UNIT-1

WAVE OPTICS

Wave Optics:

Interference of light-Principle of Superposition-Interference in thin films (reflected light)-Newton‟s Rings-Determination of Wavelength- Applications of Interference, Diffraction-Fraunhofer Diffraction-Single slit Diffraction -Diffraction Grating – Grating Spectrum - Polarization-Polarization by reflection, refraction and double refraction-Nicol‟s Prism-Half wave and Quarter wave plate- applications of Polarization.

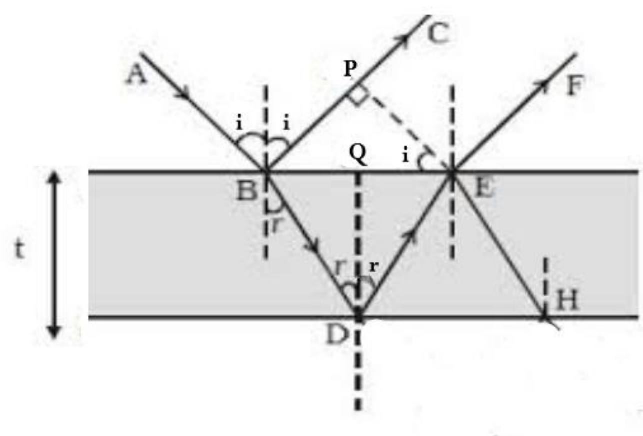
Interference: Modification of intensity of light when two or more light waves superpose each other is called interference

Principle of superposition: When two or more light waves superpose each other the resultant displacement at any point is the algebraic sum of individual displacements.

$$y = y_1 + y_2$$

Interference in thin films by reflection :

Consider a thin film of thickness 't' and refractive index 'μ'. Lt AB be the incident ray on the film. BC is reflected ray from the upper surface of the film and BD the refracted beam. DE is the emergent ray.



The optical path difference between the two reflected light rays (BC and EF) is given by

$$= \mu(BD + DE) - BP \quad \text{- - - (1)}$$

From the $\Delta BDQ \; and \; \Delta EDQ$, $cosr = \frac{t}{BD} = \frac{t}{DE} \;\; or \;\; BD = DE = \frac{t}{cosr}$

Substituting in equation (1)

∴ Path difference = $\frac{2\mu t}{cosr} - BP$

From $\Delta BPE, BP = BE \sin i = (BQ + QE) \sin i$

Also from ΔBDQ, $\mathrm{BQ} = \mathrm{t} \tan r = QE$
Or $BP = 2t \tan r \sin i$

$\therefore$ Path difference = $\frac{2\mu t}{cos r} - \frac{2\mu t\, sin^2 r}{\cos r} = 2\mu t \frac{(1-sin^2 r)}{\cos r}$

Or **Path difference = $\boldsymbol{2\mu t cos\, r}$**

As a ray reflected at a surface backed by a denser medium suffers an abrupt phase change of π which is equivalent to a path difference $\Delta = \frac{\lambda}{2}$

$\therefore$ **Effective Path difference = $\boldsymbol{2\mu t cos\, r - \frac{\lambda}{2}}$ ----------(2)**

(i) Condition for bright band

The film will appear bright if path difference $\boldsymbol{2\mu t cos\, r - \frac{\lambda}{2}} = \boldsymbol{n}\lambda$

Or $$\boldsymbol{2\mu t cos\, r = (2n+1)\frac{\lambda}{2} \quad where\;\; n = 0, 1, 2, 3 \ldots ..}$$

(ii) Condition for dark band

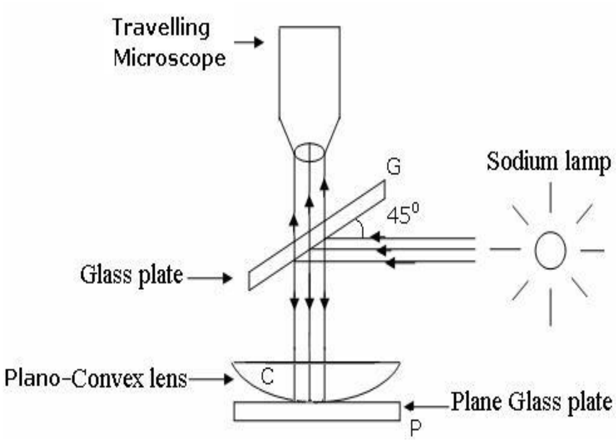
The film will appear dark if path difference $\boldsymbol{2\mu t cos\, r - \frac{\lambda}{2}} = \boldsymbol{(2n+1)\frac{\lambda}{2}}$

$$ie.,\; \boldsymbol{2\mu t cos\, r = (n+1)\lambda} \quad where\;\; n = 0,1,2,3 \ldots ..$$
$$or$$
$$\boldsymbol{2\mu t cos\, r = n\lambda}\; where\; n = 1,2,3 \ldots \ldots$$

Newton's Rings:

1. The experimental arrangement consists of a planoconvex lens L of large radius of curvature R.
2. This lens with its convex surface is placed on a plane glass plate P.
3. Light from an extended monochromatic source such as sodium lamp falls on a glass plate G held at an angle 45° with the vertical.
4. The glass plate G reflects normally a part of the incident light towards the air film enclosed by the lens L and the glass plate P.
5. A part of the incident light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of the plate.
6. These two reflected rays interfere and give rise to an interference pattern in the form of circular rings.

7. These rings are localised in the air film, and can be seen with a microscope focussed on the film.



The path difference between the superposing waves = $(2\mu t\cos r + \lambda/2)$.

For air film $\mu = 1$ and for normal incidence r = 0,

$\therefore$Path difference = $(2t + \lambda/2)$ …………….. (1)

At centre t=0 path difference = $(\lambda/2)$

which is the condition of minimum intensity. Thus the central spot is dark.

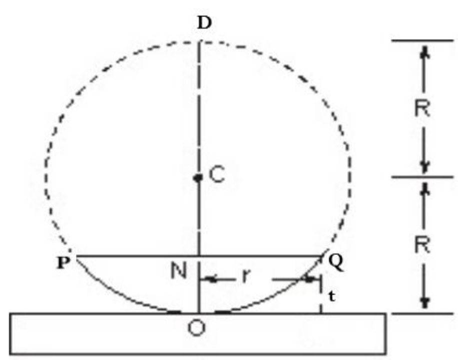
For nth order maximum, we have

$$2t + \lambda/2 = n\lambda \text{ ……………………. (2)}$$

For nth order minimum, we have

$$2t + \lambda/2 = (2n+1)\frac{\lambda}{2} \text{ ……………………. (3)}$$

Determination of wavelength of monochromatic light:



Let **'r'** be the radius of Newton's ring-corresponding to the constant film thickness **'t'**.

From the geometry of the figure,

$$NP \times NQ = NO \times ND$$

Substituting the values

$$r \times r = t \times (2R - t) = 2Rt - t^2 \approx 2Rt$$

$$\therefore \; r^2 = 2Rt \;\text{ or }\; t = r^2/2R \;\;\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From eq. (2)

$$2t + \lambda/2 = n\lambda$$

$$\therefore 2\frac{r^2}{2R} + \frac{\lambda}{2} = n\lambda$$

$$D^2 = 4(2n-1)\frac{\lambda}{2}R$$

Diameter of bright ring $D = 2\sqrt{(2n-1)\frac{\lambda}{2}R}$ **………………..(5)**

From equation (3) we have

$2t = n\lambda$ for dark ring

Substituting $t = r^2/2R$

$$2\frac{r^2}{2R} = n\lambda$$

or $\quad r^2 = n\lambda R$

or $\quad D^2 = 4n\lambda R$

D is diameter of the ring $\quad D = 2\sqrt{n\lambda R}$ **……………………..(6)**

If D_n and D_{n+p} the diameters of n^{th} and $(n+p)^{th}$ dark rings respectively, then from eq.

$$D_n^2 = 4n\lambda R$$

and $\quad D^2_{n+p} = 4(n+p)R\lambda$

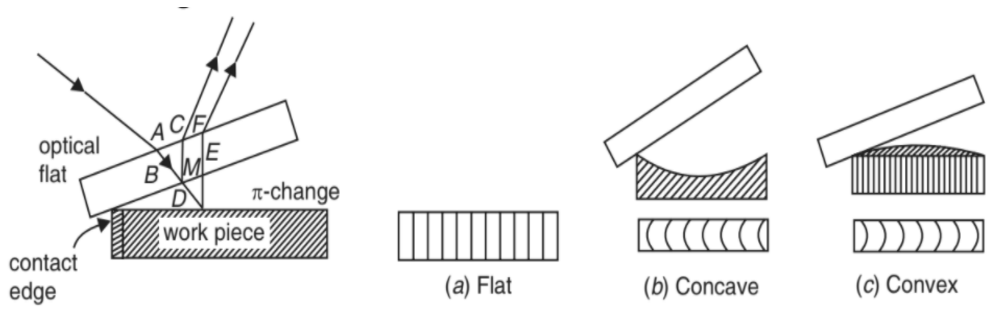
$\therefore \quad D^2_{n+p} - D_n^2 = 4pR\lambda$

or $\quad$ **Wavelength of monochromatic light** $\lambda = \dfrac{D^2_{n+p} - D_n^2}{4pR}$

Applications of Interference:

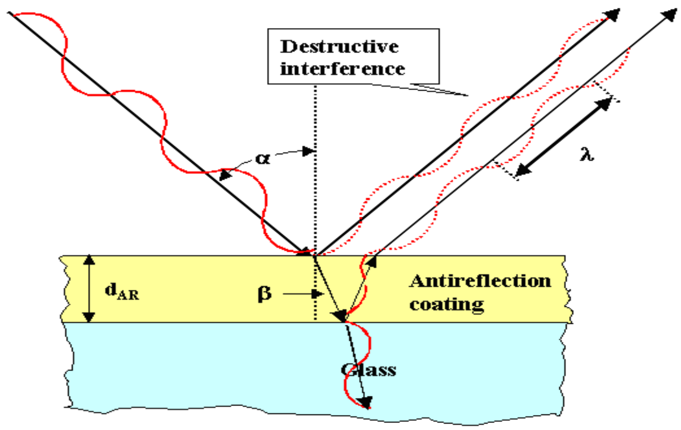
1. **Testing of flatness of surfaces:**

   The smoothness of a surface can be inspected by keeping an optically flat on the component at an angle and illuminating with monochromatic light. The air wedge formed between the component and optical flat produces straight and equidistant fringes if the surface is smooth. If fringes are curved towards the contact edge, the surface is concave and if the fringes curve away it is convex.



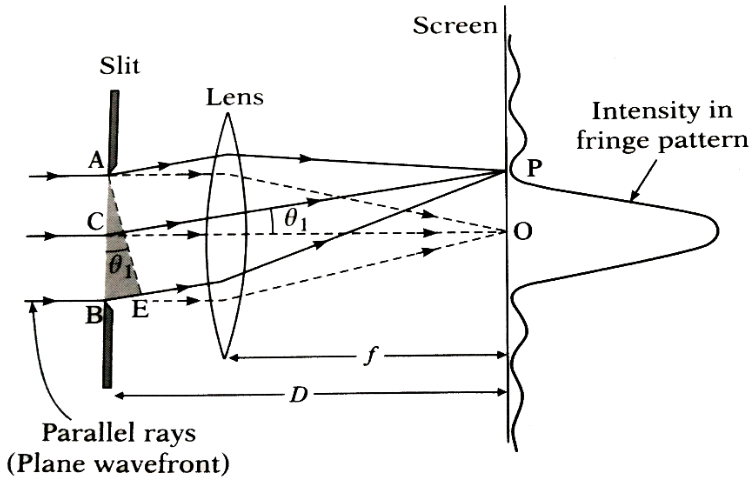
2. **Anti reflecting coatings(AR Coatings):**

   Antireflection coatings are thin transparent coatings of optical thickness of one quarter wavelength given on a surface in order to suppress reflections from the surface. The waves with equal amplitude reflected from the top and bottom surfaces of the thin film are in opposite phase such that their overlapping leads to destructive interference.



The film should adhere well and scratch proof. MgF_2 is widely used AR coating.

Fraunhofer diffraction at single slit:



Let AB represent a narrow slit of width e .

Let a plane wavefront of monochromatic light of wavelength λ propagating normally to the slit be incident on it.

Let the diffracted light be focused by means of a convex lens on a screen placed in the focal plane of the lens.

According to Huygens principle, every point of the wavefront in the plane of the slit is a source of secondary spherical wavelets, which spread out to the right in all directions.

The secondary wavelets travelling normally to the slit, are brought to focus at O by the lens. The secondary wavelets travelling at an angle θ with the normal are focused at a point P on the screen. The point P is of the minimum intensity depending upon the path difference between the secondary waves originating from the corresponding points of the wavefront.

In order to find out intensity at P.

The path difference between secondary wavelets from A and B in direction θ.

$$\mathrm{BE} = \mathrm{AB}\sin\theta = \mathrm{e}\sin\theta$$

and corresponding phase difference = $\frac{2\pi}{\lambda}.\mathrm{e}\sin\theta$.

Let us consider that the width of the slit is divided into n equal parts and the amplitude of the wave from each part is a. The phase difference between any two consecutive waves from these parts would be

$$\frac{1}{\mathrm{n}}(\text{Total phase}) = \frac{1}{\mathrm{n}}\left(\frac{2\pi}{\lambda}\mathrm{e}\sin\theta\right) = \mathrm{d}(\text{say})$$

Using the method of vector addition of amplitudes as discussed in the previous article, the resultant amplitude R is given by

$$\mathrm{R} = \mathrm{a}\frac{\sin \mathrm{nd}/2}{\sin \mathrm{d}/2} = \mathrm{a}\frac{\sin(\pi\mathrm{e}\sin\theta/\lambda)}{\sin(\pi\mathrm{e}\sin\theta/\mathrm{n}\lambda)}$$

$$= a\frac{\sin\alpha}{\sin\alpha/n} \text{ where } \alpha = \pi e\sin\theta/\lambda$$

$$= a\frac{\sin\alpha}{\alpha/n} \qquad (\because \frac{\alpha}{n} \text{ is very small})$$

$$= na\frac{\sin\alpha}{\alpha}$$

$$= A\frac{\sin\alpha}{\alpha} \text{(where } n \equiv \infty, a \to 0 \text{ but product na = A remains finite)}$$

Now the intensity is given by

$$I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{- - - (1)}$$

Principal maximum. The expression for resultant amplitude R can be written in ascending powers of α as

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

If the negative terms vanish, the value of R will be maximum, i.e., $\alpha = 0$

$$\therefore \alpha = \frac{\pi e\sin\theta}{\lambda} = 0 \text{ or } \sin\theta = 0 \text{ or } \theta = 0 \qquad \text{- - - (2)}$$

Now maximum value of R is A and intensity is proportional to A^2. The condition $\theta = 0$ means that this maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is a known as principal maximum.

Minimum intensity positions. The intensity will be minimum when $\sin\alpha = 0$. The values of α which satisfy this equation are $\alpha = \pm\pi, \pm 2\pi, 3\pi, \pm 4\pi, ...$ etc. $= \pm m\pi$

or $$\frac{\pi e\sin\theta}{\lambda} = \pm m\pi \text{ or } e\sin\theta = \pm m\lambda \text{ - - - (3)}$$

where m = 1, 2, 3, ... etc.

In this way we obtain the points of minimum intensity on either side of the principal maximum. The value of m = 0 is not admissible, because for this value $\theta = 0$ and this corresponds to principal maximum.

Secondary maxima. In addition to principal maximum at $\alpha = 0$, there are weak secondary maxima between equally spaced minima. The positions can be obtained with the rule of finding maxima and minima of a given function in calculus. Differentiating the expression of I with respect to α and equating to zero, we have

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

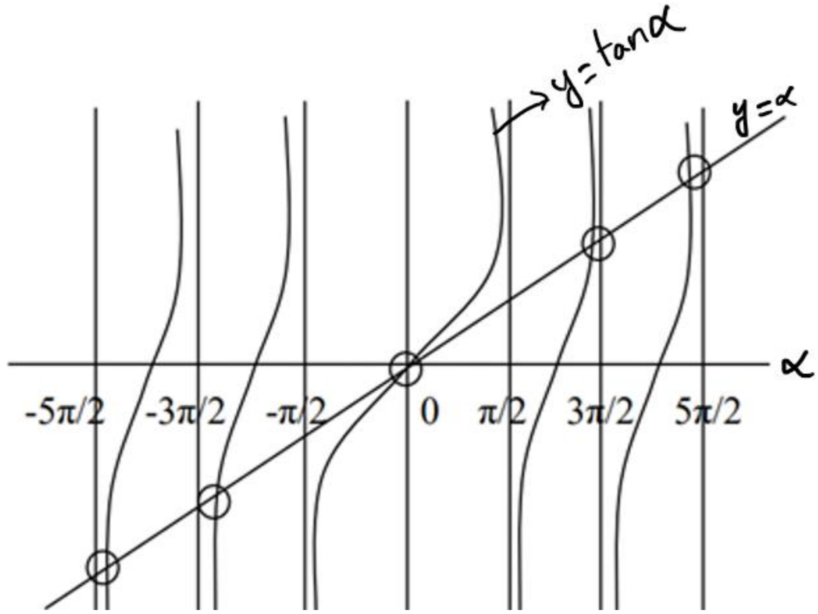
or $\quad A^2\,\frac{2\sin\alpha}{\alpha}.\frac{(\alpha\cos\alpha - \sin\alpha)}{\alpha^2} = 0$

either $\sin\alpha = 0$ or $(\alpha\cos\alpha - \sin\alpha) = 0$

The equation $\sin\alpha = 0$ gives the values of α (except 0) for which the intensity is zero on the screen. Hence the positions of maxima are given by the roots of the equation

$\alpha\cos\alpha - \sin\alpha = 0$ or $\alpha = \tan\alpha$ - - - (4)

The values of α satisfying the above equation are obtained graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$ on the same graph.



The points of intersection of the two curves gives the values of α which satisfy equation (4). The plots of $y = \alpha$ and $y = \tan\alpha$ are shown in figure.

The points of intersections are

$\alpha = 0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2},$ etc.

$\alpha = 0,$ gives principal maximum

Substituting approximate values of α in equation (1), we ge the intensities in various maxima as

$I_0 = A^2$ (Principal maximum)

$$I_1 = A^2\left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2 = \frac{A^2}{22}\text{ App.}$$

(1st Subsidiary maximum)

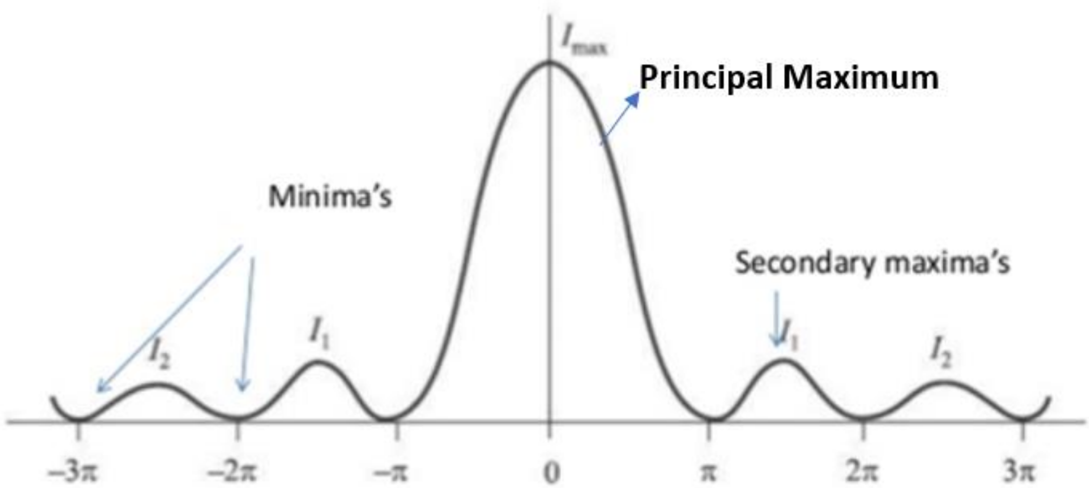
$$I_2 = A^2\left[\frac{\sin(5\pi/2)}{(5\pi/2)}\right]^2 = \frac{A^2}{62}\text{ App.}$$

(2nd Subsidiary maximum)

And so on.

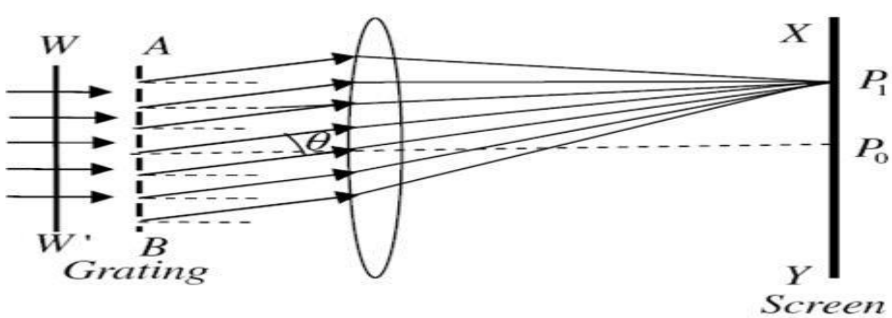
From the expressions of I_0, I_1, I_2, it is evident that most of the incident light is concentrated in the principal maximum.

Intensity distribution graph. A graph showing the variation of intensity with α is shown in figure. The diffraction pattern consists of a central principal maximum occurring in the direction of incident rays.



There are subsidiary maxima of decreasing intensity on either sides of it at positions $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$ ... It should be noted that subsidiary maxima do not fall exactly mid-way between two minima, but they are displaced towards the centre of the pattern, of course, the displacement decreases as the order of maximum increases.

PLANE DIFFRACTION GRATING (Diffraction at N parallel slits)



An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced.

If there are N slits, then we have diffracted waves, one each from the middle points of the slits. If 'e' is width of each slit and 'd' the distance between any two successive slits, the path difference between two consecutive slits is $(e+d)\sin\theta$. Therefore there is a corresponding phase difference $\partial = (2\pi/\lambda)(e+d)\sin\theta$ between the two consecutive waves.

Amplitude due to each slit = $a = A\frac{Sin\alpha}{\alpha}$

Resultant amplitude = $R = a\frac{SinN\partial/2}{Sin\partial/2} = A\frac{Sin\alpha}{\alpha}\left[\frac{SinN\pi(e+d)sin\theta/\lambda}{Sin\pi(e+d)sin\theta/\lambda}\right]$

$$R = A\frac{Sin\alpha}{\alpha}\ \frac{SinN\beta}{Sin\,\beta} \qquad \text{where } \beta = \frac{\pi}{\lambda}(e+d)sin\theta$$

$$\therefore Intensity\ I = A^2\ \frac{Sin^2\alpha}{\alpha^2}\ \frac{Sin^2N\beta}{sin^2\beta}$$

Principal maxima. The intensity would be maximum when $sin\beta = 0\ or\ \ \beta = \pm n\pi$

$$\therefore \frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

or $\quad (e+d)\sin\theta = \pm n\lambda \qquad$ --- $\quad$ (3)

where $\quad$ n = 0,1, 2, 3...

n = 0 corresponds to zero order maximum. For n = 1, 2, 3... etc. we obtain first, second, third etc., principal maxima respectively. The $\pm$ sign shows that there are two principal maxima of the same order lying on either side of zero order maximum.

According to L'Hospital's rule $\lim\limits_{\beta\to\pm n\pi}\frac{sinN\beta}{sin\beta} = \lim\limits_{\beta\to\pm n\pi}\frac{NcosN\beta}{cos\beta} = \pm N$

$$\therefore Intensity\ I = A^2\ \frac{Sin^2\alpha}{\alpha^2}\ N^2$$

Minima. A series of minima occur, when

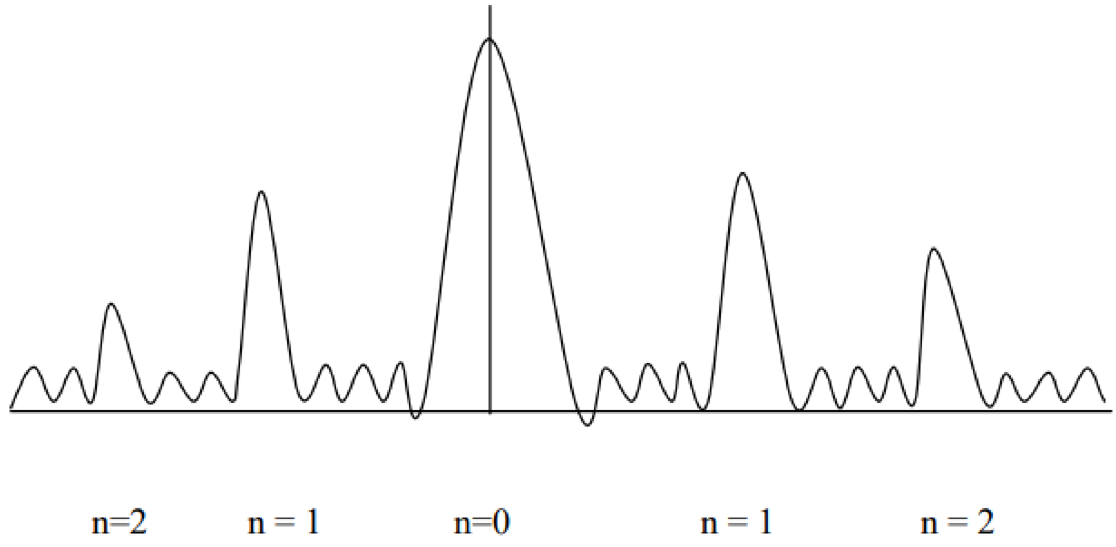
$$N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$N(e+d)\sin\theta = \pm m\lambda, \qquad \text{---} \quad (4)$$

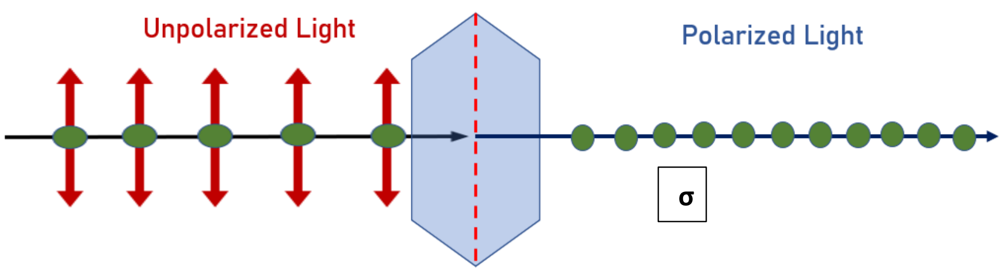
Where m has all integral values except 0, N, 2N, ... nN, because for these values $\sin\beta$ becomes zero and we get principal maxima. Thus m = 1, 2, 3 ... (N-1). Hence there are adjacent principal maxima.

As there are (N-1) minima between two adjacent principal maxima, there must be (N-2) other maxima between two principal maxima. These are known as secondary maxima.

As N increases the intensity of secondary maxima relative to principal maxima decreases and becomes negligible when N becomes large.

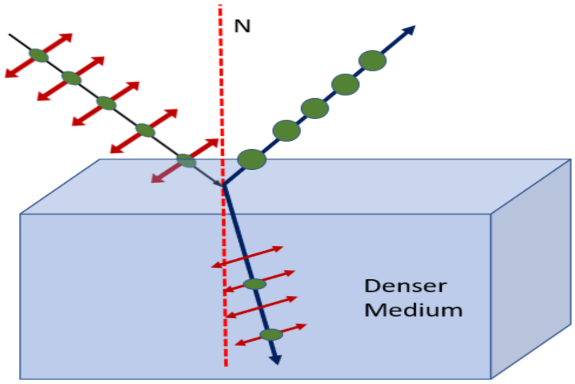


Polarization of Light:
Confining light to only one direction of vibration is called polarization.



Light is an electromagnetic wave. All the electric vectors can be resolved into two components, one with vibration parallel to plane of paper called π or arrow components and vibration perpendicular to plane of paper called σ component. When unpolarized light with both σ and π components is passed through a Tourmaline crystal, the coming out beam has only σ component. The π component is eliminated and beam is said to be polarized. The vibration is confined to only one plane called plane of vibration. The plane in which vibration is eliminated is called plane of polarization.

Polarization by reflection:



When unpolarized light is incident on a medium of refractive index μ at an angle called Brewster angle or angle of polarization (i_P) , the reflected beam is plane polarized. This is called polarization by reflection. Also the reflected and refracted rays will be perpendicular to each other.

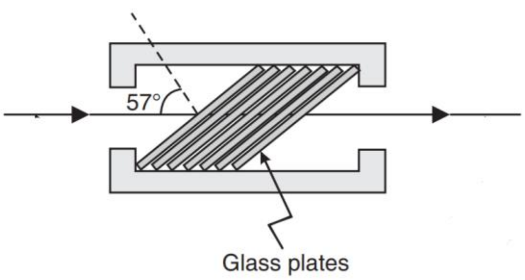
From Snell's law $\mu = \frac{Sini_p}{\sin r} = \frac{Sini_p}{\sin(90-i_p)} = \frac{Sini_p}{cosi_p} = tani_p$

This is called Brewster's law

Brewster's Law statement

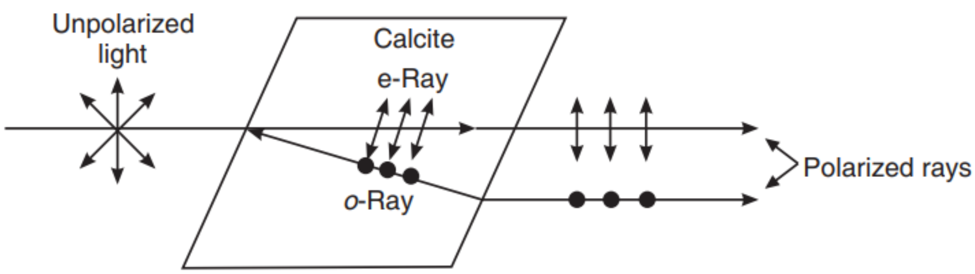
When angle of incident is equal to angle of polarization, the reflected ray and refracted ray will be perpendicular to each other.

Polarization by refraction:



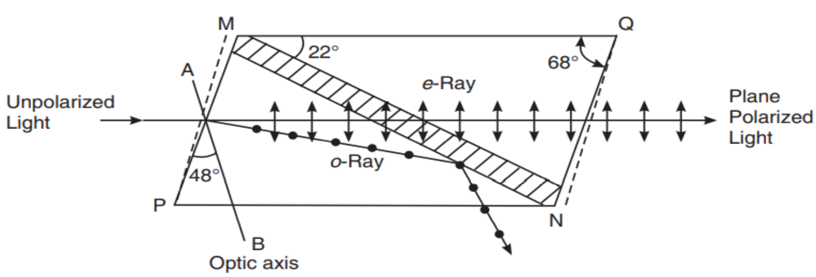
When light is incident on a denser medium like glass with an angle of incidence equal to polarizing angle, the reflected ray is completely polarized but refracted beam is partially polarized. So to make the refracted beam completely polarized we use a pile of glass plates. A stack of 15 glass plates supported in a tube of suitable size, inclined at an angle of 33^0 to the axis of tube is called pile of glass plates. Unpolarized light is incident at Brewster's angle and after passing through these set of glass plates light gets completely polarized parallel to the plane of incidence.

Polarization by double refraction:



When a beam of unpolarized light is incident on the surface of an anisotropic crystal such as calcite or quartz, it is found that it will separate into two rays that travel in different directions. This phenomenon is called birefringence or double refraction. The two rays are known as ordinary ray (o-ray) and extraordinary ray (e-ray), which are linearly polarized in mutually perpendicular directions. A single linearly polarized ray is obtained in practice through elimination of one of the two polarized rays.

Nicol Prism



Construction: A Nicol prism is made from calcite crystal. A rhomb of calcite crystal about three times as long as it is thick, is obtained by cleavage from the original crystal. The ends of the rhombohedron are ground until they make an angle of 68° instead of 71° with the longitudinal edges. This piece is then cut into two along a plane perpendicular both to the principal axis and to the new end surfaces MP and QN. The two parts of the crystal are then cemented together with canada balsam, whose refractive index lies between the refractive indices of calcite for the o-ray and e-ray. $\mu_o = 1.66$, $\mu_e = 1.486$ and μ of canada balsam = 1.55. The refractive index for e-ray depends upon the direction in which e-ray is propagating in the crystal.

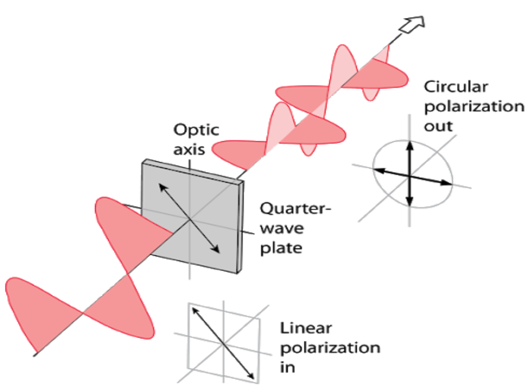
Working: Unpolarized light is made to fall on the crystal as shown. The ray after entering the crystal suffers double refraction and splits up into o-ray and e-ray. The values of the refractive indices and the angles of incidence at the canada balsam layer are such that the e-ray is transmitted while the o-ray is internally reflected. The face where the o-ray is incident is blackened so that the o-ray is completely absorbed. Then we get only the plane-polarized e-ray coming out of the Nicol. Thus, the Nicol works as a polarizer.

Quarter Wave Plate:

A quarter wave plate is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness adjusted such that it introduces a quarter-wave (l/4) path difference (or a phase difference of 90°) between the e-ray and o-ray propagating through it.
When a plane polarized light wave is incident on a negative birefringent crystal having the optic axis parallel to its refracting face, the wave splits into e-wave and o-wave .The two waves travel along the same direction but with different velocities. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them. Thus,

$$(\mu_o - \mu_e)d = \frac{\lambda}{4}$$

$$d = \frac{\lambda}{4[\mu_o - \mu_e]}$$



A quarter wave plate introduces a phase difference δ, between e-ray and o-ray

$$\delta = \left(\frac{2\pi}{\lambda}\right)\Delta = \frac{\pi}{2} = 90°$$

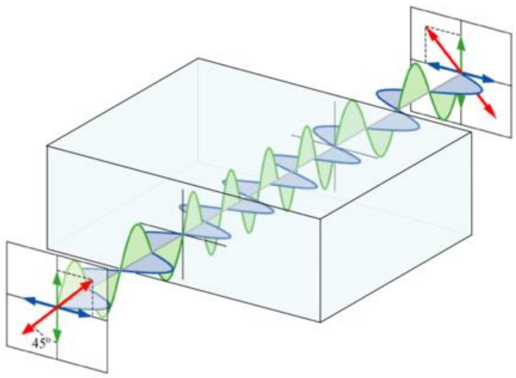
A quarter-wave plate is used for producing elliptically or circularly polarized light.

Half wave plate :

A half wave plate is a thin plate of birefringent crystal having the optic axis parallel to its refracting faces and its thickness chosen such that it introduces a half-wave (l/2) path difference (or a phase difference of 180°) between e-ray and o-ray. When a plane polarized light wave is incident on a birefringent crystal having the optic axis parallel to its refracting faces, it splits into two waves: o- and e-waves. The two waves travel along the same direction inside the crystal but with different velocities. As a result, when they emerge from the rear face of the crystal, an optical path difference would be developed between them.

$$\Delta = (\mu_o - \mu_e)d = \frac{\lambda}{2}$$

$$d = \frac{\lambda}{2(\mu_o - \mu_e)}$$



A half wave plate introduces a phase difference δ, between e-ray and o-ray given by

$$\delta = \left(\frac{2\pi}{\lambda}\right)\Delta = \pi = 180°$$

A half-wave plate rotates the plane of polarization of the incident plane polarized light through an angle 2θ.

Applications of polarization:

(1) Sunglasses:

The phenomenon of polarization is utilized in making sunglasses, which will drastically reduce the glare. Polarized sunglasses contain polarizing filters that are oriented vertically with respect to the frames . As the reflected light is partially polarized, light waves having their electric field vectors oriented in the same direction as the polarizing lenses and light waves having their electric field vectors oriented parallel to the reflecting surface are blocked by the lenses. Thus, polarized sunglasses eliminate the glare from an illuminated surface.

(2) Photography

Polarization by scattering occurs as light passes through our atmosphere. The scattered light often produces a glare in the skies. In photography, this partial polarization of scattered light produces a washed-out sky. The problem is overcome by the use of a polarizing filter fitted to the camera. As the filter is rotated, the partially polarized light is blocked and the glare is reduced. Thus, a vivid blue sky as the backdrop of a beautiful foreground is captured using polarizing filters.

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