Units – 2: Sampling Distribution

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2.1 Basics

Population: A collection of objects (collection of numbers, measurements, observations etc) is called a Population

The number of objects in a population is called its **Size** and is denoted by N
If N is finite then the population is called **Finite population**
If N is infinite then the population is called **Infinite population**

Examples:
(1) Heights of the students in a University
(2) Marks obtained by the students of SSC in Mathematics
(3) Scores of candidates obtained in a competitive exam
(4) The set of outcomes when a coin tossed 1000 times
(5) Collection of all even numbers

Parameters: The statistical measures (Mean, Median, Variance etc.) about a population are called Parameters.
The Mean, Variance and Standard deviation of a population are respectively denoted by the symbols μ, σ^2 and σ

Sample: A finite sub collection from a population is called a Sample
The number of objects in a sample is called its **Size** and is denoted by n

If $n \geq 30$ then the sample is called **large sample**

If $n < 30$ then the sample is called **small sample**

Statistics: The statistical measures (Mean, Median, Variance etc.) about a sample are called statistics.
The Mean, Variance and Standard deviation of a sample are respectively denoted by the symbols $\bar{x}, s^2$ and s

Examples:

(1) For the population of 'the Heights of the students in a University', the heights of the students in class of 40 is a sample
(2) For the population of 'the Marks obtained by the students of SSC in Mathematics', the marks of the students of a particular school is a sample
(3) For the population of 'the Scores of candidates obtained in a competitive exam', the scores of the candidates from a particular college is a sample
(4) For the population of 'the set of outcomes when a coin tossed 1000 times', a collection of 10 outcomes is a sample.
(5) For the population of 'the Collection of all even numbers', a collection of 20 even numbers is a sample

Random Sample: A sample of size n taken from a population is called a random sample if the probability of any choice of n objects from the population is same.

Sampling: Collecting samples from a given population is called sampling

Large Sampling: Collecting large samples from a given population is called large sampling.

Small Sampling: Collecting small samples from a given population is called small sampling.

Sampling with replacement: Collecting samples in which the objects may repeat, from a given population is called sampling with replacement.

Sampling without replacement: Collecting samples in which the objects do not repeat, from a given population is called sampling without replacement.

Number of Samples:

(i) The number of samples of size n without replacement, taken from a finite population of size N is ${}^{N}C_{n}$

(ii) The number of samples of size n with replacement, taken from a finite population of size N is N^{n}

(iii) The number of samples of size n with or without replacement, taken from an infinite population is ∞

Examples:

(1) Consider the finite population $\{1,2,3,4\}$ of size $N = 4$

The number of samples of size $n = 2$ without replacement is given by ${}^{N}C_{n} = {}^{4}C_{2} = 6$

The samples are $\{1,2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2,3\}$, $\{2,4\}$, $\{3,4\}$

The means of these 6 samples are respectively 1.5, 2, 2.5, 2.5, 3, 3.5

The frequency distribution of these sample means is given as follows.

Sample mean $\bar{x}$	1.5	2	2.5	3	3.5
Frequency f	1	1	2	1	1

This frequency distribution is called the **sampling distribution of the mean (SDM)**

Note:

Here drawing a sample of size $n = 2$ without replacement from the above population is a random experiment with the possible outcomes $\{1,2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2,3\}$, $\{2,4\}$, $\{3,4\}$

That is, the sample space $S = \{\{1,2\}, \{1, 3\}, \{1, 4\}, \{2,3\}, \{2,4\}, \{3,4\}\}$

If $\overline{X}$ is the random variable which gives the mean the sample, then the range of $\overline{X}$ is $\{1.5, 2, 2.5, 3, 3.5\}$

(2) Consider the finite population $\{1,2,3,4\}$ of size $N = 4$

The number of samples of size $n = 2$ with replacement is given by $N^{n} = 4^{2} = 16$

The samples are $\{1,1\}$, $\{1, 2\}$, $\{1, 3\}$, $\{1,4\}$,

$\{2,1\}, \{2, 2\}, \{2, 3\}, \{2,4\},$

$\{3,1\}, \{3, 2\}, \{3, 3\}, \{3,4\},$

$\{4,1\}, \{4, 2\}, \{4, 3\}, \{4,4\}$

The means of these 16 samples are respectively given follows.

1, 1.5, 2, 2.5,

1.5, 2, 2.5, 3,

2, 2.5, 3, 3.5,

2.5, 3, 3.5, 4

The frequency distribution of these sample means is given as follows

Sample mean $\bar{x}$	1	1.5	2	2.5	3	3.5	4
Frequency f	1	2	3	4	3	2	1

This frequency distribution is the **sampling distribution of the mean (SDM)**

2.2 Sampling Distribution of the Mean (SDM):

The frequency distribution of the means of all random samples of fixed size, taken from a population is called the Sampling Distribution of the Mean (SDM). It is denoted by $\bar{X}$.

2.2.1 Sampling Distribution of the Mean with σ (SDM with σ):

Theorem: If $\bar{X}$ is the random variable which gives the mean a random sample of size n, taken from a population having mean μ and variance σ^2, then

(i) Mean of SDM $\bar{X}$ is given by $\mu_{\bar{X}} = \mu$ or $E[\bar{X}] = \mu$

(ii) For infinite population or sampling with replacement,

Variance of SDM $\bar{X}$ is given by $\sigma_{\bar{X}}^2 = \dfrac{\sigma^2}{n}$ or $V[\bar{X}] = \dfrac{\sigma^2}{n}$

(iii) For finite population of size N and sampling without replacement,

Variance of SDM $\bar{X}$ is given by $\sigma_{\bar{X}}^2 = \dfrac{\sigma^2}{n}\left(\dfrac{N-n}{N-1}\right)$

Note:

(1) The value of $\dfrac{N-n}{N-1}$ is called the **finite population correction factor**

(2) The standard deviation of $\bar{X}$ is called **Standard error of the mean;** that is,

Standard error (SE), $\sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}}$

(3) The value of $0.6745\times\frac{\sigma}{\sqrt{n}}$ is called the **Probable error of the mean;** that is,

$$\text{Probable error (PE)} = 0.6745\times\frac{\sigma}{\sqrt{n}}$$

Central Limit Theorem: If $\overline{X}$ is the random variable which gives the mean a random sample of size n, taken from a population having mean μ and variance σ^2, then $Z=\frac{\overline{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$.

2.2.2 Sampling Distribution of the Mean with unknown σ (SDM with unknown σ):

t- distribution:

If we do not know the value of σ, then we cannot use the Central limit theorem $Z=\frac{\overline{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$.

In this case we use sample standard deviation 's' in place of population standard deviation σ so that we have a random variable different from Z. This new random variable is denoted by t; that is $t=\frac{\overline{X}-\mu}{\left(\frac{s}{\sqrt{n}}\right)}$.

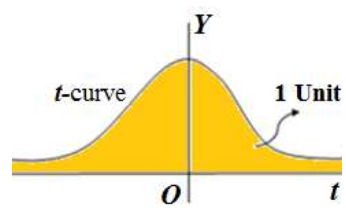
The probability distribution corresponding to this random variable t is called t-distribution with parameter $n-1$. This parameter is known as degrees of freedom, denoted by ν; that is, $\nu=n-1$.

Note: In the t-distribution, for the sample $\{x_1, x_2, x_3, \cdots x_n\}$,

(i) Sample mean $\bar{x}$ is given by $\bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i$

(ii) Sample variance is given by $s^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$

Properties of t- distribution:



(1) The curve given by t-distribution is called t-curve
(2) The t-curve is continuous and above the X-axis (or t- axis)
(3) The curve is symmetric about the Y-axis
(4) The t-curve is similar to Z-curve (standard normal curve)
(5) The area between t - axis and the curve from $-\infty$ to ∞ is 1 unit

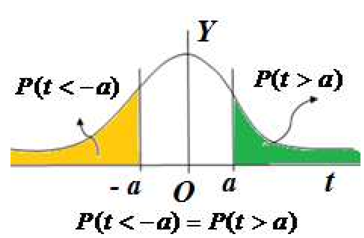
(6) The mean of t is 0 and the variance of t is greater than 1; that is, $\mu_t=0$ and $\sigma_t^2>1$

(7) As $n\to\infty$, variance of t tends to 1; that is, $\sigma_t^2\to 1$. In other words, as $n\to\infty$, $t\to Z$

(8) If n is large ($n\geq 30$), then we can write Z in place of t; that is, $Z=\frac{\overline{X}-\mu}{\left(\frac{s}{\sqrt{n}}\right)}$

(9) $P(t<-a)=P(t>a)$

(10) $P(t\leq 0)=P(t\geq 0)=\frac{1}{2}$

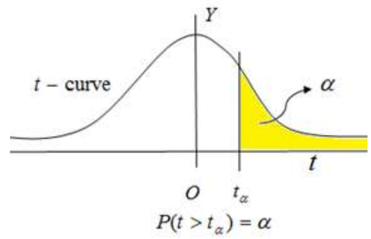


t_α - Notation:

If $\alpha \geq 0$ then t_α is a point on t – axis such that $P(t > t_\alpha) = \alpha$ or $P(t < -t_\alpha) = \alpha$

That is, the area between t – axis and the curve from t_α to ∞ is α

(or the area between t – axis and the curve from $-\infty$ to $-t_\alpha$ is α)



Note:

(1) $t_1 = -\infty$, $t_0 = \infty$, $t_{\frac{1}{2}} = 0$

(2) $t_\alpha + t_{1-\alpha} = 0$ or $-t_\alpha = t_{1-\alpha}$ or $t_\alpha = -t_{1-\alpha}$

Testing a claim using t-distribution: To test a given claim using t-distribution, we follow the rule given below.

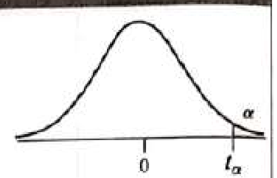
(i) If $|t| < t_{0.005}$ then the claim is accepted

(ii) If $|t| > t_{0.005}$ then the claim is rejected

t_α - Table:

In this table the values of t_α are available for different values of α and ν

Table 4 Values of t_α



ν	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.00833$	$\alpha = 0.00625$	$\alpha = 0.005$	ν
1	3.078	6.314	12.706	31.821	38.204	50.923	63.657	1
2	1.886	2.920	4.303	6.965	7.650	8.860	9.925	2
3	1.638	2.353	3.182	4.541	4.857	5.392	5.841	3
4	1.533	2.132	2.776	3.747	3.961	4.315	4.604	4
5	1.476	2.015	2.571	3.365	3.534	3.810	4.032	5
6	1.440	1.943	2.447	3.143	3.288	3.521	3.707	6
7	1.415	1.895	2.365	2.998	3.128	3.335	3.499	7
8	1.397	1.860	2.306	2.896	3.016	3.206	3.355	8
9	1.383	1.833	2.262	2.821	2.934	3.111	3.250	9
10	1.372	1.812	2.228	2.764	2.870	3.038	3.169	10
11	1.363	1.796	2.201	2.718	2.820	2.891	3.106	11
12	1.356	1.782	2.179	2.681	2.780	2.934	3.055	12
13	1.350	1.771	2.160	2.650	2.746	2.896	3.012	13
14	1.345	1.761	2.145	2.624	2.718	2.864	2.977	14
15	1.341	1.753	2.131	2.602	2.694	2.837	2.947	15

Example:

(i) For $\nu = 7$ (or $n = 8$) and $\alpha = 0.025$, $t_\alpha = t_{0.025} = 2.365$

(ii) For $\nu = 10$ (or $n = 11$) and $\alpha = 0.05$, $t_\alpha = t_{0.05} = 1.812$

(iii) For $\nu = 15$ (or $n = 16$) and $\alpha = 0.005$, $t_\alpha = t_{0.005} = 2.947$

2.3 Sampling Distribution of the Variance (SDV):

The frequency distribution of the variances of all random samples of fixed size, taken from a population is called the Sampling Distribution of the Variance (SDV). It is denoted by S^2.

Example: Consider the finite population $\{1,2,3,4\}$ of size $N=4$

The number of samples of size $n=2$ without replacement is given by ${}^{N}C_n = {}^{4}C_2 = 6$

The samples are $\{1,2\}$, $\{1,3\}$, $\{1,4\}$, $\{2,3\}$, $\{2,4\}$, $\{3,4\}$

The means of these 6 samples are respectively 1.5, 2, 2.5, 2.5, 3, 3.5

The variance $= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ or $\frac{1}{n}\sum_{i=1}^{n}x_i^2 - (\bar{x})^2$

The variances of these 6 samples are respectively given as below.

$$\frac{1}{2}(1^2+2^2)-(1.5)^2 = 2.5-2.25 = 0.25$$
$$\frac{1}{2}(1^2+3^2)-(2)^2 = 5-4 = 1$$
$$\frac{1}{2}(1^2+4^2)-(2.5)^2 = 8.5-6.25 = 2.25$$
$$\frac{1}{2}(2^2+3^2)-(2.5)^2 = 6.5-6.25 = 0.25$$
$$\frac{1}{2}(2^2+4^2)-(3)^2 = 10-9 = 1$$
$$\frac{1}{2}(3^2+4^2)-(3.5)^2 = 12.5-12.25 = 0.25$$

The frequency distribution of these sample variances is given as follows

Sample variance s^2	0.25	1	2.25
Frequency f	3	2	1

Which is the sampling **distribution of the Variance (SDV)**

2.3.1 χ^2 – Distribution:

Theorem: If S^2 is the random variable which gives the variance a random sample of size n, taken from a normal population having variance σ^2, then $\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2}$ is a random variable having the Chi-square distribution with the parameter $\nu = n-1$.

Properties of χ^2 – Distribution:

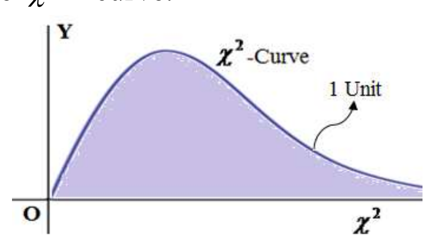
(i) The curve given by the probability density function is called the χ^2 – curve.

(ii) The χ^2 – curve is lies in the 1st quadrant

(iii) It is not symmetric about any axis and has the following shape.

(iv) It depends on the value of ν

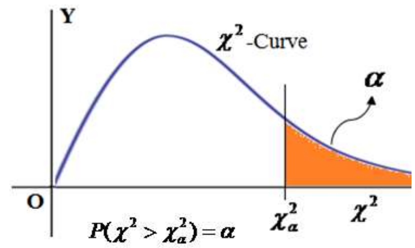
(v) $P(\chi^2 > 0) = 1$; That is, the area between χ^2 – axis and the curve from 0 to ∞ is 1



χ^2_α **– Notation:**

If $\alpha \geq 0$ then χ^2_α is a point on χ^2 – axis such that $P(\chi^2 > \chi^2_\alpha) = \alpha$

That is, the area between χ^2 – axis and the curve from χ^2_α to ∞ is α



χ^2_α **- Table:**

In this table the values of χ^2_α are available for different values of α and ν

Table 5 Values of χ^2_α

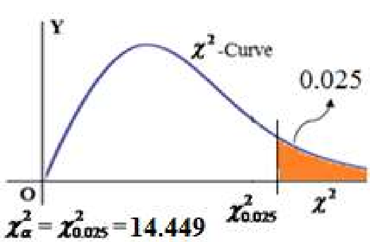
ν	$\alpha = 0.995$	$\alpha = 0.99$	$\alpha = 0.975$	$\alpha = 0.95$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$	ν
1	0.0000393	0.000157	0.000982	0.00393	3.841	5.024	6.635	7.879	1
2	0.0100	0.0201	0.0506	0.103	5.991	7.378	9.210	10.597	2
3	0.0717	0.115	0.216	0.352	7.815	9.348	11.345	12.838	3
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860	4
5	0.412	0.554	0.831	1.145	11.070	12.833	15.086	16.750	5
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548	6
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801	15

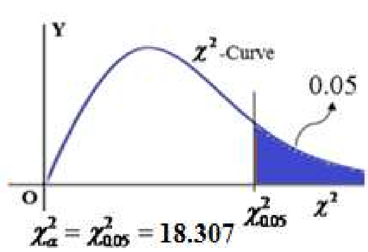
Example:

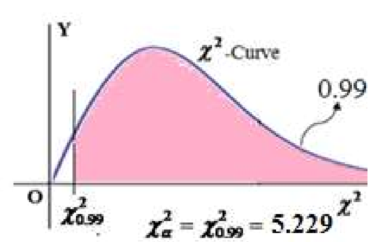
(i) For $\nu = 6$ $(\text{or } n = 7)$ and $\alpha = 0.025$, $\chi^2_\alpha = \chi^2_{0.025} = 14.449$

(ii) For $\nu = 10$ $(\text{or } n = 11)$ and $\alpha = 0.05$, $\chi^2_\alpha = \chi^2_{0.05} = 18.307$

(iii) For $\nu = 15$ $(\text{or } n = 16)$ and $\alpha = 0.99$, $\chi^2_\alpha = \chi^2_{0.99} = 5.229$







Testing a claim using χ^2-distribution: To test a given claim using χ^2-distribution, we follow the rule given below.

(i) If $\chi^2 < \chi^2_{0.005}$ then the claim is accepted

(ii) If $\chi^2 > \chi^2_{0.005}$ then the claim is rejected

2.3.2 $F-$Distribution:

Theorem: If S_1^2 and S_2^2 are the variances of independent random samples of sizes n_1 and n_2, taken from two normal populations having the same variance, then $F = \dfrac{S_1^2}{S_2^2}$ is a random variable having the $F-$ distribution with the parameters $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$.

Note: If $F = \dfrac{S_2^2}{S_1^2}$, then the parameters are in the order of $\nu_2 = n_2 - 1$ and $\nu_1 = n_1 - 1$

Properties of $F-$Distribution:

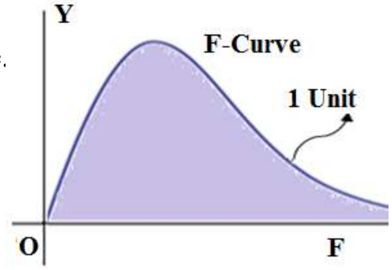
(i) The curve given by the probability density function is called the $F-$curve.

(ii) The $F-$curve is lies in the 1st quadrant

(iii) It is not symmetric about any axis and has the following shape.

(iv) It depends on the values of ν_1 and ν_2

(v) $P(F > 0) = 1$; That is, the area between F – axis and the curve from 0 to ∞ is 1



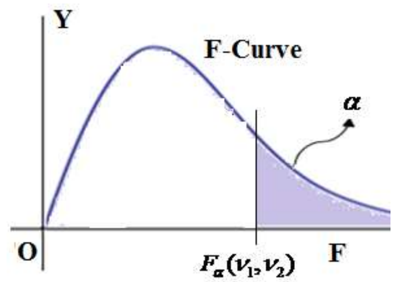
F_α – Notation:

If $\alpha \geq 0$ then F_α is a point on F – axis such that $P(F > F_\alpha) = \alpha$

That is, the area between F – axis and the curve from F_α to ∞ is α.

The value of F_α corresponding to ν_1 and ν_2 is denoted by $F_\alpha(\nu_1, \nu_2)$

Note: $F_\alpha(\nu_1, \nu_2) = \dfrac{1}{F_{1-\alpha}(\nu_2, \nu_1)}$



$F_{0.05}$ – **Tables:** In this table the values of $F_{0.05}(\nu_1, \nu_2)$ are available for different values of ν_1 and ν_2

$F_{0.01}$ – **Tables:** In this table the values of $F_{0.01}(\nu_1, \nu_2)$ are available for different values of ν_1 and ν_2

Table 6(a) Values of $F_{0.05}$

ν_2 = Degrees of Freedom for Denominator	ν_1 = Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.50	2.47	2.38	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71

ν_2 = Degrees of Freedom for Denominator	ν_1 = Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,240	6,261	6,287	6,313	6,339	6,366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.57	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.36	2.27	2.17

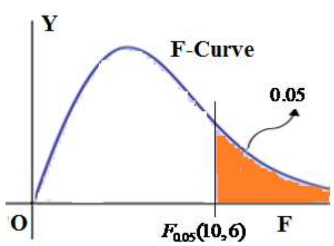
Example:

(i) For $\nu_1 = 10, \nu_2 = 6$ and $\alpha = 0.05$, $F_{0.05}(\nu_1, \nu_2) = F_{0.05}(10, 6) = 4.06$

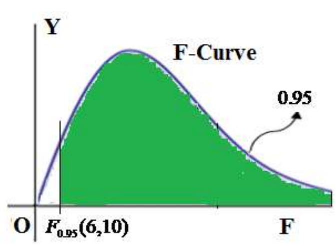
(ii) For $\nu_1 = 6, \nu_2 = 10$ and $\alpha = 0.95$, $F_{0.95}(\nu_1, \nu_2) = F_{0.95}(6, 10) = \dfrac{1}{F_{1-0.95}(10, 6)} = \dfrac{1}{F_{0.05}(10, 6)} = \dfrac{1}{4.06}$

(iii) For $\nu_1 = 9, \nu_2 = 13$ and $\alpha = 0.01$, $F_{0.01}(\nu_1, \nu_2) = F_{0.01}(9, 13) = 4.19$

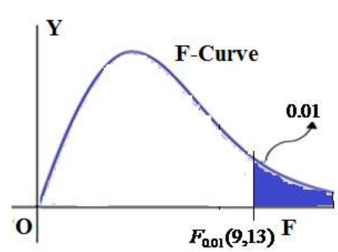
(iv) For $\nu_1 = 13, \nu_2 = 9$ and $\alpha = 0.99$, $F_{0.99}(\nu_1, \nu_2) = F_{0.99}(13, 9) = \dfrac{1}{F_{1-0.99}(9, 13)} = \dfrac{1}{F_{0.01}(9, 13)} = \dfrac{1}{4.19}$



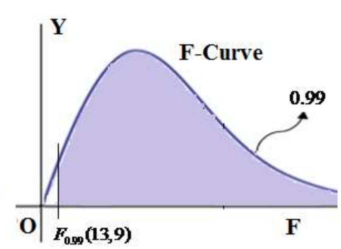
$F_{0.05}(10,6) = 4.06$



$F_{0.95}(6,10) = \dfrac{1}{F_{0.05}(10,6)} = \dfrac{1}{4.06} = 0.2463$



$F_{0.01}(9,13) = 4.19$



$F_{0.99}(13,9) = \dfrac{1}{F_{0.01}(9,13)} = \dfrac{1}{4.19} = 0.2387$