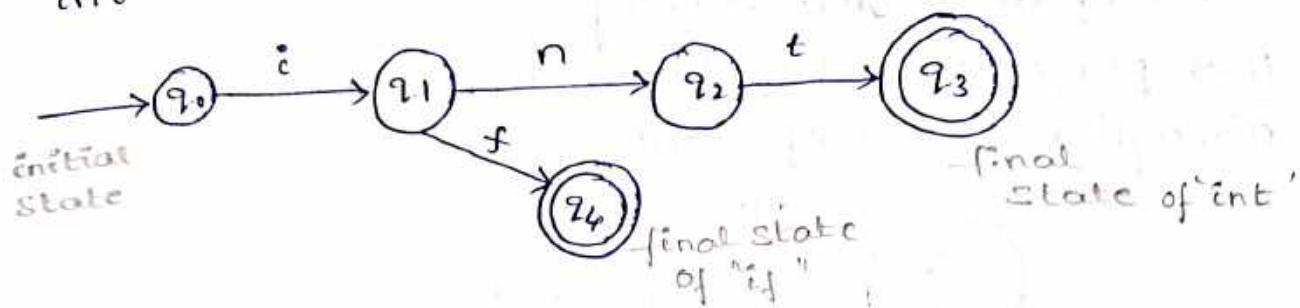


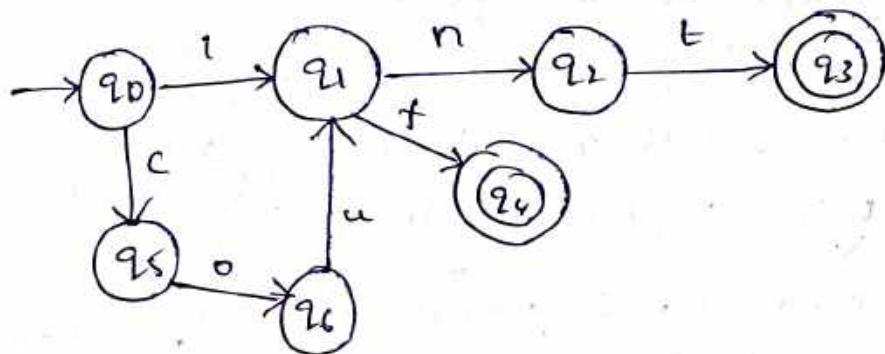
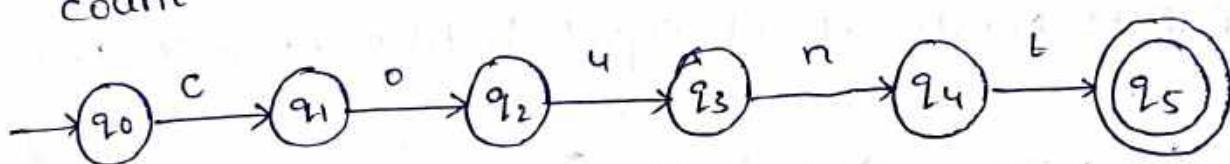
Unit-1

→ states:

int



count

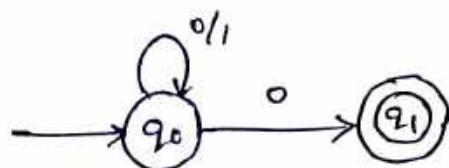


Q:- A string contains '0' at last position

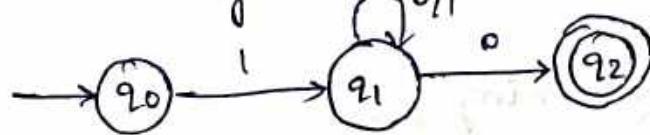
If the question is given as above it means the alphabet set contains only $\{1, 0\}$

'0' can be one such string. if we don't bother about the input character then there will be no change in state. It will be at self state.

$\{0, 00, 10\}$



Q:- string starts with '1' and ends with '0'

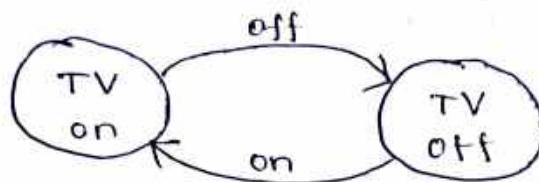


Automata:- It describes abstract model of emission

Ex: elevator, escalator etc..

Basics of automata theory:-

that performs computation on inputs by moving through a series of states.



Alphabet:- Finite set of symbols like english char, digits, special characters etc... (images, other languages)
→ It is represented with Σ .

Ex:- Binary alphabet $\Sigma = \{0, 1\}$

for TV mentioned before $\Sigma = \{\text{on}, \text{off}\}$

If machine contains octal number supporting system $\Sigma = \{0 \text{ to } 7\} = \{0, 1, 2, 3, 4, 5, 6, 7\}$

String:- It is an ordered sequence of characters from an alphabet which can be represented by w .

Ex:- If $\Sigma = \{0, 1\}$ $w = \{0, 1, 01, 10, 00, 11, 100, \dots\}$

The length of the string can be represented by $|w|$. $|w_1| = 1$ $|w_3| = 2$ $|w_4| = 2$

Language:- It is a set of strings of symbols from alphabet set, with condition based.

$L = \{w_1, w_2, w_3, w_4, \dots\} = \{0, 1, 10, 01, \dots\}$

Applications of Finite automatas-

1. Welding machines

2. Traffic lights

3. Video games

4. Text passing

5. Regular expression matching

8. protocol analysis

3. Natural language processing (NLP) etc.

→ The empty string with zero occurrences of symbols is represented by " ϵ ". (epsilon)

so $L = \{\epsilon, w_1, w_2, w_3, \dots\}$

→ Σ^* - it is a set of strings including empty string over the alphabet Σ .

$\Sigma^* = \{\epsilon, 0, 1, 10, 00, 01, 11, 100, \dots\}$

also called as universal language.

→ difference b/w L & Σ^* is L takes string based on certain conditions but Σ^* has all possible strings including ϵ .

→ Σ^+ - it is a set of non-empty strings except empty string i.e., ϵ .

$\Sigma^+ = \{0, 1, 10, 00, 01, \dots\}$

$$\boxed{\Sigma^* = \Sigma^+ \cup \epsilon}$$

Ex:- Set of strings over the $\Sigma = \{0, 1\}$ with equal no. of 0's and equal no. of 1's.

$\Sigma = \{0, 1\}$.

$\Sigma^* = \{\epsilon, 0, 1, 00, 11, 01, 10, 000, 001, 010, 011, 100, 101, 110, 111, 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, \dots\}$

$L = \{01, 10, 0011, 0101, 0110, 1001, 1010, \epsilon, \dots\}$

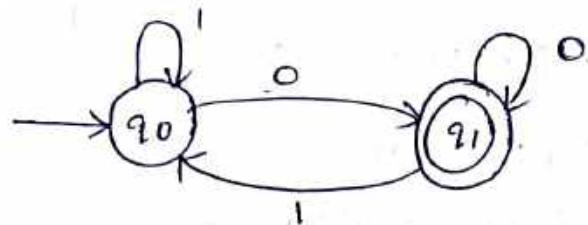
→ if ϵ is acceptable string the initial state will be final state

Ex:- Construct L with strings which should ends with '0'.

$$L = \{ 0, 00, 10, 000, 110, 100, 010, \dots \}$$

$$\Sigma^* = L \cup L'$$

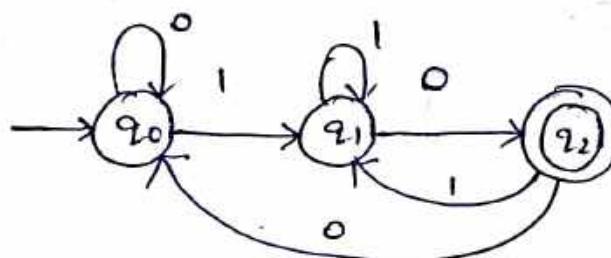
$$L' = \{ \epsilon, 1, 11, 01, 001, 011, 111, 101, \dots \}$$



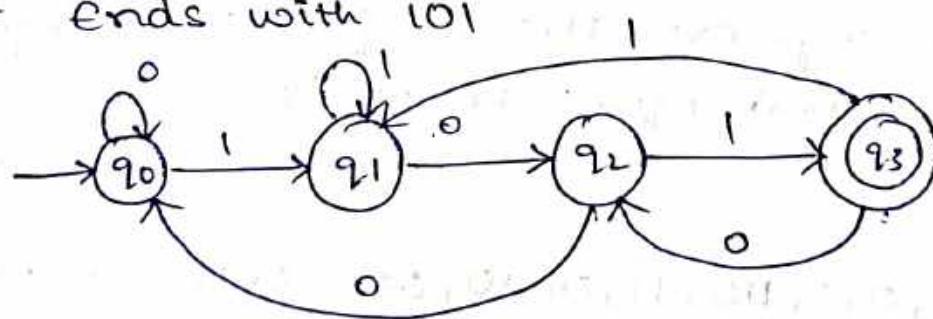
Ex:- construct L for strings ends with 10

$$L = \{ 10, 010, 110, 0010, 1110, 0110, 1010, \dots \}$$

$$L' = \{ \epsilon, 0, 1, 00, 11, 01, 001, 101, 111, 000, \dots \}$$



Ex:- ends with 101



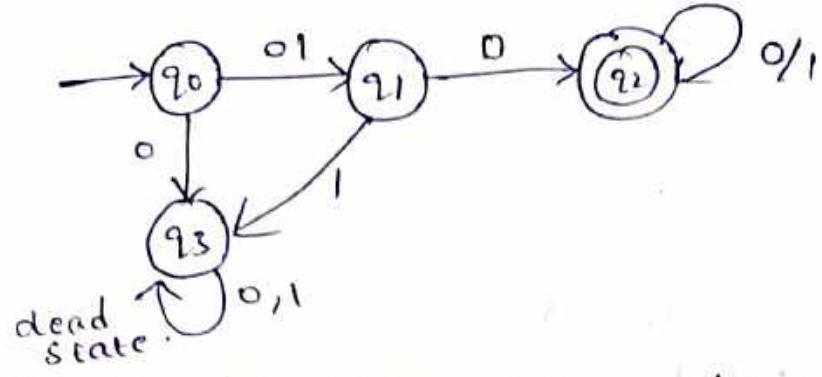
$$L = \{ 101, 0101, 1101, 00101, 11101, 01101, \dots \}$$

$$L' = \{ \epsilon, 010, 001, 111, 1010, \dots \}$$

Ex:- starts with 10

$$L = \{ 10, 101, 100, 1011, 1000, 1001, 1010, \dots \}$$

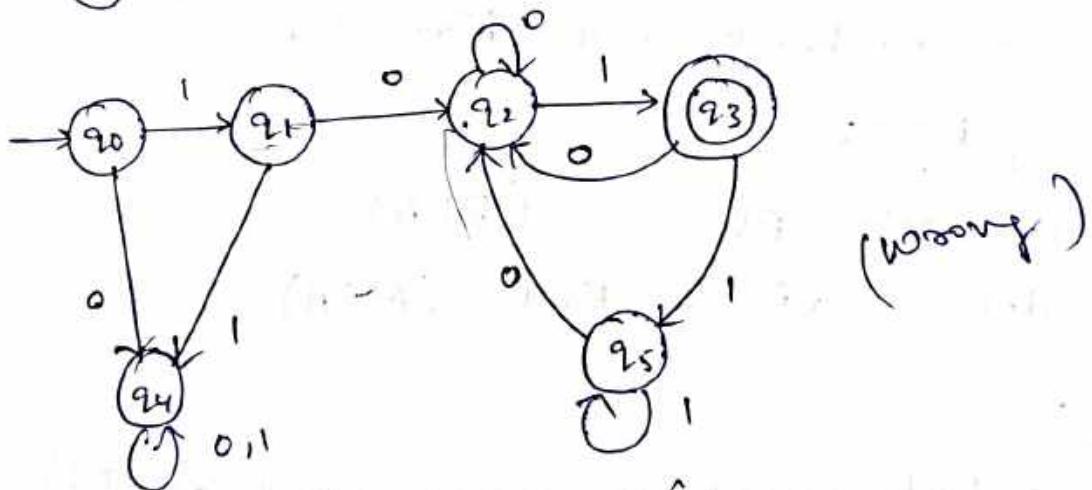
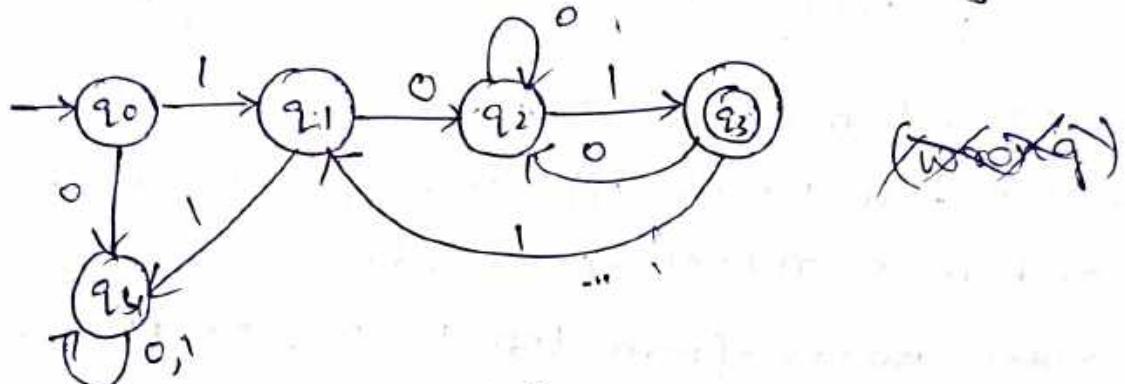
$$L' = \{ \epsilon, 0, 1, 01, 11, 00, 111, \dots \}$$



Ex1 - strings with 10 ends with 01

$$L = \{101, 1001, 10101, 10001, \dots\}$$

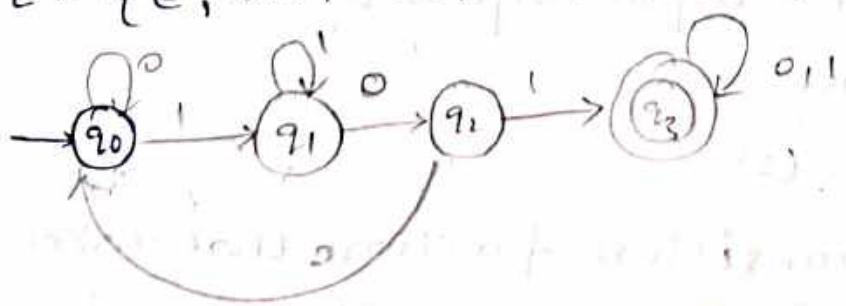
$$L' = \{\epsilon, 010, 0110, 01110, \dots\}$$



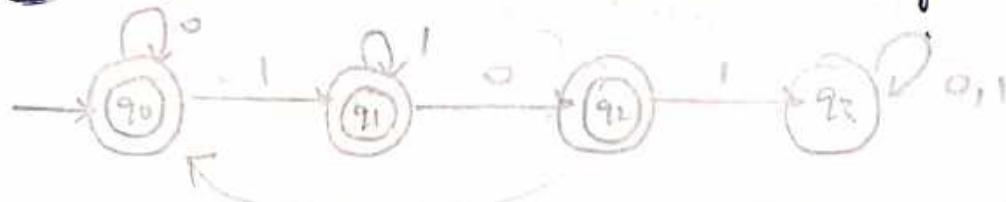
Ex2 Substring is 101

$$L = \{101, 0101, 1101, 1010, 1011, \dots\}$$

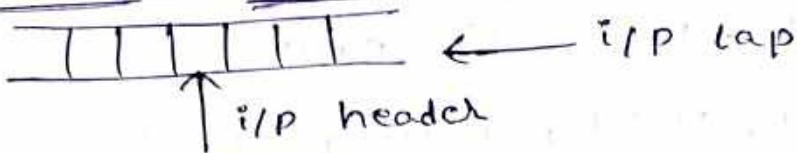
$$L' = \{\epsilon, 010, 001, 011, \dots\}$$



Ex:- not contains 101 as substring.



→ Finite automata (or) FSA



Basic Structure
of finite automata

finite control machine contains all control transitions. It takes i/p, changes state if required and moves forward.

⇒ Header moves from left to right. It doesn't move in backward direction.

* Types of FSA :-

① Deterministic FSA (DFA)

② Non-deterministic FSA (NFA)

① DFA :-

It is a 5-tuple machine represented as 'M'.

$$M = (Q, \Sigma, \delta, q_0, F)$$

[Here each & every i/p symbol should be used in DFA]

Q - finite set of states

Σ - finite set of input symbols

q_0 - initial state

F - final state(s)

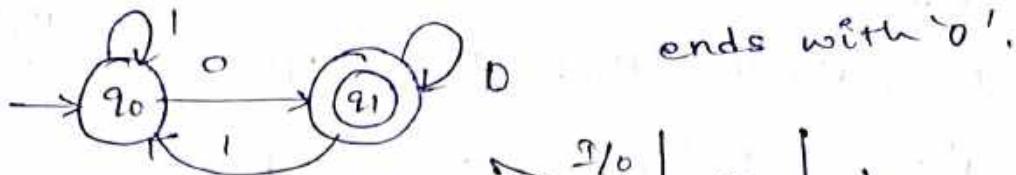
δ - it is a transition function that takes 2 arguments which are state and i/p symbol then returns a state as output.

$$\text{Ex:- } \delta(q_0, 0) = q_1$$

→ Mapping function for DFA :-

$$Q \times \Sigma \rightarrow Q$$

→ Transition table construction :-



ends with '0'.

$$\delta(q_0, 0) = q_1$$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_1, 0) = q_1$$

$$\delta(q_1, 1) = q_0$$

states	0		1
	q0	q1	q0
q0	q1	q0	q0
q1	q1	q0	q0

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{q_0, q_1\}, \Sigma = \{0, 1\}, q_0 = q_0, F = q_1$$

check for 1010

$$\delta(q_0, 1010) \vdash \delta(q_0, 010)$$

$$\vdash \delta(q_1, 10)$$

$$\vdash \delta(q_0, 0)$$

$$\vdash q_1 \rightarrow F = q_1 \text{ So it is acceptable i/p.}$$

check for 101

$$\delta(q_0, 101) \vdash \delta(q_0, 01)$$

$$\vdash \delta(q_1, 1)$$

$$\vdash q_0 \rightarrow F \neq q_0 \text{ So it is not-acceptable string}$$

Steps to answer:-

1. Construction of DFA

2. Write the tuples with transition functions

3. Write the transition table.

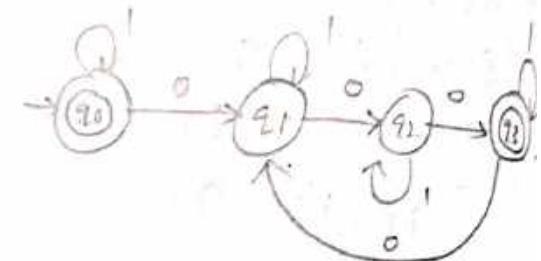
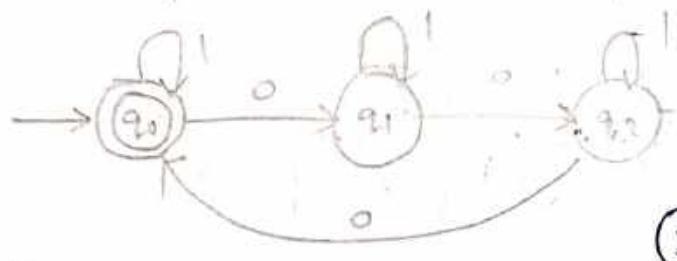
4. Acceptance string example.

5. Non-acceptance string example.

Ex: Construct the FA which accepts set of strings where no. of 0's in every string is in multiples of 3. over the Σ alphabet set $\Sigma = \{0, 1\}$.

$L = \{ \in, 000, 1000, 10001, 0100, 0010, 0001, \dots \}$

$L = \{ 0^3, 0^6, 0^9, \dots \}$



$$\textcircled{1} \quad M = (Q, \Sigma, \delta, q_0, F)$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0 \quad F = q_0$$

$$\textcircled{2} \quad \delta(q_0, 0) = q_1$$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_1, 1) = q_1$$

$$\delta(q_2, 0) = q_0$$

$$\delta(q_2, 1) = q_2$$

States	I/P	
	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_0	q_2

$\textcircled{3}$ check for 10001

$$\delta(q_0, 10001) \vdash \delta(q_0, 0001)$$

$$\vdash \delta(q_1, 001)$$

$$\vdash \delta(q_2, 01)$$

$$\vdash \delta(q_0, 1)$$

$\textcircled{4}$ 110

$$\delta(q_0, 110) \vdash \delta(q_0, 10)$$

$$\vdash \delta(q_0, 0)$$

$$\vdash q_1 \neq F$$

(not acceptable)

$$\vdash q_0 = F$$

(acceptable)

ex:- string having even no's 0's.



$$L = \{ \epsilon, 1, 11, 00, 010, 100, 001, \dots \}$$

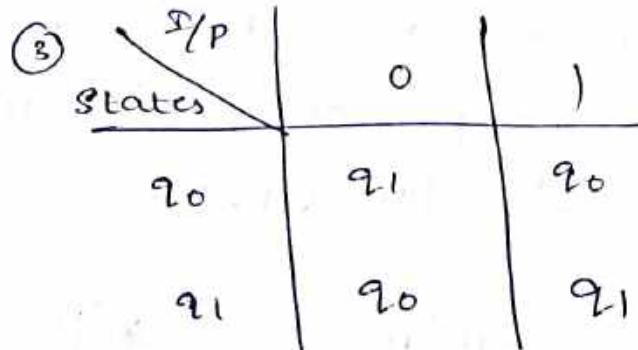
$$L' = \{ 000, 1000, 0100, \dots \}$$

① $M = (Q, \Sigma, \delta, q_0, F)$

$$Q = \{ q_0, q_1 \}$$

$$\Sigma = \{ 0, 1 \}$$

$$q_0 = q_0 \quad F = q_0$$



② $\delta(q_0, 0) = q_1$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_1, 0) = q_0$$

$$\delta(q_1, 1) = q_1$$

④ check for

$$10011$$

$$\delta(q_0, 10011) \vdash \delta(q_0, 0011)$$

$$\vdash \delta(q_1, 011)$$

$$\vdash \delta(q_0, 11)$$

$$\vdash \delta(q_0, 1)$$

$$\vdash \delta(q_0) = F$$

(acceptable)

$$\vdash \delta(q_1, 1100)$$

$$\vdash \delta(q_1, 100)$$

$$\vdash \delta(q_1, 00)$$

$$\vdash \delta(q_0, 0)$$

$$\vdash q_1 \notin F$$

(unacceptable).

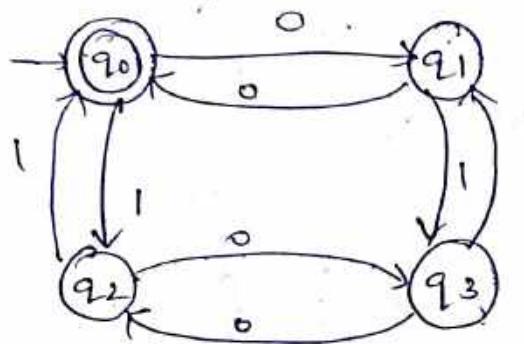
→ conditions for DPA

1. The each & every i/p alphabet over Σ should be used in transition functions.
2. Used only once (i/p alphabet).

Ex:- Even no. of 0's and even no. of 1's.

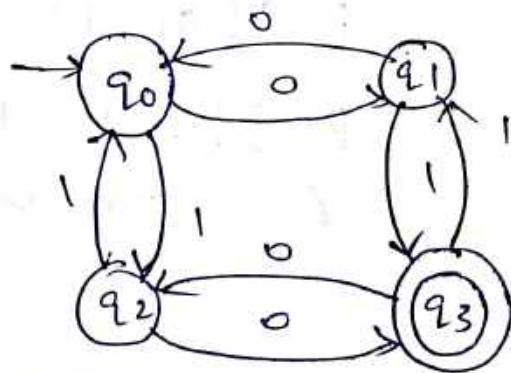
$$L = \{ \epsilon, 0011, 00111, 1100, \dots \}$$

$$L' = \{ 10, 01, 1000, \dots \}$$

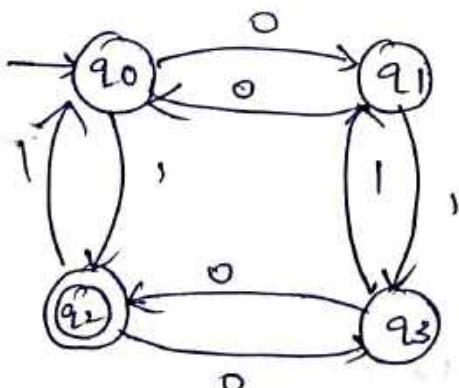


→ odd no. of 0's odd no. of 1's

$$L = \{ 01, 10, 0111, 0001, \dots \}$$



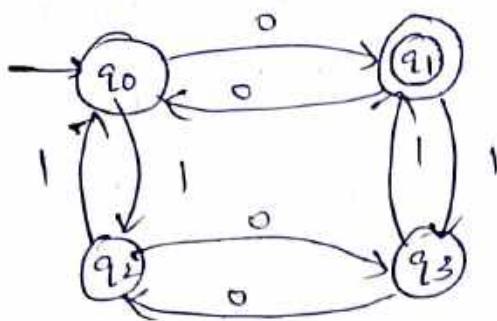
→ even no. of 0's odd no. of 1's.



$$L = \{ 1, 001, 111, 010, \dots \}$$

→ odd no. of 0's even no. of 1's

$$L = \{0, 000, 11000, 110, 101, \dots\}$$



Assignment:-

① string ends with aab or aaba

② Construct language $L = \{(ab)^n / n \geq 0\}$

over $\Sigma = \{a, b\}$

$$L = \{\epsilon, ab, abab, ababab, \dots\}$$

$$L' = \{a, b, aa, bb, \dots\}$$

③ $L = \{w \in \{0,1\}^* / \begin{cases} 3^{\text{rd}} \text{ symbol is } 0 \\ 5^{\text{th}} \text{ symbol is } 1 \end{cases}\} \quad [\star \text{ means } \geq 0]$

$$L = \{00\underline{0}01, 11\underline{0}11, 11\underline{0}01, \dots\}$$

② NFA :-

It is a 5 tuple machine represented as 'M'.

$$M = (Q, \Sigma, \delta, q_0, F)$$

Q - finite set of states

Σ - alphabet set

q_0 - initial state

F - final set of states

Transition function that takes states as argument and returns power set of Q is called as δ .

→ mapping function

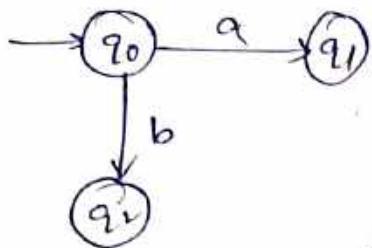
$$Q \times (\Sigma \cup \epsilon) \rightarrow 2^Q$$

$$Q \times \Sigma^* \rightarrow 2^Q$$

→ Conditions for NFA

1. Input Symbols can be used any no. of times.
2. each & every alphabet over Σ need not be used. $\rightarrow \epsilon$ can be used as input.

Ex:-



$$\delta(q_0, a) = \epsilon$$

$$\delta(q_0, a) = q_0$$

$$\delta(q_0, a) = q_1$$

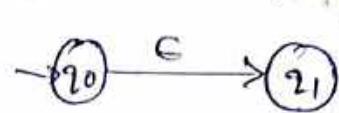
$$\delta(q_0, a) = q_2$$

$$\delta(q_0, a) = \{q_0, q_1\}$$

$$\delta(q_0, a) = \{q_0, q_2\}$$

$$\delta(q_0, a) = \{q_1, q_2\}$$

$$\delta(q_0, a) = \{q_0, q_1, q_2\}$$



no. of states = 3 (Q)

So max. no. of combinations = $2^Q = 2^3 = 8$.

Ex:- $Q = \{q_0, q_1\}$.

$$\delta(q_0, Q) = \epsilon$$

$$\delta(q_0, a) = q_0$$

$$\delta(q_0, a) = q_1$$

$$\delta(q_0, a) = \{q_0, q_1\}$$

Ex:- Design NFA for language $L = \{ \text{all the strings over } \Sigma = \{0, 1\} \text{ which has at least 2 consecutive 0's or 1's} \}$

has at least 2 consecutive 0's or 1's

$$L = \{00, 11, 100, 111, 000, 011, \dots\} \quad L' = \{\epsilon, 101, 010, \dots\}$$



Ex :- ends with 01

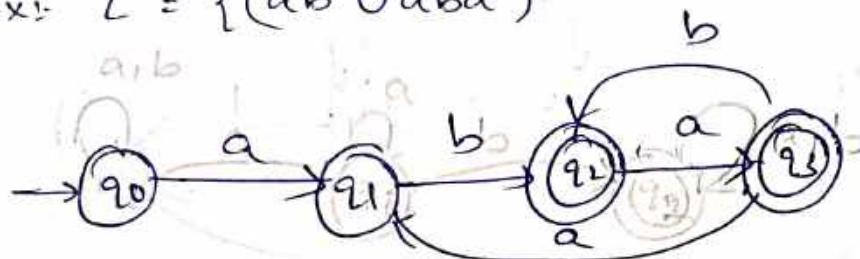
$$L = \{01, 101, 001, 0001, 0101, 1001, 1101, \dots\}$$

$$L' = \{\epsilon, 0, 1, 10, 11, 00, 100, \dots\}$$



0101
01101
1001

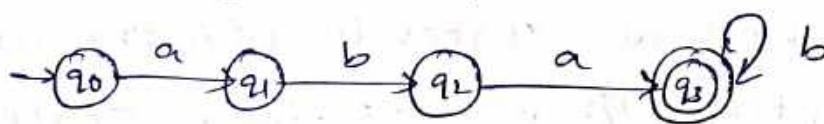
Ex :- $L = \{(ababa)^*\}^{multiple \ times.}$



abababab
acceptable

Ex :- Construct NFA for language

$$L = \{abab^n \mid n \geq 0\} \quad L = \{aba, abab, ababb, \dots\}$$



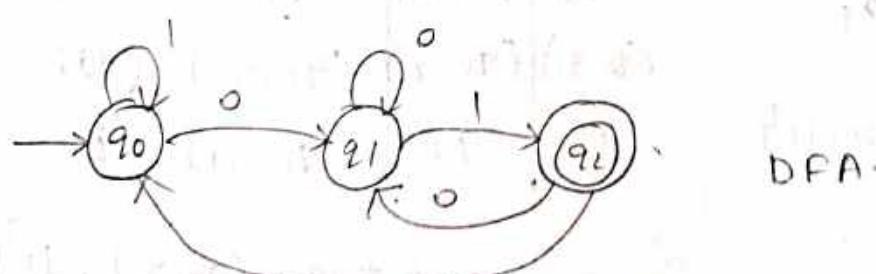
Ex :- Designing NFA & DFA accepting all strings ending with 01 over $\Sigma = \{0, 1\}$.

$$L = \{01, 101, 001, 1101, 0001, 1001, 0101, \dots\}$$

$$L' = \{\epsilon, 0, 1, 10, 11, 00, 111, 000, \dots\}$$



NFA



DFA

* Converting from NFA to DFA :-

Transition table - NFA

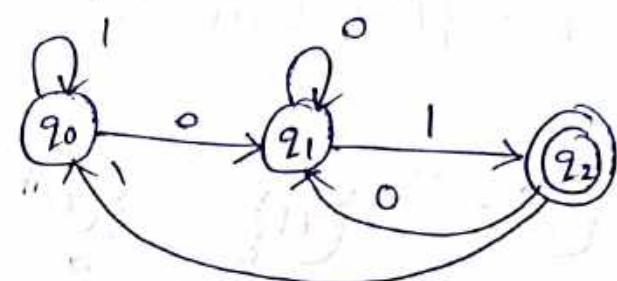
i/p States	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	\emptyset	q_2
(q_2)	\emptyset	\emptyset

Transition table for DFA

i/p states	0	1
$q_0 \rightarrow q_0$	(q_0, q_1)	q_0
q_1	(q_0, q_1)	(q_0, q_2)
(q_2)	(q_0, q_2)	(q_0, q_1)

rename

i/p States	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_0



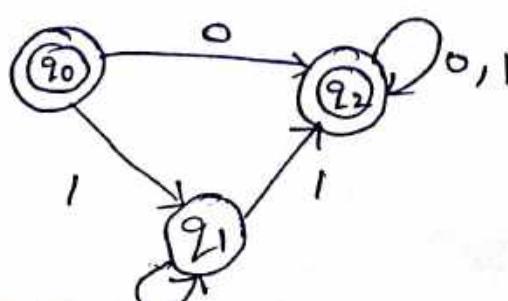
To determine final state, first consider final (q_2) state in NFA. Then all states in DFA transition table which contain q_2 will be final states.

Ex:- Convert following NFA to DFA.



i/p States	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_1
q_1	q_1	$\{q_0, q_1\}$

i/p States	0	1
$\rightarrow q_0$	(q_0, q_1)	q_1
(q_0, q_1)	(q_0, q_1)	(q_0, q_1)

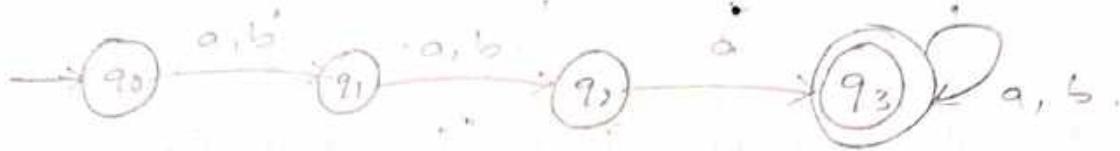


$$\begin{aligned}
 Q &= \{q_0, (q_0, q_1), q_1\} \\
 Q' &= \{q_0, q_2, q_1\} \\
 F &= \{q_0, (q_0, q_1)\} \\
 &= \{q_0, q_1\}
 \end{aligned}$$

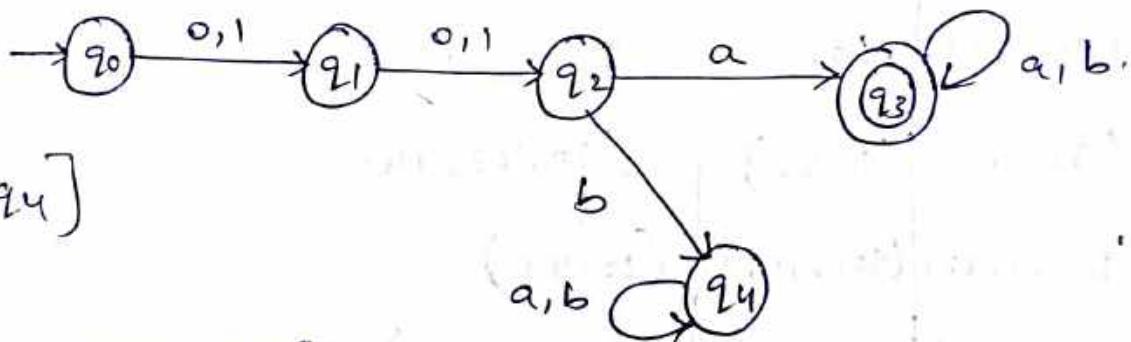
Ex:- NFA - 3rd character should be 'a'

$$\Sigma = \{a, b\}$$

L = {aaa, bba, aba, baa, aaaa, bbab,
abab, baab,
baaa, ...}.



if r states	a	b	r/p states	a	b
$\rightarrow q_0$	q_1	q_1	$\rightarrow q_0$	q_1	q_1
q_1	q_2	q_2	q_1	q_2	q_2
q_2	q_3	\emptyset	q_2	q_3	\emptyset
(q_3)	q_3	q_3	(q_4) \emptyset q_3	\emptyset	q_3



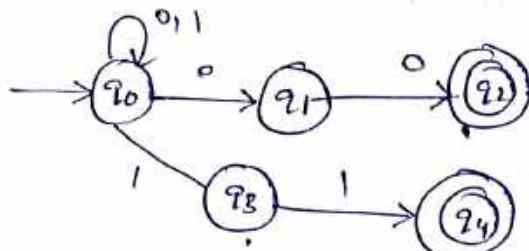
* Steps for conversion :-

1. Start with initial state.

2. After finding the transition of initial state only the resultant states into the list until no new state is added to the list

3. declare the states as final if it has atleast one final state of NFA.

Ex:-



i/p States	0	1	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_3\}$	q_0	(q_0, q_1)
q_1	q_2	\emptyset	(q_0, q_1)	(q_0, q_1, q_2)
q_2	\emptyset	\emptyset	(q_0, q_3)	(q_0, q_1, \emptyset)
q_3	\emptyset	q_4	(q_0, q_1, q_2)	$(q_0, q_1, q_2, \emptyset)$
q_4	\emptyset	\emptyset		

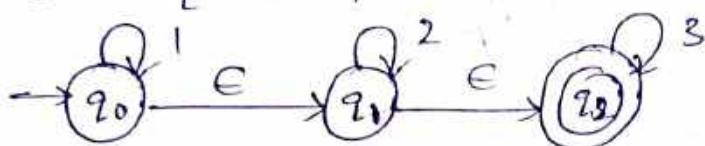
i/p States	0	1
$\rightarrow q_0$	(q_0, q_1)	(q_0, q_3)
q_1	(q_0, q_1)	(q_0, q_1, q_2)
q_2	(q_0, q_3)	(q_0, q_1, q_4)
q_3	(q_0, q_1, q_2)	(q_0, q_3)
q_4	(q_0, q_3, q_4)	(q_0, q_3, q_4)

* NFA with ϵ -moves:-

ex:- $L = \{1^* 2^* 3^* / \Sigma = \{1, 2, 3\}\}$

$$L = \{\epsilon, 1, 2, 3, 12, 13, 23, 112, \dots\}$$

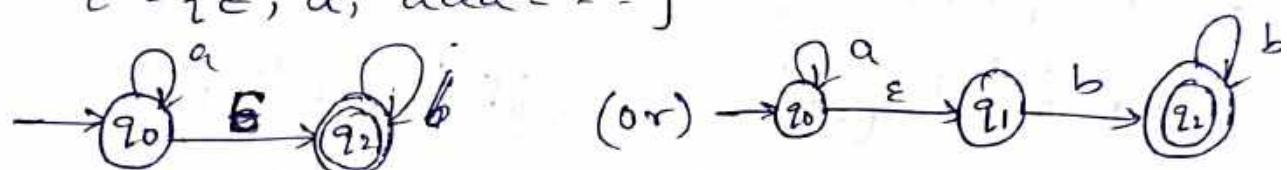
$$L' = \{21, 31, 32, \dots\}$$



ex:- $L = \{a^* b b^* / \Sigma = \{a, b\}\}$

$$L = \{b, ab, abb, bb, aab, aabb, aabbb, \dots\}$$

$$L' = \{\epsilon, a, aaa, \dots\}$$

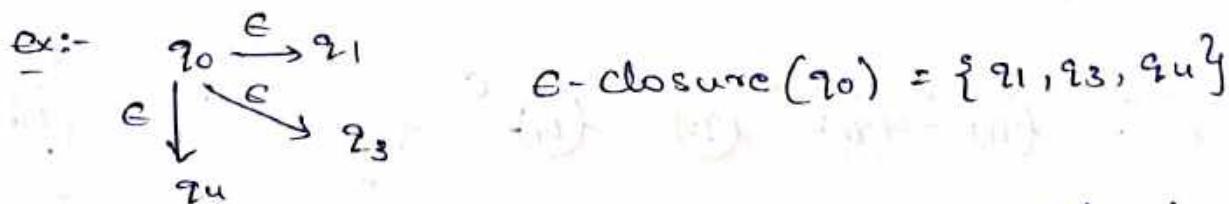


If 211 is i/p, it reaches q_2 by using ϵ , but it will not be accepted because the i/p string will not become empty.

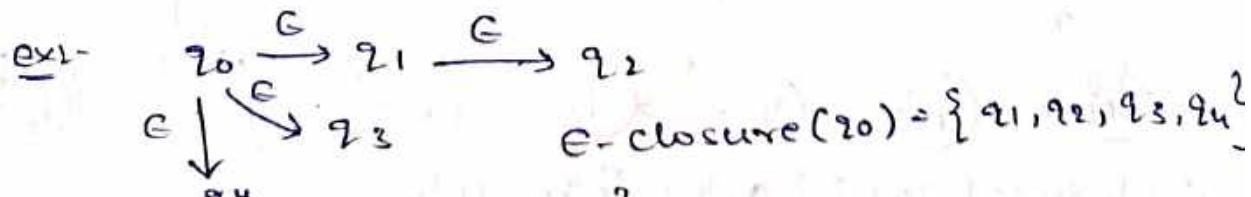
* ϵ -closure:- It is a set of all states which are reachable from state P on null transition (ϵ transition)

1. ϵ -closure of $P = x$, Here $x \in Q$ [x is a set of states]

$$\epsilon\text{-closure}(P) = x, x \in Q \quad [x \text{ is a set of states}]$$



2. If there exists $\epsilon\text{-closure}(P) = q$ and $S(q, \epsilon) = r$ and then $\epsilon\text{-closure}(P) = \{q, r\}$



3. $\epsilon\text{-closure}(P) = \{P, q, r\}$

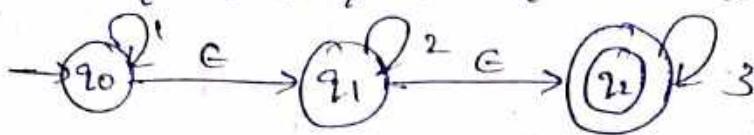
$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2, q_3, q_4\}$$

For last diagram

$$\epsilon\text{-closure}(q_1) = \{q_2\} = \{q_2\} = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_3\}.$$

Ex 2- $L = \{1^* 2^* 3^* / \Sigma = \{1, 2, 3\}\}$

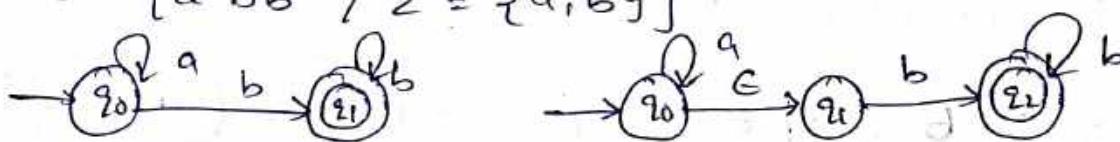


$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = \{q_2\} = \{q_2\} = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_3\}$$

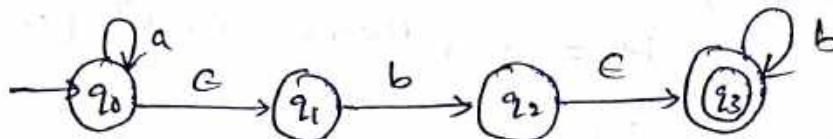
Ex 3- $L = \{a^* b b^* / \Sigma = \{a, b\}\}$



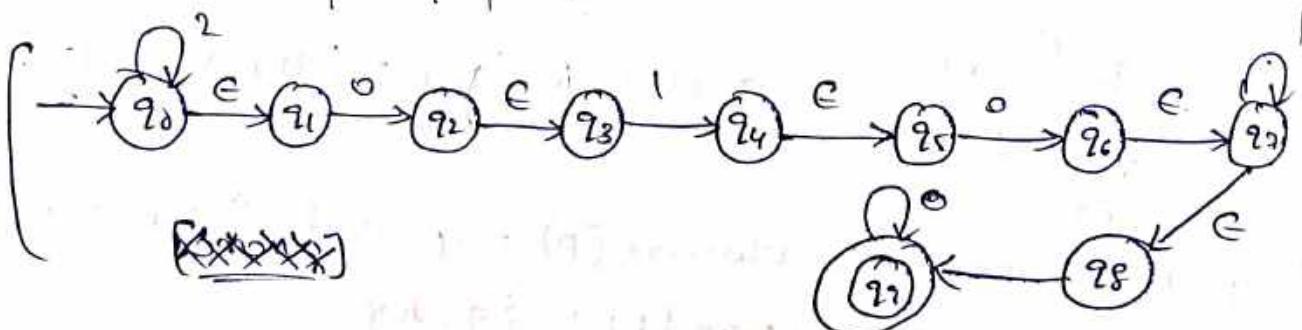
$$\epsilon\text{-closure}(q_0) = \{q_1\} = \{q_1\} = \{q_0, q_1\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$



Ex 4- $L = \{2^* 0101^* 0^*\}$



(or)



$$\epsilon\text{-closure}(q_0) = \{q_1\} = \{q_0, q_1\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\}$$

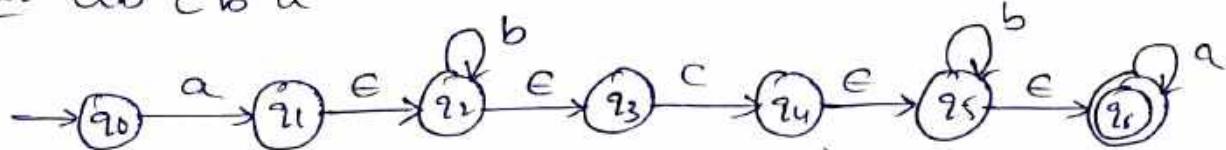
$$\epsilon\text{-closure}(q_3) = \{q_3\}$$

$$\epsilon\text{-closure}(q_4) = \{q_4, q_5, q_6\}$$

$$\epsilon\text{-closure}(q_5) = \{q_5, q_6\}$$

$$\epsilon\text{-closure}(q_6) = \{q_6\}$$

Ex:- $ab^*c b^*a^*$.



$$\epsilon\text{-closure}(q_0) = \{q_0\}$$

$$\epsilon\text{-closure}(q_1) = \{q_1, q_2, q_3\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2, q_3\}$$

$$\epsilon\text{-closure}(q_3) = \{q_3\}$$

$$\epsilon\text{-closure}(q_4) = \{q_4, q_5, q_6\}$$

$$\epsilon\text{-closure}(q_5) = \{q_5, q_6\}$$

$$\epsilon\text{-closure}(q_6) = \{q_6\}$$

* Conversion of NFA with ϵ -moves to without

ϵ -moves:

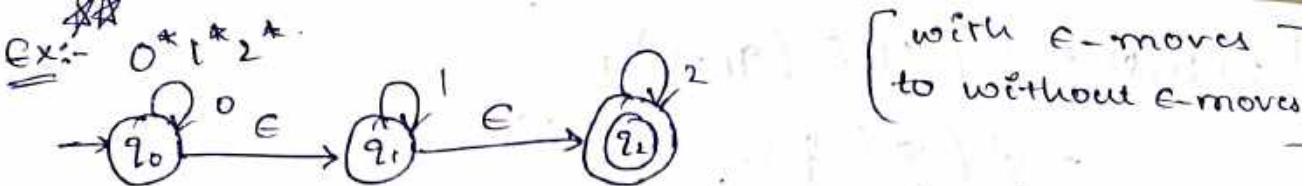
→ ϵ -closure denoted by δ'

$$\delta'(q, a) = \epsilon\text{-closure}(\delta(\delta'(q), a))$$

$$\begin{aligned} \text{Ex:- } \delta'(q_0, a) &= \delta'(\delta(\delta'(q_0), a)) \\ &= \delta'(\delta(q_0, a)) \\ &= \delta'(q_1) = \{q_1, q_2, q_3\} \end{aligned}$$

$$\begin{aligned} \delta'(q_0, b) &= \delta'(\delta(\delta'(q_0), b)) \\ &= \delta'(\delta(q_0, b)) \\ &= \delta'(\emptyset) = \emptyset \end{aligned}$$

$$\begin{aligned} \delta'(q_1, a) &= \delta'(\delta(\delta'(q_1), a)) \\ &= \delta'(\delta(\{q_1, q_2, q_3\}, a)) \\ &= \end{aligned}$$



[with ε-moves -
to without ε-moves -]

$$\epsilon\text{-closure } (q_0) = \{q_0\} = \{q_1, q_2\} = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure } (q_1) = \{q_2\} = \{q_1, q_2\}$$

$$\epsilon\text{-closure } (q_2) = \{q_2\}.$$

$$\begin{aligned}\delta'(q_0, 0) &= \delta'(\delta(\delta'(q_0, \epsilon), 0)) \\ &= \delta'(\delta(\{q_0, q_1, q_2\}, 0)) \\ &= \delta'(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)) \\ &= \delta'(q_0 \cup \emptyset \cup \emptyset) \\ &= \delta'(q_0) \\ &= \{q_0, q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\delta'(q_0, 1) &= \delta'(\delta(\delta'(q_0, \epsilon), 1)) \\ &= \delta'(\delta(\{q_0, q_1, q_2\}, 1)) \\ &= \delta'(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)) \\ &= \delta'(\emptyset \cup q_1 \cup \emptyset) \\ &= \delta'(q_1) \\ &= \{q_1, q_2\}\end{aligned}$$

$$\begin{aligned}\delta'(q_0, 2) &= \delta'(\delta(\delta'(q_0, \epsilon), 2)) \\ &= \delta'(\delta(\{q_0, q_1, q_2\}, 2)) \\ &= \delta'(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2)) \\ &= \delta'(\emptyset \cup \emptyset \cup q_2) \\ &= \delta'(q_2) \\ &= \{q_2\}\end{aligned}$$

$$\begin{aligned}
 s'(q_1, 0) &= s'(s(s'(q_1, \epsilon), 0)) \\
 &= s'(s(\{q_1, q_2\}, 0)) \\
 &= s'(s(q_1, 0) \cup s(q_2, 0)) \\
 &= s'(\emptyset \cup \emptyset) \\
 &= s'(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 s'(q_1, 1) &= s'(s(s'(q_1, \epsilon), 1)) \\
 &= s'(s(\{q_1, q_2\}, 1)) \\
 &= s'(s(q_1, 1) \cup s(q_2, 1)) \\
 &= s'(q_1 \cup \emptyset) \\
 &= s'(q_1) \\
 &= \{q_1, q_2\}
 \end{aligned}$$

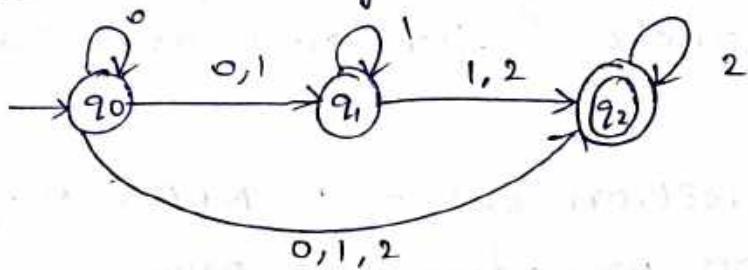
$$\begin{aligned}
 s'(q_1, 2) &= s'(s(s'(q_1, \epsilon), 2)) \\
 &= s'(s(\{q_1, q_2\}, 2)) \\
 &= s'(s(q_1, 2) \cup s(q_2, 2)) \\
 &= s'(\emptyset \cup q_2) \\
 &= s'(q_2) \\
 &= \{q_2\}
 \end{aligned}$$

$$\begin{aligned}
 s'(q_2, 0) &= s'(s(s'(q_2, \epsilon), 0)) \\
 &= s'(s(\{q_2\}, 0)) \\
 &= s'(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}s'(q_2, 1) &= s'(s(s'(s(q_2, \epsilon), 1))) \\&= s'(s(q_2, 1)) \\&= s'(\emptyset)\end{aligned}$$

$$\begin{aligned}s'(q_2, 2) &= s'(s(s'(s(q_2, \epsilon), 2))) \\&= s'(s(q_2, 2)) \\&= s'(q_2) \\&= \{q_2\}\end{aligned}$$

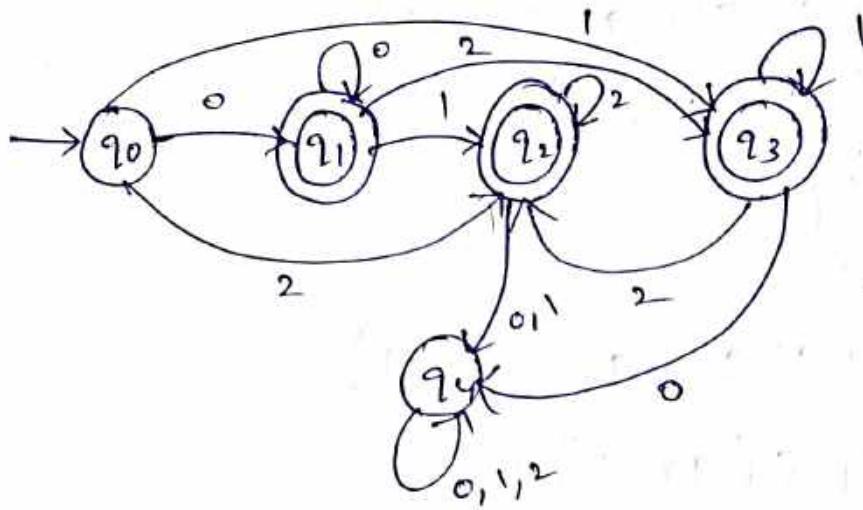
Transition diagram of NFA.



conversion to DFA.

States \ i/p	0	1	2
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	$\{\emptyset\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_2	\emptyset	\emptyset	$\{q_2\}$

States \ i/p	0	1	2	
q_0	(q_0, q_1, q_2)	(q_1, q_2)	(q_2)	$q_0 \leftarrow$
(q_0, q_1, q_2)	(q_0, q_1, q_2)	(q_1, q_2)	(q_2)	q_1
(q_1, q_2)	\emptyset	(q_1, q_2)	q_2	q_3
q_2	\emptyset	\emptyset	q_2	q_2
\emptyset	\emptyset	\emptyset	\emptyset	q_4



→ Convert with ϵ -moves to DFA

i. Draw diagram for equivalent language

ii. Calculate ϵ -closure of each & every state.

3. Compute or calculate δ' for each and every alphabet

4. Draw the transition table of NFA & diagram

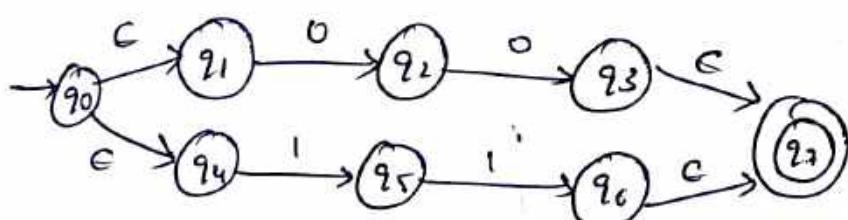
5. Start conversion procedure of DFA.

6. Transition table of DFA

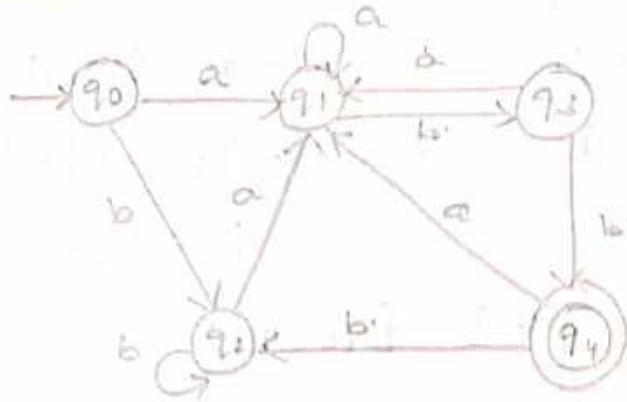
7. Transition diagram of DFA

8. Accepting string & non-accepting string procedure.

Ex:- $(00 + 11)$



* Minimisation of DFA :-



Equivalence method :-

⇒ 0-equivalence - Π_0 : Here we keep all final states in one set and non-final states in one set.

$$\{q_4\} \quad \{q_0, q_1, q_2, q_3\}$$

⇒ 1-equivalence - Π_1 ,

$$\{q_4\} \quad \{q_0, q_1, q_2, q_3\}$$

$$\delta(q_0, a) = q_1, \delta(q_0, b) = q_2$$

$$\delta(q_1, a) = q_1, \delta(q_1, b) = q_3$$

$$/\!/\! \quad \delta(q_0, a) = q_1 \quad \delta(q_0, b) = q_2$$

$$\delta(q_3, a) = q_1 \quad \delta(q_3, b) = q_4$$

$$\{q_1, q_2\} \quad \{q_4\}$$

$$\text{so } q_0 \neq q_3$$

These transitions o/p belong to same set. So

$$q_0 \equiv q_1$$

$$/\!/\! \quad \delta(q_0, a) = q_1 \quad \delta(q_0, b) = q_2$$

$$\delta(q_2, a) = q_1 \quad \delta(q_2, b) = q_2$$

$$q_0 \equiv q_2$$

$$\{q_4\} \quad \{q_3\} \quad \{q_0, q_1, q_2\}$$

\Rightarrow 2-equivalence- Π_2

$\{q_0\}$ $\{q_3\}$ $\{q_0, q_1, q_2\}$

$\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_2$ $\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_1$

$\delta(q_1, a) = q_1$ $\delta(q_1, b) = q_3$ $\delta(q_2, a) = q_1$ $\delta(q_2, b) = q_1$

$q_0 \neq q_1$

$q_0 \equiv q_2$

$\{q_4\}$ $\{q_3\}$ $\{q_1\}$ $\{q_0, q_2\}$

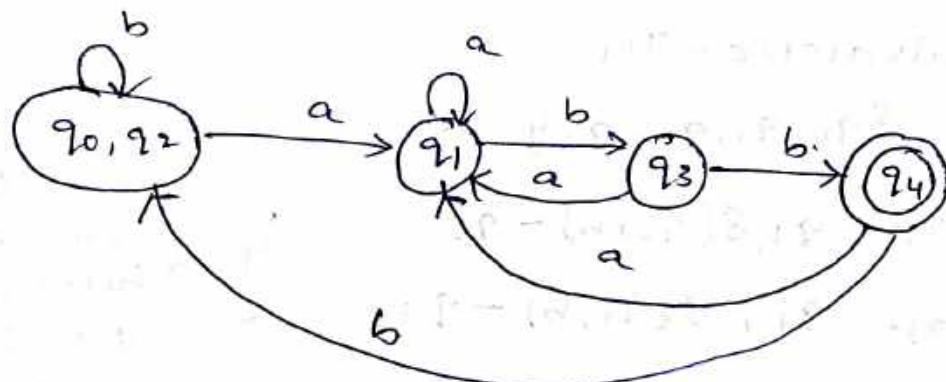
\Rightarrow 3-equivalence- Π_3

$\{q_4\}$ $\{q_3\}$ $\{q_1\}$ $\{q_0, q_2\}$

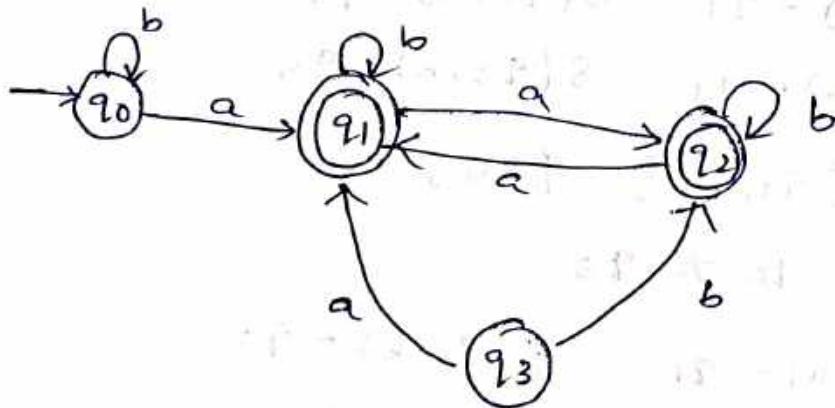
$\delta(q_0, a) = q_1$ $\delta(q_0, b) = q_2$

$\delta(q_2, a) = q_1$ $\delta(q_2, b) = q_2$

$q_0 \equiv q_2$

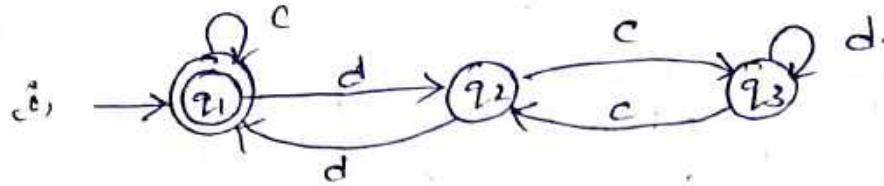


HW

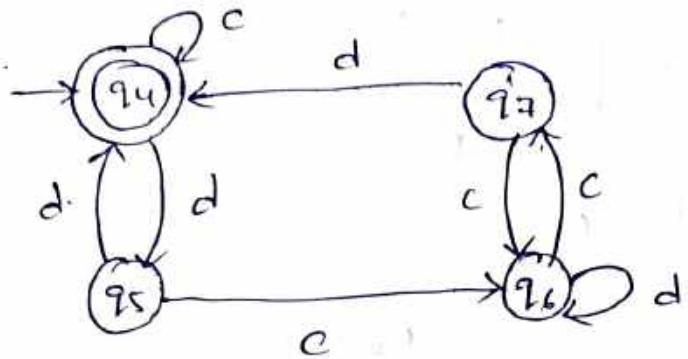


* Equivalence of finite automata

Ex:-



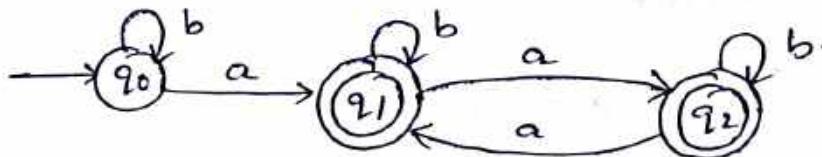
ii,



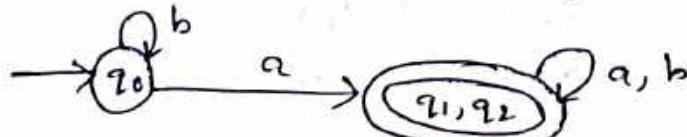
State	c	d.
$\{q_1, q_4\}$	(q_1, q_4) F F	(q_2, q_5) NF NF
$\{q_2, q_5\}$	(q_3, q_6) NF NF	(q_1, q_4) F F
$\{q_3, q_6\}$	(q_2, q_7) NF NF	(q_3, q_6) NF NF
$\{q_2, q_7\}$	(q_3, q_6) NF NF	(q_1, q_4) F F

Both DFA's
are
equivalent

→ i)



ii)



After renaming.



i/p State	a	b
(q_0, q_3)	(q_1, q_4) F F	(q_0, q_3) NP NP
(q_1, q_4)	(q_2, q_4) F F	(q_1, q_4) F F
(q_2, q_4)	(q_1, q_4) F F	(q_2, q_4) F F

Both DFA's
are
equivalent.

* Moore Machine:-



The o/p is associated with each state is called Moore machine.

Moore machine tuple

$$M = \{ Q, \Sigma, \delta, q_0, \Delta, \lambda \}$$

Q - finite set of states

Δ - finite set of o/p's

Σ - finite set of i/p

λ - mapping function

δ - Transition function

$$\lambda: Q \rightarrow \Delta$$

q_0 - initial state

$$\rightarrow Q = \{ q_0, q_1 \} \quad \Sigma = \{ 0, 1 \}$$

$$\delta(q_0, 0) = q_1 \quad \delta(q_1, 0) = q_1$$

$$\delta(q_0, 1) = q_0 \quad \delta(q_1, 1) = q_0$$

$$q_0 = q_0 \quad \Delta = \{ 0, 1 \}$$

$$\lambda: Q \rightarrow \Delta$$

Ex:- Construct Moore machine for %3
(modulo 3).

$$\Sigma = \{ 0, 1 \}$$

$$\Delta = \{ 0, 1, 2 \}$$

$$000 - 0$$

$$001 - 1$$

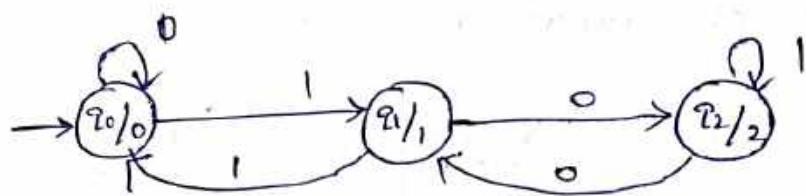
$$010 - 2$$

$$011 - 0$$

$$100 - 1$$

$$101 - 2$$

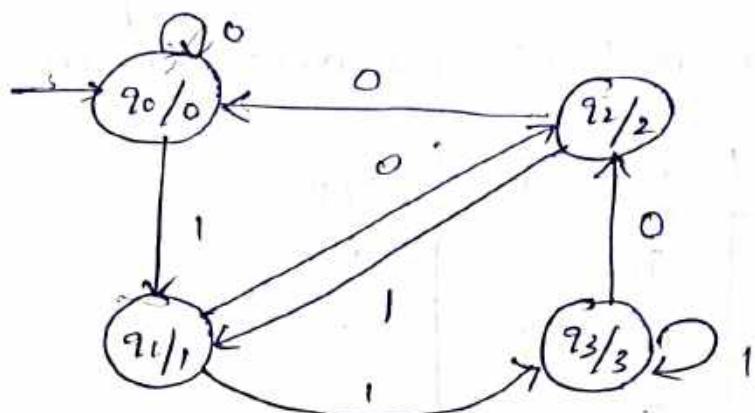
$$110 - 0$$



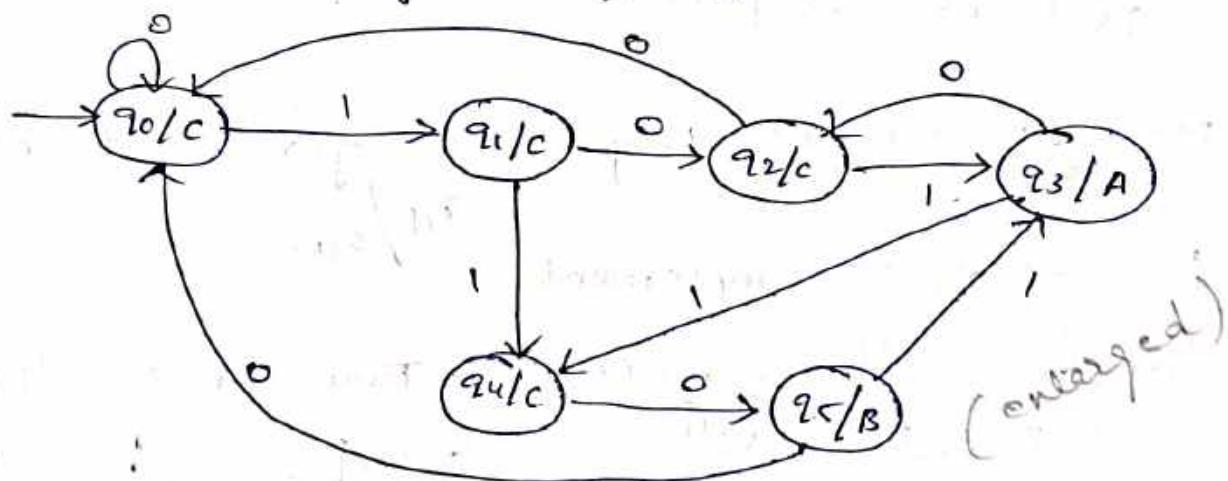
Ex:- Modulo 4.

$000 - 0$
 $001 - 01$
 $010 - 2$
 $011 - 3$
 $100 - 0$
 $101 - 1$
 $110 - 2$
 $111 - 3$

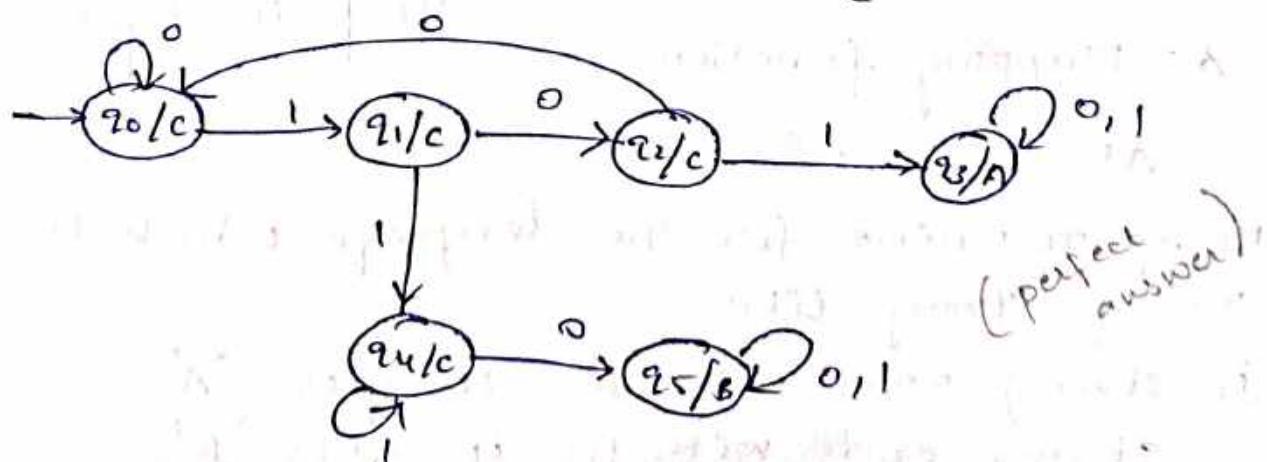
$$\Delta = \{0, 1, 2, 3\}$$



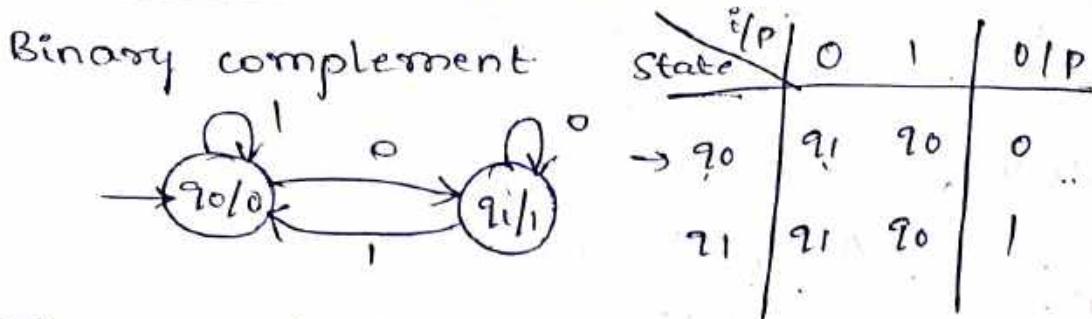
Ex:- If the substring ends with 101 which gives o/p 'A'. If the string ends with 110 which gives o/p 'B' otherwise 'C'.



Ex:- If the string has substring 101 o/p 'A'
if 110 o/p 'B' otherwise 'C'.



* Transition table of moore machine:-

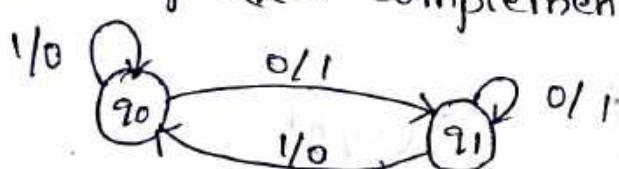


Transition table for above example.

state	i/p		o/p
	0	1	
$\rightarrow q_0$	q_0	q_1	C
q_1	q_2	q_4	C
q_2	q_0	q_3	C
q_3	q_3	q_3	A
q_4	q_5	q_4	C
q_5	q_5	q_5	B

* Mealy machine:- Outputs are given to transitions

Binary equiv. complement



i/p / o/p.

Transition state/table

6 tuples

$$M = \{Q, \Sigma, \delta, q_0, \Delta, \lambda\}$$

λ = mapping function

$$\lambda: Q \times \Sigma \rightarrow \Delta$$

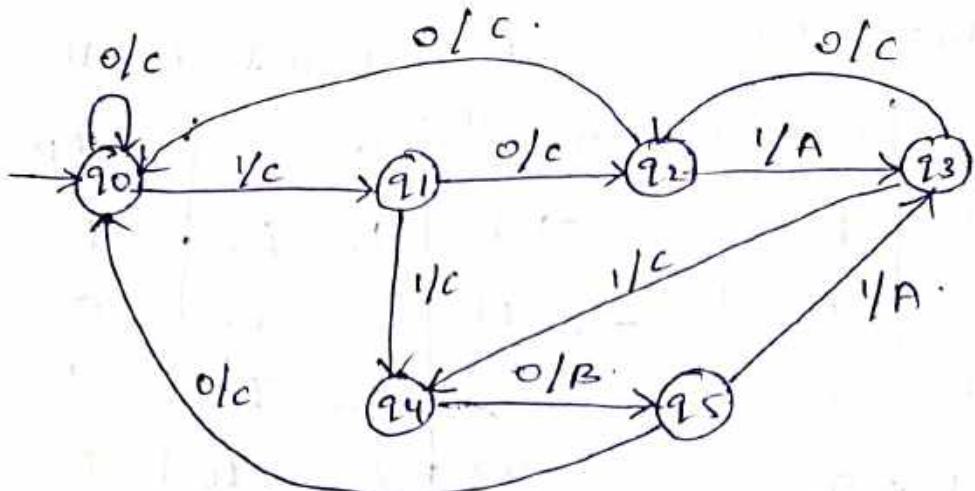
state	i/p		o/p	1	0/p
	0	1			
$\rightarrow q_0$	q_1	1	q_0	0	0
q_1	q_1	1	q_0	0	0

Mealy machine for the language which is having strings like.

i) String ends with 101 it goes 'A'

String ends with 110 it goes 'B'.

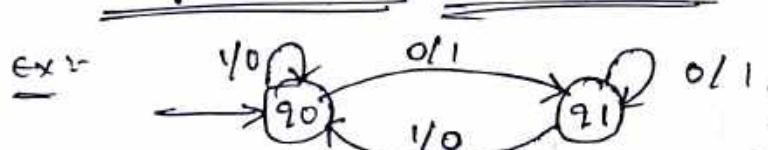
Otherwise 'C'.



Transition table:-

i/p	0	0/p	1	0/p
state				
q0	q0	C	q1	C
q1	q2	C	q4	C
q2	q0	C	q3	A
q3	q2	C	q4	C
q4	q5	B	q4	C
q5	q0	C	q3	A

* Mealy to Moore Conversion:



Transition table-Mealy

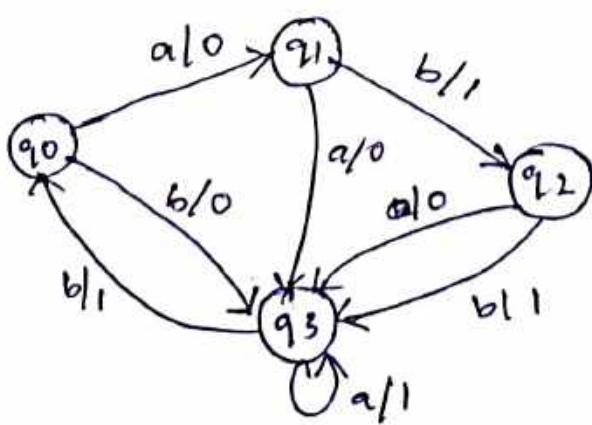
i/p	0	0/p	1	0/p
state				
q0	q1	1	q0	0
q1	q1	1	q0	0

Transition table-moore.

i/p	0	1	0/p
state			
q0	q1	q0	0
q1	q1	q0	1



Ex:-

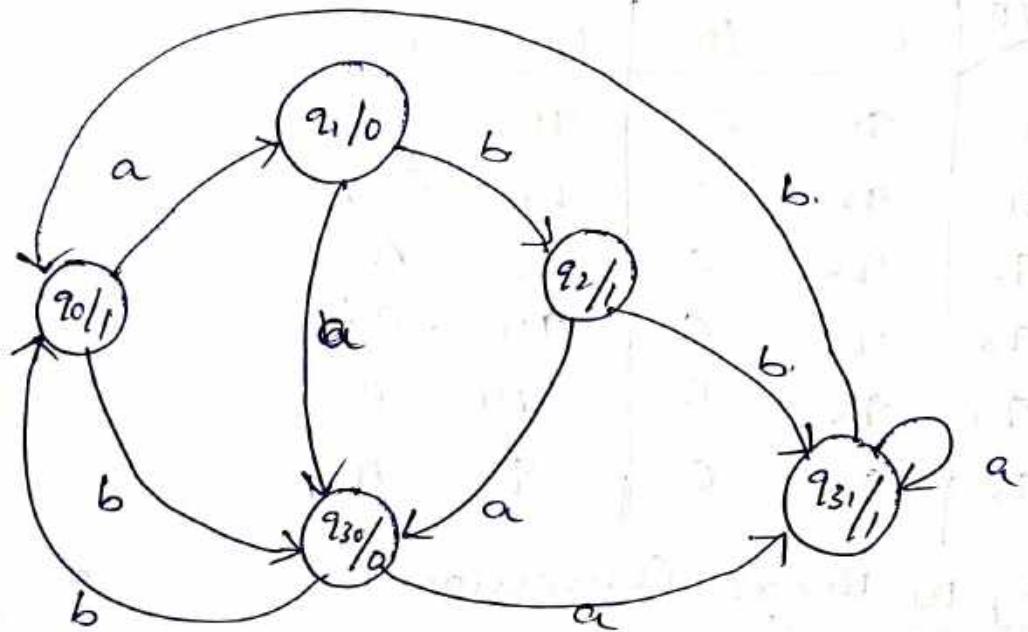


Transition table - Holey

state	i/p	a	o/p	b	o/p
$\rightarrow q_0$		q_1	0	q_3	0
q_1		q_3	0	q_2	1
q_2		q_3	0	q_3	1
q_3		q_3	1	q_0	1

Transition table - Moore

state	i/p	a	b	o/p
$\rightarrow q_0$		q_1	q_{30}	1
q_1		q_{30}	q_2	0
q_2		q_{30}	q_{31}	1
q_{30}		q_{31}	q_0	0
q_{31}		q_{31}	q_0	1

Ex:-

	a	o/p	b	o/p
q_0	q_2	1	q_3	0
q_1	q_0	0	q_1	1
q_2	q_1	1	q_2	0
q_3	q_2	0	q_0	1

* chomsky Hierarchy

