6.1 Confidence Intervals and Hypothesis Tests

3. Make an assessment about the chances that your hypothesis test conclusion (which will be reject or don't reject) is wrong. There two ways you can be wrong (only one will apply):

Type I error: Since we chose a 95% confidence interval, the probability of getting an $\bar{x}$ outside of the confidence interval (alternatively stated, in the rejection region) is 5%. So even if μ = 2, were still true, in 5% of the trials, we could reject H_o, when we shouldn't have.

Type II error: Given the parameters of the problem, the rejection region is calculated as:

$$\bar{x} - 1.96 \times SE, \bar{x} + 1.96 \times SE$$

Where $\bar{x}$ is 2 and the standard error (SE) is 0.1414 (calculated in the 6_1.py submission).

$$2 - 1.96 \times 0.1414, 2 + 1.96 \times 0.141 = (1.722, 2.278)$$

If we want to test the probability of a type II error with $\bar{x}$ = 2.2, we would have to compute the probability that 2.2 falls inside the confidence interval (outside of the rejection region).

$$P(1.722 \leq \bar{x} \leq 2.278)$$

The scores for the rejection region can be computed the following way:

$$Z = \frac{1.722 - 2.2}{0.1414} = -3.38$$

$$Z = \frac{2.278 - 2.2}{0.1414} = 0.55$$

These scores are approximately equivalent to 0.00036 and 0.7088, respectively, so the probability of the 2.2 $\bar{x}$ falling inside this region, is approximately:

$$0.7088 - 0.00036 = 0.7084$$

Or 70.84%, which is a very high chance of getting a type II error. The method proposed in the problem has a high probability of failing to reject H_0, even when it should be rejected.

4. Suppose you found x=2.1. In this case you should not reject $H0$, but it could be that μ=2.2 was used to generate the n values. Given this, what are the chances that your non–rejection is a Type II error?

I believe this can be calculated using the formula from above, but replacing 2 with 2.1 to obtain the bounds of the rejection region:

$$2.1 - 1.96 \times 0.1414, 2.1 + 1.96 \times 0.141 = (1.822, 2.378)$$

And like before, we can compute the probability of a type II error and the z scores:

$$P(1.822 \leq \bar{x} \leq 2.378)$$

$$Z = \frac{1.822 - 2.1}{0.1414} = -1.96$$

$$Z = \frac{2.378 - 2.1}{0.1414} = 1.96$$

These nearly exactly coincide with the 95% confidence interval. I think I am missing something from this question, perhaps confusing how x and $\bar{x}$ are being distinguished.

5. Explain how the confidence interval and the results of parts 2. and 3. are related.

   The rejection region defined in part 3, is anything that falls outside of the confidence interval. 5% of the time, μ will not be trapped by the confidence interval, and will fall in the rejection region, thus resulting in a type I error.