*Types of Turing Machines:-

Non-deterministic Turing Machines

A non-deterministic turing Machines is a machine for which like non-deterministic finite automata, at any current state and for the tape symbol it is reading, there may be different possible actions to be performed. Here an action means a combination of writing a symbol on the tape, moving the tape head and going to a next state. One action could be just changing the state without modifying the cell content. One action could be not changing the state and changing the cell content. One action could be changing both the state and cell content. In all actions, it may move left or right.

For example, $L = \{W|W / W \in \{a,b\}^*\}$.

It must findout the midpoint by, pairing off symbols from the two ends of x.

* Any language accepted by a non-deterministic Turing Machine is also accepted by deterministic Turing Machine

* Transition function (δ):- $Q \times \Gamma \longrightarrow 2^{Q \times \Gamma \times \{L,R\}}$

Turing Machines with two-dimensional tapes

Turing machines with two-dimensional tape is a kind of TM that has one finite control, one read-write head and one two-dimensional tape. The cells in the tape is two-dimensional i.e., the tape has the top end and the left end, but extends indefinitely to the right and down. It is divided into rows of small squares. For any TM of this type, there is an equivalent TM with a one-dimensional tape that is equally powerful.

* To simulate a two-dimensional tape with a one-dimensional tape, first we map the squares of two-dimensional tape to those of a one-dimensional tape diagonally as shown in the following tables.

Two-dimensional tape:-

v	v	v	v	v	v	v	...	...
h	1	2	6	7	15	16	...	...
h	3	5	8	14	17	26	...	...
h	4	9	13	18	25	...	...	...
h	10	12	19	24	...	...	...	...
h	11	20	23	...	...	...	...	...
h	21	22	<.7	...	...	...	...	...
...	...	...	...	...	...	...	...	...

Here, the numbers indicate the correspondence between the squares of two tapes: square numbered i in two-dimensional tape is mapped to square numbered i in one-dimensional tape. Symbols h and v are not in tape alphabet and they are used to mark the left and the top end of the tape respectively.

Equivalent one-dimensional tape:-

v	1	v	2	3	h	4	5	6	v	7	8	9	10	h	11	...	...

Head of two-dimensional tape moves one square up, down, left or right.

* Some TM's with a one-dimensional tape can simulate every move of TM with a two-dimensional tape. Hence they are at least as powerful as TMs with two-dimensional tape.
* Since TMs with a two-dimensional tape obviously can simulate TMs with a one-dimensional tape, it can be said that they are equally powerful
* Transition function (δ):- $Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, T, B\}$

Turing Machines with Multiple Tapes:-

* This kind of TM has one finite control and more than one tape each with its own read/write head.
* It is denoted by a 7-tuple.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

* It's transition function is a partial function

$$\delta : Q \times \Gamma^n \longrightarrow (Q \times \{h\}) \times \Gamma^n \times \{L, R\}^n$$

A configuration for this kind of TM must show the current state the machine is in and the state of each tape.
It can be proved that any language accepted by a n-tape TM can be accepted by a one-tape TM and that any function computed by n-tape TM can be computed by a one-tape TM. Since the converses are obviously true, one can say that one-tape TMs are as powerful as n-tape TMs.

Turing Machines with Multiple heads:-

* This kind of TMs has one finite control and one tape, but more than one read/write heads. In each state, only one of the heads is allowed to read and write.
* It is denoted by a 7-tuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

* The transition function is a partial function:

$$\delta : Q \times \{H_1, H_2, \ldots, H_n\} \times \Gamma \rightarrow (Q \cup \{h\}) \times \Gamma \times \{L, R\}.$$

Where $H_1, H_2, \ldots, H_n$ denote the tape heads.

* It can be easily seen that this type of TMs are as powerful as one-tape TMs.

Turing Machines with Infinite tape

* This is a kind of TM that have one finite control and one tape which extends infinitely in both directions.
* It turns out that this type of TMs are also as powerful as one-tape TMs whose tape has left end.

* Two-way infinite tapes:-

A_{-4}	A_{-3}	A_{-2}	A_{-1}	A_0	A_1	A_2	A_3	A_4	...