Normal distribution: A continuous random variable X having probability density function (pdf) $f(x)=\dfrac{1}{b\sqrt{2\pi}}\,e^{-\frac{(x-a)^2}{2b^2}}$ for all x, where a,b constants and $b>0$ is called *Normal random variable or Normal variate* with parameters a and b and this distribution is called *Normal Distribution*.

Note:

(1) Normal distribution is a continuous distribution and its range is $(-\infty,\infty)$

(2) The Mean, Variance and Standard deviation of this Normal distribution are given by

(i) Mean, $\mu=\int_{-\infty}^{\infty} x\, f(x)\,dx=a$

(ii) Variance, $\sigma^2=\int_{-\infty}^{\infty} x^2\, f(x)\,dx-\mu^2=b^2$

(iii) Standard deviation, $\sigma=\sqrt{\text{Variance}}=b$

(3) As the parameters a and b of this Normal distribution are coincide with its Mean μ and Standard deviation σ, here afterwards we write the probability density function (pdf) of a Normal random variable with parameters μ and σ as follows.

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{for all } x$$

Normal Curve or Bell Curve: The curve given by $f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}\,e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ for all x, is called the *Normal curve* with parameters μ and σ

Properties of Normal distribution:

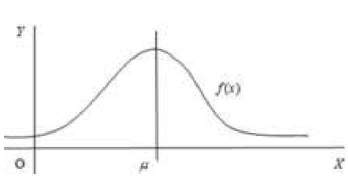
(1) Normal curve is continuous and above the *X*-axis

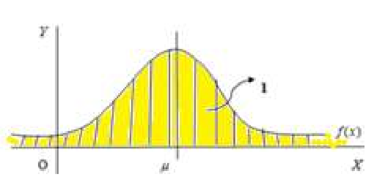
(2) The curve is symmetric about the mean μ of the distribution (that is, symmetric about the line $x=\mu$)

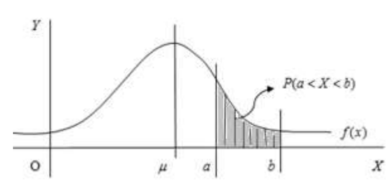
(3) The area between *X*-axis and the Normal curve from $-\infty$ to ∞ is 1 unit

(4) The curve is in Bell shape and so it is also known as **Bell curve**

(5) $P(a\le X\le b)=\int_a^b f(x)\,dx=$ The area between *X*-axis and the curve from a to b







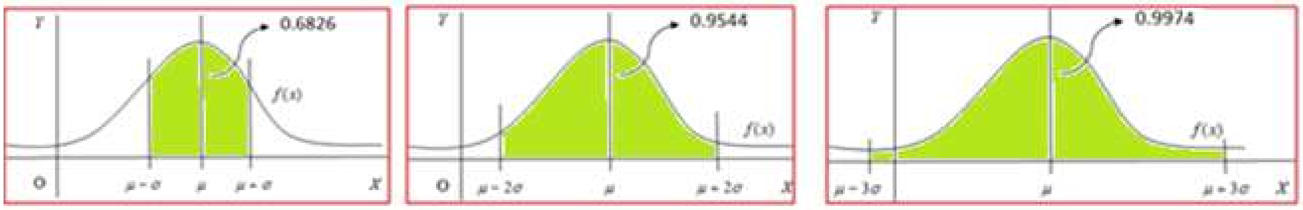
(6) $P(X=a)=P(a\le X\le a)=\int_a^a f(x)dx=0$

(7) $P(a\le X\le b)=P(a<X<b)=P(a<X\le b)=P(a\le X<b)$

(8) $P(X<\mu)=P(X>\mu)=\dfrac{1}{2}$

(9) The mean, median and mode are coincide

(10) $P(\mu - \sigma < X < \mu + \sigma) = 0.6826$
$P(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9544$
$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9974$



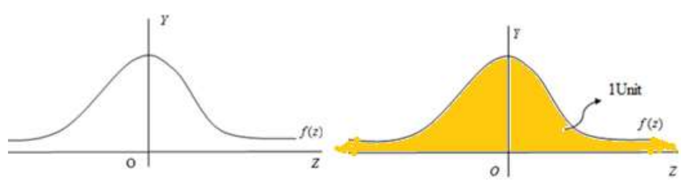
$P(\mu - \sigma < X < \mu + \sigma)$ 0.6826 $\quad$ $P(\mu - 2\sigma < X < \mu + 2\sigma)$ 0.9544 $\quad$ $P(\mu - 3\sigma < X < \mu + 3\sigma)$ 0.9974

Standard Normal distribution: A normal distribution with mean 0 and standard deviation 1 is called *Standard normal distribution*. A random variable that follows standard normal distribution is called *Standard normal random variable* or *Standard normal variate* and is generally denoted by Z .

The probability density function (pdf) of Z is given by $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ for all z

Standard Normal Curve or Standard bell curve: The curve given by $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ for all z, is called the Standard normal curve

Properties of Standard normal distribution:
(1) Standard normal curve is continuous and above the X-axis (or Z - axis)
(2) The curve is symmetric about the Y-axis
(3) The area between Z - axis and the curve from $-\infty$ to ∞ is 1 unit

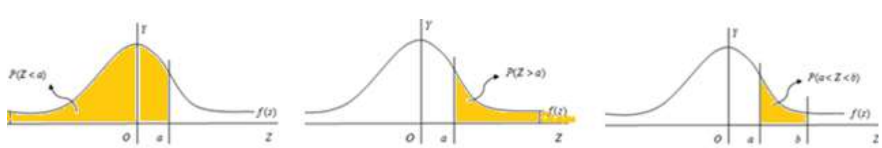


(4) Mean of Z is 0 and Variance of Z is 1

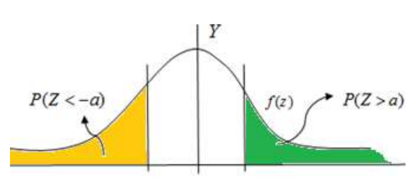
(5) $P(Z < a) = \int_{-\infty}^{a} f(z)\,dz =$ The area between Z - axis and the curve from $-\infty$ to a

(6) $P(Z > a) = \int_{a}^{\infty} f(z)\,dz =$ The area between Z - axis and the curve from a to ∞

(7) $P(a < Z < b) = \int_{a}^{b} f(z)\,dz =$ The area between Z - axis and the curve from a to b



$P(Z < a)$ $\quad$ $P(Z > a)$ $\quad$ $P(a < Z < b)$



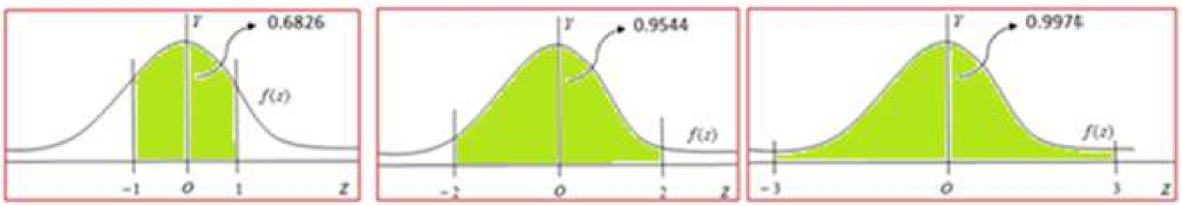
(8) $P(Z < -a) = P(Z > a)$

(9) $P(Z \le 0) = P(Z \ge 0) = \dfrac{1}{2}$

(10) $P(-1 < Z < 1) = 0.6826$

$P(-2 < Z < 2) = 0.9544$

$P(-3 < Z < 3) = 0.9974$



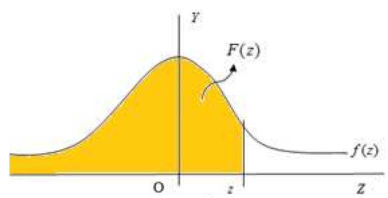
$P(-1 < Z < 1) = 0.6826$ $\quad$ $P(-2 < Z < 2) = 0.9544$ $\quad$ $P(-3 < Z < 3) = 0.9974$

Cumulative Distributive Function (CDF) of Z :

It is given by $\quad F(z) = P(Z \le z) = \int_{-\infty}^{z} f(z)\,dz$ for all z

Properties of CDF:

(1) $F(z) = P(Z \le z) = \int_{-\infty}^{z} f(z)\,dz$ for all z, The area between Z - axis and the curve from $-\infty$ to z



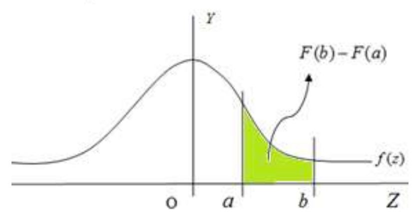
$F(z) = P(Z \le z) = \int_{-\infty}^{z} f(z)dz$

(2) $0 \le F(z) \le 1$ for all z

(3) $F(-\infty) = 0,\ F(\infty) = 1$ and $F(0) = \dfrac{1}{2}$

(4) If $a < b$ then $F(a) \le F(b)$

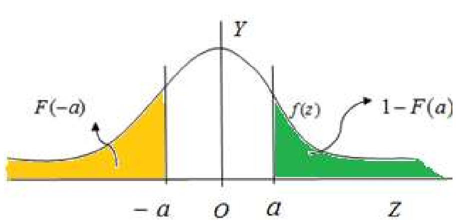
(5) * If $a \le b$ then $F(b) - F(a) = P(a \le Z \le b) = \int_{a}^{b} f(z)\,dz$



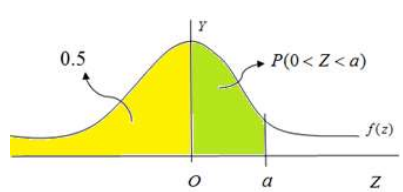
$F(b) - F(a) = P(a \le Z \le b)$

(6)* $F(a) + F(-a) = 1$ or $F(-a) = 1 - F(a)$

(7) If $a > 0$ then $F(a) = 0.5 + P(0 < Z < a)$



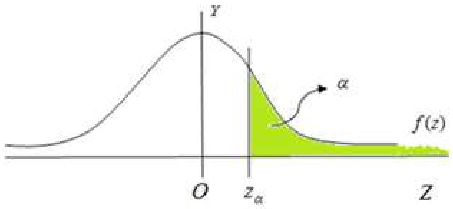
$F(-a) = 1 - F(a)$



$F(a) = 0.5 + P(0 < Z < a)$

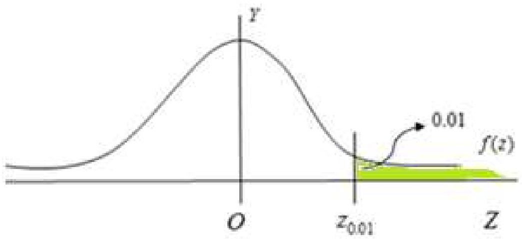
(8) Relation between X and Z : If X is a Normal random variable with mean μ and variance σ^2 then $Z = \dfrac{X - \mu}{\sigma}$ becomes the standard normal random variable.

z_α - **Notation:** If $\alpha \geq 0$ then z_α is a point on $Z-$axis such that $P(Z > z_\alpha) = \alpha$ *or* $F(z_\alpha) = 1 - \alpha$; that is, the area between Z - axis and the curve from z_α to ∞ is α

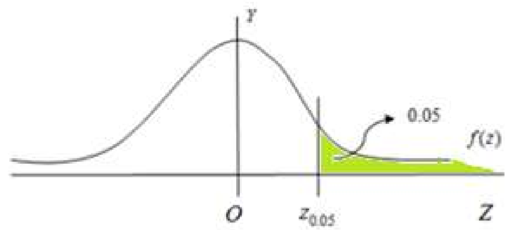


$P(Z > z_\alpha) = \alpha$ *or* $F(z_\alpha) = 1 - \alpha$

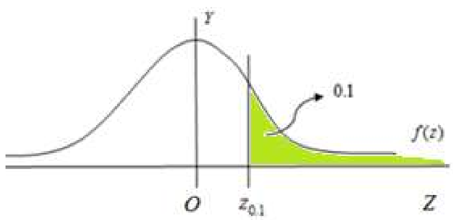
For example,



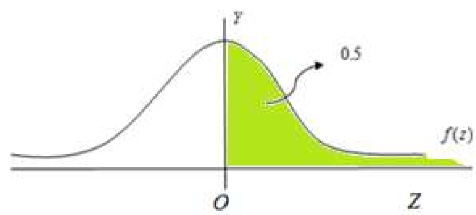
$P(Z > z_{0.01}) = 0.01$ *or* $F(z_{0.01}) = 0.99$



$P(Z > z_{0.05}) = 0.05$ *or* $F(z_{0.05}) = 0.95$



$P(Z > z_{0.1}) = 0.1$ *or* $F(z_{0.1}) = 0.9$



$z_{0.5} = 0$

$P(Z > z_{0.5}) = 0.5$ *or* $F(z_{0.5}) = 0.5$

Note:

(1)* $z_{0.01} = 2.33$

(2)* $z_{0.05} = 1.645$

(3)* $z_{0.005} = 2.575$

(4)* $z_{0.025} = 1.96$

(5) $z_1 = -\infty$

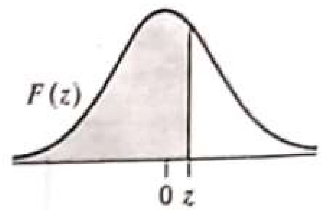
(6) $z_0 = \infty$

(7) $z_{0.5} = 0$

(8) $z_\alpha + z_{1-\alpha} = 0$ or $-z_\alpha = z_{1-\alpha}$

Standard normal distribution table: In this table the values of $F(z)$ are available for different values of z. In the first column the values of z with first decimal place and in the top row the value of second decimal place of z are available.

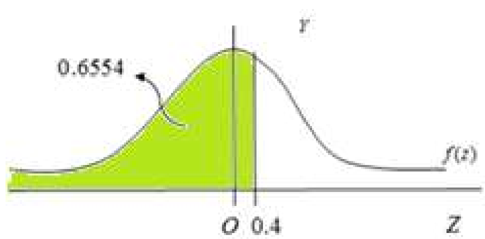
$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\, dt$$



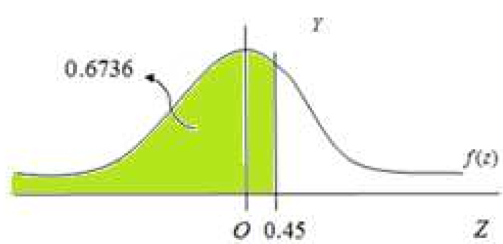
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5973	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621

From the table, we have

(i) $F(0.4) = 0.6554,\ F(0.45) = 0.6736$



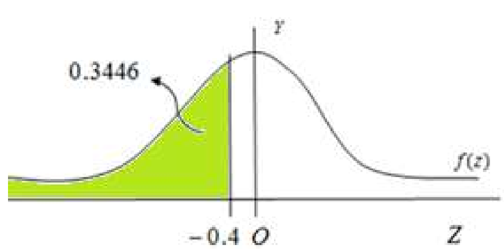
$F(0.4) = 0.6554$



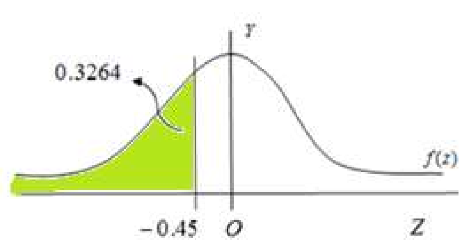
$F(0.45) = 0.6736$

(ii) $F(-0.4) = 1 - F(0.4) = 1 - 0.6554 = 0.3446$

$F(-0.45) = 1 - F(0.45) = 1 - 0.6736 = 0.3264$



$F(-0.4) = 1 - F(0.4) = 1 - 0.6554 = 0.3446$



$F(-0.45) = 1 - F(0.45) = 1 - 0.6736 = 0.3264$