Example:

28.1.

$f(x) = x^3 - 4x - 9$.

$f(1) = -12$ $\quad f(2) = -9 < 0$ $\quad f(3) = 6 > 0$

root lies b/w 2 & 3

$x_1 = \frac{2+3}{2} = 2.5$

$f(x_1) = -3.37 < 0$

root lies b/w x_1 & 3

$x_2 = 2.75$ $\quad f(x_2) > 0$

root lies b/w x_1 & x_2

$x_3 = 2.625$ $\quad f(x_3) < 0$

root lies b/w x_3 & x_2

$x_4 = 2.6875$ $\quad f(x_4) < 0$

root lies b/w x_2 & x_4

$x_5 = 2.71875$ $\quad f(x_5) > 0$

root lies b/w x_4 & x_5

$x_6 = 2.7031$ $\quad f(x_6) < 0$

root lies b/w x_6 & x_5

$x_7 = 2.7109$ $\quad f(x_7) > 0$

root lies b/w x_7 & x_6

$x_8 = 2.707$ $\quad f(x_8) > 0$

root lies b/w x_8 & x_6

$x_9 = 2.705$ $\quad f(x_9) < 0$

root lies b/w x_9 & x_8

$x_{10} = 2.706$ $\quad f(x_{10}) < 0$

root lies b/w x_{10} & x_8

$x_{11} = 2.7065$

$\therefore$ The root is 2.7065

2) $\sin x = 1/x$ b/w $x = 1$ & $x = 1.5$

$f(x) = \sin x - 1/x$

$f(1) < 0$ $f(1.5) > 0$

root lies b/w 1 & 1.5

$x_1 = 1.25$ $f(x_1) > 0$

root lies b/w 1 & 1.25

$x_2 = 1.125$ $f(x_2) > 0$

root lies b/w 1 & 1.125

$x_3 = 1.0625$ $f(x_3) < 0$

root lies b/w x_2 & x_3

$x_4 = 1.0937$ $f(x_4) < 0$

root lies b/w x_4 & x_2

$x_5 = \cancel{1.125}$ 1.10937 $f(x_5) < 0$

root lies b/w x_5 & x_2

$x_6 = 1.11719$ $f(x_6) > 0$

root lies b/w x_6 & x_5

$x_7 = 1.11328$

root is 1.11328

Q8.3. $x^3 - 2x - 5 = 0$

$f(1) = -6$ $f(2) = -1 < 0$ $f(3) = 16 > 0$

root lies b/w 2 & 3

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

$x_1 = 2.0588$

$f(x_1) < 0$

root lies b/w x_1 & 3

$x_2 = 2.0813$

$f(x_2) < 0$

root lies b/w x_2 & 3

$x_3 = 2.0896$ $f(x_3) < 0$

root lies b/w x_3 & 3

$x_4 = 2.0927$ $f(x_4) < 0$

root lies b/w x_4 & 3

$x_5 = 2.0939$ $f(x_5) < 0$

root lies b/w x_5 & 3

$x_6 = 2.0943.$ $f(x_6) < 0$

root lies b/w x_6 & 3

$x_7 = 2.0945$

$\therefore$ The root is $\boxed{2.0945}$

Q.4, $f(x) = \cos x - xe^x$

$f(0) > 0$ $f(1) < 0$

root lies b/w 0 & 1

$x_1 = 0.31467$

$f(x_1) > 0$

root lies b/w x_1 & 1

$x_2 = 0.4467$

$f(x_2) > 0$

root lies b/w x_2 & 1.

$x_3 = 0.494$

$f(x_3) > 0$

root lies b/w x_3 & 1

$x_4 = 0.50995$

$f(x_4) > 0$

root lies b/w x_4 & 1

$x_5 = 0.5152$

$f(x_5) > 0$

root lies b/w x_5 & 1

$x_6 = 0.5169$

$f(x_6) > 0$

root lies b/w x_6 & 1

$x_7 = 0.51748$

$f(x_7) > 0$

root lies b/w x_7 & 1

$x_8 = 0.51767$

$f(x_8) > 0$

root lies b/w x_8 & 1

$x_9 = 0.5177$

$\therefore$ The root is

0.5177

28.5 $x \log_{10} x = 1.2$

$f(x) = x \log x - 1.2$

$f(0)$ n.d. $f(1) < 0$ $f(2) < 0$ $f(3) > 0$

root lies b/w 2 & 3

$x_1 = 2.72102$ $f(x_2) < 0$

root lies b/w x_1 & 3

$x_2 = \underline{2.74021}$ $f(x_2) < 0$

root lies b/w x_2 & 3

$x_3 = \underline{2.74024}$

root is $x_3 = 2.7402$

28.6 $x = (32)^{1/4}$

$x^4 = 32$

$x^4 - 32 = 0$

$f(1) < 0$ $f(2) < 0$ $f(3) > 0$

root lies b/w 2 & 3

$x_1 = 2.2462$ $f(x_1) < 0$

root lies b/w x_1 & 3

$x_2 = 2.335$ $f(x_2) < 0$

root lies b/w x_2 & 3

$x_3 = 2.3645$ $f(x_3) < 0$

root lies b/w x_3 & 3

$x_4 = \underline{2.3770}$ $f(x_4) < 0$

root lies b/w x_4 & 3

$x_5 = \underline{2.3779}$ root is 2.3779

28.7

$x^4 - x - 10 = 0$

$f(0) < 0 \quad f(1) < 0 \quad f(2) > 0$

$f(x) = x^4 - x - 10$

$f'(x) = 4x^3 - 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \left(\frac{x_0^4 - x_0 - 10}{4x_0^3 - 1}\right) = 1.871$

let us take $x_0 = 2$

$x_1 = 1.871$

$x_2 = 1.856$

$x_3 = 1.856$

root is 1.856

28.8

$f(x) = 3x - \cos x - 1$

$f(0) < 0 \quad f(1) > 0$

$f'(x) = 3 + \sin x$

let us take $x_0 = 0.6$

$x_1 = 0.6071$

$x_2 = 0.60715$

$\therefore$ The root is 0.60715

28.9

$f(x) = x \log x - 1.2$

$f'(x) = 1 + \log x$

$f(1) < 0 \quad f(2) < 0 \quad f(3) > 0$

~~$f'(x) = x + \log x$~~

let $x_0 = 2$

$x_1 = 2.81$

$x_2 = 2.741$

$x_3 = 2.74064$

$x_4 = 2.74065$

$x_5 = 2.74065$

$\therefore$ The root is 2.74065

→ $\cancel{x = \frac{1}{N}}$ $\qquad f(x) = \cancel{x - 1/N}$

$\cancel{x - 1/N = 0}$ $\qquad f'(x) =$

$x = 1/N \qquad N = 1/x$

$f(x) = 1/x - N = 0$

2, 3, 5, 6, 12, 14, 15, 17, 20, $f'(x) = -x^{-2}$

24, 26, 28, 35, $\quad x_{n+1} = x_n - \dfrac{f(x)}{f'(x_n)}$

37, 38, 39, 44,
48, 51, 52, 53, 58,
59, 62, 64, 65, 66
CE-5,

$= x_n - \dfrac{(1/x_n - N)}{-x_n^{-2}}$

$= x_n + (1/x_n - N)x_n^2$

$= x_n + x_n - Nx_n^2$

$= 2x_n - Nx_n^2$

$= x_n(2 - Nx_n)$

$x_2 \ldots \cancel{x_1^2}$ →

$1/31$

$\sqrt{31}$

→ $1/3$ $\quad 1/N \quad x_{n+1} = x_n(2 - Nx_n)$

$\sqrt{N} \quad x_{n+1} = \frac{1}{2}(x_n + N/x_n)$

$1/\sqrt{N} \quad x_{n+1} = \frac{1}{2}(x_n + 1/Nx_n)$

$\sqrt[k]{N} \quad x_{n+1} = 1/k\left[(k-1)x_n + N/x_n^{k-1}\right]$

→ $1/\sqrt{14}$

$x = 1/\sqrt{14}$

$x^2 = 1/14$

$f(x) = x^2 - 1/14 = 0$

$f'(x) = 2x,$

root lie b/w 0 & 1

$f(0) = -1/14$

$f(1) = 13/14$

$x_0 = 1.5$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = \quad + 0.778$

$x_1 = 0.773$

$x_2 = 0.432$

$x_3 = 0.298$

$x_4 = 0.268$

$x_5 = 0.267$

$x_6 = 0.2672$

→ $1/31$

$x = 1/31$ → for $1/N$ $\quad x_{n+1} = x_n(2 - Nx_n)$.

$f(x) =$ $\quad x_{n+1} = x_n(2 - 31x_n)$

let $x_0 = 0.03$

$x_1 = x_0(2 - 31x_0) = 0.0321$

$x_2 = x_1(2 - 31x_1) = 0.032257$

$x_3 = x_2(2 - 31x_2) = 0.03226$

$x_2 = x_3 \quad \therefore \ 1/31 = 0.03226$.

→ $\sqrt{5}$ $\quad$ for $\sqrt{N}$ $\quad x_{n+1} = \frac{1}{2}(x_n + N/x_n)$

$x_{n+1} = \frac{1}{2}(x_n + 5/x_n)$

let $x_0 = 2$

$x_1 = \frac{1}{2}(x_0 + 5/x_0) = 2.25$

$x_2 = \frac{1}{2}(x_1 + 5/x_1) = 2.2361$

$x_3 = \frac{1}{2}(x_2 + 5/x_2) = 2.2361$

$x_2 = x_3 \quad \sqrt{5} = 2.2361$

→ $\sqrt[3]{24}$ $\quad \sqrt[k]{N}$, $\quad x_{n+1} = 1/k\left[(k-1)x_n + N/x_n^{k+1}\right]$

$x_0 = 3$ $\quad x_1 = 1/3\left[2x_0 + 24/x_0^4\right] = 2.88889$

$x_2 = 1/3\left[2x_1 + 24/x_1^4\right] = 2.88451$

$x_3 = 1/3\left[2x_2 + 24/x_2^4\right] = 2.8845$

$x_2 = x_3 \quad \sqrt[3]{24} = 2.8845$

$\rightarrow$ 5) $(30)^{-1/5}$

$N = 30,\ r = -5$

$$x_{n+1} = \frac{1}{5}\left(-6x_n + 30/x_n^{-6}\right) = \frac{x_n}{5}\left(6 - 30x_n^5\right)$$

$x_0 = 1/2.$

$$x_1 = \frac{x_0}{5}\left(6 - 30x_0^5\right) = 0.50625$$

$$x_2 = \frac{x_1}{5}\left(6 - 30x_1^5\right) = 0.506495$$

$$x_3 = \frac{x_2}{5}\left(6 - 30x_2^5\right) = 0.506496.$$

$x_2 = x_3$

$(30)^{-1/5} = 0.5065$

Exercise 28.1

1)

i) $x^3 - 2x - 5 = 0$

using bisection method $x_0 = \frac{a+b}{2}.$

$f(x) = x^3 - 2x - 5$

x	2	3
$f(x)$	-1	16

Root lies b/w 2 & 3 $x_0 = \frac{2+3}{2} = 2.5,\ f(x_0) > 0$

Root lies b/w 2 & 2.5 $x_1 = 2.25,\ f(x_1) > 0$

Root lies b/w 2 & 2.25 $x_2 = 2.125,\ f(x_2) > 0$

Root lies b/w 2 & 2.125 $x_3 = 2.0625,\ f(x_3) < 0$

Root lies b/w 2.0625 & 2.125 $x_4 = 2.0938,\ f(x_4) < 0$

Root lies b/w 2.0938 & 2.125 $x_5 = 2.1094,\ f(x_5) > 0$

Root lies b/w. 2.0938 & 2.1094 $x_6 = 2.1016,\ f(x_6) > 0$

Root lies b/w 2.0938 & 2.1016 $x_7 = 2.0977,\ f(x_7) > 0$

Root lies b/w 2.0938 & 2.0977 $x_8 = 2.0957,\ f(x_8) > 0$

Root lies b/w 2.0938 & 2.0957 $x_9 = 2.0947,\ f(x_9) > 0$

Root lies b/w 2.0938 & 2.0947 $x_{10} = 2.0942$

$\therefore$ The root is 2.094

2 → $x^3 - x^2 - 1 = 0$

$f(x) = x^3 - x^2 - 1$

x	0	1	2
$f(x)$	-1	-1	3

$x = \frac{a+b}{2}$

Using bisection method

Root lies b/w 1 & 2 $x_1 = 1.5$ $f(x_1) > 0$

Root lies b/w 1 & 1.5 $x_2 = 1.25$ $f(x_2) < 0$

Root lies b/w 1.25 & 1.5 $x_3 = 1.375$ $f(x_3) > 0$

Root lies b/w 1.375 & 1.25 $x_4 = 1.3125$ $f(x_4) < 0$

Root lies b/w 1.3125 & 1.375 $x_5 = 1.3438$ $f(x_5) > 0$

Root lies b/w 1.3125 & 1.3438 $x_6 = 1.3281$ $f(x_6) > 0$

Root lies b/w 1.3125 & 1.3281 $x_7 = 1.3203$ $f(x_7) < 0$

Root lies b/w 1.3203 & 1.3281 $x_8 = 1.3242$ $f(x_8) < 0$

Root lies b/w 1.3242 & 1.3281 $x_9 = 1.3262$ $f(x_9) > 0$

Root lies b/w 1.3242 & 1.3262 $x_{10} = 1.3252$ $f(x_{10}) > 0$

Root lies b/w 1.3242 & 1.3252 $x_{11} = 1.325$

∴ The root is 1.325.

iii) $f(x) = x^3 - x - 11$

Root lies b/w 2 & 3

Root lies between.

a	b	$x=\frac{a+b}{2}$	$f(x)$
2	3	2.5	>0
2	2.5	2.25	<0
2.25	2.5	2.375	>0
2.25	2.375	2.3125	<0
2.3125	2.375	2.3438	<0
2.3438	2.375	2.3594	<0
2.3594	2.375	2.3672	<0
2.3672	2.375	2.3711	<0
2.3711	2.375	2.373	<0
2.373	2.375	2.374	>0
2.373	2.374	2.3735	<0
2.3735	2.374	2.3738	

$\therefore$ The root is 2.373.

iv) $2x^3+x^2-20x+12=0$.

x	0	1
$f(x)$	12	-5

Root lies between

a	b	$x=\frac{a+b}{2}$	$f(x)$
0	1	0.5	>0
0.5	1	0.75	<0
0.5	0.75	0.625	>0
0.625	0.75	0.6875	<0
0.625	0.6875	0.6562	<0

0.625	0.6562	0.6406	>0
0.6406	0.6562	0.6484	<0
0.6406	0.6484	0.6445	>0
0.6445	0.6484	0.6465	>0
0.6465	0.6484	0.6475	>0
0.6475	0.6484	0.6479	

∴ The root is 0.647.

(2) i) $\cos x = xe^x$

$f(x) = \cos x - xe^x$

x	0	1
$f(x)$	1	-2.1

Root lies between

a	b	$x = \frac{a+b}{2}$	$f(x)$
0	1	0.5	>0
0.5	1	0.75	<0
0.5	0.75	0.625	<0
0.5	0.625	0.5625	<0
0.5	0.5625	0.5312	<0
0.5	0.5312	0.5156	>0
~~0.0065~~ 0.5156	0.5312	0.5234	<0
0.5156	0.5234	0.5195	<0
0.5156	0.5195	0.5176	>0
0.5176	0.5195	0.5186	<0
0.5176	0.5186	0.5181	<0

∴ The root is 0.518

ii) $x\log x = 1.2$ (2 and 3)

$f(x) = x\log x - 1.2$

Root lies b/w		$x = \frac{a+b}{2}$	$f(x)$
a	b		
2	3	2.5	<0
2.5	3	2.75	>0
2.5	2.75	2.625	<0
2.625	2.75	2.6875	<0
2.6875	2.75	2.7188	<0
2.7188	2.75	2.7344	<0
2.7344	2.75	2.7422	>0
2.7344	2.7422	2.7383	<0
2.7383	2.7422	2.7402	<0
2.7402	2.7422	2.7412	<0
2.7412	2.7422	2.7417	

∴ The root is 2.741.

iii) $f(x) = e^x x - 2$ (1, 1.4)

Root lies b/w		$x = \frac{a+b}{2}$	$f(x)$
a	b		
1	1.4	1.2	>0
1	1.2	1.1	<0
1.1	1.2	1.15	>0
1.1	1.15	1.125	<0
1.125	1.15	1.1375	<0
1.1375	1.15	1.1437	<0

1.1437	1.15	1.1469	>0
1.1437	1.469	1.1453	<0
1.1453	1.1469	1.1461	<0
1.1461	1.1469	1.1465	

$\therefore$ The root is 1.146.

iv) $f(x) = e^x - 4\sin x$

x	0	1
$f(x)$	1	-0.6

root lies b/w a	b	$x = \frac{a+b}{2}$	$f(x)$
0	1	0.5	<0
0	0.5	0.25	>0
0.25	0.5	0.375	<0
0.25	0.375	0.3125	>0
0.3125	0.375	0.3438	>0
0.3438	0.375	0.3594	>0
0.3594	0.375	0.3672	>0
0.3672	0.375	0.3711	<0
0.3672	0.3711	0.3691	>0
0.3691	0.3711	0.3701	>0
0.3701	0.3711	0.3706	

$\therefore$ The root is 0.370

③ Regula falsi method

i) $f(x) = x^3 - 5x + 1 = 0$

x	0	1
$f(x)$	1	-3

$$\boxed{x = \frac{a \cdot f(b) - b f(a)}{f(b) - f(a)}}$$

a	b	x	f(x)
0	1	0.25	<0
0	0.25	0.2025	<0 -0.0044
0	0.2025	0.2017	<0
0	0.2017.		≈0 -0.000296

The root is 0.2017.

ii) $x^3 - 4x - 9$

$f(x) = x^3 - 4x - 9$

x	0	1	2	3
$f(x)$	-9	-12	-9	6

$$\boxed{x = \frac{a \cdot f(b) - b f(a)}{f(b) - f(a)}}$$

a	b	x	f(x)
2	3	2.6	<0
2.6	3	2.6933	<0
2.6933	3	2.7049	<0
2.7049	3	2.7063	<0
2.7063	3	2.7065	

∴ The root is 2.706

iii) $x^3 - x^2 - 1 = 0$

x	0	1	2
$f(x)$	-1	-1	3

a	b	x	f(x)
1	2	1.25	<0
1.25	2	1.3766	<0
1.3766	2	1.4309	<0
1.4309	2	1.4524	<0
1.4524	2	1.4606	<0
1.4606	2	1.4637	<0
1.4637	2	1.4649	<0
1.4649	2	1.4653	<0
1.4653	2	1.4655	

∴ The root is 1.465

iv) $x^6 - x^4 - x^3 - 1 = 0.$

$$\boxed{x = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

x	0	1	2
$f(x)$	-1	-2	39

a	b	x	f(x)
1	2	1.0488	<0
1.0488	2	1.0959	<0
1.0959	2	1.1406	<0
1.1406	2	1.182	<0
1.182	2	1.2195	<0
1.2195	2	1.2528	<0
1.2528	2	1.2816	<0
1.2816	2	1.306	<0
1.306	2	1.3263	<0
1.3263	2	1.3429	<0

1.3429	2	1.3562	<0
1.3562	2	1.3669	<0
1.3669	2	1.36699	

∴ The root is 1.3669

4) $xe^x = 2$.

$f(x) = xe^x - 2$

x	0	1
$f(x)$	-2	0.71

$$\boxed{x = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

a	b	x	f(x)
0	1	0.7358	<0
0.7358	1	0.8395	<0
0.8395	1	0.8512	<0
0.8512	1	0.8525	<0
0.8525	1	0.8526	

∴ The root is 0.852

→ $2x - \log x = 7$

$f(x) = 2x - \log x - 7$

x	0	1	2	3	4
$f(x)$	0.0	-5	-3.3	-1.40	

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

a	b	x	f(x)
3	4	3.7878	<0
3.7878	4	3.7883	<0
3.7893	4	3.7889	

∴ The root is 3.788

iii) $\cos x = 3x - 1$

$f(x) = \cos x - 3x + 1 = 0$

x	0	1
$f(x)$	2	-1.4

$$\boxed{x = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

a	b	x	f(x)
0	1	0.5781	>0
0.5781	1	0.606	>0
0.606	1	0.6069	

∴ The root is 0.606.

iv) $3x + \sin x - e^x = 0$

x	0	1
$f(x)$	-1	1.1

$f(x) = 3x + \sin x - e^x$

$$\boxed{x = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

a	b	x	f(x)
0	1	0.471	>0
0	0.471	0.3723	>0
0	0.3723	0.3616	>0
0	0.3616	0.3610	

∴ The root is 0.361

5) $x = \sqrt[4]{12}$

$x^4 = 12$

$f(x) = x^4 - 12$

x	1	2
$f(x)$	-11	4

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

a	b	x	$f(x)$
1	2	1.733	<0
1.733	2	1.847	<0
1.844	2	1.8597	<0
1.8597	2	1.8611	<0
1.8611	2	1.8612	

$\therefore$ The root is 1.861

6) i) $x^3 - 3x + 1 = 0$

1-3+.

$f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3x$

x	0	1
$f(x)$	1	-1

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$x_0 = \frac{0+1}{2} = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.333$$

$x_2 = 0.3472$

$x_3 = 0.3473$

$\therefore$ The root is 0.347.

8-

$\rightarrow x^3 - 2x - 5$

$f'(x) = 3x^2 - 2.$

x	0	1	2	3
$f(x)$	-5	-6	-1	16

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_0 = \dfrac{2+3}{2} = 2.5$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2.1642$

$x_2 = 2.0971$

$x_3 = 2.0946$

$x_4 = 2.094$

$\therefore$ The root is 2.094.

$\rightarrow x^3 - 5x + 3 = 0$

8 -10 +3

$f'(x) = 3x^2 - 5.$

x	0	1	2
$f(x)$	3	-1	1

$x_0 = 1.5$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2.1428$

$x_2 = 1.9006$

$x_3 = 1.8385$

$x_4 = 1.8342$

$x_5 = 1.834$

$\therefore$ The root is 1.834

$\rightarrow 3x^3 - 9x^2 + 8 = 0$

$x_0 = \dfrac{1+2}{2} = 1.5$

$f'(x) = 9x^2 - 18x$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1.185$

$x_2 = 1.2257$

$x_3 = 1.22607$

$x_4 = 1.226$

$\therefore$ The root is 1.226.

(7)

i) $x^4+x^3-7x^2-x+5=0$ $\qquad (2,3)$

$f'(x)=4x^3+3x^2-14x-1$ $\qquad x_0=2.5$

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$x_1=2.203$

$x_2=2.0825$

$x_3=2.0614$

$x_4=2.06085$

$x_5=2.0608$

$\therefore$ The root is 2.0608

ii) $x^5-5x^2+3=0$

$f'(x)=5x^4-10x$

x	0	1
$f(x)$	3	-1

$x_0=0.5$

$x_1=0.88$

$x_2=0.8206$

$x_3=0.82147$

$x_4=0.8214$

$\therefore$ The root is 0.821

8) $x^3-21x+3500=0$

$f'(x)=3x^2-21$

x	0	-15	-16
$f(x)$	3500	440	-260

$x_0=-15.5$

$x_1=-15.6452$

$x_2=-15.6439$

$x_3=-15.643$

$\therefore$ The root is -15.643

a) $xe^x - 2 = 0$

x	0	1
$f(x)$	-2	0.71

$x_0 = 0.5$

$f'(x) = e^x + xe^x$

$x_1 = 0.975$

$x_2 = 0.863$

$x_3 = 0.8526$

$x_4 = 0.8526$

$\therefore$ The root is 0.8526

ii) $x^2 + 4\sin x = 0$

$f'(x) = 2x + 4\cos x$

x	0	1	-1	-2
$f(x)$	0	1	-2.3	0.362

$x_0 = \frac{0 + (-1)}{2} = -0.5$

$x_1 = -2.37$

$x_2 = -1.47$

$x_3 = -1.05$

$x_4 = -0.905$

$x_5 = -0.877$

$x_6 = -0.8767$

$x_7 = -0.876$

$\therefore$ The root is -0.876.

iii) $x\sin x + \cos x = 0$

$f'(x) = \sin x + x\cos x - \sin x$

$= x\cos x$

x	0	2	3
$f(x)$	1	1.4	-0.1

$x_0 = 2.5$

$x_1 = 2.847$

$x_2 = 2.799$

$x_3 = 2.7983$

$x_4 = 2.798$

$\therefore$ The root is 2.798

iv) $e^x = x^2 + \cos 25x$

$f(x) = e^x - x^2 - \cos 25x$

$f'(x) = e^x - 2x + 25\sin 25x$

x	-1	0
$f(x)$	1.6	0

$x_0 = -0.5$

$x_1 = -0.303$
$x_2 = -0.286$
$x_3 = -0.2847$
$x_4 = \underline{-0.284}$

$\therefore$ The root is -0.284

(10)

i) $\cos x = xe^x$

$f(x) = xe^x - \cos x$

$f'(x) = xe^x + e^x + \sin x$

x	0	1
$f(x)$	-1	2.17

$x_0 = 0.5$

$x_1 = 0.518$
$x_2 = 0.5177$
$x_3 = \underline{0.517}$

$\therefore$ The root is 0.517

ii) $x\log x = 12.34$

$f'(x) = \log x + 1$

x	1	11	12
$f(x)$	-12.34	-0.8	0.6

$x_0 = \frac{23}{2} = 11.5$

$x_1 = 6.925$
$x_2 = 6.563$
$x_3 = 6.5602$
$x_4 = \underline{6.560}$

$\therefore$ The root is 6.560.

iii) $10^x + x - 4$

$f'(x) = 10^x \log 10 + 1 = 10^x + 1$

x	0	1
$f(x)$	-3	7

$x_0 = 0.5$

$x_1 = 0.540$

$x_2 = \underline{0.5391}$

$x_3 = \underline{0.539}$

$\therefore$ The root is 0.539

iv) $x + \log x - 3.375$

$f'(x) = 1 + 1/x$

x	1	2	3
$f(x)$	-2.3	-1	0.1

$x_0 = 2.5$

$x_1 = \underline{2.4705}$

$x_2 = \underline{2.470}$

$\therefore$ The root is 2.470.

(12) $x = \sqrt{13}$

$f(x) = x^2 - 13$

$f'(x) = 2x$

x	3	4
$f(x)$	-4	3

$x_0 = 3.5$

$x_1 = 3.607$

$x_2 = \underline{3.6055}$

$x_3 = \underline{3.605}$

$\therefore$ The root is 3.605

(13) $x = \sqrt{10}$

$f(x) = x^2 - 10$

$f'(x) = 2x$

x	3	4
$f(x)$	-1	6

$x_0 = 3.5$

$x_1 = 3.178$

$x_2 = \underline{3.1623}$

$x_3 = \underline{3.162}$

$\therefore$ The root is 3.162

(13) $x = \sqrt[3]{17}$

$f(x) = x^3 - 17$

$f'(x) = 3x^2$

let $x_0 = 2$

$x_1 = 2.75$
$x_2 = 2.582$
$x_3 = 2.5713$
$x_4 = 2.5712$

$\therefore$ The root is 2.571

(14)

~~f(x~~ $x = (11)^{1/4}$

let ~~x = 0.5~~
$x = 1$

$f(x)\ x^4 - 11$
$f'(x) = 4x^3$

$x_1 = 3.5$
$x_2 = 2.689$
$x_3 = 2.158$
$x_4 = 1.892$
$x_5 = 1.825$
$x_6 = 1.8211$
$x_7 = 1.821$

$\therefore$ The root is 1.821

ii) $x = (32)^{1/4}$

$f(x) = x^4 - 32$
$f'(x) = 4x^3$

Let $x_0 = 2.5$

$x_1 = 2.387$
$x_2 = 2.3784$
$x_3 = 2.378$

$\therefore$ The root is 2.378 //

(15) $x = 1/18$

$f(x) = 18x - 1$
$f'(x) = 18$

x	0	1
$f(x)$	-1	17

$x_0 = 0.5$

$x_1 = 0.0556$
$x_2 = 0.055$

$\therefore$ The root is 0.055

$x = 1/\sqrt{15}$

$x^2 = 1/15$

$f(x) = 15x^2 - 1$

$f'(x) = 30x$

x	0	1
$f(x)$	-1	14

$x_0 = 0.5$

$x_1 = 0.3167$

$x_2 = 0.2636$

$x_3 = 0.2583$

$x_4 = \underline{0.2582}$

$\therefore$ The root is 0.258.

$\dashv$ $x = (28)^{-1/4}$

$28x^4 = 1$

$f(x) = 28x^4 - 1$

$f'(x) = 112x^3$

x	0	1
$f(x)$	-1	27

$x_1 = 0.4464$

$x_2 = 0.4352$

$x_3 = 0.4347$

$x_4 = \underline{0.434}$

$\therefore$ The root is 0.434

ex 29.1

i) $\Delta \tan^{-1}x = \tan^{-1}(x+h) - \tan^{-1}x$

$= \tan^{-1}\left(\dfrac{x+h-x}{1+(x+h)x}\right)$

$= \tan^{-1}\left(\dfrac{h}{1+hx+x^2}\right)$

ii) $\Delta(e^x \log 2x)$

$= e^{x+h}\log 2(x+h) - e^x \log 2x$

$= e^{x+h}\log 2(x+h) - e^{x+h}\log 2x + e^{x+h}\log 2x - e^x \log 2x$

$$= e^{x+h}\log\frac{x+h}{x} + (e^{x+h} - e^{x})\log 2x$$

$$= e^{x}\left[e^{h}\log\left(1+\frac{h}{x}\right) + (e^{h}-1)\log 2x\right]$$

iii) $$\Delta\left(\frac{x^2}{\cos 2x}\right) = \frac{(x+h)^2}{\cos 2(x+h)} - \frac{x^2}{\cos 2x}$$

$$= \frac{(2hx+h^2)\cos 2x + 2x^2\sin(h)\sin(2x+h)}{\cos 2(x+h)\cos 2x}$$

iv) $$\Delta^2\cos 2x = \Delta(\cos 2(x+h) - \cos 2x)$$

$$= \Delta\cos 2(x+h) - \Delta\cos 2x$$

$$= [\cos 2(x+2h) - \cos 2(x+h)] - [\cos 2(x+h) - \cos 2x]$$

$$= -2\sin(2x+3h)\sin h + 2\sin(2x+h)\sin h$$

$$= -2\sin h\,(\sin(2x+3h) - \sin(2x+h))$$

$$= -2\sin h\,[2\cos(2x+2h)\sin h] = -4\sin^2 h\cos(2x+2h)$$

29.2

$$\Delta^2\left(\frac{5x+12}{x^2+5x+6}\right)$$

$$= \Delta^2\left(\frac{2}{x+2} + \frac{3}{x+3}\right)$$

$$= \Delta\left(\Delta\left(\frac{2}{x+2}\right) + \Delta\left(\frac{3}{x+3}\right)\right) = \Delta\left(2\left(\frac{1}{x+3} - \frac{1}{x+2}\right) + 3\left(\frac{1}{x+4} - \frac{1}{x+3}\right)\right)$$

$$= -2\Delta\left(\frac{1}{(x+2)(x+3)}\right) - 3\Delta\left(\frac{1}{(x+3)(x+4)}\right)$$

$$= -2\left(\frac{1}{(x+3)(x+4)} - \frac{1}{(x+2)(x+3)}\right) - 3\left(\frac{1}{(x+4)(x+5)} - \frac{1}{(x+3)(x+4)}\right)$$

$$= \frac{2(5x+16)}{(x+2)(x+3)(x+4)(x+5)}$$

ii) $\Delta(ab^{(x)}) = a\Delta(b^{(x)}) = a(b^{(x+1)} - b^{(x)}) = ab^{(x)}(b-1)$

$\Delta^2(ab^{(x)}) = \Delta(\Delta ab^{(x)}) = a(b-1)\Delta(b^{(x)})$

$= a(b-1)(b^{(x+1)} - b^{(x)}) = a(b-1)^2 b^{(x)}$.

iii) $\Delta e^x = e^{x+1} - e^x = (e-1)e^x$

$\Delta^2 e^x = \Delta(\Delta e^x) = \Delta((e-1)e^x)$

$= (e-1)(e-1)e^x = (e-1)^2 e^x$

$\Delta^3 e^x = (e-1)^3 e^x$. ... $\Delta^n e^x = (e-1)^n e^x$.//

Exercise 29.1

8) i) $\Delta(x + \cos x)$

$= (x+1) - x + \cos(x+1) - \cos x$

$= 1 + \cos x \cos 1 - \sin x \sin 1 - \cos x$

$= 1 - \cos x(1 - \cos 1) - \sin x \sin 1$

$= 1 - \cos x \, 2\sin^2(1/2) - \sin x \, 2\sin 1/2 \cos 1/2$

$= 1 - 2\sin 1/2 (\cos x \sin 1/2 + \sin x \cos 1/2)$

$= 1 - 2\sin 1/2 \sin(x + 1/2)$.

ii) $\Delta \tan^{-1}\left(\frac{n-1}{n}\right)$

$\tan^{-1}\left(\frac{n+1-1}{n+1}\right) - \tan^{-1}\left(\frac{n-1}{n}\right)$

$\tan^{-1}\left(\frac{n}{n+1}\right) - \tan^{-1}\left(\frac{n-1}{n}\right)$

$\tan^{-1}\left(\frac{\frac{n}{n+1} - \frac{n-1}{n}}{1 + \frac{n-1}{n+1}}\right)$

$= \tan^{-1}\left(\frac{\frac{n^2 - (n^2-1)}{n(\cancel{n+1})}}{\frac{n+\cancel{1}+n-\cancel{1}}{\cancel{n+1}}}\right)$

$= \tan^{-1}\left(\frac{1}{2n^2}\right)$

iv) $\Delta^2\left(\frac{1}{x^2+5x+6}\right)$

$= \Delta^2\left(\frac{1}{(x+2)(x+3)}\right)$

$= \Delta\left(\frac{1}{(x+3)(x+4)} - \frac{1}{(x+2)(x+3)}\right)$

$= \Delta\left(\left(\frac{1}{x+3}\right)\left(\frac{\cancel{x}+2-\cancel{x}-4}{(x+2)(x+4)}\right)\right)$

$= \Delta\left(\frac{-2}{(x+2)(x+3)(x+4)}\right)$

$= -2\left(\frac{1}{(x+3)(x+4)(x+5)} - \frac{1}{(x+2)(x+3)(x+4)}\right)$

$= \frac{-2}{(x+3)(x+4)}\left(\frac{\cancel{x}+2-\cancel{x}-5}{(x+2)(x+5)}\right)$

$= \frac{6}{(x+2)(x+3)(x+4)(x+5)}$

2) i) $\Delta\left(e^{3x}\log 2x\right)$

$= e^{3(x+1)}\log 2(x+1) - e^{3x}\log 2x$

$= e^{3(x+1)}\log 2(x+1) - e^{3(x+1)}\log 2x + e^{3(x+1)}\log 2x - e^{3x}\log 2x$

$= e^{3(x+1)}\log\frac{x+1}{x} + \left(e^{3x}e^{3} - e^{3x}\right)\log 2x$

$= e^{3x}e^{3}\log\frac{x+1}{x} + e^{3x}\left(e^{3}-1\right)\log 2x$

$= e^{3x}\left[e^{3}\log\left(1+\frac{h}{x}\right) + \left(e^{3}-1\right)\log 2x\right]$

ii) $\Delta(2^x/x!)$

$2^{x+1}/(x+1)! - 2^x/x!$

$\frac{2^x 2}{(x+1)x!} - \frac{2^x}{x!} \Rightarrow \frac{2^x}{x!}\left(\frac{2}{x+1} - 1\right)$

$= \frac{2^x}{x!}\left(\frac{2-x-1}{x+1}\right)$

$= \frac{2^x}{x!}\left(\frac{1-x}{x+1}\right)$

$= \frac{2^x(1-x)}{(x+1)!}$

29.7

i) $hD = \log(1+\Delta) = -\log(1-\nabla) = \sinh^{-1}(\mu\delta)$

$e^{hD} = E = 1+\Delta$; $hD = \log(1+\Delta)$ $\quad E^{-1} = 1-\nabla$

$hD = \log E = -\log(E^{-1}) = -\log(1-\nabla)$ $\quad \nabla = 1-E^{-1}$

$\mu = \frac{1}{2}(E^{1/2}+E^{-1/2}) \qquad \delta = E^{1/2} - E^{-1/2})$

$\mu\delta = \frac{1}{2}(E-E^{-1}) = \frac{1}{2}(e^{hD}-e^{-hD}) = \sinh(hD)$

$\Rightarrow hD = \sinh^{-1}(\mu\delta)$.

ii) $(E^{1/2}+E^{-1/2})(1+\Delta)^{1/2} = 2+\Delta$.

$(E^{1/2}+E^{-1/2})(E)^{1/2} \Rightarrow \quad E+1 = 1+\Delta+1 = 2+\Delta$

~~iii) $\frac{1}{2}\delta^2 + \delta\sqrt{\quad}$~~

$hD \quad \sinh^{-1}(\mu\delta)$

29.8)

x	2	3	4	5	6
y	45.0	49.2	54.1	–	67.4

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
2	45.0				
		4.2			
3	49.2		0.7		
		4.9		$a-59.7$	
4	54.1		$a-59.0$		$240.2-4a$
		$a-54.1$		$180.5-3a$	
5	a		$121.5-a$		
		$67.4-a$			
6	67.4				

$\Delta^4 y = 0$ $\qquad a = 60.05$ //

29.9

x	45	50	55	60	65
y	3	–	2	–	-2.4

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
45	3			
		$a-3$		
50	a		$5-2a$	
		$2-a$		$3a+b-9$ (1)
55	2		$b+a-4$	
		$b-2$		$3.6-a-3b$ (2)
60	b		$-0.4-2b$	
		$-2.4-b$		
65	-2.4			

Solve (1) & (2) $\qquad a = 2.925$, $y_7 = 0.225$

29.10

$y_{10} = 3$, $y_{11} = 6$, $y_{12} = 11$, $y_{13} = 18$, $y_{14} = 27$, y_4

x	y
10	
11	
12	
13	
14	

29.4

$y_1+y_7=-7845$, $y_2+y_6=686$, $y_3+y_5=1088$, y_2

let Start with y_1 instead of y_0

$\Delta^6 y_1=0$

$(E-1)^6 y_1=0$

$(E^6-6E^5+15E^4-20E^3+15E^2-6E+1)y_1$

$= y_7-6y_6+15y_5-20y_4+15y_3-6y_2+y_1$

$=(y_1+y_7)-6(y_6+y_2)+15(y_3+y_5)=20y_4$

$\frac{1}{20}\left(-7845+686(-6)+15(1088)\right)=y_4$

$y_4=571.4$

Ex-29.2

i) $\left(\frac{\Delta^2}{E}\right)u_x \quad = \Delta^2 E^{-1}u_x \quad = \Delta^2 u_{x-1}$

$=\Delta(\Delta(u_{x-1}))$

$=\Delta(u_x-u_{x-1})$

$=u_{x+1}-u_x-(u_x-u_{x-1})$

$=u_{x+1}-2u_x+u_{x-1}$

$\frac{\Delta^2 u_x}{Eu_x} = \frac{\Delta(u_{x+1}-u_x)}{u_{x+1}}$

$=\frac{u_{x+2}-u_{x+1}-u_{x+1}+u_x}{u_{x+1}}$

$=\frac{u_{x+2}-2u_{x+1}+u_x}{u_{x+1}}$

2) i) $\frac{\Delta^2}{E} \sin x$

$\Delta^2 E^{-1} \sin x$

$\Rightarrow \Delta^2 \sin(x-1)$

$\Delta(\sin x - \sin(x-1))$

$\rightarrow \Delta(\sin x - \sin x \cos 1 + \cos x \sin 1)$

$\rightarrow \sin(x+1) - \sin x - \sin(x+1)\cos 1 + \sin x \cos 1 + \cos(x+1)\sin 1 - \cos x \sin 1$

$\rightarrow \sin x \cos 1 + \cancel{\cos x} \sin 1 - \sin x - \sin x \cos^2 1 - \cancel{\cos x} \sin 1 \cos 1 + \sin x \cos 1 + \cancel{\cos x \cos} \sin 1 - \sin x \sin^2 1 - \cancel{\cos x} \sin 1$

$\Rightarrow 2\sin x \cos 1 - \sin x - \sin x(1) +$

$\rightarrow 2\sin x \cos 1 - 2\sin x$

$\rightarrow 2\sin x (\cos 1 - 1)$

$\rightarrow -2\sin x \, 2\sin^2 1/2$

$\rightarrow -4 \sin x \sin^2 1/2$

ii) $\left(\frac{\Delta^2}{E}\right) x^4$

$\Delta^2 E^{-1} x^4$

$\Delta^2 (x-1)^4$

$\Delta[x^4 - (x-1)^4]$

$[(x+1)^4 - x^4 - (x)^4 + (x-1)^4]$

$[(x+1)^4 - 2x^4 + (x-1)^4] \quad = 12x^2 + 2.$

iii) $\left(\frac{\Delta 2}{E}\right) \sin(x+h) + \frac{\Delta^2 \sin(x+h)}{E \sin(x+h)}$

$\downarrow$

$\Delta^2(\sin x) + \frac{\Delta^2 \sin(x+h)}{\sin(x+2h)}$

$\downarrow$

$\rightarrow (\sin(x+h) - \sin x)$

$\rightarrow \Delta(\sin x \cos h + \cos x \sin h - \sin x)$

$\rightarrow \sin(x+h)\cos h - \sin x\cos h + \cos(x+h)\sin h - \cos x\sin h$

$- \sin(x+h) + \sin x$

$\sin x\cos^2 h + \cos x\sin h\cos h - \sin x\cos h$

$+ \cos x\cos h\sin h - \sin x\sin^2 h$

$- \cos x\sin h \quad - \sin x\cos h - \cos x\sin h + \sin x$

$\rightarrow \sin x\cos 2h + 2\cos x\cos h\sin h - 2\sin x\cos h - 2\cos x\sin h$

$+ \sin x$

$\rightarrow \sin x\cos 2h + \cos x\sin 2h - 2(\sin(x+h)) + \sin x$

$\rightarrow \sin(x+2h) - 2\sin(x+h) + \sin x.$

$\Delta^2 \sin(x+h)$

$\Delta(\sin(x+2h) - \sin(x+h))$

$\Delta(\sin x\cos 2h + \cos x\sin 2h - \sin x\cos h - \cos x\sin h)$

$\rightarrow \sin(x+h)\cos 2h - \sin x\cos 2h + \cos(x+h)\sin 2h - \cos x\sin 2h$

$- \sin(x+h)\cos h + \sin x\cos h - \cos(x+h)\sin h + \cos x\sin h$

$\rightarrow \sin x\cos h\cos 2h + \cos x\sin h\cos 2h - \sin x\cos 2h + \cos x\cos h\sin 2h$

$- \sin x\sin h\sin 2h - \cos x\sin 2h - \sin x\cos^2 h - \cos x\sin h\cos$

$+ \sin x\cos h - \cos x\cos h\sin h + \sin x\sin^2 h + \cos x\sin h.$

$\rightarrow \cos h\sin(x+2h) + \sin h\cos(x+2h) - \sin(x+2h)$

$+ \sin(x+h) - \cos x\sin 2h - \sin\cos 2h$

$\rightarrow \sin(x+3h) - \sin(x+2h) + \sin(x+h) - \sin(x+2h)$

$\rightarrow \sin(x+3h) - 2\sin(x+2h) + \sin(x+h)$

Overall

$$\sin(x+2h) - 2\sin(x+h) + \sin x. + \frac{\sin(x+3h) - 2\sin(x+2h) + \sin(x+h)}{\sin(x+2h)}$$

$$\left(\sin(x+2h)-2\sin(x+h)+\sin x\right)+E\frac{\left[\sin(x+2h)-2\sin(x+h)+\sin x\right]}{\sin(x+2h)}$$

$$\rightarrow \left(\sin(x+2h)-2\sin(x+h)+\sin x\right)\left(1+\frac{E}{\sin(x+2h)}\right)$$

$$\rightarrow \quad '' \quad \left(1+\frac{1}{\sin(x+h)}\right)$$

$$\rightarrow \left(\sin(x+2h)-2\sin(x+h)+\sin x\right)\left(\frac{\sin(x+h)+1}{\sin(x+h)}\right)$$

iii) $(\Delta+\nabla)^2(x^2+x)$

$(\Delta^2+2\Delta\nabla+\nabla^2)(x^2+x)$

$\Delta^2(x^2+x)+2\Delta\nabla(x^2+x)+\nabla^2(x^2+x)$

$\Delta((x+1)^2-x^2+(x+1)-x) \quad + 2\Delta\left(x^2+x-(x-1)^2-(x-1)\right) \quad \nabla(x^2+x-(x-1)^2-(x-1))$

$\Delta(2x+2) \qquad x^2-\cancel{x^2}-\cancel{1}+2x+\cancel{x}-\cancel{x}+1 \qquad \nabla(2x)$

$2 \qquad 2\Delta(2x) \qquad 2(x-(x-1))$

$4(x+1-x) \qquad 2(x-x+1)$

$4 \qquad 2$

$\rightarrow 2+4+2 \rightarrow 8$

3)

i) $\nabla = 1-e^{-hD}$ $\qquad E=e^{hD}$

$\nabla = 1-E^{-1}$

$= 1-e^{-hD}$

ii) $D=\frac{2}{h}\sinh^{-1}\left(\frac{\delta}{2}\right)$

$\delta = E^{1/2}-E^{-1/2}$

$\delta = \frac{e^{(hD/2)}-E^{(-hD/2)}}{2}\times 2$

$\frac{\delta}{2} = \sinh\left(\frac{hD}{2}\right)$

$\rightarrow \sinh^{-1}\left(\frac{\delta}{2}\right)=\frac{hD}{2}$

$$\boxed{D=\frac{2}{h}\sinh^{-1}(\delta/2)}$$

iii) $(1+\Delta)(1-\nabla)=1$.

$E \times E^{-1} = 1$

iv) $\Delta - \nabla = \nabla\Delta = \delta^2$

$(E-1)-(1-E^{-1})$

$E-1-1+E^{-1}$

$\rightarrow E+E^{-1}-2.$

$\rightarrow (E^{1/2}-E^{-1/2})^2 = \delta^2$

$\nabla\Delta \rightarrow$

$(E-1)(1-E^{-1})$

$E-1-1+E^{-1}$

$E+E^{-1}-2$

$\rightarrow (E^{1/2}-E^{-1/2})^2 = \delta^2$

4 i) $\delta = \Delta(1+\Delta)^{-1/2} = \nabla(1-\nabla)^{-1/2}$

$= (E-1)(E)^{-1/2}$

$= (E^{1/2}-E^{-1/2}) = \delta$

$= (1-E^{-1})(E^{-1})^{-1/2}$

$= (E^{1/2}-E^{-1/2})$

$= \delta$

ii) $\mu^2 = 1+\delta^2/4$.

$= \frac{4+(E^{1/2}-E^{-1/2})^2}{4} = \frac{4+E+E^{-1}-2}{4} = \left(\frac{E^{1/2}+E^{-1/2}}{2}\right)^2$

$= \mu^2$

iii) $\delta(E^{1/2}+E^{-1/2}) = \Delta E^{-1}+\Delta$

$(E^{1/2}-E^{-1/2})(E^{1/2}+E^{-1/2})$

$= E-E^{-1} = \cancel{E^{-1}E_1}$

$= 1+\Delta-E^{-1} = \cancel{E-E^{-1}}+$

$= \nabla+\Delta$

$= \Delta E^{-1}+\Delta$

iv) $\nabla = \Delta E^{-1} = E^{-1}\Delta = 1 - E^{-1}.$

$(E-1)E^{-1}$

$(1-E^{-1})$

$= \nabla$

$E^{-1}(E-1)$

$= 1 - E^{-1}$

$= \nabla$

$= \nabla$

5) i) $\mu\delta = \frac{1}{2}(\Delta + \nabla)$

$\frac{1}{2}(E - E^{-1})$

$\frac{1}{2}(E - 1 + 1 - E^{-1})$

$\frac{1}{2}((E-1) + (1-E^{-1}))$

$= \frac{1}{2}(\Delta + \nabla)$

ii) $1 + \delta^2/2 = \sqrt{1 + \delta^2\mu^2}$

$= 1 + \frac{(E^{1/2} - E^{-1/2})^2 (E^{1/2} + E^{-1/2})^2}{4}$

$= 1 + \frac{(E - E^{-1})^2}{4}$

$= 4 + E^2 + E^{-2} - 2$

$= \frac{E^2 + E^{-2} + 2}{4}$

$= \sqrt{\left(\frac{E + E^{-1}}{2}\right)^2}$

$= \frac{E + E^{-1} + 2 - 2}{2}$

$= 1 + \frac{(E^{1/2} - E^{-1/2})^2}{2}$

$= 1 + \delta^2/2.$

iii) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$=$ WKT $\nabla = \Delta E^{-1}$

$= E - E^{-1}$

$= E - 1 + 1 - E^{-1}$

$= \Delta + \nabla$

iv) $\nabla^2 = h^2D^2 - h^3D^3 + \frac{7}{12}h^4D^4 - \ldots$

$\nabla^2 = (1 - E^{-1})^2$

$= 1 + E^{-2} - 2E^{-1}$

$= 1 + e^{-2hD} - 2e^{-hD}$

$e^x \to 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$

$= \cancel{1} + \cancel{1} - 2hD + \frac{\cancel{4}h^2D^2}{2} + \frac{48h^3D^3}{36} \ldots$

$= h^2D^2 - h^3D^3 + \ldots \quad -\cancel{2} + \cancel{2hD} - \cancel{\frac{2h^2D^2}{2}}$

6) i) $\nabla^r f_k = \Delta^r f_{k-r}$

$\nabla = \Delta E^{-1}$

$(\Delta E^{-1})^r f_k$

$\Delta^r E^{-r} f_k = \Delta^r f_{k-r}$

ii) $\Delta f_k^2 = (f_k + f_{k+1}) \Delta f_k$

$(f_{k+1})^2 - (f_k)^2$

$(f_{k+1} + f_k)(f_{k+1} - f_k)$

$= (f_{k+1} + f_k) \Delta f_k.$

iv) $E^{1/2} = (1 + \delta^2/4)^{1/2} + \delta/2$

$= (\mu^2)^{1/2}$

$= \mu + \delta/2$

$= \dfrac{E^{1/2} + E^{-1/2}}{2} + \dfrac{E^{1/2} - E^{-1/2}}{2}$

$= E^{1/2}$

a)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		2			
1	3		4		
		6		$a-19$	
2	9		$a-15$		$124-4a$
		$a-9$		$105-3a$	
3	a		$90-2a$		
		$81-a$			
4	81				

$\Delta^4 y = 0$

$124 = 4a$

$a = 31$

(c)

10)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	6				
		$a-6$			
1.5	a		$16-2a$		
		$10-a$		$3a-16$	
2	10		a		$b-4a-14$…
		10		$b-a-30$	
2.5	20		$b-30$		$81.5-4b+$…
		$b-20$		$51.5-3b$	
3	b		$21.5-2b$		$6b-71$…
		$1.5-b$		$3b-19.5$	
3.5	1.5		$2+b$		
		3.5			
4	5				

$\Delta^5 y$

$95.5-5b+5a$ — (1)

$10b-152.5-a$ — (2)

Solve (1) & (2)

$a=0.222 \qquad b=22.022$

11)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	5			
		6		
1	11		5	
		11		2
2	22		7	
		18		$a-65$
3	40		$a-58$	
		$a-40$		$238-3a$
4	a		$180-2a$	
		$140-a$		$3a+b-460$
5	140		$a+b-280$	
		$b-140$		
6	b			

$\Delta^4 y$	$\Delta^5 y$
$a-67$	
	$370-5a$ — (1)
$303-4a$	
	$10a+b-1001$ — (2)
$6a+b-698$	

Solve (1) & (2)

$a=74$

$b=261$ //