

Qn1-4

e.g. 28.13

$$x+4y+z = -5, \quad x+y-6z = -12; \quad 3x+y-2z = 4$$

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & -6 \\ 3 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$$

$$R_2 - R_1, \quad R_3 - 3R_1, \quad \begin{bmatrix} 1 & 4 & -1 \\ 0 & -3 & -5 \\ 0 & -13 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -7 \\ 19 \end{bmatrix}$$

$$R_3 - \frac{13}{3}R_2, \quad \begin{bmatrix} 1 & 4 & -1 \\ 0 & -3 & -5 \\ 0 & 0 & 71/3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -7 \\ 148/3 \end{bmatrix}$$

$$\frac{7}{3}x = \frac{148}{3} \quad x = \frac{148}{7},$$

$$3y = 7 - 52 = 7 - 5\left(\frac{148}{7}\right) = -3422 \rightarrow$$

$$y = -1.1408$$

$$z = -5 - 4y + x = -5 + 4(1.1408) + 2.084 \rightarrow 1.6429$$

$$\underline{28.14} \quad 10x - 7y + 3z + 5u = 6, \quad -6x + 8y - 2z - 4u = 5$$

$$3x + y + 4z + 11u = 2, \quad 5x - 9y - 2z + 4u = 7$$

$$\begin{bmatrix} 10 & -7 & 3 & 5 \\ -6 & 8 & -1 & -4 \\ 3 & 1 & 4 & 11 \\ 5 & -9 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 2 \\ 7 \end{bmatrix}$$

$$R_1 \leftarrow R_1 \div 10$$

$$\begin{bmatrix} 1 & -0.7 & 0.3 & 0.5 \\ -6 & 8 & -1 & -4 \\ 3 & 1 & 4 & 11 \\ 5 & -9 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 0.6 \\ 5 \\ 2 \\ 7 \end{bmatrix}$$

$$R_2 \leftarrow R_2 + 6R_1$$

$$\begin{bmatrix} 1 & -0.7 & 0.3 & 0.r \\ 0 & 3.8 & 0.8 & r \\ 0 & 1 & 4 & r \\ 5 & -9 & -2 & 9 \end{bmatrix} \begin{pmatrix} q \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 0.6 \\ 8.6 \\ 2 \\ 7 \end{pmatrix}$$

$$R_3 \leftarrow R_3 - 3R_1$$

$$R_4 \leftarrow R_4 - 5R_1$$

$$\begin{bmatrix} 1 & -0.7 & 0.3 & 0.r \\ 0 & 3.8 & 0.8 & r \\ 0 & 3.1 & 3.1 & 9.r \\ 0 & -5.r & -3.r & 1.r \end{bmatrix} \begin{pmatrix} q \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 0.6 \\ 8.6 \\ 8.6 \\ 0.2 \end{pmatrix}$$

$$R_2 \leftarrow R_2 \times 0.263$$

$$\begin{bmatrix} 1 & -0.7 & 0.3 & 0.r \\ 0 & 1 & 0.261 & 0.263 \\ 0 & 3.1 & 3.1 & 9.r \\ 0 & -5.r & -3.r & 1.r \end{bmatrix} \begin{pmatrix} q \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 0.6 \\ 2.263 \\ 2.263 \\ 0.2 \end{pmatrix}$$

$$R_1 \leftarrow R_1 + 0.7R_2$$

$$R_3 \leftarrow R_3 - 3.1R_2$$

$$R_4 \leftarrow R_4 + 5.5R_2$$

$$\begin{bmatrix} 1 & 0 & 0.442 & 0.316 \\ 0 & 1 & 0.211 & -0.263 \\ 0 & 0 & 2.442 & 10.316 \\ 0 & 0 & -2.342 & 0.073 \end{bmatrix} \begin{pmatrix} q \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 2.184 \\ 2.263 \\ -6.816 \\ 16.447 \end{pmatrix}$$

$$R_4 \leftarrow R_4 + 2.342 \times R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1.57 \\ 0 & 1 & 0 & -1.17 \\ 0 & 0 & 1 & 4.21r \\ 0 & 0 & 0 & 9.92r \end{bmatrix} \begin{pmatrix} q \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 3.43 \\ 2.849 \\ -2.78r \\ 9.92r \end{pmatrix}$$

$$R_4 \leftarrow R_4 - 9.92r$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1.57 \\ 0 & 1 & 0 & -1.17 \\ 0 & 0 & 1 & 4.21r \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} q \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 3.43 \\ 2.849 \\ -2.78r \\ 1 \end{pmatrix}$$

$$R_1 \leftarrow R_1 + 1 \cdot R_2 \quad R_2 \leftarrow R_2 + 1 \cdot R_4 \quad R_3 \leftarrow R_3 - 4 \cdot R_4$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 5 \\ y \\ -7 \\ 1 \end{pmatrix}$$

$$x=5, \quad y=u, \quad z=-7, \quad u=1,$$

$$x+y+z=9$$

$$2x+3y+4z=13$$

$$3x+4y+5z=40$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 4 \\ 3 & 4 & 5 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 9 \\ 13 \\ 40 \end{pmatrix}$$

$$R_2 - 2R_1, \quad R_3 - 3R_1$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 9 \\ -5 \\ 13 \end{pmatrix}$$

$$R_3 + \frac{R_2}{5}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 12/5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 9 \\ -5 \\ 12 \end{pmatrix}$$

$$12/5 z = 12$$

$$z = 5$$

$$-5y + 2z = -5$$

$$-5y = -15$$

$$y = 3$$

$$x + y + z = 9$$

$$x + 3 + 5 = 9$$

$$x = 1$$

$$x = 1$$

$$y = 3$$

$$z = 5$$

eg 28.3

$$\begin{array}{l} x_1 + x_2 + x_3 + x_4 = 4 \\ x_1 + x_2 + 6x_3 + x_4 = -5 \end{array} \quad \begin{array}{l} x_1 + 7x_2 + x_3 + x_4 = 12 \\ x_1 + x_2 + x_3 + 4x_4 = -6 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 1 & 1 & 6 & 1 & -5 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 4 \\ -5 \end{array} \right]$$

$$R_2 \leftarrow R_2 - R_1/5 \quad R_3 \leftarrow R_3 - R_1/5 \quad R_4 \leftarrow R_4 - R_1/5$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 0.8 & 0.8 & 0.8 & -1 \\ 0 & 0.8 & 5.8 & 0.8 & -1.2 \\ 0 & 0.8 & 0.8 & 3.8 & -1.8 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 4 \\ -1 \\ -5.8 \\ -6.8 \end{array} \right]$$

$$R_3 \leftarrow R_3 - 0.118 R_2 \quad R_4 \leftarrow R_4 - 0.118 R_2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 0.8 & 0.8 & 0.8 & -1 \\ 0 & 0 & 5.7 & 0.7 & -1.2 \\ 0 & 0 & 0.7 & 3.7 & -7.18 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 4 \\ -1 \\ -7.18 \\ -8.118 \end{array} \right]$$

$$R_4 \leftarrow R_4 - 0.124 R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 0.8 & 0.8 & 0.8 & -1 \\ 0 & 0 & 5.7 & 0.7 & -1.2 \\ 0 & 0 & 0 & 3.6 & -7.237 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 4 \\ -1 \\ -7.237 \\ -7.237 \end{array} \right]$$

$$3.6x_4 = -7.237 \quad x_4 \approx -2$$

$$5.7x_3 + 0.7x_4 = -7.118$$

$$x_3 \approx -1$$

$$0.8x_2 + 0.8x_3 + 0.8x_4 = 11.2$$

$$x_2 \approx 2$$

$$5x_4 + x_1 + x_3 + x_4 \approx 4$$

$$x_1 \approx 1$$

$$\rightarrow x + y + z = 9$$

$$2x - 3y + 4z = 13$$

$$3x + 4y + 5z = 40$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - 2R_1 \quad R_3 \leftarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 13 \end{bmatrix}$$

$$R_3 \leftarrow R_3 + 0.2R_2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -5 & 2 \\ 0 & 0 & 2.4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 12 \end{bmatrix}$$

$$2.4z = 12$$

$$z = \frac{12}{2.4} \times 10 = 5$$

$$-5y + 2z = -5$$

$$-5y + 10 = -5$$

$$-5y = -15 \quad | :(-5)$$

$$y = 3$$

$$x + y + z = 9$$

$$x + 3 + 5 = 9$$

$$x = 1$$

\rightarrow

$$2x - y + 3z = 9$$

$$x + y + z = 6$$

$$x - y + z = 2$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ -5 \end{bmatrix}$$

$$R_3 \leftarrow 4R_3 + R_2$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & 1 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 17 \end{bmatrix}$$

$$-5z = 17$$

$$z = 17/(-5) = -3.4$$

$$4y - z = 3$$

$$4y = 6.4$$

$$y = 1.6$$

$$2x - y + 3z = 9.$$

$$2x - 1.6 + 3(-3.4) = 9$$

$$2x = 9 + 8.6$$

$$2x = 0.4$$

$$x = 0.2$$

→

$$2x + 3y + 7z = 52$$

$$2x + y - z = 0$$

$$x + y + z = 9$$

$$R_1 \leftarrow R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 2.5 & 3.5 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ 0 \\ 9 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - 2R_1; \quad R_3 \leftarrow R_3 - R_1$$

$$\begin{bmatrix} 1 & 2.5 & 3.5 \\ 0 & -4 & -8 \\ 0 & -1.5 & -2.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ -52 \\ -17 \end{bmatrix}$$

$$R_2 \leftarrow R_2 / 4$$

$$\begin{bmatrix} 1 & 2.5 & 3.5 \\ 0 & 1 & 2 \\ 0 & -1.5 & -2.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 26 \\ 13 \\ -17 \end{bmatrix}$$

$$R_1 \leftarrow R_1 - 2.5 R_2$$

$$R_3 \leftarrow R_3 + 1.5 R_2$$

$$\begin{bmatrix} 1 & 0 & -1.5 \\ 0 & 1 & 2 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6.5 \\ 13 \\ 2.5 \end{bmatrix}$$

$$R_3 \leftarrow R_3 / 2$$

$$\begin{bmatrix} 1 & 0 & -1.5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6.5 \\ 13 \\ 1.5 \end{bmatrix}$$

$$R_1 \leftarrow R_1 + 1.5 R_3$$

$$R_2 \leftarrow R_2 + 2 R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

$$x = 1, y = 3, z = 5$$

$$\rightarrow x + 3y + 3z = 16$$

$$x + 4y + 3z = 18$$

$$x + 3y + 4z = 19$$

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ 19 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - R_1$$

$$R_3 \leftarrow R_3 - R_1$$

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 2 \\ 3 \end{bmatrix}$$

$$R_1 \leftarrow 3R_2 + 3R_3 - R_1$$

$$x = -1$$

$$y = 2$$

$$z = 3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$2x_1 + x_2 + 5x_3 + x_4 = 5$$

$$x_1 + x_2 - 3x_3 + 4x_4 = -1$$

$$3x_4 + 6x_2 + 2x_3 + x_4 = 8$$

$$2x_1 + 2x_2 + 2x_3 - 3x_4 = 2$$

$$\left[\begin{array}{cccc|c} 2 & 1 & 5 & 1 \\ 1 & 1 & -3 & 4 \\ 3 & 6 & -2 & 1 \\ 2 & 2 & 2 & -3 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 5 \\ -1 \\ 8 \\ 2 \end{array} \right]$$

$$R_1 \leftarrow R_1 / 2$$

$$\left[\begin{array}{cccc|c} 1 & 0.5 & 2.5 & 0.5 \\ 1 & 1 & -3 & 4 \\ 3 & 6 & -2 & 1 \\ 2 & 2 & 2 & -3 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 2.5 \\ -1 \\ 8 \\ 2 \end{array} \right]$$

$$R_2 \leftarrow R_2 - R_1, \quad R_3 \leftarrow R_3 - 3R_1 \quad R_4 \leftarrow R_4 - 2R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0.5 & 2.5 & 0.5 \\ 0 & 0.5 & -5.5 & 3.5 \\ 0 & 4.5 & -9.5 & -0.5 \\ 0 & 1 & -3 & -4 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 2.5 \\ -3.5 \\ 0.5 \\ -3 \end{array} \right]$$

$$R_2 \leftarrow R_2 \times 2$$

$$\left[\begin{array}{cccc|c} 1 & 0.5 & 2.5 & 0.5 \\ 0 & 1 & -11 & 7 \\ 0 & 4.5 & -9.5 & -0.5 \\ 0 & 1 & -3 & -4 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 2.5 \\ -7 \\ 0.5 \\ -3 \end{array} \right]$$

$$R_1 \leftarrow R_1 - 0.5R_2$$

$$R_3 \leftarrow R_3 - 4.5R_2 \quad R_4 \leftarrow R_4 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 8 & -3 \\ 0 & 1 & -11 & 7 \\ 0 & 0 & 40 & -32 \\ 0 & 0 & 8 & -4 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 6 \\ -7 \\ 32 \\ 4 \end{array} \right]$$

$$R_3 \leftarrow R_3 / 40$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 8 & -3 \\ 0 & 1 & -11 & 7 \\ 0 & 0 & 1 & -0.8 \\ 0 & 0 & 8 & -1 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right] = \left[\begin{array}{c} 6 \\ -7 \\ 0.8 \\ 1 \end{array} \right]$$

$$R_1 \leftarrow R_1 - 8R_3 \quad R_2 \leftarrow R_2 + 11R_3 \quad R_4 \leftarrow R_4 - 8R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 3.4 \\ 0 & 1 & 0 & -1.8 \\ 0 & 0 & 1 & -0.8 \\ 0 & 0 & 0 & -4.6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 1.8 \\ 0.8 \\ -2.4 \end{bmatrix}$$

$$R_4 \leftarrow R_4 \times -0.2174$$

$$\begin{bmatrix} 1 & 0 & 0 & 3.4 \\ 0 & 1 & 0 & -1.8 \\ 0 & 0 & 1 & -0.8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 1.8 \\ 0.8 \\ 0.5217 \end{bmatrix}$$

$$R_1 \leftarrow R_1 - 3.4R_4$$

$$R_2 \leftarrow R_2 + 1.8R_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2.1739 \\ 2.7391 \\ 1.2174 \\ 0.5217 \end{bmatrix}$$

$$x_1 = -2.1739, x_2 = 2.7391, x_3 = 1.2174, x_4 = 0.5217$$

→ LU decomposition Crout's method:

$$10x+y+2z=62$$

$$2x+10y+2z=13$$

$$2x+2y+10z=14$$

$$\begin{bmatrix} 10 & 1 & 2 \\ 2 & 10 & 2 \\ 2 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 62 \\ 13 \\ 14 \end{bmatrix}$$

$$A = \begin{bmatrix} 10 & 1 & 2 \\ 2 & 10 & 2 \\ 2 & 2 & 10 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 62 \\ 13 \\ 14 \end{bmatrix}$$

$$A = LU$$

$$\begin{bmatrix} 10 & 1 & 2 \\ 2 & 10 & 2 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{22} & l_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{11}l_{12} & l_{11}l_{13} \\ l_{21} & l_{21}l_{12} + l_{22} & l_{21}l_{13} + l_{22}l_{23} \\ l_{31} & l_{31}l_{12} + l_{32} & l_{31}l_{13} + l_{32}l_{23} + l_{33} \end{bmatrix}$$

$l_{11} = 10$ $l_{21} = 2$ $l_{31} = 2$

$$l_{11}l_{12} = 1 \rightarrow l_{12} = 1/10 \quad l_{11}l_{13} = 1 \quad l_{13} \rightarrow 1/10$$

$$l_{21}l_{12} + l_{22} = 0 \rightarrow l_{22} = 4/9/10 \quad l_{21}l_{13} + l_{22}l_{23} = 1$$

$$l_{31}l_{12} + l_{32} = 2 \rightarrow l_{32} = 9/10 \quad l_{31}l_{13} + l_{32}l_{23} + l_{33} = 1 \quad l_{23} = 4/49$$

$$A = LU \quad l_{33} = 10$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 2 & 4/9/10 & 0 \\ 2 & 9/10 & 4/49 \end{bmatrix} \begin{bmatrix} 1 & 1/10 & 1/10 \\ 0 & 1 & 4/49 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix}$$

$$AX = B$$

$$A = LU$$

$$LUx = B$$

$$Ux = y \quad \text{and} \quad Ly = B$$

$$Ly = B$$

$$\begin{bmatrix} 10 & 0 & 0 \\ 2 & 9/8 & 0 \\ 2 & 1/8 & 9/6131 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 67 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{By comparing } y_1 = 1.2, \quad y_2 = 1.0816, \quad y_3 = 1$$

$$Ux = B$$

$$\begin{bmatrix} 1 & 0.1 & 0.1 \\ 0 & 1 & 0.0816 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.0816 \\ 1 \end{bmatrix}$$

$$x = 1, \quad y = 1, \quad z = 1$$

$$2x - 2y = 1 \quad , \quad y = 1, x = 1$$

By Crammer

$$\rightarrow x + 2y + 3z = 14$$

$$2x + 3y + 4z = 20$$

$$3x + 4y + z = 14$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$$

$$A = I\ U$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & l_{12} & l_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} + l_{22}U_{23} \\ l_{31} & l_{31}U_{12} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix}$$

$$l_{11} = 1, l_{21} = 2, l_{31} = 3$$

$$l_{11}U_{12} = 2, \quad U_{12} = 2, \quad l_{11}U_{13} = 3, \quad U_{13} = 3$$

$$l_{21}U_{12} + l_{22} = 3 \rightarrow l_{22} = -1, \quad l_{21}U_{13} + l_{22}U_{23} = 4, \quad U_{23} = 2$$

$$l_{31}U_{12} + l_{32} = 4, \quad l_{32} = -2, \quad l_{31}U_{13} + l_{32}U_{23} + l_{33} = 1, \quad l_{33} = 1$$

$$AX = B, \quad A = I\ U \quad I\ O\ x = B \quad O\ x = y \quad U\ y = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 20 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix}$$

$$\text{By comparing} \quad y_1 = 14, \quad y_2 = 8, \quad y_3 = 3$$

$$U\ x = y$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \\ 3 \end{bmatrix} \quad \begin{array}{l} x = 3 \\ y = 2 \\ z = 1 \end{array}$$

$$2x + y + 4z = 12$$

$$8x - 3y + 2z = 20$$

$$4x + 11y - z = 33$$

$$A = \begin{pmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow B = \begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

$$A = LU$$

$$\begin{pmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{pmatrix}$$

$$U_{11} = 2, \quad U_{12} = 1, \quad U_{13} = 4$$

$$L_{21}U_{11} = 8, \quad L_{21} = 4$$

$$L_{21}U_{12} + U_{22} = -3$$

$$L_{21}U_{13} + U_{23} = 2$$

$$L_{31}U_{11} = 4, \quad L_{31} = 2$$

$$L_{31}U_{12} + L_{32}U_{22} = 11, \quad L_{32} = -9/2$$

$$L_{31}U_{13} + L_{32}U_{23} + U_{33} = -1, \quad U_{33} = -27$$

$$A = LU$$

$$Ux = y \quad AX = B \quad A = LU \quad LUX = B$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -1.2857 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 20 \\ 33 \end{pmatrix}$$

By comparing

$$y_1 = 7.2, \quad y_2 = -28, \quad y_3 = -27$$

$$Ux = y$$

$$\begin{pmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ -28 \\ -27 \end{pmatrix}$$

by comparing

$$x = 1, \quad y = 2, \quad z = 3$$

$$+ 2x_1 + x_2 + x_3 = -1$$

$$2x_2 - x_3 + x_4 = 1$$

$$x_1 + 2x_3 - x_4 = -1$$

$$x_1 + x_2 + 2x_4 = 3$$

$$A = \begin{pmatrix} 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 0 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad B = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 3 \end{pmatrix}$$

$$A = 10$$

$$\begin{pmatrix} 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 0 & 2 \end{pmatrix} \rightarrow \begin{pmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{pmatrix} \quad \begin{aligned} u_{11} &= 1_1 u_{12} + u_{22} \\ u_{12} &= 1_2 u_{13} + u_{23} \\ u_{13} &= 1_3 u_{14} + u_{24} \\ u_{14} &= 1_4 u_{12} + 1_2 u_{13} + 1_3 u_{14} + 1_4 u_{23} \end{aligned}$$

Comparing $u_{11} = 2$ $u_{12} = -1$ $u_{13} = 1$ $u_{14} = 1$
 $u_{21} = 0$ $u_{22} = 2$ $u_{23} = -1$ $u_{24} = 1$
 $u_{31} = 1/2$ $u_{32} = 1/4$ $u_{33} = 3/4$ $u_{34} = -5/4$

$$u_{11} = 1/2 \quad u_{12} = 3/4 \quad u_{13} = 1/4 \quad u_{14} = 10/2$$

$$Ax = B$$

$$10x = B \quad 10x = y \quad 1y = B$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0.5 & 0.25 & 1 & 0 \\ 0.5 & 0.75 & 0.1429 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix}$$

By comparing $y_1 = -1$ $y_2 = 1$ $y_3 = -0.75$ $y_4 = 4.8571$

$$Ux = y$$

$$\left(\begin{array}{cccc} 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & 1.75 & -1.25 \\ 0 & 0 & 0 & 1.428 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \right) = \left(\begin{array}{c} 1 \\ 1 \\ 0.75 \\ 4.833 \end{array} \right)$$

$$x_4 = 3.4$$

$$x_3 = 2$$

$$x_2 = -0.2$$

$$x_1 = -1.61$$

eg 28.19

$$10x + y - z = 11.19$$

$$x + 10y + z = 28.08$$

$$-x + y + 10z = 35.61$$

$$x = \frac{1}{10}(11.19 - y + z), \quad y = \frac{1}{10}(28.08 - x - z), \quad z = \frac{1}{10}(35.61 + x - y)$$

$$x_0 = y_0 = z_0 = 0$$

$$x_1 = \frac{11.19}{10} = 1.119 \quad y_1 = \frac{28.08}{10} = 2.808 \quad z_1 = \frac{35.61}{10} = 3.561$$

$$x_2 = \frac{1}{10}(11.19 - y_1 + z_1) = 1.19$$

$$y_2 = \frac{1}{10}(28.08 - x_1 - z_1) = 2.24$$

$$z_2 = \frac{1}{10}(35.61 + x_1 - y_1) = 3.39$$

$$x_3 = \frac{1}{10}(11.19 - y_2 + z_2) = 1.22$$

$$y_3 = \frac{1}{10}(28.08 - x_2 - z_2) = 2.35$$

$$z_3 = \frac{1}{10}(35.61 + x_2 - y_2) = 3.45$$

$$x_4 = \frac{1}{10}(11.19 - y_3 + z_3) = 1.23$$

$$y_4 = \frac{1}{10}(28.08 - x_3 - z_3) = 2.34$$

$$z_4 = \frac{1}{10}(35.61 + x_3 - y_3) = 3.45.$$

$$x_5 = \frac{1}{10} (11.19 - y_4 + z_4) = 1.23$$

$$y_5 = \frac{1}{10} (28.08 - x_4 - z_4) = 2.34$$

$$z_5 = \frac{1}{10} (35.61 + x_4 - y_4) = 3.45$$

$$x = 1.23 \quad y = 2.34 \quad z = 3.45$$

$$\rightarrow 20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

$$x = \frac{1}{20} (17 - y + 2z) \quad y = \frac{1}{20} (-18 - 3x + z) \quad z = \frac{1}{20} (25 - 2x + 3y)$$

$$x_0 = y_0 = z_0 = 0$$

$$x_1 = \frac{17}{20} = 0.85 \quad y_1 = \frac{-18}{20} = -0.9 \quad z_1 = \frac{25}{20} = 1.25$$

$$x_2 = \frac{1}{20} (17 - y_1 + 2z_1) = 1.02$$

$$y_2 = \frac{1}{20} (-18 - 3x_1 + z_1) = -0.965$$

$$z_2 = \frac{1}{20} (25 - 2x_1 + 3y_1) = 1.1515$$

$$x_3 = \frac{1}{20} (17 - y_2 + 2z_2) = 1.0134$$

$$y_3 = \frac{1}{20} (-18 - 3x_2 + z_2) = -0.9954$$

$$z_3 = \frac{1}{20} (25 - 2x_2 + 3y_2) = 1.0032$$

$$x_4 = \frac{1}{20} (17 - y_3 + 2z_3) = 1.009$$

$$y_4 = \frac{1}{20} (-18 - 3x_3 + z_3) = -1.0018$$

$$z_4 = \frac{1}{20} (25 - 2x_3 + 3y_3) = 0.993$$

$$x_5 = \frac{1}{20} (17 - y_4 + 2z_4) = 1.0008$$

$$y_5 = \frac{1}{20} (-18 - 3x_4 + z_4) = -1.0002$$

$$z_5 = \frac{1}{20} (25 - 2x_4 + 3y_4) = 0.9996$$

$$x_6 = \frac{1}{20} (17 - y_5 + 2z_5) = 1.00$$

$$y_6 = \frac{1}{20} (-18 - 3x_5 + z_5) = -1.000$$

$$z_6 = \frac{1}{20} (25 - 2x_5 + 3y_5) = 1.000$$

$x=1, y=-1, z=1$

Q28.2)

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

$$x = \frac{1}{20} (17 - y + 2z), y = \frac{1}{20} (-18 - 3x + z), z = \frac{1}{20} (25 - 2x + 3y)$$

$$x_1 = \frac{1}{20} (17 - y_0 + 2z_0) = 0.85$$

$$y_1 = \frac{1}{20} (-18 - 3x_1 + z_1) = -1.027$$

$$z_1 = \frac{1}{20} (25 - 2x_1 + 3y_1) = 1.0109$$

$$x_2 = \frac{1}{20} (17 - y_1 + 2z_1) = 1.002$$

$$y_2 = \frac{1}{20} (-18 - 3x_2 + z_2) = -0.9998$$

$$z_2 = \frac{1}{20} (25 - 2x_2 + 3y_2) = 0.9998$$

$$x_3 = \frac{1}{20} (17 - y_2 + 2z_2) = 1.000$$

$$y_3 = \frac{1}{20} (-18 - 3x_3 + z_3) = -1.000$$

$$z_3 = \frac{1}{20} (25 - 3x_3 + 2y_3) = 1.000$$

$x=1, y=-1, z=1$

$$\begin{aligned} \rightarrow 10x_4 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_4 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_4 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

$$x_1 = 0.3 + 0.2x_2 + 0.1x_3 + 0.1x_4$$

$$x_2 = 1.5 + 0.2x_1 + 0.1x_3 + 0.1x_4$$

$$x_3 = 2.7 + 0.1x_1 + 0.1x_2 + 0.2x_4$$

$$x_4 = -0.9 + 0.1x_2 + 0.1x_3 + 0.2x_4$$

①

$$x_2 = 0, x_3 = 0, x_4 = 0 \rightarrow x_1 = 0.3$$

$$x_1 = 0.3, x_3 = 0, x_4 = 0 \rightarrow x_2 = 1.5$$

$$x_1 = 0.3, x_2 = 1.5, x_4 = 0 \rightarrow x_3 = 2.886$$

$$x_1 = 0.3, x_2 = 1.5, x_3 = 2.886 \rightarrow x_4 = -0.1368$$

②

$$x_2 = 1.5, x_3 = 2.886, x_4 = -0.1368$$

$$x_1 = 0.8869, x_3 = 2.886, x_4 = -0.1368 \rightarrow x_2 = 1.9523$$

$$x_1 = 0.8869, x_2 = 1.9523, x_4 = -0.1368 \rightarrow x_3 = 2.9566$$

$$x_1 = 0.8869, x_2 = 1.9523, x_3 = 2.9566 \rightarrow x_4 = -0.0248$$

③

$$x_2 = 1.9523, x_3 = 2.9566, x_4 = -0.0248 \rightarrow x_1 = 0.9836$$

$$x_1 = 0.9836, x_3 = 2.9566, x_4 = -0.0248 \rightarrow x_2 = 1.9899$$

$$x_1 = 0.9836, x_2 = 1.9899, x_4 = -0.0248 \rightarrow x_3 = 2.9924$$

④

$$x_1 = 0.9968, x_2 = 1.9982, x_3 = 2.9987, x_4 = -0.0042$$

$$x_1 = 0.9994, x_2 = 1.9997, x_3 = 2.9997, x_4 = -0.0008$$

⑤

$$x_1 = 0.9999, x_2 = 1.9999, x_3 = 2.9999, x_4 = -0.0001$$

$$\text{Sekr } x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 0.$$

Ex-28.4

$$\rightarrow 5x - y + z = 10$$

$$2x + 4y = 12$$

$$x + y + 5z = -1$$

Start with $(2, 3, 0)$.

$$x_{k+1} = \frac{1}{5}(10 + y_k - z_k)$$

$$y_{k+1} = \frac{1}{4}(12 - 2x_k - 0z_k)$$

$$z_{k+1} = \frac{1}{5}(-1 - x_k - y_k)$$

$$\textcircled{1} \quad x_1 = \frac{1}{5}(10 + 3 - 0) = \frac{13}{5} = 2.6$$

$$y_1 = \frac{1}{4}(12 - 2(2)) - 0(0) = \frac{8}{4} = 2$$

$$z_1 = \frac{1}{5}(-1 - 2 - 3) = \frac{-6}{5} = -1.2$$

$$\textcircled{2} \quad x_2 = \frac{1}{5}[10 + 2 - (-1.2)] = \frac{13.2}{5} = 2.64$$

$$y_2 = \frac{1}{4}[12 - 2(2.6) - 0(-1.2)] = \frac{6.8}{4} = 1.7$$

$$z_2 = \frac{1}{5}[-1 - 2.6 - 2] = \frac{-5.6}{5} = -1.12$$

$$\textcircled{3} \quad x_3 = \frac{1}{5}[10 + (1.7) - (-1.12)] = \frac{12.82}{5} = 2.564$$

$$y_3 = \frac{1}{4}[12 - 2(2.64) - 0(-1.12)] = \frac{6.72}{4} = 1.68$$

$$z_3 = \frac{1}{5}[-1 - (2.64) - (1.68)] = \frac{-5.34}{5} = -1.068$$

$$\textcircled{4} \quad x_4 = \frac{1}{5}[10 + 1.68 - (-1.068)] = \frac{12.748}{5} = 2.55$$

$$y_4 = \frac{1}{4}[12 - 2(2.55) - 0(-1.068)] = \frac{6.872}{4} = 1.718$$

$$z_4 = \frac{1}{5}[-1 - (2.55) - (1.718)] = \frac{-5.244}{5} = -1.049$$

$$\textcircled{5} \quad x_5 = \frac{1}{5}[10 + (1.718) - (-1.049)] = \frac{12.769}{5} = 2.553$$

$$y_5 = \frac{1}{4}[12 - 2(2.553) - 0(-1.049)] = \frac{6.901}{4} = 1.725$$

$$z_5 = \frac{1}{5}[-1 - (2.553) - (1.725)] = \frac{-5.268}{5} = -1.054$$

$$x_0 = \frac{1}{5} [10 + (0.72r) - (-1.056)] = \underline{\underline{2.116}}$$

$$y_0 = \frac{1}{4} [12 - 2(2.116)] = \underline{\underline{1.723}}$$

$$z_0 = \frac{1}{4} [-1 - (2.116) - (1.723)] = \underline{\underline{-1.056}}$$

$$x_1 = \frac{1}{5} [10 + (1.723) - (-1.056)] = \underline{\underline{2.116}}$$

$$y_1 = \frac{1}{4} [12 - 2(2.116)] = \underline{\underline{1.722}}$$

$$z_1 = \frac{1}{4} [-1 - (2.116) - (1.723)] = \underline{\underline{-1.056}}$$

$$\therefore x = 2.116, y = 1.722, z = 1.056$$

$$27x + 6y - z = 85$$

$$x + y + 5z = 110$$

$$6x + 15y + 2z = 72$$

$$x_{k+1} = \frac{1}{27} (85 - 6y_k + z_k)$$

$$y_{k+1} = \frac{1}{15} (50 - 6x_k - 0z_k)$$

$$z_{k+1} = \frac{1}{54} (110 - x_k - y_k) \quad (x, y, z) \rightarrow (0, 0, 0)$$

$$(1) x_1 = \frac{1}{27} (85 - 6(0) + 0) = \frac{85}{27} = 3.148$$

$$y_1 = \frac{1}{15} [72 - 6(0) - 2(0)] = \frac{72}{15} = 4.8$$

$$z_1 = \frac{1}{54} [110] = \frac{110}{54} = 2.037$$

$$x_2 = \frac{1}{27} [85 - 6(3.148) + 2.037] = 2.157$$

$$y_2 = \frac{1}{15} [72 - 6(3.148) + 2(2.037)] = 3.269$$

$$z_2 = \frac{1}{54} [110 - (3.148) - (4.8)] = 1.89$$

$$\textcircled{3} \quad x_3 = \frac{1}{27} [85 - 6(3.269) + (1.89)] = 2.492$$

$$y_3 = \frac{1}{15} [72 - 6(2.157) - 2(1.89)] = 3.681$$

$$z_3 = \frac{1}{54} [110 - (2.157) - (3.269)] = 1.937$$

$$\textcircled{4} \quad x_4 = \frac{1}{27} [85 - 6(3.681) + (1.937)] = 2.401$$

$$y_4 = \frac{1}{15} [72 - 6(2.492) - 2(1.937)] = 3.545$$

$$z_4 = \frac{1}{54} [110 - (2.492) - (3.681)] = \frac{1}{54} [103.823] = 1.923$$

$$\textcircled{5} \quad x_5 = \frac{1}{27} [85 - 6(3.583) + (1.923)] = 2.432$$

$$y_5 = \frac{1}{15} [72 - 6(2.432) - 2(1.923)] = 3.583$$

$$z_5 = \frac{1}{54} [110 - (2.432) - (3.583)] = 1.927$$

$$\textcircled{6} \quad x_6 = \frac{1}{27} [85 - 6(3.583) + (1.927)] = 2.423$$

$$y_6 = \frac{1}{15} [72 - 6(2.432) - 2(1.927)] = 3.57$$

$$z_6 = \frac{1}{54} [110 - (2.432) - (3.583)] = 1.926$$

$$\textcircled{7} \quad x_7 = \frac{1}{27} [85 - 6(3.57) + (1.926)] = 2.426$$

$$y_7 = \frac{1}{15} [72 - 6(2.423) - 2(1.926)] = 3.574$$

$$z_7 = \frac{1}{54} [110 - (2.423) - (3.571)] = 1.926$$

$$\therefore x = 2.426$$

$$y = 3.574$$

$$z = 1.926$$

Geometrische Reihe

$$\begin{aligned} \rightarrow 2x + y + 6z &= 9 \\ 8x + 3y + 2z &= 13 \\ x + 5y + z &= 7 \end{aligned}$$

$$x_{k+1} = 1/8 (13 - 3y_k - 2z_k)$$

$$y_{k+1} = 1/5 (7 - x_{k+1} - z_k)$$

$$z_{k+1} = 1/6 (9 - 2x_{k+1} - y_{k+1})$$

$$\textcircled{1} \quad x_1 = 1/8 (13) = 1.625$$

$$y_1 = \frac{(5 - 3 \cdot 1.625)}{5} = 1.075$$

$$z_1 = \frac{(4 \cdot 1.625)}{6} = 0.779$$

$$\textcircled{2} \quad x_2 = 1/8 (13 - 3(1.075) - 2(0.779)) = \frac{8.212}{8} = 1.027$$

$$y_2 = 1/5 (7 - (1.027) - (0.779)) = \frac{5.194}{5} = 1.039$$

$$z_2 = 1/6 (9 - 2(1.027) - (0.779)) = \frac{5.907}{6} = 0.985$$

$$\textcircled{3} \quad x_3 = 1/8 (13 - 3(1.039) - 2(0.985)) = \frac{7.915}{8} = 0.989$$

$$y_3 = 1/5 (7 - (0.989) - (0.985)) = \frac{5.026}{5} = 1.005$$

$$z_3 = 1/6 (9 - 2(0.989) - (1.005)) = \frac{6.040}{6} = 1.003$$

$$\textcircled{4} \quad x_4 = 1/8 (13 - 3(1.005) - 2(1.003)) = \frac{7.949}{8} = 0.992$$

$$y_4 = 1/5 (7 - 0.992 - 1.003) = \frac{5.005}{5} = 1$$

$$z_4 = 1/6 (9 - 2(0.992) - 1) = \frac{6.005}{6} = 1.001$$

$$\textcircled{5} \quad x_5 = 1/8 (13 - 3(1) - 2(1.001)) = \frac{7.998}{8} = 1$$

$$y_5 = 1/5 (7 - (1) - (1.001)) = \frac{4.999}{5} = 1$$

$$z_5 = 1/6 (9 - 2(1) - (1)) = \frac{6.001}{6} = 1$$

$$\textcircled{6} \quad x_6 = 1/8 (13 - 3(1) - 2(1)) = \frac{8}{8} = 1$$

$$y_6 = 1/5 (7 - (1) - (1)) = \frac{5}{5} = 1$$

$$z_6 = 1/6 (9 - 2(1) - (1)) = \frac{6}{6} = 1$$

$$x = 1, \quad y = 1, \quad z = 1$$

$$83x + 11y - 4z = 95$$

$$7x + 52y + 13z = 104$$

$$3x + 8y + 29z = 71$$

$$x_{k+1} = \frac{1}{83}(95 - 11y_k + 4z_k)$$

$$y_{k+1} = \frac{1}{52}(104 - 7x_{k+1} - 13z_k)$$

$$z_{k+1} = \frac{1}{29}(71 - 3x_{k+1} - 8y_{k+1}) = \begin{pmatrix} x_0, y_0, z_0 \\ (0, 0, 0) \end{pmatrix}$$

$$\textcircled{1} x_1 = \frac{1}{83}(95 - 11(0) + 4(0)) = \frac{95}{83} \approx 1.145$$

$$y_1 = \frac{1}{52}(104 - 7(1.145) - 13(0)) = \frac{95.985}{52} \approx 1.849$$

$$x_1 = \frac{1}{29}(71 - 3(1.145) - 8(1.849)) = \frac{1}{29}(52.799) = 1.824$$

$$\textcircled{2} x_2 = \frac{1}{83}(95 - 11(1.849) + 4(1.824)) = \frac{1}{83}(81.977) = 0.988$$

$$y_2 = \frac{1}{52}(104 - 7(0.988) - 13(1.824)) = \frac{73.418}{52} = 1.412$$

$$x_2 = \frac{1}{29}(71 - 3(0.988) - 8(1.412)) = \frac{56.742}{29} = 1.952$$

$$\textcircled{3} x_3 = \frac{1}{83}(95 - 11(1.412) + 4(1.952)) = \frac{1}{83}(87.297) = 1.052$$

$$y_3 = \frac{1}{52}(104 - 7(1.052) - 13(1.952)) = \frac{71.202}{52} = 1.369$$

$$x_3 = \frac{1}{29}(71 - 3(1.052) - 8(1.369)) = \frac{56.891}{29} = 1.962$$

$$\textcircled{4} x_4 = \frac{1}{83}(95 - 11(1.369) + 4(1.962)) = \frac{87.785}{83} = 1.058$$

$$y_4 = \frac{1}{52}(104 - 7(1.058) - 13(1.962)) = \frac{71.094}{52} = 1.367$$

$$x_4 = \frac{1}{29}(71 - 3(1.058) - 8(1.367)) = \frac{56.889}{29} = 1.962$$

$$\textcircled{5} x_5 = \frac{1}{83}(95 - 11(1.367) + 4(1.962)) = 87.808 / 83 = 1.058$$

$$y_5 = \frac{1}{52}(104 - 7(1.058) - 13(1.962)) = 71.092 / 52 = 1.367$$

$$x_5 = \frac{1}{29}(71 - 3(1.058) - 8(1.367)) = 56.889 / 29 = 1.962$$

$$x = 1.06, y = 1.37, z = 1.96$$

$$1.2x + 2.1y + 4.2z = 9.9$$

$$5.3x + 6.1y + 4.7z = 21.6$$

$$9.2x + 8.3y + z = 15.2$$

$$x_{k+1} = \frac{1}{1.2} (9.9 - 2.1y_0 + 4.2z_0)$$

$$y_{k+1} = \frac{1}{6.1} (21.6 - 5.3x_{k+1} + 4.7z_0)$$

$$z_{k+1} = \frac{1}{1} (15.2 - 9.2x_{k+1} - 8.3y_{k+1})$$
$$(x_0, y_0, z_0) = (0, 0, 0)$$

$$\textcircled{1} \quad x_1 = \frac{1}{1.2} (9.9) = 8.25$$

$$y_1 = \frac{1}{6.1} [21.6 - 5.3(8.25) - 4.7(0)] = \frac{-22.125}{6.1} = -3.627$$

$$z_1 = \frac{1}{1} [15.2 - 9.2(8.25) - 8.3(-3.627)] = \frac{1}{1} (-30.595) = -30.595$$

\textcircled{2}

$$x_2 = \frac{1}{1.2} [9.9 - 2.1(-3.627) - 4.2(-30.595)] = \frac{146.018}{1.2} = 121.682$$

$$y_2 = \frac{1}{6.1} [21.6 - 5.3(121.682) - 4.7(-30.595)] = \frac{1}{6.1} (-479.53) = -78.609$$

$$z_2 = \frac{1}{1} [15.2 - 9.2(121.682) - 8.3(-78.609)] = (-451.818)$$

\textcircled{3}

$$x_3 = \frac{1}{1.2} [9.9 - 2.1(-78.609) - 4.2(-451.818)] = \frac{2072.612}{1.2} = 1727.172$$

$$y_3 = \frac{1}{6.1} [21.6 - 5.3(1727.172) - 4.7(-451.818)] = \frac{1}{6.1} (-7008.89) = -1148.999$$

$$z_3 = \frac{1}{1} [15.2 - 9.2(1727.172) + 8.3(-1148.999)] = -6338.130$$

$$\begin{aligned} 5x + 2y + z &= 12 \\ x + 4y + 2z &= 15 \\ x + 2y + 5z &= 20 \end{aligned}$$

$$x_{k+1} = 1/5 (12 - 2y_k - z_k)$$

$$y_{k+1} = 1/4 (15 - x_{k+1} - 2z_k)$$

$$z_{k+1} = 1/5 (20 - x_{k+1} - 2y_{k+1})$$

$$(x_0, y_0, z_0) = (0, 0, 0)$$

$$\textcircled{1} \quad \frac{x_1}{1/5} [12 - 2(0) - (0)] = 1/5 (12) = 2.4$$

$$y_1 = 1/4 (15 - (2.4) - (0)) = 1/4 (12.6) = 3.15$$

$$z_1 = 1/5 (20 - (2.4) - 2(3.15)) = 1/5 (11.3) = 2.26$$

$$\textcircled{2} \quad x_2 = 1/5 (12 - 2(3.15) - (2.26)) = 1/5 (3.44) = 0.688$$

$$y_2 = 1/4 (15 - (0.688) - 2(2.26)) = 1/4 (9.792) = 2.448$$

$$z_2 = 1/5 (20 - (0.688) - 2(2.448)) = \frac{14.416}{5} = 2.883$$

$$\textcircled{3} \quad x_3 = 1/5 (12 - 2(2.448) - (2.883)) = 1/5 (4.221) = 0.844$$

$$y_3 = 1/4 (15 - (0.844) - 2(2.883)) = 1/4 (8.389) = 2.092$$

$$z_3 = 1/5 (20 - (0.844) - 2(2.092)) = 1/5 (14.961) = 2.992$$

$$\textcircled{4} \quad x_4 = 1/5 (12 - 2(2.092) - (2.992)) = \frac{4.813}{5} = 0.963$$

$$y_4 = 1/4 (15 - (0.963) - 2(2.992)) = 8.053/4 = 2.013$$

$$z_4 = 1/5 (20 - (0.963) - 2(2.013)) = 1/5 (15.011) = 3.002$$

$$\textcircled{5} \quad x_5 = 1/5 (12 - 2(2.013) - (3.002)) = 4.981/5 = 0.994$$

$$y_5 = 1/4 (15 - (0.994) - 2(3.002)) = 8.001/4 = 2$$

$$z_5 = 1/5 (20 - (0.994) - 2(2)) = 15.005/5 = 3.001$$

$$\textcircled{6} \quad x_6 = 1/5 [12 - 2(2) - 3 \cdot 001] = 4998/5 \approx 1$$

$$y_6 = 1/4 [15 - 1 - 2(3 \cdot 001)] = 7998/4 \approx 2$$

$$z_6 = 1/5 [20 - (1) - 2(2)] \approx 15 \cdot 001/5 \approx 3$$

$$\textcircled{7} \quad x_7 = 1/5 [12 - 2(2) - 3] = 5 \cdot 001/5 \approx 1$$

$$y_7 = 1/4 [15 - (1) - 2(3)] = 7999/4 \approx 2$$

$$z_7 = 1/5 [20 - (1) - 2(2)] \approx 15/5 = 3$$

$$x=1, y=2, z=3, \text{ all } N \in \{(0), (1), (2), (3)\}$$