HK Simulations

March 17, 2021

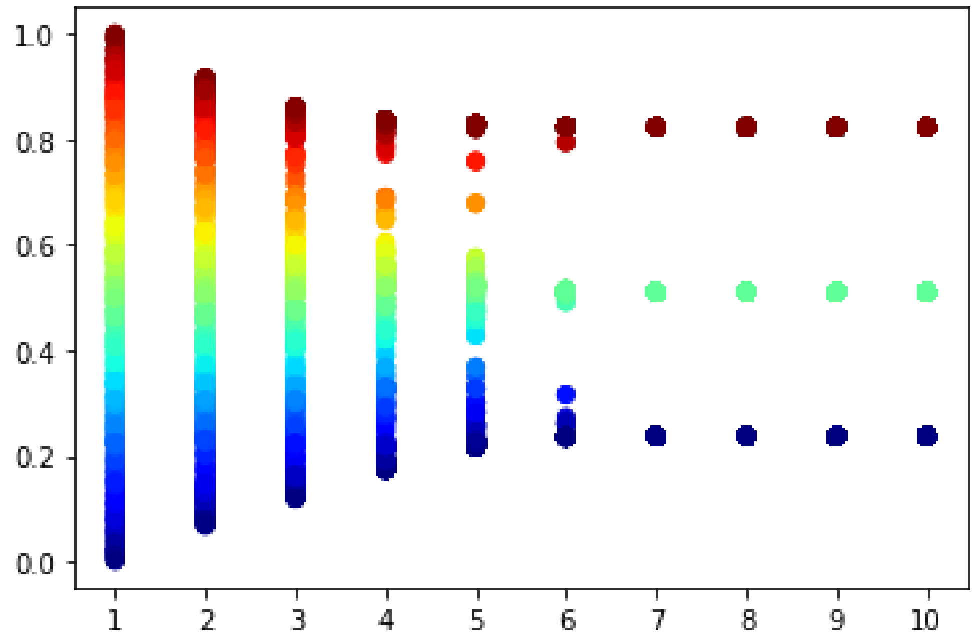
[4]:
```
import numpy as np
import matplotlib.pyplot as plt
from collections import defaultdict
from matplotlib.patches import Polygon
np.set_printoptions(linewidth=400)
```

0.0.1 HK Simulation functions

[5]:
```
# try np.abs(a - neighbor) with l2 norm.
def compute_next_state(agents, eps):
    next_agents = np.zeros(agents.shape[0])
    for i, a in enumerate(agents):
        next_agents[i] = np.mean([neighbor for neighbor in agents if np.abs(a -␣
↪neighbor) <= eps])
    return next_agents

def simulate(agents, eps=0.1, num_iters=10):
    curr_agents = agents
    history = [curr_agents]
    for _ in range(num_iters-1):
        next_agents = compute_next_state(curr_agents, eps)
        history.append(next_agents)
        curr_agents = next_agents

    return history

def simulate_to_convergence(agents, eps):
    curr_agents = agents
    prev_agents = np.zeros_like(agents)
    history = [curr_agents]
    while not np.allclose(prev_agents, curr_agents):
        next_agents = compute_next_state(curr_agents, eps)
        history.append(next_agents)
        prev_agents = curr_agents
        curr_agents = next_agents
    return history
```

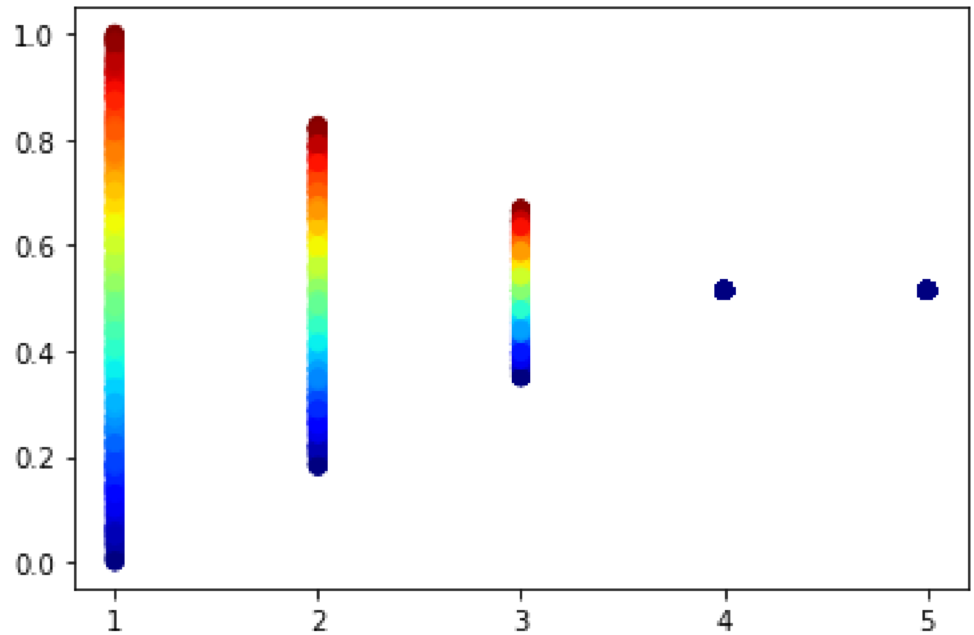
[6]:

```
initial_agents = np.random.rand(625)
history = simulate(initial_agents, eps=0.15, num_iters=10)

for t, agents in enumerate(history):
    plt.scatter([t+1] * len(agents), agents, c=agents, cmap='jet')


plt.xticks(range(1, len(history) + 1))
plt.show()
```



[7]:

```
initial_agents = np.random.rand(625)
history = simulate_to_convergence(initial_agents, eps=0.35)

for t, agents in enumerate(history):
    plt.scatter([t+1] * len(agents), agents, c=agents, cmap='jet')

plt.xticks(range(1, len(history) + 1))
plt.show()
```



0.1 Algorithm

[22]:
```
# Finds the minimum epsilon / confidence bound that leads to a consensus␣
↪convergence.
# Assumes agents are within the 0-to-1 range.
def find_min_eps_normalized(agents, target_clusters):
    high = 1.0
    low = 0.0
    num_iters = 0
    diffs = []
    while np.abs(low - high) >= 10**(-6):
        num_iters += 1
        diffs.append(
            (num_iters, np.abs(low - high))
        )
        mid = (high + low) / 2.0
        num_clusters = np.unique(simulate_to_convergence(agents, eps=mid)[-1]).
↪shape[0]

        if num_clusters <= target_clusters:
            high = mid
        else:
            low = mid

    return low, high, diffs
```

```
def find_min_eps(agents, target_num_clusters):
    # First, we normalize the agents to the 0-to-1 range.
    max_agents = max(agents)
    agents_norm = np.array([elem / max_agents for elem in agents])
    assert max(agents_norm) <= 1.0

    # Find the solution confidence bound for the normalized agents.
    low_n, high_n, diffs = find_min_eps_normalized(agents_norm,␣
 ↪target_num_clusters)
    low, high = max(agents) * low_n, max(agents) * high_n

    return low, high, diffs
```

0.1.1 Test Algorithm

[23]:
```
t1 = np.random.rand(10)
t2 = 100*np.random.rand(10)
mixed_agents = np.concatenate([t1, t2])
```

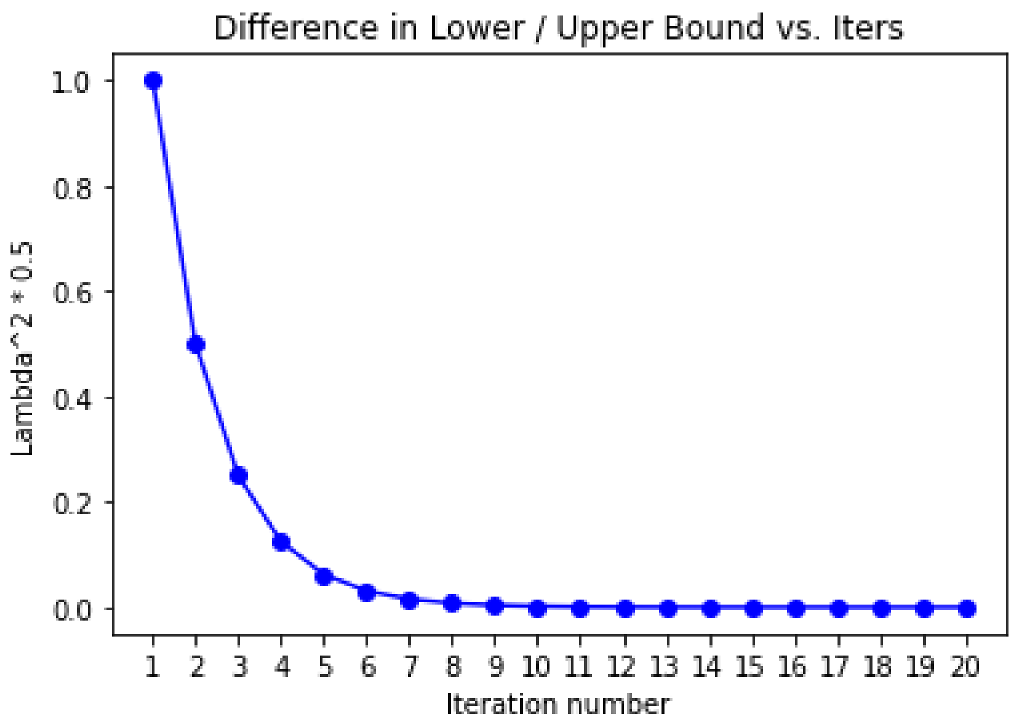
[24]:
```
low_eps, high_eps, diffs = find_min_eps(mixed_agents, target_num_clusters=3)
print("Confidence bound leads to non-consensus convergence: ", low_eps)
print("Confidence bound leads to consensus convergence: ", high_eps)
```

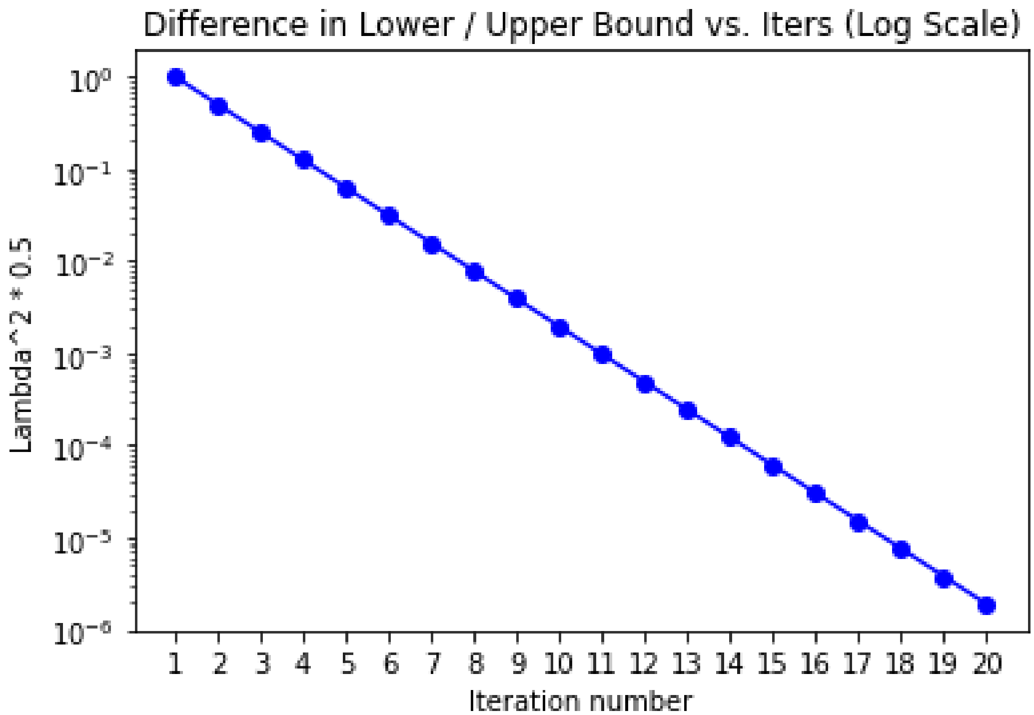
```
Confidence bound leads to non-consensus convergence:  20.37982269176631
Confidence bound leads to consensus convergence:  20.379916987354978
```

0.1.2 Graph Difference in Lower / Upper Bound vs. Iters

[34]:
```
iters, diff_terms = list(zip(*diffs))

# Normal Plot.
plt.title("Difference in Lower / Upper Bound vs. Iters")
plt.plot(iters, diff_terms, color="blue", marker="o")
plt.xticks(iters)
plt.xlabel("Iteration number")
plt.ylabel("Lambda^2 * 0.5")
plt.show()

# Log-scale Plot.
plt.title("Difference in Lower / Upper Bound vs. Iters (Log Scale)")
plt.plot(iters, diff_terms, color="blue", marker="o")
plt.yscale('log')
plt.xticks(iters)
```

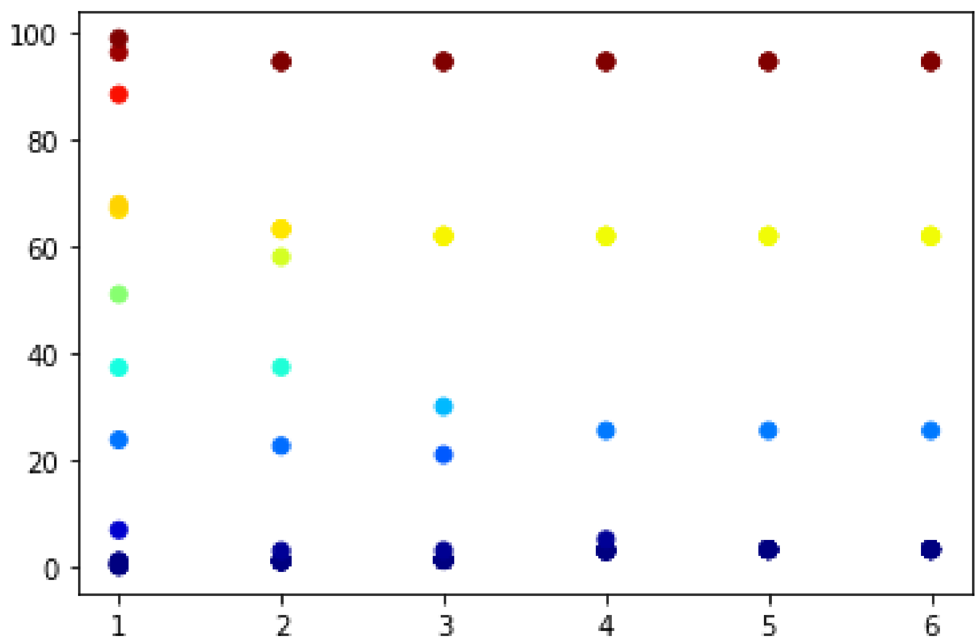
```
plt.xlabel("Iteration number")
plt.ylabel("Lambda^2 * 0.5")
plt.show()
```





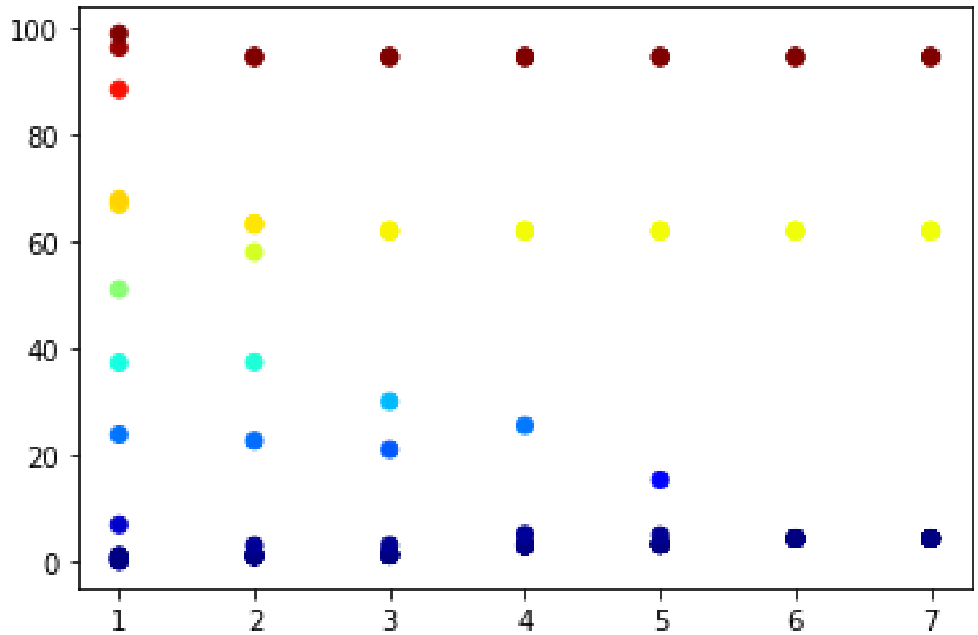
0.1.3 Test lower-bound confidence bound

[28]:
```
history = simulate_to_convergence(mixed_agents, eps=low_eps)
for t, agents in enumerate(history):
    plt.scatter([t+1] * len(agents), agents, c=agents, cmap='jet')

plt.xticks(range(1, len(history) + 1))
plt.show()
```



0.1.4 Test upper-bound confidence bound

[29]:
```
history = simulate_to_convergence(mixed_agents, high_eps)
for t, agents in enumerate(history):
    plt.scatter([t+1] * len(agents), agents, c=agents, cmap='jet')

plt.xticks(range(1, len(history) + 1))
plt.show()
```



0.2 Simulations

0.2.1 First set of simulations

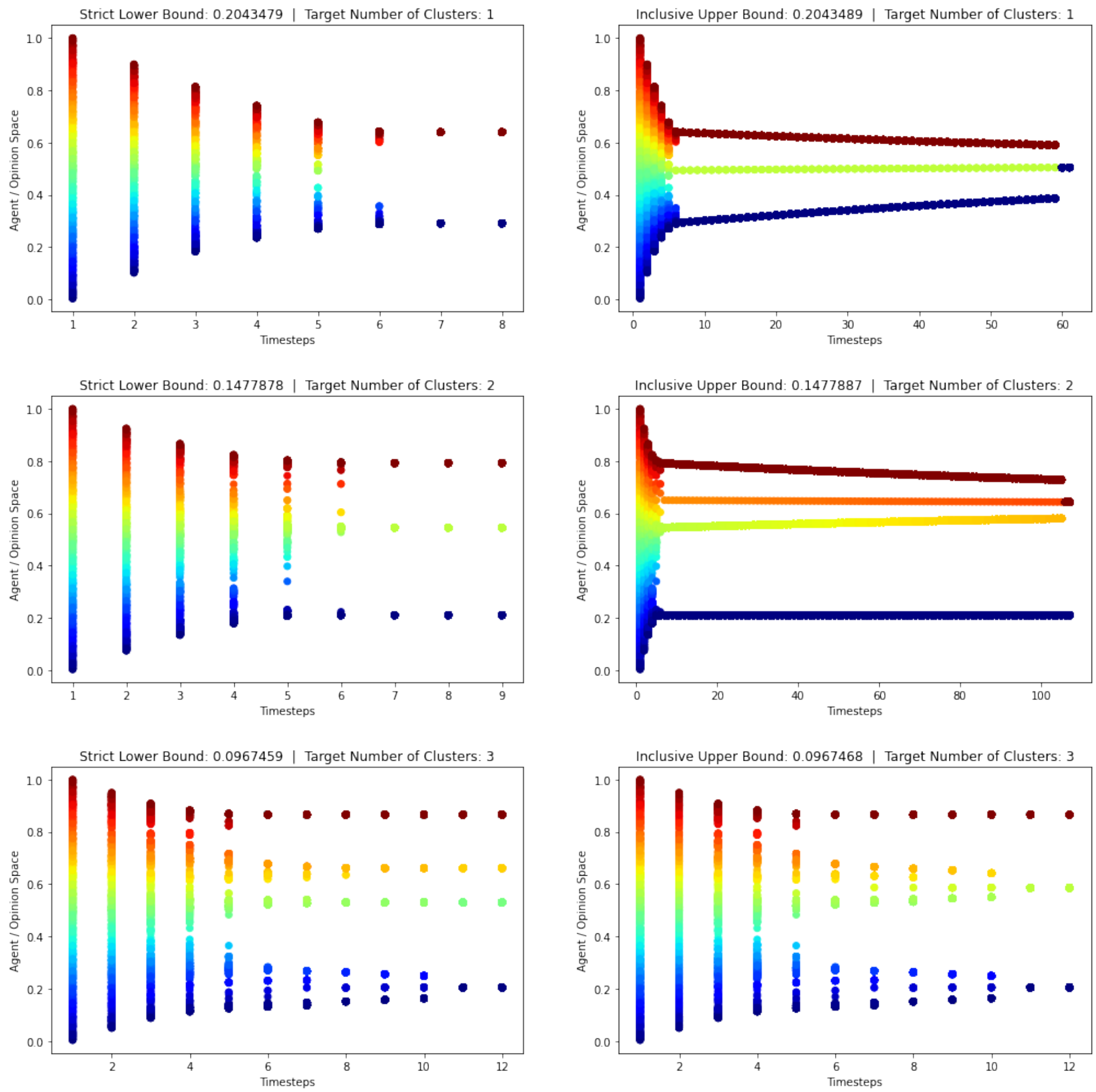
[41]:
```
target_clusters = [1, 2, 3]
lower_eps = []
upper_eps = []
simulations_history = []
diffs_history = []
agents = np.random.rand(626)
```

[42]:
```
# We run the algorithm for each target number of clusters to find a strict lower␣
↪bound &
# inclusive upper bound on the minimum confidence bound necessary for the agents␣
↪to
# converge to the target number of clusters
for target in target_clusters:
    low_eps, high_eps, diffs = find_min_eps(agents, target_num_clusters=target)
    lower_eps.append(low_eps)
    upper_eps.append(high_eps)
    diffs_history.append(diffs)
```

[43]:
```
# With each set of determined bounds, use both bounds and simulate the HK system␣
↪with the agents
# to convergence and record the agent positions at each timestep.
```

```
low_eps_history_simulations = []
high_eps_history_simulations = []
for i in range(len(target_clusters)):
    low_eps = lower_eps[i]
    high_eps = upper_eps[i]

    low_history = simulate_to_convergence(agents, low_eps)
    high_history = simulate_to_convergence(agents, high_eps)

    low_eps_history_simulations.append(low_history)
    high_eps_history_simulations.append(high_history)
```

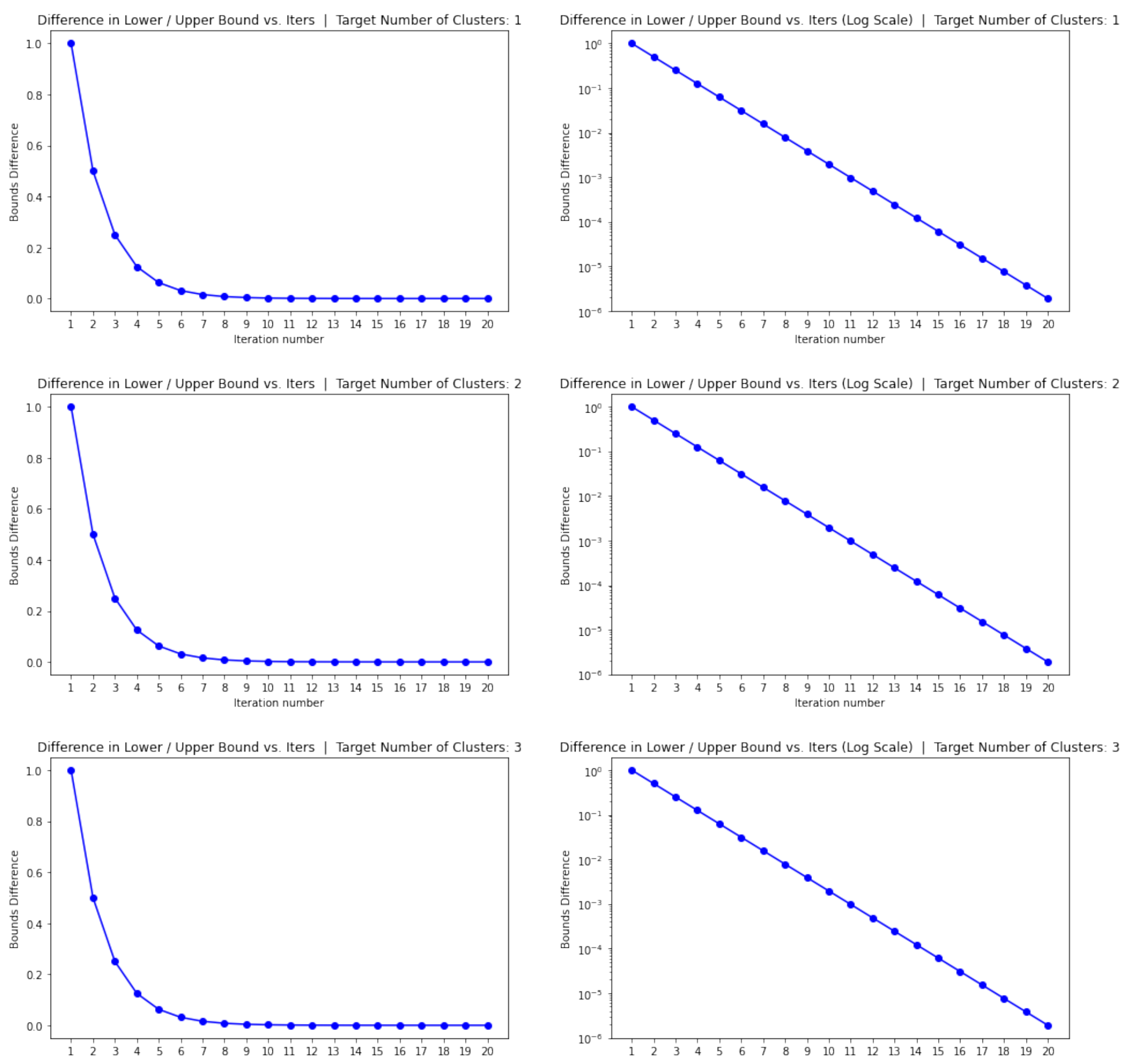
[44]:
```
plt.figure(figsize=(14, 14))

subplot_index = [elem for elem in range(1, len(target_clusters) * 2 + 1, 2)]

for i, target in enumerate(target_clusters):
    low_history = low_eps_history_simulations[i]
    low_eps = lower_eps[i]
    high_history = high_eps_history_simulations[i]
    high_eps = upper_eps[i]

    plt.subplot(3, 2, subplot_index[i])
    for t, agents_h in enumerate(low_history):
        plt.scatter([t+1] * len(agents_h), agents_h, c=agents_h, cmap='jet')
        plt.ylabel("Agent / Opinion Space")
        plt.xlabel("Timesteps")
        plt.title(f"Strict Lower Bound: {round(low_eps, 7)}  |  Target Number of␣
↪Clusters: {target}")

    plt.subplot(3, 2, subplot_index[i] + 1)
    for t, agents_h in enumerate(high_history):
        plt.scatter([t+1] * len(agents_h), agents_h, c=agents_h, cmap='jet')
        plt.ylabel("Agent / Opinion Space")
        plt.xlabel("Timesteps")
        plt.title(f"Inclusive Upper Bound: {round(high_eps, 7)}  |  Target␣
↪Number of Clusters: {target}")

plt.tight_layout(h_pad=3, w_pad=5)
plt.show()
```



0.2.2 Convergence Plot

[48]:
```python
plt.figure(figsize=(14, 14))
subplot_index = [elem for elem in range(1, len(target_clusters) * 2 + 1, 2)]
for i, diff_history in enumerate(diffs_history):
    iters, diff_terms = list(zip(*diff_history))

    # Normal Plot.
    plt.subplot(3, 2, subplot_index[i])
    plt.title(f"Difference in Lower / Upper Bound vs. Iters  |  Target Number of Clusters: {i + 1}")
    plt.plot(iters, diff_terms, color="blue", marker="o")
```

```
    plt.xticks(iters)
    plt.xlabel("Iteration number")
    plt.ylabel("Bounds Difference")

    # Log-scale Plot.
    plt.subplot(3, 2, subplot_index[i] + 1)
    plt.title(f"Difference in Lower / Upper Bound vs. Iters (Log Scale)  | Target Number of Clusters: {i + 1}")
    plt.plot(iters, diff_terms, color="blue", marker="o")
    plt.yscale('log')
    plt.xticks(iters)
    plt.xlabel("Iteration number")
    plt.ylabel("Bounds Difference")

plt.tight_layout(h_pad=3, w_pad=5)
plt.show()
```



0.2.3 Second Set of Simulations

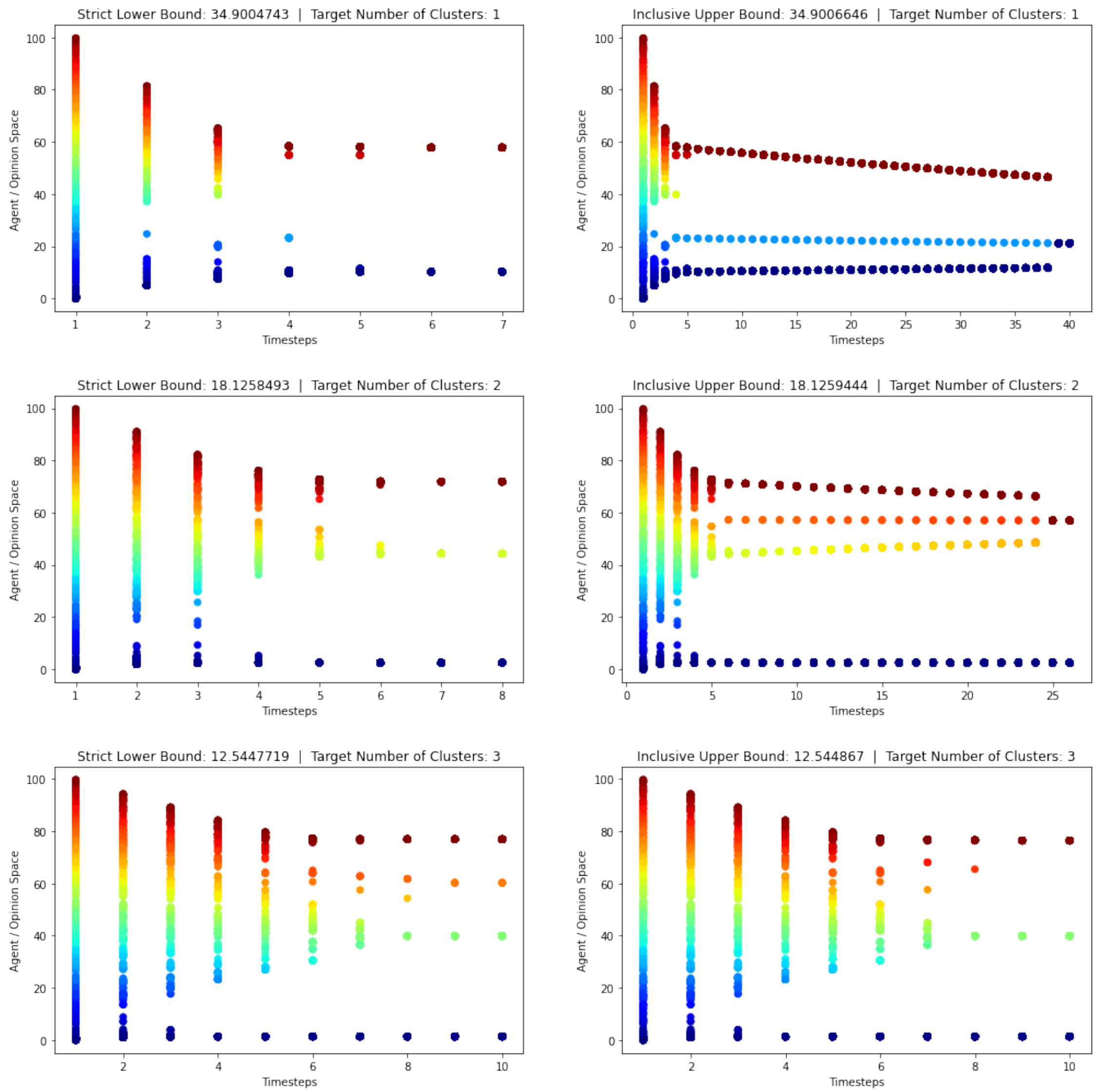
[89]:
```
target_clusters = [1, 2, 3]
lower_eps = []
upper_eps = []
simulations_history = []
diffs_history = []

agents_01 = np.random.rand(323)
agents_100 = 100 * np.random.rand(323)
agents = np.concatenate([agents_01, agents_100])
```

[90]:
```
# We run the algorithm for each target number of clusters to find a strict lower␣
↪bound &
# inclusive upper bound on the minimum confidence bound necessary for the agents␣
↪to
# converge to the target number of clusters
for target in target_clusters:
    low_eps, high_eps, diffs = find_min_eps(agents, target_num_clusters=target)
    lower_eps.append(low_eps)
    upper_eps.append(high_eps)
    diffs_history.append(diffs)
```

[91]:
```
# With each set of determined bounds, use both bounds and simulate the HK system␣
↪with the agents
# to convergence and record the agent positions at each timestep.
low_eps_history_simulations = []
high_eps_history_simulations = []
for i in range(len(target_clusters)):
    low_eps = lower_eps[i]
    high_eps = upper_eps[i]

    low_history = simulate_to_convergence(agents, low_eps)
    high_history = simulate_to_convergence(agents, high_eps)

    low_eps_history_simulations.append(low_history)
    high_eps_history_simulations.append(high_history)
```

[92]:
```
plt.figure(figsize=(14, 14))

subplot_index = [elem for elem in range(1, len(target_clusters) * 2 + 1, 2)]

for i, target in enumerate(target_clusters):
    low_history = low_eps_history_simulations[i]
```

```
    low_eps = lower_eps[i]
    high_history = high_eps_history_simulations[i]
    high_eps = upper_eps[i]

    plt.subplot(3, 2, subplot_index[i])
    for t, agents_h in enumerate(low_history):
        plt.scatter([t+1] * len(agents_h), agents_h, c=agents_h, cmap='jet')
        plt.ylabel("Agent / Opinion Space")
        plt.xlabel("Timesteps")
        plt.title(f"Strict Lower Bound: {round(low_eps, 7)}  |  Target Number of␣
↪Clusters: {target}")

    plt.subplot(3, 2, subplot_index[i] + 1)
    for t, agents_h in enumerate(high_history):
        plt.scatter([t+1] * len(agents_h), agents_h, c=agents_h, cmap='jet')
        plt.ylabel("Agent / Opinion Space")
        plt.xlabel("Timesteps")
        plt.title(f"Inclusive Upper Bound: {round(high_eps, 7)}  |  Target␣
↪Number of Clusters: {target}")

plt.tight_layout(h_pad=3, w_pad=5)
plt.show()
```



0.2.4 Third Set of Simulations

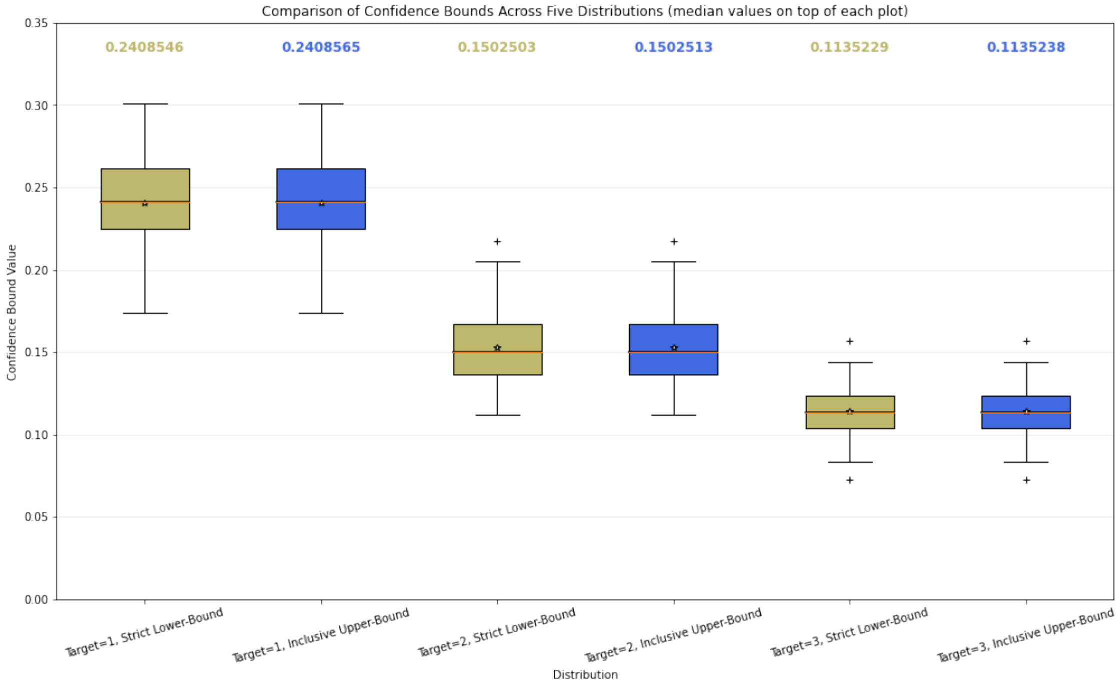
[100]:
```
set_of_agents = [np.random.rand(100) for _ in range(100)]
target_clusters = [1, 2, 3]
```

[102]:
```
low_eps = { 1: [], 2: [], 3:[] }
high_eps = { 1: [], 2: [], 3:[] }

for agents_i in set_of_agents:
    for target in target_clusters:
        low_eps_i, high_eps_i = find_min_eps(agents_i,␣
 ↪target_num_clusters=target)
```

```
        low_eps[target].append(low_eps_i)
        high_eps[target].append(high_eps_i)
```

[131]:
```
simulation_labels = [
    'Target=1, Strict Lower-Bound', 'Target=1, Inclusive Upper-Bound',
    'Target=2, Strict Lower-Bound', 'Target=2, Inclusive Upper-Bound',
    'Target=3, Strict Lower-Bound', 'Target=3, Inclusive Upper-Bound'
]

data = [
    low_eps[1], high_eps[1],
    low_eps[2], high_eps[2],
    low_eps[3], high_eps[3],
]

fig, ax1 = plt.subplots(figsize=(15, 11))
fig.subplots_adjust(left=0.075, right=0.95, top=0.9, bottom=0.25)

bp = ax1.boxplot(data, notch=0, sym='+', vert=1, whis=1.5)
plt.setp(bp['boxes'], color='black')
plt.setp(bp['whiskers'], color='black')
plt.setp(bp['fliers'], color='red', marker='+')

ax1.yaxis.grid(True, linestyle='-', which='major', color='lightgrey',
               alpha=0.5)

ax1.set(
    axisbelow=True,
    title='Comparison of Confidence Bounds Across Five Distributions (median␣
 ↪values on top of each plot)',
    xlabel='Distribution',
    ylabel='Confidence Bound Value',
)

box_colors = ['darkkhaki', 'royalblue']
num_boxes = len(data)
medians = np.empty(num_boxes)
for i in range(num_boxes):
    box = bp['boxes'][i]
    box_x = []
    box_y = []
    for j in range(5):
        box_x.append(box.get_xdata()[j])
        box_y.append(box.get_ydata()[j])
    box_coords = np.column_stack([box_x, box_y])

    ax1.add_patch(Polygon(box_coords, facecolor=box_colors[i % 2]))
```

```
    med = bp['medians'][i]
    median_x = []
    median_y = []
    for j in range(2):
        median_x.append(med.get_xdata()[j])
        median_y.append(med.get_ydata()[j])
        ax1.plot(median_x, median_y, 'k')
    medians[i] = median_y[0]
    ax1.plot(np.average(med.get_xdata()), np.average(data[i]),
             color='w', marker='*', markeredgecolor='k')

# Set the range + labels of the axes.
ax1.set_ylim(0.0, 0.35)
ax1.set_xticklabels(simulation_labels,
                    rotation=15, fontsize=10)

# Plot median values.
pos = np.arange(num_boxes) + 1
upper_labels = [str(round(s, 7)) for s in medians]
weights = ['bold', 'semibold']
for tick, label in zip(range(num_boxes), ax1.get_xticklabels()):
    k = tick % 2
    ax1.text(
        pos[tick],
        .95,
        upper_labels[tick],
        transform=ax1.get_xaxis_transform(),
        horizontalalignment='center', size='large',
        weight=weights[k],
        color=box_colors[k]
    )


plt.show()
```


Comparison of Confidence Bounds Across Five Distributions (median values on top of each plot)
0.2408546
0.2408565
0.1502503
0.1502513
0.1135229
0.1135238
Confidence Bound Value
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Target=1, Strict Lower-Bound
Target=1, Inclusive Upper-Bound
Target=2, Strict Lower-Bound
Target=2, Inclusive Upper-Bound
Target=3, Strict Lower-Bound
Target=3, Inclusive Upper-Bound
Distribution