Unit-3

Regular Grammars

* Regular Grammar tuples are.

$$G = (V, T, P, S)$$

(NT)
V = set of non-terminal symbols or variables

T = set of terminal symbols (Σ)

P = Production rules

S = Initial non-terminal symbol.

Ex:- $\underbrace{aa}_{A}\underbrace{bb}_{B}\underbrace{cc}_{C}\underbrace{aa}_{A}$

$V = \{S, A, B, C\}$

$T = \{a, b, c\}$

$P = \quad S \rightarrow ABCA$

$\quad A \rightarrow aa$

$\quad B \rightarrow bb$

$\quad C \rightarrow cc$

$S = S$

Ex:- The string ends with 'a'

$T = \{a\}$

$L = \{a, aa, aaa, \ldots\}$

$S \rightarrow a / aA$

$A \rightarrow aA / \epsilon$

Ex:- starts with a $T = \{a, b\}$

$L = \{a, aa, ab, aaa, abb, \ldots\}$

$S \rightarrow a / aA$

$A \rightarrow [\cancel{a/b/AA}]\ aA / bA / \epsilon \Rightarrow (a+b)^{a}$

universal expression.

Ex:- starts with a ends with b

$L = \{ab, aab, abb, \ldots\}$

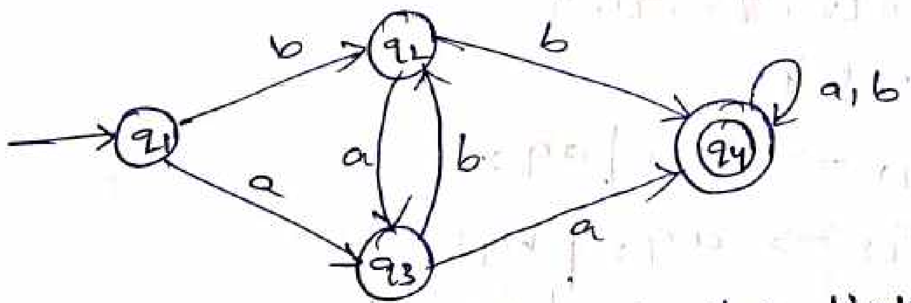
$S \rightarrow [\cancel{ab}]/ aAb$

$A \rightarrow aA / bA / \epsilon$

→ P : $A \rightarrow Aa / a$ — left linear grammar.

$A \rightarrow aA / a$ — Right linear grammar.

Ex:- L = Substring aa/bb.



consider out going links. If the link reaches final state write only terminal. We can get Right linear grammar from automata

$V = \{q_1, q_2, q_3, q_4\}$

$T = \{a, b\}$

$S = \{q_1\}$

P:
$q_1 \rightarrow bq_2 / aq_3$
$q_2 \rightarrow aq_3 / bq_4 / b$
$q_3 \rightarrow bq_2 / aq_4 / a$
$q_4 \rightarrow aq_4 / bq_4 / a / b$

Let us consider the string "aaba".

Start with S

$q_1 \rightarrow aq_3$
$\rightarrow aaq_4$ ∴ string is derivable.
$\rightarrow aabq_4$
$\rightarrow aaba$

for abab

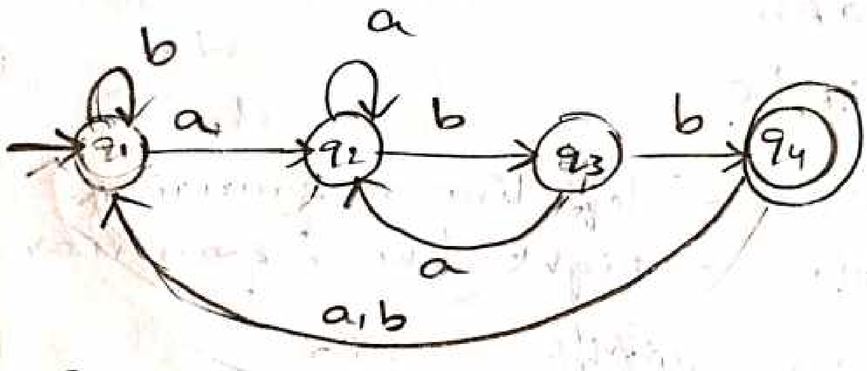
$q_1 \to aq_3$

$\to abq_2$

$\to abaq_3$

$\to abab q_2$ $\Rightarrow$ still there are non-termin... so string is not derivabl...

ex:- $(a+b)^* abb$.



$V = \{q_1, q_2, q_3, q_4\}$

$T = \{a, b\}$

P: $q_1 \to bq_1 \mid aq_2$

$q_2 \to aq_2 \mid bq_3$

$q_3 \to aq_2 \mid bq_4 \mid b$

$q_4 \to aq_1 \mid bq_1 \mid$

ababb

$q_1 \to aq_2$

$\to abq_3$

$\to abaq_2$

$\to ababq_3$

$\to ababb \to$ acceptable.

abab.

$q_1 \to aq_2$

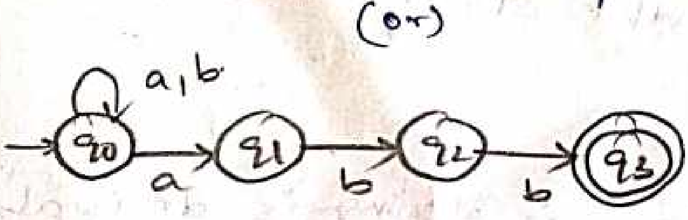
$\to abq_3$

$\to abaq_2$

$\to abab q_3$

not acceptable

(or)



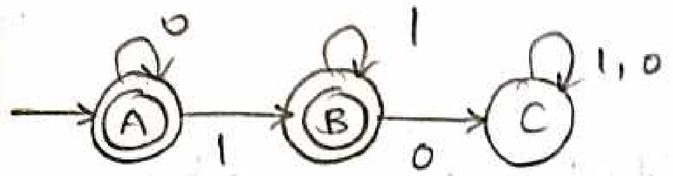
P: $q_0 \rightarrow aq_0 | bq_0 | aq_1$

$q_1 \rightarrow bq_2$

$q_2 \rightarrow bq_3 | b$

$q_3 \rightarrow e$

Ex:-



Language - 0^*1^*

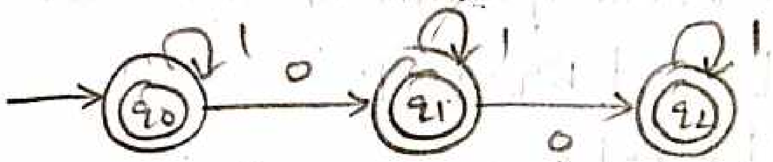
$V = \{A, B, C\}$ $T = \{0, 1\}$

P: $A \rightarrow 0A, 1B | 0 | 1$

$B \rightarrow 1B, 0C | 1$

$C \rightarrow 0C | 1C |$

Ex:-



$V = \{q_0, q_1, q_2\}$ $T = \{0, 1\}$

P: $q_0 \rightarrow 1q_0 | 0q_1 | 0 | 1$

$q_1 \rightarrow 1q_1 | 0q_2 | 0 | 1$

$q_2 \rightarrow 1q_2 | 1$

Language:- string with atmost 2 0's

* <u>Right linear grammar to left linear grammar</u>:-

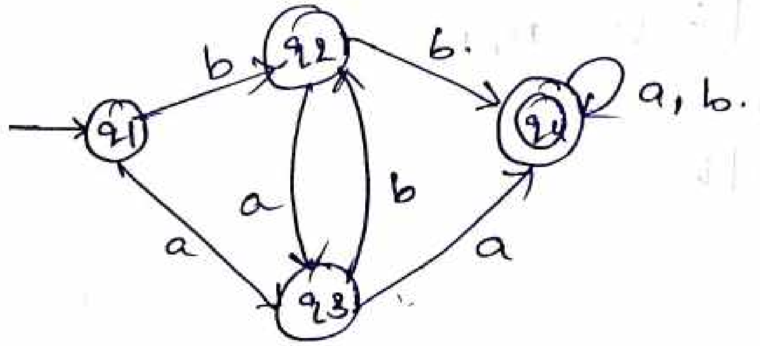
1, Interchange the initial & final states

2, Change the directions of transition links

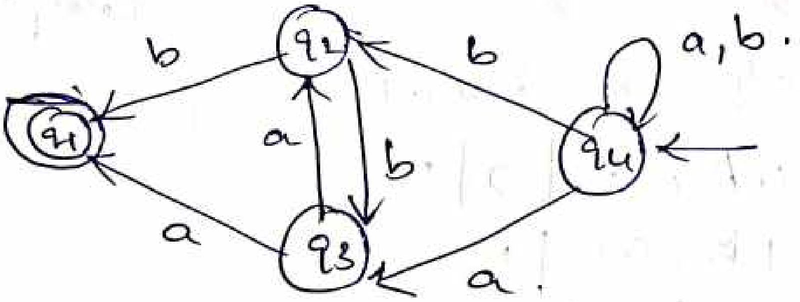
3, Construct right linear grammar for the given finite automata.

4, Now, construct left linear form Right linear.

Ex:-



Step-1 & 2



Step-3:- Right linear grammar after conversion

$q_4 \to aq_4 \mid bq_4 \mid bq_2 \mid aq_3$

$q_2 \to bq_1 \mid bq_3 \mid b$

$q_3 \to aq_2 \mid aq_1 \mid a$

$q_1 \to \lambda$ [null production rule]

Step-4:- Left linear grammar for initial automata.

$q_4 \to q_4a \mid q_4b \mid q_2b \mid q_3a$

$q_2 \to q_1b \mid q_3b \mid b$

$q_3 \to q_2a \mid q_1a \mid a$

$q_1 \to \lambda$

Right linear grammar for initial automata:-

$q_1 \to bq_2 \mid aq_3$

$q_2 \to bq_4 \mid aq_3 \mid b$

$q_3 \to bq_2 \mid aq_4 \mid a$

$q_4 \to aq_4 \mid bq_4 \mid a \mid b$

consider string bb

right linear verification

bb ~~$q_1 \to bb$~~

$q_1 \to bq_2$

$\to bb$

acceptable

left linear verification

$q_1 \to q_2 b.$

$\to bb$

acceptable.

* left linear to right linear

1, Construct right linear grammar equivalent to given left linear grammar. [interchange terminal & non terminal]

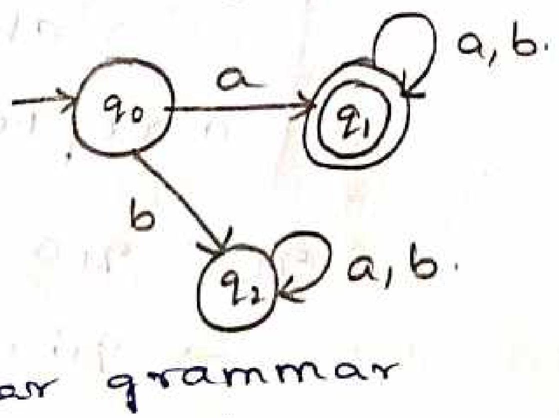
2, Construct finite automata from above intermediate right linear grammar.

3, Interchange initial & final states

4, change direction of transition links.

5, Construct exact right linear grammar form the above automata.

Ex:- Construct the left linear grammar for the language which accepts the strings starts with 'a'. $\Sigma = \{a, b\}$



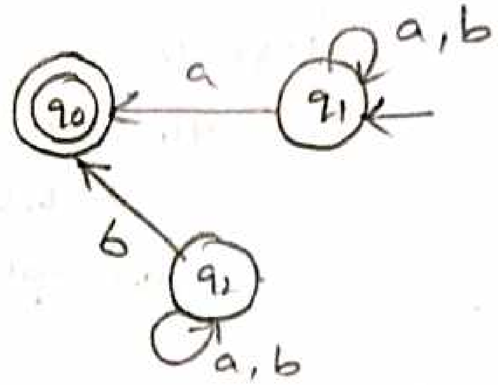
right linear grammar

$q_0 \to aq_1 | bq_2 | a$

$q_1 \to aq_1 | bq_1 | a | b$

$q_2 \to aq_2 | bq_2 | a | b$

Step-1,2:-



Step-3 :- intermediate right linear grammar

$q_2 \rightarrow aq_2 \mid bq_2 \mid bq_0 \mid b$

$q_1 \rightarrow aq_1 \mid bq_1 \mid aq_0 \mid a$

$q_0 \rightarrow \lambda$

Step-4 :- left linear grammar.

$q_2 \rightarrow q_2a \mid q_2b \mid q_0b \mid b$

$q_1 \rightarrow q_1a \mid q_1b \mid q_0a \mid a$

$q_0 \rightarrow \lambda$

consider string aba

Right linear verification

$q_0 \rightarrow aq_1$

$\rightarrow abq_1$

$\rightarrow aba$

acceptable

left linear verification

~~$q_2 \rightarrow q_2a$~~

~~$\rightarrow q_1ba$~~

~~$\rightarrow aba$~~

~~acceptable.~~

$q_1 \rightarrow q_1a$

$\rightarrow q_1ba$

$\rightarrow aba$

acceptable