

Unit-II

Find y when $x = 160 \text{ & } x = 410$

Eg - 29.13.

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63							
150	13.03	> 2.4	> -0.34					
200	15.04	> 2.01	> -0.24	> 0.15				
250	16.81	> 1.77	> -0.24	> 0.08	> -0.07	> 0.02		
300	18.42	> 1.61	> -0.16	> 0.03	> -0.05			> -0.02
350	19.90	> 1.48	> -0.13	> 0.02	> -0.01	> 0.04		
400	21.27	> 1.32	> -0.11					

$$P = \frac{x - x_0}{h} = \frac{160 - 100}{50} = 1.2 ;$$

$$P = \frac{x - x_n}{h} = \frac{10 - 50}{50} = 0.2$$

$$\begin{aligned} f(160) &= y_0 + P\Delta y + \frac{P(P-1)}{2!} \Delta^2 y + \dots \\ &= 10.63 + 2.88 + (-0.0668) + (-0.00048) + (-0.0001008) + \\ &\quad (-0.000016128) \\ &= 13.457, \end{aligned}$$

$$\begin{aligned} f(410) &= y_n + P\Delta y_n + \frac{P(P+1)}{2!} \Delta^2 y + \dots \\ &= 21.27 + 0.274 + (-0.0132) + 0.00176 + (-0.000704) \\ &= 21.53 \end{aligned}$$

Eg 29.14

Studenten b/w marks 40 and 45

Mark
Students

	30-40	40-50	50-60	60-70	70-80
	31	42	51	35	31

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
<40	31					
<50	73	> 42				
<60	124	> 51	> 9	> -25		
<70	159	> 35	> -16	> 12	> 37	
<80	190	> 31	> -4			

$$P = \frac{x - x_0}{h} = \frac{40 - 40}{20} = 0.5$$

$$f(4.8) = y_0 + P\Delta y + \frac{P(P-1)}{2!} \Delta^2 y + \dots$$

$$= 47.8$$

$$\approx 48$$

No of students get marks b/w 40 & 45 = 48 - 31 = 12

eg 29.15 find cubic polynomial eq.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1	> 1	> -2	
1	2	> -1	> 12	
2	1	> -1	> 10	
3	10	> 9		

$$P = \frac{x-x_0}{h} = \frac{x-0}{1} = x$$

$$f(x) = y_0 + P\Delta y + \frac{P(P-1)}{2!} \Delta^2 y + \dots$$

$$= 1 + x + \frac{(x-1)x}{2}(-2) + x \frac{(x-1)(x-2)}{6} \frac{12}{x+5}$$

$$= 1 + x - x(x-1) + 2x(x-1)(x-2)$$

$$= 2x^3 - 7x^2 + 6x + 1$$

$$P = \frac{x-x_0}{h} = \frac{4-3}{1} = 1$$

$$f(u) = y_n + P\Delta y_n + \frac{P(P+1)}{2!} \Delta^2 y_n + \dots$$

$$= 10 + 9 + \frac{1(2)}{2} (10) + \frac{1(2)(3)}{3 \times 2!} \approx 12$$

$$= 41$$

eg 29.16 find 1st & 10th term

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
3	4.8	> 3.6			
4	8.4	> 2.5			
5	14.5	> 6.1	> 3	> 0.5	> 0
6	23.6	> 9.1	> 3.5	> 0.5	> 0
7	36.2	> 12.6	> 4	> 0.5	> 0
8	52.8	> 16.6	> 4.5	> 0.5	> 0
9	73.9	> 21.1			

$$P = \frac{x-x_0}{h} = \frac{1-3}{1} = -2$$

$$f(1) = y_0 + p\Delta y + \frac{p(p-1)}{2!} \Delta^2 y + \dots$$

$$= 4.87 - 2 \times 3.6 + \frac{(-2)(-2-1)(2-1)}{2} + \frac{(-2)(-2-1)(2-2)}{6} \approx 0.1$$

$$\approx 31$$

$$p = \frac{x-x_0}{h} = \frac{10-9}{1} = 1$$

$$f(10) = y_0 + p\Delta y + \frac{p(p-1)}{2!} \Delta^2 y + \dots$$

$$= 73.9 + 21.1 + \frac{2}{2} \cdot 4.5 + \frac{23}{3.2} \approx 0.7$$

$$= 100$$

Eg Recurrence 29.3.

Find y at $x=10$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	9			
15	36	> 21	> -16	
25	35	> 5	> 2	> 18
35	42	> 7		

$$p = \frac{x-x_0}{h} = \frac{10-5}{10} = 0.5$$

$$f(10) = y_0 + p\Delta y + \frac{p(p-1)}{2!} \Delta^2 y + \dots$$

$$= 9 + (0.5)21 + \frac{(0.5)(0.5-1)}{2}(-16) + 0.5 \frac{(0.5-1)(0.5-2)}{6} \approx 0.1$$

$$= 22.625 \approx 23$$

2. Using Newton interpolation value of $\exp(1.85)$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.7	5.474	> 0.576					
1.8	6.030	> 0.636	> 0.06	> 0.007			
1.9	6.686	> 0.703	> 0.067	> 0.007	> 0		
2.0	7.389	> 0.777	> 0.074	> 0.008	> 0.001		
2.1	8.166	> 0.859	> 0.082	> 0.008	> 0		
2.2	9.022	> 0.949	> 0.09	> 0.008	> 0		
2.3	9.974						

$$P = \frac{x - x_0}{h} = \frac{1.85 - 1.8}{0.1} = 0.5$$

$$\begin{aligned}
 f(1.85) &= y_0 + P(\Delta y) + \frac{P(P-1)}{2!} \Delta^2 y + \dots \\
 &= 6.030 + (0.5)(0.636) + \frac{(0.5)(0.5-1)(0.067)}{2} + \frac{(0.5)(0.5-1)(0.5-2)}{6}(0.001) \\
 &\quad + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(0.008) \\
 &= 6.359
 \end{aligned}$$

3)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
40	184	> 20		
50	204	> 22	> 2	> 0
60	226	> 24	> 2	
70	250	> 26	> 2	> 0
80	270	> 28	> 2	> 0
90	304			

$$P = \frac{x - x_0}{h} = \frac{x - 40}{10}$$

$$\begin{aligned}
 f(40) &= y_0 + P(\Delta y) + \frac{P(P-1)}{2!} \Delta^2 y + \dots \\
 &= 184 + \frac{(x-40)}{10} \times 20 + \frac{(x-40)(x-50)}{10 \times 10 \times 2} \times 2
 \end{aligned}$$

$$= 0.01x^2 + 1.1x + 124$$

$$P = \frac{4340}{10} = 0.3$$

$$f(43) = y_0 + P\Delta y + \frac{P(P-1)}{2!} \Delta^2 y$$

$$= 184 + (0.3)(20) + \frac{(0.3)(0.3-1)}{2} \times 2$$

$$= 189.79$$

4)

Find s_{MS2}

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
45	0.7031			
50	0.7660	> 0.0589		
55	0.8192	> 0.0531	> -0.0055	
60	0.8660	> 0.0468	> -0.0063	> -0.0005

$$P = \frac{52-50}{5} = 0.4$$

$$- P(52) = 0.7660 + 0.4 \times 0.0531 + \frac{0.4(0.4-1)}{2} \times (-0.0063) = 0.78779$$

(i) Find y at $x=46$.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
45	114.84				
50	96.16	> -18.68			
55	83.32	> -12.84	> 5.84		
60	74.48	> -8.84	> 4	> -1.84	
65	68.48	> -6	> 2.84	> -1.16	> 0.68

$$P = \frac{46-45}{5} = 0.2$$

$$f(46) = 114.84 + (0.2)(-18.68) + \frac{(0.2)(0.2-1)}{2} \times 5.84 + \frac{(0.2)(0.2-1)(0.2-2)}{3!} \times 2.84 - \frac{(0.2)(0.2-1)(0.2-2)(0.2-3)}{4!} \times (-1.16) = 110.5286$$

7) Find $f(22)$ & $f(42)$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
20	354	>-22	>-19			
25	332	>-41	>10	>29	>-37	
30	291	>-31	>2	>-8	>8	>45
35	260	>-29	>2	>0		
40	231	>-27	>2			
45	204					

$$P = \frac{22-20}{5} = 0.4$$

$$f(2) = 354 + 0.4(-22) + \frac{0.4(0.4-1)(-19)}{2} + \frac{0.4(0.4-1)(0.4-2)(29)}{6} + \\ \frac{0.4(0.4-1)(0.4-2)(0.4-3)(-37)}{24} + \frac{0.4(0.4-1)(0.4-2)(0.4-3)(0.4-4)}{120} \times 45 \\ = 352.72 = 352$$

$$P = \frac{42-45}{5} = -0.6$$

$$f(42) = 204 + (-0.6)(-27) + \frac{(-0.6)(-0.6+1)(2)}{2} + \frac{(-0.6)(-0.6+1)(-0.6+2)(0.6+2)}{24} + \\ \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)(-0.6+4)}{120} \times 45 \\ = 218.66 \\ = 219$$

	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
10		0.9848					
20		0.9397	>-0.0451				
30		0.8660	>-0.0737	>-0.02816			
40		0.7660	>-0.1	>-0.0263	>0.0023	>0.0008	
50		0.6428	>-0.1232	>-0.0232	>0.0031	>0.0005	>-0.0003
60		0.5000	>-0.1428	>-0.0196	>0.0036	>0.0008	>0.0003
70		0.3420	>-0.158	>-0.0152	>0.0044	>0.0005	>-0.0013
80		0.1737	>-0.1683	>-0.0103	>0.0049	>0.0005	

$\Delta^6 y$ $\Delta^7 y$

$$0.0006 > 0.0012$$

$$-0.0006$$

$$P = \frac{25-20}{10} = 0.5$$

$$f(25) = 0.9847 + 0.5(-0.0737) + \frac{0.5(0.5+1)(-0.0263)}{2} + \frac{0.5(0.5+1)(0.5+2)}{6} \\ + \frac{0.5(0.5+1)(0.5+2)(0.5+3)}{24} (0.0005) + \frac{0.5(0.5+1)(0.5+2)(0.5+3)(0.5+4)}{(0.0031)}$$

$$\approx 0.9063$$

$$P = \frac{73-80}{10} = -0.7$$

$$f(73) = 0.1737 + (-0.7)(-0.1683) + \frac{(-0.7)(-0.7+1)(-0.7+2)}{2} (-0.0103) \\ + \frac{(-0.7)(-0.7+1)(-0.7+2)(-0.7+3)}{6} (0.0049) \\ \approx 0.2923$$

(1)

$$\begin{array}{r} x \ 0 \ 1 \ 2 \\ y \ 0 \ 1 \ 2 \end{array}$$

$$x \quad y \quad \Delta y \quad \Delta^2 y \quad P = \frac{x_0}{1} = x$$

$$\begin{array}{r} 0 \ 0 \\ 1 \ 5 \\ 2 \ 8 \end{array} > 5 > -8$$

$$f(x) = y_0 + P\Delta y + \frac{P(P-1)}{2!} \Delta^2 y = 0 + x(5) + \frac{x(x-1)}{2}(-8)$$

$$= 5x - 4x(x-1) = 5x - 4x^2 + 4x$$

$$= -4x^2 + 9x$$

$$\rightarrow \begin{array}{r} x \ 4 \ 6 \ 8 \ 10 \\ y \ 1 \ 3 \ 8 \ 16 \end{array}$$

$$\begin{array}{r} x \ 4 \ 6 \ 8 \ 10 \\ y \ 1 \ 3 \ 8 \ 16 \\ \Delta y \ 2 \ 5 \ 8 \\ \Delta^2 y \ 3 \ 3 \ 0 \\ \Delta^3 y \ 0 \ 0 \ 0 \end{array}$$

$$f(x) = 1 + \frac{x-4}{2} \cdot 2 + \frac{(x-4)(x-6)}{2 \cdot 2 \cdot 2} \cdot 3$$

$$= \frac{3x^2 - 22x + 48}{8}$$

$$P = \frac{1}{2} = 0.5$$

$$f(x) = 1 + 0.5(2) + 0.5 \cdot \frac{(0.5-1)}{2!} \times 3 = 1.625$$

$$\rightarrow f(0.6)$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0.1	0.003					
0.3	0.067	> 0.064	> 0.017			
0.5	0.148	> 0.081	> 0.019	> 0.002	> 0.001	
0.7	0.248	> 0.1	> 0.022	> 0.003	> 0.001	> 0
0.9	0.370	> 0.122	> 0.022	> 0.004	> 0.001	> 0
1.1	0.518	> 0.148	> 0.026	> 0.005	> 0.001	
1.3	0.697	> 0.179	> 0.031			

$$P = \frac{0.6 - 0.5}{0.2} = 0.5$$

$$f(0.6) = 0.148 + 0.5(0.1) + \frac{0.5(0.5-1)}{2} \times 0.022 + \frac{0.5(0.5-1)(0.5-2)}{3!} \times 0.004 \\ + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24} \times 0.001 \approx 0.19546$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
1	1.5	> 0.5	> 0.2	> 0	> 0.4
2	2.2	> 0.7	> 0.2	> 0.4	
3	3.1	> 0.9	> 0.6		
4	4.6	> 1.5			

$$P = \frac{3.8 - 4}{1} = -0.2$$

$$f(3.8) = 4.6 + (-0.2)(1.5) + \frac{(-0.2)(0.2+1)(0.6)}{2} + \frac{(-0.2)(-0.2+1)}{(0.2+2)(0.4)} \times 0.4 \\ + \frac{(-0.2)(-0.2+1)(-0.2+2)(-0.2+3)}{24} \times 0.4 \\ = 4.21820$$

$$(6) \quad u_1 = 40, \quad u_3 = 45, \quad u_5 = 54 \quad \text{find } u_2 \text{ & } u_4$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
u_1	40				
u_2	a	$>a-40$			
u_3	45	$>45-a$	$>85-2a$	$>3a+b-175$	$>364-4a-4b$
u_4	b	$>b-45$	$>a+b-90$	$>189-a-3b$	
u_5	54	$>54-b$	$>99-2b$		
		$\Delta^3 y = 0$	$\Delta^4 y = 0$		
				$3a+b-175 = 0$	$364-4a-4b = 0$
				$a=42, b=49$	

$$(7) \quad u_1 = 10, \quad u_2 = 8, \quad u_3 = 10, \quad u_4 = 50 \quad \text{find } u_0 \text{ & } u_5$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
u_1	10					
u_0	a	$>a-10$	$>18-2a$			
u_1	8	$>8-a$	$>a-6$	$>3a-24$		
u_2	10	>2	$>b-12$	$>b-a-6$	$>b-4a+18$	
u_3	b	$>b-10$	$>-3b+72$	$>a-4b+78$	$>5a-5b+60$	
u_4	50	$>50-b$	$>60-2b$			
		$\Delta^4 y = 0$	$\Delta^5 y = 0$			
				$-4a+b+18 = 0$	$5a-5b+60 = 0$	
				$u_3 = 22$		
				$u_0 = 10$		

Eg - 29.24

Lagrange formula

x	5	7	11	13	17
y	150	392	1492	2366	5202

$$y = \frac{(x-x_1)(x-x_2) \dots (x-x_n)}{(x_0-x_1)(x_0-x_2) \dots (x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2) \dots (x-x_n)}{(x_1-x_0)(x_1-x_2) \dots (x_1-x_n)} y_1 + \dots$$

$$\begin{aligned}
f(a) &= \frac{(a-7)(a-11)(a-13)(a-17)}{(5-7)(5-11)(5-13)(5-17)} \times 150 + \frac{(a-5)(a-11)(a-13)(a-17)}{(7-5)(7-11)(7-13)(7-17)} \times 392 \\
&\quad + \frac{(a-5)(a-7)(a-13)(a-17)}{(11-5)(11-7)(11-13)(11-17)} \times 1492 + \frac{(a-5)(a-7)(a-11)(a-17)}{(13-5)(13-7)(13-11)(13-17)} \times 2366 \\
&\quad + \frac{(a-5)(a-7)(a-11)}{(17-5)(17-7)(17-11)(17-13)} \times 5202 = 897.823
\end{aligned}$$

Eg 29.25

x	0	2	5
f(x)	2	3	12

$$\begin{aligned}
y &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \dots \\
&= \frac{(x+1)(x+2)(x+5)}{(-1)(-2)(-5)} x^2 + \frac{x(x+2)(x+5)}{1 \cdot (-1)(-4)} x^3 + \frac{x(x+1)(x+5)}{2 \cdot 1 \cdot (-3)} x^4 + \frac{x(x+1)(x+2)}{5 \cdot 4 \cdot 3} x^5 \\
&= x^3 + x^2 - x + 2
\end{aligned}$$

$$f(3) = 35.$$

eg-29.26.

x	0	1	3	6
y	18	10	-18	90

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1, \dots$$

$$= \frac{(x-1)(x-2)(x-6)}{-1 \times 3 \times -6} \times 18 + \frac{x(x-3)(x-6)}{1 \times (-2) \times (-5)} \times 10 + \frac{x(x-1)(x-6)}{3 \times 2 \times -3} (-18)$$

$$+ \frac{x(x-1)(x-3)}{6 \times 5 \times 3} \times 90$$

$$y = 2x^3 - 10x^2 + 18$$

$$\left(\frac{dy}{dx} \right)_{x=2} = 6x^2 - 20x$$

$$= 6(4) - 20(2)$$

$$= -16,$$

eg 29.27. Using Lagrange formula exp the fun

$\frac{3x^2+x+1}{(x-1)(x-2)(x-3)}$ as sum of partial func

$$\rightarrow \text{let } y = 3x^2 + x + 1$$

x	1	2	3
y	5	15	31

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1, \dots$$

$$= \frac{(x-2)(x-3)}{(-1)(-1)} \times 5 + \frac{(x-1)(x-3)}{(1)(-1)} \times 15 + \frac{(x-1)(x-2)}{(1)(1)} \times 3,$$

$$3x^2 + x + 1 = \underline{\underline{5(x-2)(x-3)}} - 30(x-1)(x-3) + 62(x-1)(x-2)$$

$$\frac{3x^2+x+1}{(x-1)(x-2)(x-3)} = \frac{5}{2(x-1)} - \frac{15}{x-2} + \frac{31}{x-3},$$

g 29.28

t	0	1	3	4
v	21	15	12	10

$$\text{distance } (s) = \int v \cdot dt$$

$$\text{Acceleration} \approx \frac{dv}{dt}$$

$$v = (t-t_1)(t-t_2)(t-t_3)$$

$$\frac{(t_0-t_1)(t_0-t_2)(t_0-t_3)}{(t_1-t_0)(t_2-t_1)(t_3-t_0)} v_0 + \frac{(t-t_0)(t-t_2)(t-t_3)}{(t_1-t_0)(t_2-t_1)(t_3-t_0)} v_1 + \dots$$

$$= (t-1)(t-3)(t-4)$$

$$= \frac{-1(0-3)(0-4)}{1(0-2)(0-3)} \times 21 + \frac{t(t-3)(t-4)}{1(0-2)(0-3)} \times 15 + \frac{t(t-1)(t-4)}{3(0-2)(0-1)} \times 12$$

$$+ \frac{t(t-1)(t-3)}{4(0-2)(0-1)} \times 10$$

$$v = \frac{1}{12} (-5t^3 + 38t^2 - 105t + 252)$$

$$\text{distance moved} = \int_0^4 v \cdot dt$$

$$= \frac{1}{12} \int_0^4 (-5t^3 + 38t^2 - 105t + 252) dt$$

$$= 54.9$$

$$\text{Acceleration} = \frac{dv}{dt}$$

$$= \frac{1}{12} (-5t^2 + 76t - 105)$$

$$t = 4$$

$$= -3.44$$

Exercise - 29.5

1) Find y at $x=10$

x	5	6	9	12
y	12	13	14	16

$$y = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \dots$$

$$= \frac{4 \cdot 1 \cdot (-2)}{-1 \cdot 4 \cdot -7} \cdot 12 + \frac{5 \cdot 1 \cdot (-2)}{1 \cdot (-3) \cdot (-6)} \cdot 13 + \frac{5 \cdot 4 \cdot (-2)}{4 \cdot 3 \cdot (-3)} \cdot 14 + \frac{5 \cdot 4 \cdot 1}{7 \cdot 6 \cdot 3} \cdot 16$$

$$= \frac{124}{7} - \frac{65}{9} + \frac{140}{9} + \frac{160}{63} = 14.30.$$

2.	x	654	658	659	661
	y	2.8165	2.8182	2.8189	2.8202

at $x=656$

$$y = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \dots$$

$$= \frac{(-2)(-3)(-1)}{-4 \cdot (-3) \cdot (-2)} \cdot 2.8165 + \frac{2 \cdot (-3)(-5)}{4 \cdot (-1) \cdot (-3)} \cdot 2.8182 + \frac{2 \cdot (-2)(-1)}{5 \cdot 1 \cdot (-2)} \cdot 2.8189 + \frac{2 \cdot (-2)(-3)}{7 \cdot 3 \cdot 2} \cdot 2.8202$$

$$= 2.8169,$$

→ Find I at $T=16$

T	1.2	2.0	2.5	3.0
I	1.36	0.58	0.34	0.20

$$I = \frac{(t-t_0)(t-t_1)(t-t_3)}{(t_0-t_1)(t_0-t_2)(t_0-t_3)} I_0 + \frac{(t-t_0)(t-t_1)(t-t_3)}{(t_1-t_0)(t_1-t_2)(t_1-t_3)} I_1 + \dots$$

$$= \frac{-0.4(-0.9)(-1.4)}{-0.8(-1.3)(-1.8)} \cdot 1.36 + \frac{0.4(-0.9)(-1.4)}{0.8(-0.5)(-1)} \cdot 0.58 + \frac{0.4(-0.4)(-1.4)}{1.3 \cdot 0.5(-0.5)} \cdot 0.2$$

$$+ \frac{0.4 \cdot (-0.4) \cdot 0.9}{1.8 \cdot 1 \cdot 0.5} \cdot 0.2 = 0.8946.$$

→ Find y at $x=2000$

y	1997	1999	2001	2002
x	43	65	159	248

$$\begin{aligned}
 y &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \dots \\
 &= \frac{+1 \cdot (-2)}{-2 \cdot -4 \cdot -5} \times 43 + \frac{3 \cdot (-1) \cdot (-2)}{2 \cdot (-2) \cdot (-3)} 65 + \frac{3 \cdot 1 \cdot (-2)}{4 \cdot 2 \cdot (-1)} \times 159 \\
 &\quad + \frac{3 \cdot 1 \cdot (-1)}{5 \cdot 3 \cdot 1} \times 248 \\
 &= 10011
 \end{aligned}$$

x	0	1	2	5
y	2	3	12	147

Find polynomial

$$\begin{aligned}
 y &= \frac{(x-1)(x-2)(x-5)}{-1 \cdot -2 \cdot -5} x^2 + \frac{x(x-2)(x-5)}{1 \cdot (-1)(-4)} x^3 + \frac{x(x-1)(x-5)}{2 \cdot 1 \cdot 3} x^4 \\
 &\quad + \frac{x(x-1)(x-2)}{5 \times 4 \times 3} x^4 \\
 &= x^3 + x^2 - x + 2
 \end{aligned}$$

x	1	3	4	6
y	-3	9	36	132

$$\begin{aligned}
 y &= \frac{(x-3)(x-4)(x-6)}{-2 \cdot -3 \cdot -5} x(-3) + \frac{(x-1)(x-4)(x-6)}{2 \cdot 1 \cdot 3} x^9 \\
 &\quad + \frac{(x-1)(x-3)(x-6)}{3 \cdot 1 \cdot 2} x^5 + \frac{(x-1)(x-3)(x-4)}{5 \times 3 \times 2} x^{132} \\
 &= x^3 + 3x^2 - x - 6
 \end{aligned}$$

x	0	1	3	5	6	9
y	-18	0	0	-248	0	(3104)

$$\begin{aligned}
 y &= \frac{(x-1)(x-3)(x-5)(x-6)(x-9)}{-1 \cdot -3 \cdot -5 \cdot -6 \cdot -9} \times (-18) + \frac{x(x-1)(x-3)(x-5)(x-6)}{5 \cdot 4 \cdot 2 \cdot -1 \cdot -4} \times 248 \\
 &\quad + \frac{x(x-1)(x-3)(x-5)(x-6)}{9 \cdot 8 \cdot 6 \cdot 4 \cdot 3} \times 3104 \\
 &= x^5 - 9x^4 + 18x^3 - x^2 + 19x - 18.
 \end{aligned}$$

x	1	2	4	5	6
y	14	15	5	-9	

$$\begin{aligned}
 f(5) &= \frac{3 \cdot 1 \cdot (-1)}{-1 \cdot -3 \cdot -5} \times 14 + \frac{4 \cdot 1 \cdot (-1)}{1 \cdot 2 \cdot -4} \times 15 + \frac{4 \cdot 3 \cdot (-1)}{3 \cdot 2 \cdot (-2)} \times 5 \\
 &\quad + \frac{4 \cdot 3 \cdot 1}{5 \cdot 4 \cdot 2} \times 9 \\
 &= 2 \cdot 8 - 7 \cdot 5 + 5 + 2 \cdot 7 = 31.
 \end{aligned}$$

Newton's Divide Difference

eg 29.29

x	5	7	11	13	17
y	150	392	1452	2316	5202

f(9)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	150			
7	392	> 241	> 24	
11	1452	> 265	> 32	> 1
13	2316	> 457	> 42	> 1
17	5202	> 709		

$$f(x) = y_0 + (x-x_0) [x_0, x_1] + (x-x_0)(x-x_1) [x_0, x_1, x_2] + \\ (x-x_0)(x-x_1)(x-x_2) [x_0, x_1, x_2, x_3]$$

$$f(9) = (50 + (9-5) (21 + (9-5) (9-7) (24)) + (9-5) (9-7) (9-4)) \\ = (50 + 484 + 192 - 16) = 810$$

eg 29.30

x	-4	-1	0	2	5
y	1245	33	5	9	1335

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-4	1245				
-1	33	> -404	> 94		
0	5	> -28	> 14	> 3	
2	9	> 2	> 10	> 13	
5	1335	> 442	> 88		

$$f(x) = y_0 + (x-x_0) \Delta y + (x-x_0)(x-x_1) \Delta^2 y + (x-x_0)(x-x_1)(x-x_2) \Delta^3 y \\ + (x-x_0)(x-x_1)(x-x_3)(x-x_4) \Delta^4 y \\ = (1245 + (x+4)(-404)) + (x+4)(x+3) 94 + (x+4)(x+3)x(-14) + \\ (x+4)(x+3)x(x-2) 3 \\ = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

ex@ex 29.7

find x when $f(x) = 15$

①	x	5	6	9	14
	y	12	13	14	15

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)} x_1 +$$

$$= \frac{2 \times 1 \times -1}{-1 \times -2 \times -4} \times 5 + \frac{3 \times 1 \times -1}{1 \times -1 \times -3} \times 6 + \frac{3 \times 2 \times (-1)}{2 \times 1 \times -2} \times 9 + \frac{3 \times 2 \times 1}{4 \times 3 \times 2} \times 11 \\ = \frac{5}{4} - 6 + \frac{27}{2} + \frac{11}{4} = 11.511.$$

∴ find t when $A = 8t$

$$t \quad 2 \quad 5 \quad 8 \quad 14$$

$$A \quad 94.8 \quad 87.9 \quad 81.3 \quad 68.7$$

$$t = \frac{-2 \cdot 9 \times 3 \cdot 7 \times 16.3}{6.9 \times (3.5 \times 26.1)} \times 2 + \frac{-9 \cdot 8 \times 3.7 \times 16.3}{-6.9 \times 6.6 \times 19.2} \times 5 \\ t = \frac{-9.8 \times -2.9 \times 16.3}{-13.5 \times -6.6 \times 12.6} \times 8 + \frac{-9.8 \times -2.9 \times 3.7}{-26.1 \times -19.2 \times -12.6} \times 14 \\ = 6.3038$$

Q 29.31

$$x \quad 1.2 \quad 2.1 \quad 2.8 \quad 4.1 \quad 4.9 \quad 6.2 \\ y \quad 4.2 \quad 6.8 \quad 9.8 \quad 13.4 \quad 18.5 \quad 19.6$$

find x for $y = 12$

$$x = \frac{(12-6.8)(12-9.8)(12-13.4)(12-18.5)(12-19.6)}{(4.2-6.8)(4.2-9.8)(4.2-13.4)(4.2-18.5)(4.2-19.6)} \times 1.2 + \\ \frac{(12-4.2)(12-9.8)(12-13.4)(12-18.5)(12-19.6)}{(6.8-4.2)(6.8-9.8)(6.8-13.4)(6.8-18.5)(6.8-19.6)} \times 2.1 + \\ \frac{(12-4.2)(12-6.8)(12-13.4)(12-18.5)(12-19.6)}{(9.8-4.2)(9.8-6.8)(9.8-13.4)(9.8-18.5)(9.8-19.6)} \times 2.8 + \\ \frac{(12-4.2)(12-6.8)(12-9.8)(12-15.5)(12-19.6)}{(13.4-4.2)(13.4-6.8)(13.4-9.8)(13.4-15.5)(13.4-19.6)} \times 4.1 + \\ \frac{(12-4.2)(12-6.8)(12-9.8)(12-13.4)(12-19.6)}{(15.5-4.2)(15.5-6.8)(15.5-9.8)(15.5-13.4)(15.5-19.6)} \times 4.9 +$$

$$\begin{aligned}
 & (12-4.2) (12-6.8) (12-9.8) (12-18.4) (12-15.4) \times 6.2 \\
 & (19.6-4.2) (19.6-6.8) (19.6-9.8) (19.6-18.4) (19.6-15.4) \\
 = & 0.022 - 0.234 + 1.252 + 3.419 - 0.964 + 0.054 \\
 = & 3.55
 \end{aligned}$$

eg 29.32

x	30	34	38	42
y	-30	-13	3	18

$$\begin{aligned}
 x &= \frac{(13)(-3)(-18)}{(-17)(-337)(-48)} \times 30 + \frac{(30)(-3)(18)}{17(16)(-31)} \times 34 + \frac{30(13)(-18)}{33(16)(-15)} \times 42 \\
 &\quad + \frac{30(13)(-3)}{48(31)(15)} \times 42 \\
 &= -0.782 + 6.532 + 33.682 - 2.202 \\
 &= 37.23
 \end{aligned}$$

eg 30.1

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.780	10.031

find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ at $x=1.1$ & $x=1.6$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$	$\Delta^7 y$
1.0	7.989	>0.414						
1.1	8.403	>-0.378	>-0.036					
1.2	8.781	>0.348	>-0.03	>0.006				
1.3	9.129	>0.322	>-0.026	>0.004	>-0.002			
1.4	9.451	>0.299	>-0.023	>0.003	>-0.001	>0.001		
1.5	9.780	>0.281	>-0.018	>0.005	>0.002	>0.003	>0.002	
1.6	10.031							

$$a) \left(\frac{dy}{dx} \right)_{1,1} = \frac{1}{h} \left[\Delta y - \frac{1}{2} \Delta^2 y + \frac{1}{3} \Delta^3 y - \frac{1}{4} \Delta^4 y + \frac{1}{5} \Delta^5 y - \frac{1}{6} \Delta^6 y \right]$$

$$= \frac{1}{0.1} \left[0.378 + \frac{0.03}{2} + \frac{0.004}{3} + \frac{0.001}{4} + \frac{0.003}{5} \right]$$

$$= 3.9518 \text{ r/s}$$

$$\left(\frac{d^2y}{dx^2} \right)_{1,1} = \frac{1}{(0.1)^2} \left[\Delta^2 y - \Delta^3 y + \frac{11}{12} \Delta^4 y - \frac{5}{6} \Delta^5 y + \dots \right]$$

$$= \frac{1}{(0.1)^2} \left[-0.03 - 0.004 - \frac{11}{12}(0.001) - \frac{5}{6}(0.003) + \dots \right]$$

$$= -3.7416.$$

$$\left(\frac{dy}{dx} \right)_{1,6} = \frac{1}{h} \left[\Delta y + \frac{1}{2} \nabla^2 y + \frac{1}{3} \nabla^3 y + \frac{1}{4} \nabla^4 y + \frac{1}{5} \nabla^5 y + \dots \right]$$

$$= \frac{1}{0.1} \left[0.281 - \frac{0.018}{2} + \frac{0.005}{3} + \frac{0.002}{4} + \frac{0.003}{5} + \frac{0.002}{6} \right]$$

$$= 2.757$$

$$\left(\frac{d^2y}{dx^2} \right)_{1,6} = \frac{1}{h^2} \left[\nabla^2 y + \nabla^3 y + \frac{11}{12} \nabla^4 y + \frac{5}{6} \nabla^5 y + \dots \right]$$

$$= \frac{1}{(0.1)^2} \left[-0.018 + 0.005 + \frac{11}{12}(0.002) + \frac{5}{6}(0.003) + \dots \right]$$

$$= -0.7077 \text{ r/s}$$

Ex-30.1

①

x	0	1	2	3	4	5
y	4	1/8	2/15	3/7	4/6	5/2

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	4					
1	8	> 4	> 3			
2	15	> 7	> -15	> -18	> 42	
3	24	> -8	> 9	> 24	> -30	> -72
4	37	> 1	> 9	> -6		
5	59	> 4	> 3			

$$y'(0) = \frac{1}{1} [4 - 3/2 - 18/3 + \frac{42}{4} - \frac{72}{5} \dots] = -28.5$$

$$y''(0) = \frac{1}{1} [3 + 18 + \frac{4}{12} (402) - 8(-72) \dots] = -0.5$$

→

x	1.5	2.0	2.5	3	3.5	4
y	3.375	7	13.625	24	38.875	59

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375	> 3.625				
2	7	> 3				
2.5	13.625	> 6.625	> 3.75	> -6		
3	24	> 10.375	> 6.75	> -2.25	> 9	
3.5	38.875	> 14.875	> 4.5	> 0.75	> 3	
4	59	> 20.125	> 5.25			

$$\frac{dy}{dx} = \frac{1}{0.5} (3.625 - \frac{3}{2} + \frac{3.75}{3} + \frac{6}{4} + \frac{9}{5} \dots) = 13.35$$

$$\frac{d^2y}{dx^2} = \frac{1}{(0.5)^2} [3 - 3.75 - \frac{11}{12}(6) - \frac{5}{6}(9) \dots] = -5.5$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3} \left[\Delta^3 y_0 - \frac{3}{2} \Delta^4 y_0 + \dots \right]$$

$$= \frac{1}{(0.5)^3} \left[3.75 + \frac{3}{2}(6) + \dots \right] = 1024$$

x	y	Δy	δy	$\delta^2 y$	$\delta^3 y$	$\delta^4 y$	$\delta^5 y$	$\delta^6 y$
1.00	1.024	> 0.024	> -0.001	> 0.002				
1.01	1.049	> 0.024	> 0.001	> 0.001	> 0.002			
1.02	1.072	> 0.023	> 0.001	> -0.001	> 0.001	> 0		
1.03	1.095	> 0.023	> 0	> 0	> 0.001	> -0.001	> -0.002	
1.04	1.118	> 0.023	> 0	> -0.001				
1.05	1.140	> 0.022	> -0.001					

$x = 1.05$

$$\frac{dy}{dx} = \frac{1}{0.05} \left(0.004 - \frac{0.001}{2} - \frac{0.001}{3} - \frac{0.001}{4} - \frac{0.002}{5} + \dots \right)$$

$$= 0.4504$$

$$\frac{d^2y}{dx^2} = \frac{1}{(0.05)^2} \left(0.001 + 0.001 + 0.001 \times \frac{1}{12} + \frac{5}{12} \times (0.002)^2 + \dots \right)$$

$$= 1.833.$$

at $x = 1.25$

$$\frac{dy}{dx} = \frac{1}{0.05} \left(0.023 + \frac{0.001}{4} + \dots \right) = 0.461$$

$$\frac{d^2y}{dx^2} = \frac{1}{(0.05)^2} \left(\frac{11}{12} (0.001) + \dots \right) = 0.3666$$

→ find value of ~~Δx~~ cos 1.74

x	1.7	1.74	1.78	1.82	1.86
$\sin x$	0.9916	0.9857	0.9781	0.9691	0.9584

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1.7	0.9916				
1.74	0.9857	> -0.0059			
1.78	0.9781	> -0.0076	> -0.0017	> 0.0003	> -0.0006
1.82	0.9691	> -0.009	> -0.0014	> -0.0003	
1.86	0.9584	> -0.0107	> -0.0017		

$$\left(\frac{dy}{dx}\right)_{0.74} = \frac{1}{0.04} \left[-0.0076 + \frac{0.0014}{2} - \frac{0.0003}{3} \right] \\ \approx -0.175$$

-	t	0	1	2	3	4	5	6
D		0	2.5	8.5	15.5	24.5	36.5	50

find Speed at $t=5$ sec

t	x	Δx	$\Delta^2 x$	$\Delta^3 x$	$\Delta^4 x$	$\Delta^5 x$	$\Delta^6 x$
0	0						
1	2.5	> 2.5	> 3.5				
2	8.5	> 6	> 1	> -2.5	> 3.5		
3	15.5	> 7	> 2	> 1	> 0	> -3.5	> 1
4	24.5	> 9	> 3	> 1	> 0	> -2.5	> -2.5
5	36.5	> 12	> 3	> -1.5	> -2.5	> -2.5	
6	50	> 13.5	> 1.5				

$$\left(\frac{dx}{dt}\right)_{t=5} = \frac{1}{1} [12 + 3(-2.5) + 1(3) - 3 \cdot 1] \\ = 13.133$$

$$ex = 29.6$$

1. Given arguments 2 4 9 10 $f(x) = x^3 - 2x$

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$
2	4	> 26	> 15	
4	56	> 13		> 1
9	74	> 269	> 23	
10	980			

→ / find $f(5.5)$

x	0	1	4	5	6
$f(x)$	1	14	56	6	3

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1	> 13			
1	14	$> -3.167\bar{r}$	$> 0.167\bar{r}$		
4	56	> 0.33	$> -2.332\bar{r}$	> 0.1799	
5	6	> -9	> 3	$> 1.066\bar{r}$	
6	3	> -3			

$$f(x) = y_0 + (x-x_0) \Delta y + (x-x_0)(x-x_1) \Delta^2 y + (x-x_0)(x-x_1)(x-x_2) \Delta^3 y + (x-x_0)(x-x_1)(x-x_2)(x-x_3) \Delta^4 y$$

$$\begin{aligned} f(5.5) &= 1 + (5.5-0) (14 + (5.5-1) (-3.167\bar{r}) + (5.5-1)(5.5-2) (0.167\bar{r}) + (5.5-1)(5.5-2)(5.5-3) 0.1799 \\ &\quad (5.5-4) (1.066\bar{r}) + 5.5-4) (5.5-4)(5.5-5) 0.1799 \\ &= 1 + 7.15 + (-98.3956) + 6.1998 + 3.3393 \\ &= 364.35 \end{aligned}$$

→ find $f(x)$ & $f(18)$

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
4	48				
5	100	> 52	> 15		
7	294	> 97	> 21	> 1	> 0
10	900	> 202	> 27	> 1	> 0
11	1210	> 310	> 33	> 1	> 0
13	2028	> 409			

$$f(x) = y_0 + (x-x_0) \Delta y + (x-x_0)(x-x_1) \Delta^2 y + (x-x_0)(x-x_1)(x-x_2) \Delta^3 y$$

$$\begin{aligned} f(8) &= 48 + (8-4)52 + (8-4)(8-5)15 + (8-4)(8-5)(8-7), \\ &= 48 + 208 + 180 + 12 = 448 \end{aligned}$$

$$\begin{aligned} f(18) &= 48 + (15-4)52 + (15-4)(15-7)15 + (15-4)(15-7)(15-9), \\ &= 48 + 572 + 1650 + 880 = 3150 \end{aligned}$$

→ find $f(6)$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	168			
7	120	> -12		
9	72	> -4	> -2	
10	63	> -9	> 5	

$$\begin{aligned} f(x) &= y_0 + (x-x_0) \Delta y + (x-x_0)(x-x_1) \Delta^2 y + (x-x_0)(x-x_1)(x-x_2) \Delta^3 y \\ &= 168 + (x-3)(-12) + (x-3)(x-7)(-2) + (x-3)(x-7)(x-9), \\ &= x^3 - 21x^2 + 119x - 26 \end{aligned}$$

$$f(6) = 148$$

$$\rightarrow \begin{array}{ccccc} x & -1 & 0 & 1 & 3 \\ f(x) & 2 & 1 & 0 & -1 \end{array} \quad \text{find polynomial}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	2			
0	1	>-1	> 0	
1	0	>-1	> 0.415	
3	-1	>-0.5	> 0.166	

$$\begin{aligned} f(x) &= y_0 + (x-x_0) \Delta y + (x-x_0)(x-x_1) \Delta^2 y + (x-x_0)(x-x_1)(x-x_2) \Delta^3 y + \dots \\ &= 2 + (x+1)(-1) + (x+1)x(0) + (x+1)x(x-1)(0.415) \\ &= 0.415x^3 - 1.415x + 1 \end{aligned}$$

$$\rightarrow \begin{array}{c|ccccc} x & 5 & 6 & 9 & 11 \\ \hline y & 12 & 13 & 14 & 16 \end{array}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	12			
6	13	> 1	> -0.1675	
9	14	> 0.33	> 0.134	> 0.0502
11	16	> 1		

$$\begin{aligned} f(x) &= y_0 + (x-x_0) \Delta y + (x-x_0)(x-x_1) \Delta^2 y + (x-x_0)(x-x_1)(x-x_2) \Delta^3 y \\ &= 12 + (x-5) + (x-5)(x-6)(-0.1675) + (x-5)(x-6)(x-9)(0.0502) \\ &= 0.0502x^3 - 1.1715x^2 + 9.3183x - 11.579. \end{aligned}$$

$$\begin{array}{r} x \\ y = 1 + 3 + 9 + 27 + 81 \\ \hline 81 \end{array}$$

8d)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
1	3	> 2		
2	9	> 6	> 2	
3	27	> 18	> 10	
4	81			

$$\begin{aligned}
 f(3) &= 1 + 3x2 + 3x2x2 + 3x2x1x2 \\
 &= 1 + 6 + 12 + 12 \\
 &= 31
 \end{aligned}$$