Homework 1.2 Sample Space

1. An experiment involves tossing a coin 3x. What is the sample space of this experiment?

   The formula for sample space is k^n, where k is the possible outcomes, and n is the number of trials. Assuming this is a fair coin, then there are two possible outcomes, one for heads and one for tails. As stated in the problem, there are 3 trials, so the sample space for this experiment should be, $2^3 = 8$.

2. How many of the outcomes in the sample space had two heads?
   This can be determined using this formula:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

   Again, k is the possible outcomes –considering it's restricted to 2 heads—and n is the number of trials.

$$\binom{3}{2} = \frac{3!}{2!\,(3-2)!} = \frac{6}{2} = 3$$

   Those 3 possibilities are, HHT, HTH, THH

3. Define event A to be that the experiment yields two heads. Define event B as the experiment yielding two tails. What is $A \cup B$ and $A \cap B$?

   The formula for intersection is:
$$P(A \cap B) = P(A) \times P(B)$$

   But since A and B require there to be at least 4 outcomes, and the experiment only includes 3 outcomes, the intersection of A (two heads) and B (two tails) is impossible. Therefore, the intersection of A and B is impossible.

$$P(A \cap B) = 0$$

   The formula for union is:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   We already know that:

$$P(A) = 3$$

And therefore,

$$P(B) = 3$$

Because, like outcomes with two tails is identical in formula to outcomes with 2 heads, in this experiment.

And lastly, from above:

$$P(A \cap B) = 0$$

Putting these together:

$$P(A \cup B) = 3 + 3 - 0 = 6$$