1], orthogonal matching pursuit (OMP) [1], and smoothed L0 norm (SL0) [2] in terms of computation time and cosine similarity.**

What is compressed sensing?

CS models signal treatment as an underdetermined linear system [3], which can be solved by imposing certain constraints. It exploits the sparse representation of signals in the frequency domain in order to recover them from much fewer samples than required by NSST.

Why compressed sensing?

Because of the fewer samples required, CS is not as stringent in its acquisition speed and storage requirements, in contrast with standard sampling techniques. Also, due to the lower rate, CS-based systems can potentially attain higher signal-to-noise ratio [4] and larger dynamic range [5].

Methodology

1. Record an acoustic guitar signal $\mathbf{x}$ playing a single E_4 note (330 Hz) at 8 kHz sampling rate for 4 s.
2. Compressively sample the signal $\mathbf{x}$ of length N by taking M random indices, equivalent to satisfying NSST for frequencies up to $0.5M$ Hz. Define the random index sequence as $\mathbf{r} \equiv \{r_i\}_{i=0}^{M-1}$.
3. Construct the orthonormal basis $\mathbf{\Psi}$ by taking the discrete cosine transform (DCT) of an $N \times N$ identity matrix.
4. Take the $\mathbf{r}$-th row vectors of $\mathbf{\Psi}$ and stack them to construct the $M \times N$ measurement matrix $\mathbf{\Phi}$.
5. The compressed signal $\mathbf{y}$ is now subject to the constraint
$$\mathbf{y} = \mathbf{\Phi}\mathbf{x}$$
6. CS algorithms then search for the sparsest signal $\mathbf{x}$ that yields $\mathbf{y}$.
7. Time the performance of three algorithms from acquisition to reconstruction. Obtain the cosine similarity by
$$\text{similarity} = \cos\theta = \frac{\mathbf{x} \cdot \hat{\mathbf{x}}}{\|\mathbf{x}\|_2 \|\hat{\mathbf{x}}\|_2}$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ are the normalized original and reconstructed signals, respectively.

References

[1] F. Pedregosa, G. Varoquaux, A. Gramfort, et al., Scikit-learn: Machine learning in Python, arXiv:1201.0490v4 (2012).

[2] H. Mohimani, M. Babaie-Zadeh, and C. Jutten, A fast approach for overcomplete sparse decomposition based on smoothed L0 norm, arXiv:0809.2508v2 (2008).

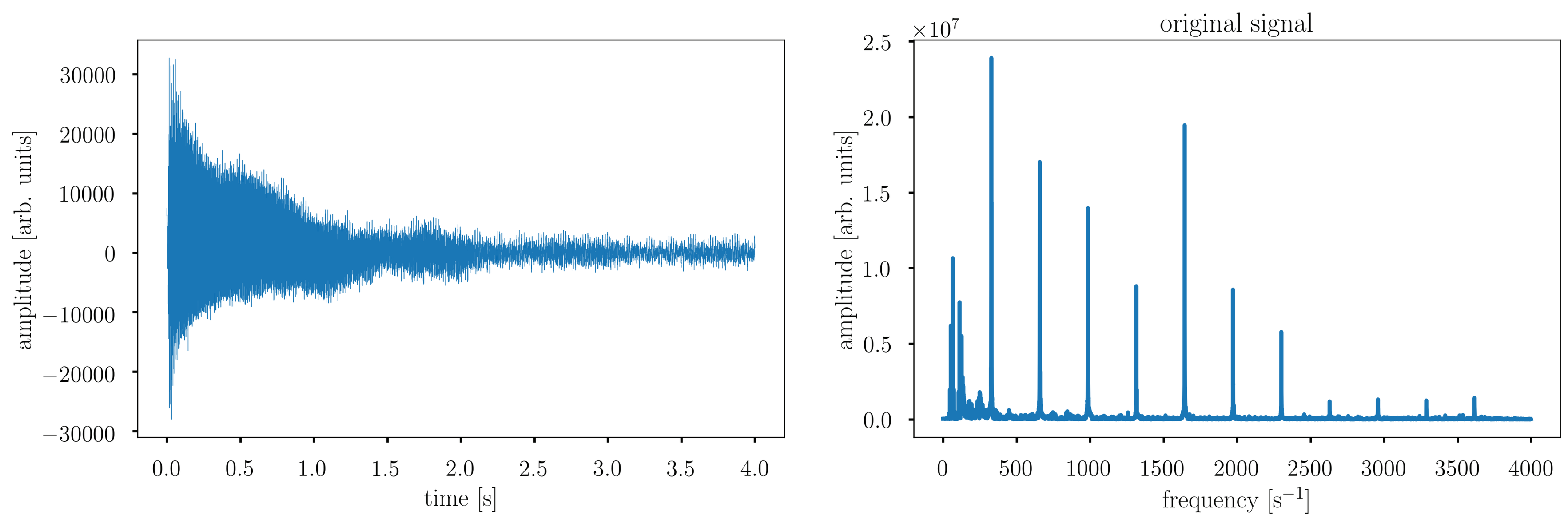
[3] E. Candès, J. Romberg, and T. Tao, Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information, arXiv:math/0409186v1 (2006).

[4] R. A. Romero, G. A. Tapang, and C. A. Saloma, in *Proceedings of the Samahang Pisika ng Pilipinas Physics Conference* (University of the Philippines Visayas, Iloilo City, Philippines, 2016), vol. 34, SPP-2016-PA-14.

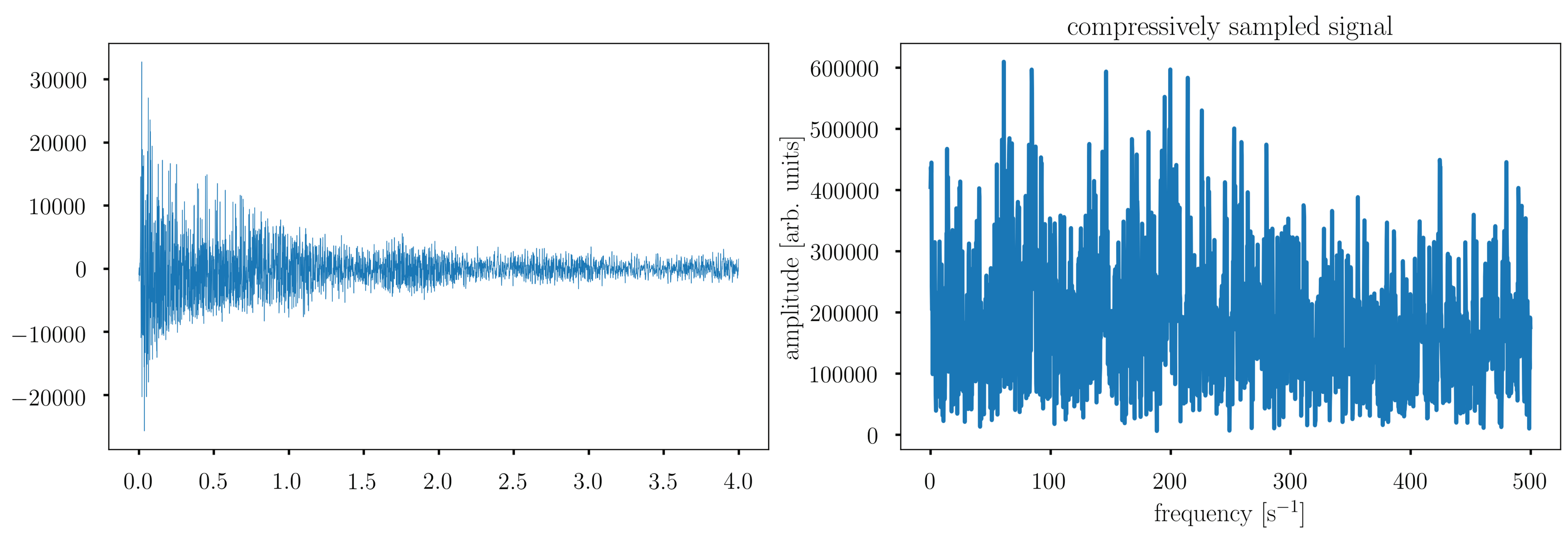
[5] M. Davenport, J. Laska, J. Treichler, and R. Baraniuk, The pros and cons of compressive sensing for wideband signal acquisition: noise folding vs. dynamic range, arXiv:1104.4842 (2011).

Results & Discussion

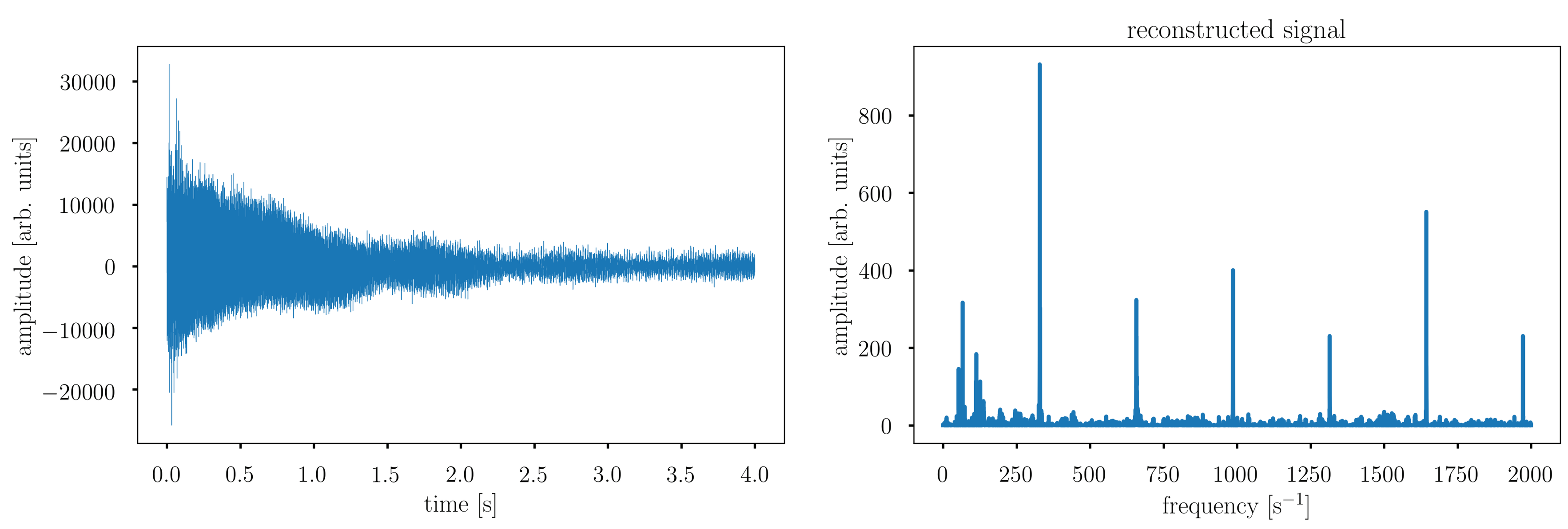
- Original signal with multiple harmonics (8 kHz sampling rate):



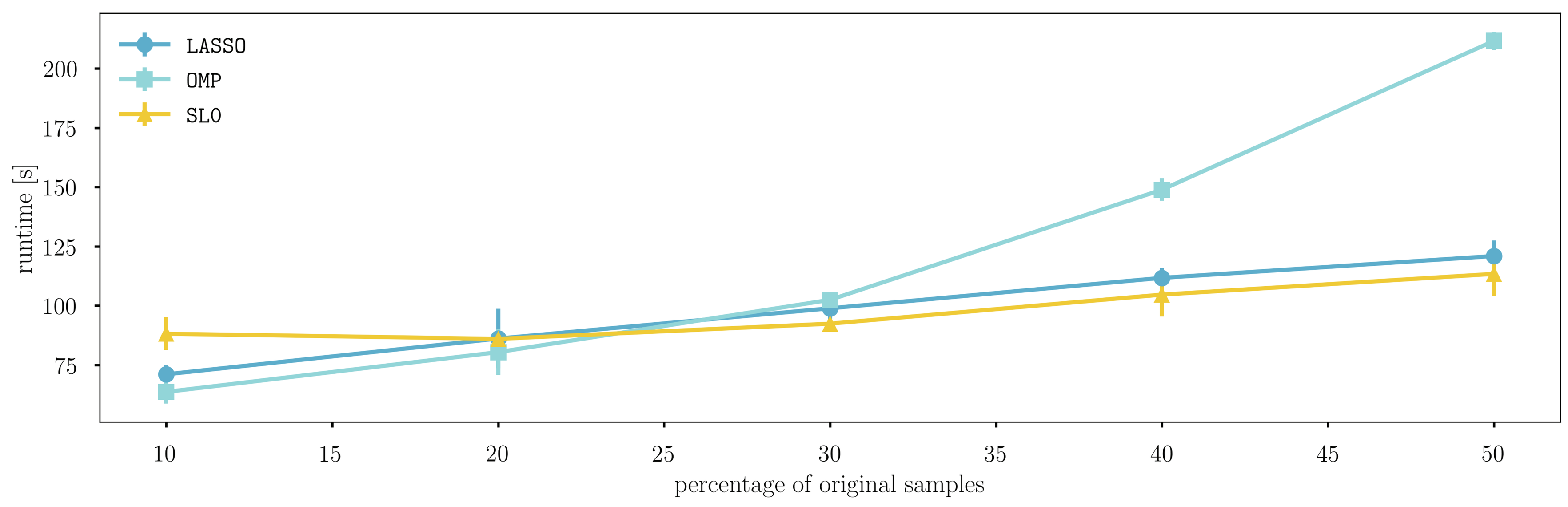
- Compressively sampled signal (1000 samples/s):



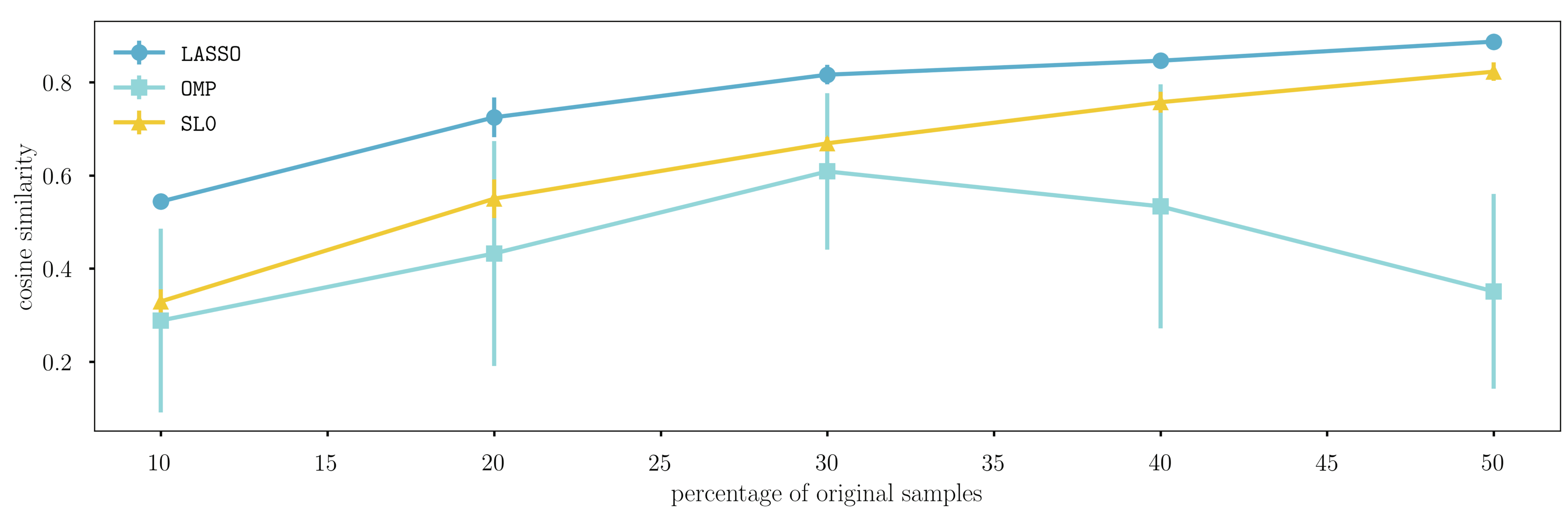
- Recovered signal using LASSO:



- Comparison of computation time:



- Comparison of reconstruction error (cosine similarity):



- No clear conclusion about runtime. However, LASSO consistently performs best in terms of reconstruction accuracy.