Unit-2 Regular Languages

• Regular Expressions:

Finite Automata

↓

Regular Language

Reg Grammar (LLR, RLG) | Regular Expression

• Regular Expression are useful for certain sets of strings in an algebric fashion.

→ Regular Expressions describes the language accepted by finite automata.

• ~~Definition of~~ **Definition of RE over Σ:**

→ Any terminal symbol or element of Σ is a regular expression.

eg: ϕ, ϵ, a in Σ

* ϕ is a R.E and denoted the Empty set {}, {ϕ}
* ϵ is a RE and denotes the {ϵ}
* a is a RE and denotes the {a}.

→ Union of 2 R.E R_1 and R_1 is a R.E. ~~$(R_1 \cup R_2)$~~

R $(R = R_1 + R_2)$

→ Concatenation of 2 R.E R_1 and R_2 is also a R.E

R $(R = R_1 \cdot R_2)$

→ Iteration of a closure of a RE is also a R.E denoted as R^*.

• Regular Sets:

→ Any set Represented by R.E is called a Regular Set

→ If a and b are elements Σ, then R.E a denotes the set {a}. then $a+b$ denotes the set {a, b}

~~thu~~ $a \cdot b$ denotes the set $\{a, b\}$. a^* denotes the set $\{\epsilon, a, aa, aaa, \ldots\}$. a^+ denotes the set $\{a, aa, aaa, \ldots\}$. $(a+b)^*$ denotes $\{\epsilon, a, b, ab, aa, \ldots\}$
$(ab)^*$ denotes $\{\epsilon, ab, abab, ababab, \ldots$

• Examples of Regular Expressions:

1) All strings of 0's and 1's.
A) RE- $(0+1)^*$

2) set of all strings of 0's and 1's ending with 00
A) RE- $(0+1)^*00$

3) set of all strings of 0's and 1's begin with '0' ends with '1'
A) RE- $0(0+1)^*1$

4) set of all strings ~~of~~ having even no. of 1's.
RE $= (1 \cdot 1)^*$

5) set of all strings ~~of~~ odd no. of 1's.
RE $= 1 \cdot (11)^*$

6) set of all strings of 0's & 1's with atleast 2 consecutive 0's.

$$(0+1)^* 00 (0+1)^*$$

7) All strings ends with 011
$(0+1)^* 011$

8) Set of all strings of 0's & 1's begin with 1 or 0 not having 2 consecutive 0's.
~~$(0+1)^*$~~ ~~$(10)^*(1)^*(10)^*$~~ $(0+\epsilon)(10+1)^*$.

9) All strings with 0's followed by 1's and 1's followed by 2's such that atleast one 0 followed by at least one 1 & atleast 1 followed by atleast 2.

$(0)^+(1)^+(2)^+$

10) All strings of 0's & 1's whose last 2 symbols are same.

$$\cancel{(0+1)^*(00)^*+(11)^*} \Rightarrow (0+1)^*(00+11)$$

11) $\Sigma = \{a, b\}$ whose length is atleast 2.

$$(a+b)(a+b)(a+b)^*$$

12) length atmost 2. $\cancel{(a+b)(a+b)}$ $\cancel{(a+\epsilon)(b+\epsilon)(a+b)}$

$$(a+b+\epsilon)(a+b+\epsilon)$$

13) $\Sigma = (a, b)$ no. of a's exactly 2.

$$b^* \underline{a} b^* \underline{a} b^*$$

14) $\Sigma = (a, b)$ atleast 2 a's : $b^* a b^* a (a+b)^*$.

15) $\Sigma = (a, b)$ starting and ending with different symbols

$$(a(a+b)^*b)+(b(a+b)^*a)$$

- <u>Identity</u> Rules for Regular Expressions:

P and Q are two regular Expression, then to simplify the RE the following identity Rules are used.

(i) $\phi + R = R$

(ii) $\epsilon + R = R + \epsilon$

(iii) $R + R = R$

(iv) $R \cdot R \neq R$

(v) $\epsilon \cdot R = R \cdot \epsilon = R$

(vi) $R \cdot R^* = R^* \cdot R = R^+$

(vii) $R^* \cdot R^* = R^*$

(viii) $\epsilon + R \cdot R^* = R^*$

(ix) $\phi \cdot R = R \cdot \phi = \phi$

(x) $\epsilon^* = \epsilon$ $\phi^* = \epsilon$

(xi) $(R^*)^* = R^*$

(xii) $(PQ)^*.P = P(QP)^*$

(xiii) $(P+Q)R = PR + QR$

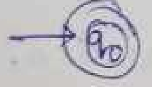
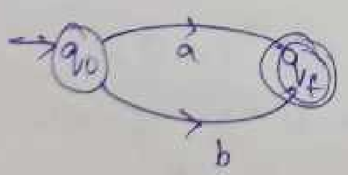
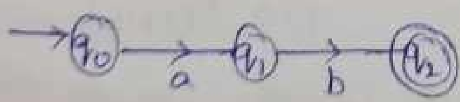
(xiv) $(P+Q)^* = (P^*.Q^*)^* = (P^*+Q^*)^*$

(xv) $(P+Q)^*.P^*.Q^* = (P+Q)^*$

- **Equivalance of FA with RE:**

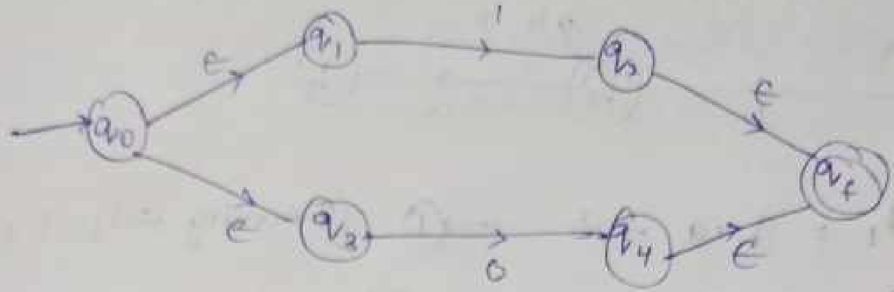
→ RE & FA are equivalant in their descriptive power, the Equivalance can be proved by mathematical induction on the size of RE ie., for any give RE, we can construct its equivalant FA that recognizes the language it describes & vice-versa.

- **Relation b/w FA, RE and Regular Sets:**

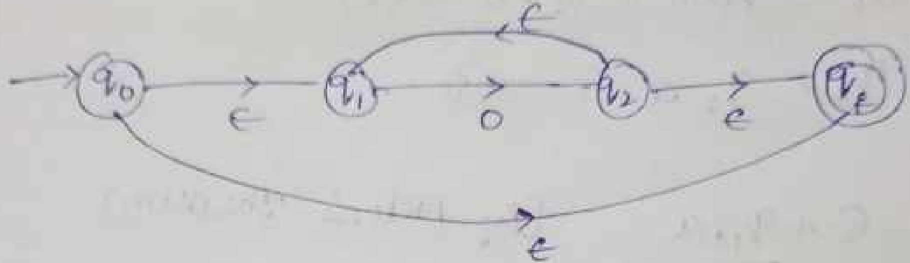
Sno	F.A	R.E	R.S
1.		ϕ	$\{\phi\}$
2.		ϵ	$\{\epsilon\}$
3.		a	$\{a\}$
4.		$a+b$	$\{a,b\}$
5.		$a \cdot b$	$\{ab\}$
6.		a^*	$\{\epsilon, a, aa, aaa \ldots\}$
7.		a^+	$\{a, aa, aaa, \ldots\}$
8.		$b^* a (a+b b^* a)^*$ $(b^* a a+b)^* b^* a$	

- Conversion of RE to ε-NFA:

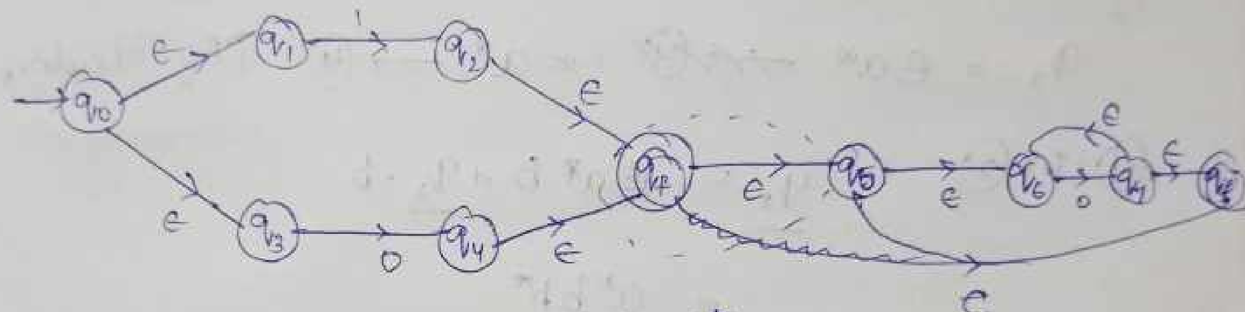
1) Construct ε-NFA for the RE $(1+0)0^*$



(1+0)

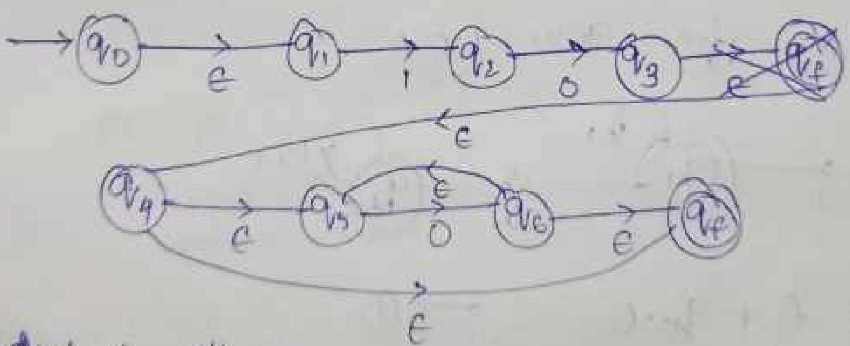


0^*



$(1+0)^*$

2) $(1.0)0^*$



- Arden's Theorem: (DFA to RE)

→ This theorem is useful to simplify RE, let C and B be a RE over ~~Σ~~ Σ and let A be unknown.

→ If C doesnot contain ε then eqⁿ for A given as $\boxed{A = B + AC}$ as a unique solⁿ.

given by $\boxed{A = BC^*}$

• Example: Construct RE for a given DFA

$q_1 = q_1 \cdot a + \epsilon \rightarrow (1)$ (Incoming edge & state)

$q_2 = q_1 \cdot b + q_2 \cdot b \rightarrow (2)$

$q_3 = q_2\, a \rightarrow (3)$

$q_1 = \epsilon + q_1 \cdot a$ [By Arden's theorem]

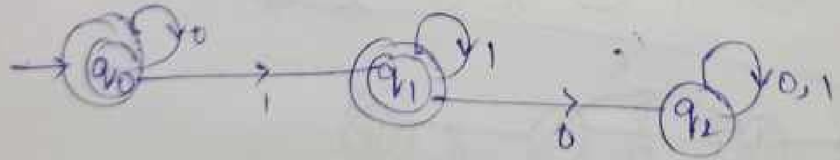
$q_1 = \epsilon a^* \cancel{\rightarrow (4)} = a^* \rightarrow (4)$ [By identity

(4) in (2) $\quad q_2 = a^* b + q_2 \cdot b$

$= a^* b . b^*$

$\boxed{RE = a^* b\, b^*}$

(Q) construct RE for given DFA.



$q_0 = \epsilon + q_0 \cdot 0 \rightarrow (1)$

$q_1 = q_0 \cdot 1 + q_1 \cdot 1 \rightarrow (2)$

$q_2 = q_1 \cdot 0 + q_2 \cdot 0 + q_2 \cdot 1 \rightarrow (3)$

$q_0 = \epsilon + q_0 \cdot 0 = \epsilon \cdot 0^* = 0^* \rightarrow (4)$

(4) in (2) $\quad q_1 = 0^* \cdot 1 + q_1 \cdot 1$

$= 0^* \cdot 1 \cdot 1^*$

final state $\Rightarrow q_0 + q_1$

$= 0^* + 0^*.1.1^*$

$= 0^*(\epsilon + 1.1^*)$ By identity Rule

$= 0^* 1^*$

(B)

$A = \epsilon + A\cdot 1 \cancel{\rightarrow (1)} + B\cdot 1 \rightarrow (1)$

$B = A\cdot 0 + B\cdot 0 \rightarrow (2)$

$\cancel{A = \epsilon + A\cdot 1 = \epsilon\cdot 1^*}$ $\qquad B = A\cdot 0\cdot 0^* \rightarrow (3)$ [By AT]

$A = (\epsilon + B\cdot 1) + A\cdot 1$

$=$

(3) in (1) $\Rightarrow A = 1\cdot A + 1\cdot A\cdot 0\cdot 0^* + \epsilon$.

$= A(1 + 1\cdot 0\cdot 0^*) + \epsilon$

$= A(1(\epsilon + 0\cdot 0^*)) + \epsilon$

$A = A(1(0^*)) + \epsilon = A10^* + \epsilon$

$A = \cancel{A\cdot 1\cdot 0^*} = (10^*)^*$

$B = A\cdot 0\cdot 0^*$

$RE = (10^*)^* 0\cdot 0^*$

(C)

$q_1 = \epsilon + q_1\cdot a \rightarrow (1)$

$q_2 = q_1\cdot b + q_2\cdot a \rightarrow (2)$

$q_3 = q_2\cdot b \rightarrow (3)$

$q_1 = \epsilon a^* = a^* \rightarrow (4)$

(4) in (2)

$q_2 = a^* b + q_2\cdot a$

$q_2 = a^* b a^* \rightarrow (5)$

(5) in (3)

$q_3 = a^* b a^* b$

Pumping Lemma:

→ It is used to a language is not Regular

→ Let $M = \{Q, \Sigma, \Delta, q_0, F\}$ be a FA with n states with a Regular language accepted by M.

→ Let $w \in L$ and their exist x, y, z such that $w = xyz$ and $xy^iz \notin L$ for each $i \geq 0$.

→ Applications:

Step-1: Assume M be the RE and n is the no. of states in FA.

Step-2: choose the string w such that $|w| \leq n$ use pumping Lemma to write $w = xyz$ with the conditions:

i) $|xy| < n$

ii) $|y| > 0$

Step-3: Find the suitable integer i such that xy^iz (doesnot (belongs to L) $\in L$ and hence L is not Regular.

example:

$L = \{0^{i^2} / i \geq 1\}$

$L = \{0^1, 0^4, 0^9, \ldots\}$

$L = 0^{i^2}$

$0^{i^2-3} \; 0^1 \; 0^2 = 0^{i^2-3}(0^1)^i 0^2$

$i=2 \Rightarrow 0^1 0^2 0^2 = \underline{0^5 \notin L}$. Hence proved.

(8) $L = \{1^p \mid p \text{ is a prime}\}$ S.T L is not Reg.

$L = \{1^2, 1^3, 1^5, 1^7, \ldots\}$

$L = 1^p$

1^{p-5} 1^2 1^3

x y z.

$= 1^{p-5} (1^2)^i (1^3)$

$x \quad y^i \quad z$.

$i = 2, \quad 1^{p-5} 1^4 1^3 = 1^{p+2}$

$p = 2, \quad 1^{2+2} = \underline{1^4 \notin L}$.

7M

- Closure properties of R.E: (10)

→ RE are closed under union operation.

→ RE are " " intersection operation

→ " " " " Concatenation.

→ " " " " Kleen closure.

→ " " " " Difference.

→ " " " " Complement

→ " " " " Reverse.

→ " " " " Symmetric Difference

→ " " " " Homomorphism.

→ " " " " Inverse Homomorphism.