2.2 Bayes' Rule

A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

- 85% of the cabs in the city are Green and 15% are Blue. A witness identified the cab as Blue. The court tested the reliability of the witness under the circumstances that existed on the night of the accident and concluded that the witness correctly identified each one of the two colors 80% of the time and failed 20% of the time.

What is the probability that the cab involved in the accident was Blue rather than Green?

From the problem, there are the following probabilities:

P(green) = 0.85
P(blue) = 0.15
P(correct_id) = 0.8
P(incorrect_id) = 0.2

We want to calculate for the probability that the witness correctly identified a blue cab. As the problem suggests, this can be done with Bayes' Rule.

P(witness_correct) = P(correct_id)P(blue)/(P(correct_id)P(blue)+P(incorrect_id)P(green))

$$P(witness\ correct) = \frac{(0.8)(0.15)}{(0.8)(0.15)+(0.2)(0.85)} = 0.413$$

The probability that the witness correctly identified a blue cab is only about 41.3%. This isn't a particularly high probability for a testimony.