What is the effect on standard error, if a sample is taken from an infinite population and its size is increased from 400 to 900?

Here $n_1 = 400$ and $n_2 = 900$

Initially, the Standard Error $SE_1 = \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n_1}} = \dfrac{\sigma}{\sqrt{400}}$

After the sample size increasing, the Standard Error $SE_2 = \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n_2}} = \dfrac{\sigma}{\sqrt{900}}$

Now consider $\dfrac{SE_2}{SE_1} = \dfrac{\sigma}{\sqrt{900}} \times \dfrac{\sqrt{400}}{\sigma} = \dfrac{2}{3}$

Therefore, the Standard Error is decreased $\dfrac{2}{3}$ times of its original value

What is the effect on standard error, if a sample is taken from an infinite population and its size is decreased from 100 to 25?

Here $n_1 = 100$ and $n_2 = 25$

Initially, the Standard Error $SE_1 = \sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n_1}} = \dfrac{\sigma}{\sqrt{100}}$

After the sample size decreasing, the Standard Error $SE_2 = \sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n_2}} = \dfrac{\sigma}{\sqrt{25}}$

Now consider $\dfrac{SE_2}{SE_1} = \dfrac{\sigma}{\sqrt{25}} \times \dfrac{\sqrt{100}}{\sigma} = 2$

Therefore, the Standard Error is increased 2 times of its original value

Sampling Distribution of the Mean with σ known

If $\bar{X}$ is the mean of a random sample of size n≥30 taken from a population with mean μ and finite variance σ^2, then

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

is a random variable, whose distribution function approaches that of the standard normal distribution as $n \to \infty$.

A random sample of size 100 is taken from an infinite population having the mean 76 and the variance 256. What is the probability that the sample mean will be between 75 and 78

Here $n = 100$, $\mu = 76$, $\sigma^2 = 256$ and $\sigma = 16$

By central limit theorem, $Z = \dfrac{\overline{X} - \mu}{\left(\dfrac{\sigma}{\sqrt{n}}\right)}$

$$P\left(75 < \overline{X} < 78\right) = P\left(\frac{75-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} < \frac{\overline{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} < \frac{78-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}\right) = P\left(\frac{75-76}{\left(\frac{16}{\sqrt{100}}\right)} < Z < \frac{78-76}{\left(\frac{16}{\sqrt{100}}\right)}\right)$$

$$= P(-0.63 < Z < 1.25)$$

$$= F(1.25) - F(-0.63)$$

$$= F(1.25) - [1 - F(0.63)]$$

$$= 0.8944 - 1 + 0.7357 \qquad = 0.6301$$

If a 1- gallon can of paint covers on an average 513 square feet with a standard deviation of 31.5 square feet, what is the probability that the mean area covered by a sample of 40 of these 1- gallon cans will be anywhere from 510 to 520 square feet

Here $n = 40$, $\mu = 513$ and $\sigma = 31.5$

By central limit theorem, $Z = \dfrac{\overline{X} - \mu}{\left(\dfrac{\sigma}{\sqrt{n}}\right)}$

$$P(510 < \overline{X} < 520) = P\left(\frac{510-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} < \frac{\overline{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} < \frac{520-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}\right) = P\left(\frac{510-513}{\left(\frac{31.5}{\sqrt{40}}\right)} < Z < \frac{520-513}{\left(\frac{31.5}{\sqrt{40}}\right)}\right)$$

$$= P(-0.6 < Z < 1.4)$$

$$= F(1.4) - F(-0.6)$$

$$= F(1.4) - [1 - F(0.6)]$$

$$= 0.9192 - 1 + 0.7258 = 0.645$$

* A random sample of size 64 is taken from a normal population with $\mu =$ 51.4 and $\sigma =$ 68. What is the prob. that the mean of a sample will

(i) exceed 52.9 (ii) fall b/w 50.5 & 52.3

(iii) less than 50.6

Sol: $n = 64$, $\mu = 51.4$, $\sigma = 68$

(i) $P(\bar{x} > 52.9) = 1 - P(\bar{x} \leq 52.9)$

$\bar{x} = 52.9 \Rightarrow z = \dfrac{52.9 - 51.4}{68/\sqrt{64}}$

$= 0.176$

$\therefore 1 - P(z \leq 0.176) = 1 - F(0.176)$

$= 1 - 0.5675$

$= 0.4325$

iii) $P(50.5 \leq \bar{x} \leq 52.3)$

$\bar{x}_1 = 50.5 \Rightarrow z_1 = \dfrac{50.5 - 51.4}{68/\sqrt{64}}$

$z_1 = -0.105$

$$\bar{x}_2 = 52.3 \Rightarrow z_2 = \frac{52.3 - 51.4}{68/8}$$

$$= 0.1085$$

$$P(50.5 \leq \bar{x} \leq 52.3) = P(-0.105 \leq z \leq 0.105)$$

$$= F(0.105) - F(-0.105)$$

$$= 0.5398 - 0.4602$$

$$= 0.0796.$$

(iii) $P(\bar{x} < 50.6) \Rightarrow 1$

$\bar{x} = 50.6 \Rightarrow z = \dfrac{50.6 - 51.4}{68/\sqrt{8}}$

$= -0.09$

$P(z < -0.09) = P(-0.09)$

$= 0.46$[illegible]41

Sampling Distribution of the Mean with σ unknown

If we do not know the value of σ , then we cannot use the Central limit theorem $Z = \dfrac{\bar{X} - \mu}{\left(\dfrac{\sigma}{\sqrt{n}}\right)}$.

In this case we use sample standard deviation 's' in place of population standard deviation σ so that we have a random variable different from Z. This new random variable is denoted by t; that is $t = \dfrac{\bar{X} - \mu}{\left(\dfrac{s}{\sqrt{n}}\right)}$.

The probability distribution corresponding to this random variable t is called t-distribution with parameter $n-1$. This parameter is known as degrees of freedom, denoted by ν; that is, $\nu = n-1$.

Note: In the t-distribution, for the sample $\{x_1, x_2, x_3, \cdots x_n\}$,

(i) Sample mean $\bar{x}$ is given by $\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$

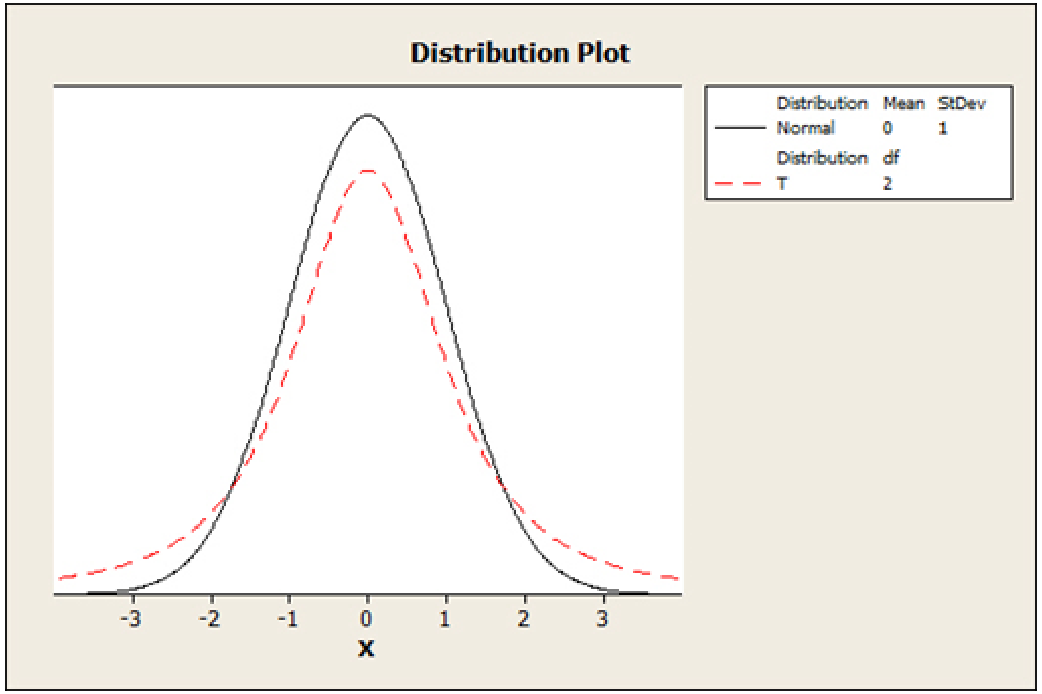
(ii) Sample variance is given by $s^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Properties of t-distribution:

1) The t-distribution is symmetrical about the mean.

2) The mean of t- distribution is ‘0’ but the variance depends on the parameter ‘**ν**’ called the number of degrees of freedom

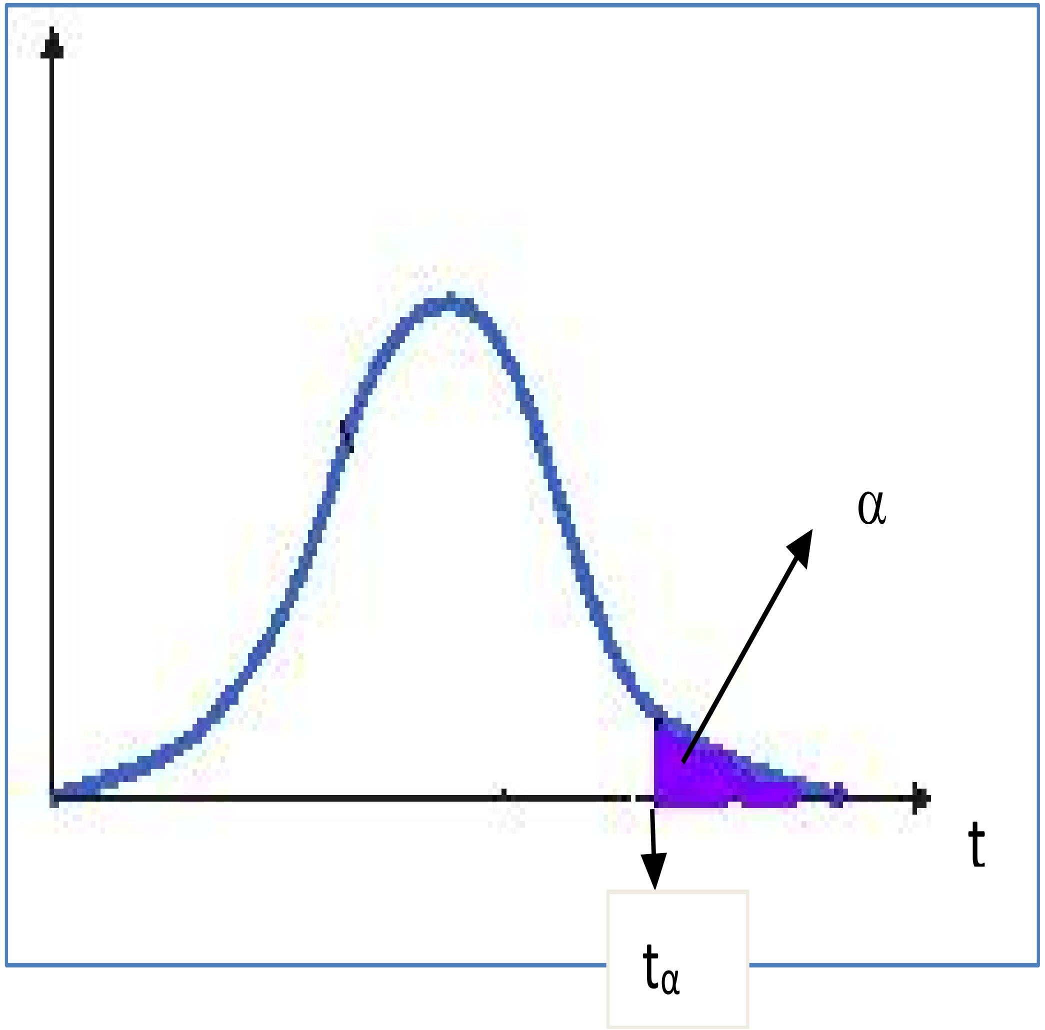
3) As n→∞ The standard deviation of t-distribution approaches to 1.



***That is the *t-distribution* approaches *standard normal distribution* as n→∞ (**ν**→∞).

t_α: t_α is value of t-distribution such that the area under the t-distribution to its right is equal to 'α'.

The t_α values for various values of 'α' and 'ν' are given in a table called t-table. We can observe from this t-table that standard normal distribution provides a good approximation to the t-distribution for samples of size 30 or more.



Note: By symmetry of t-curve we have $t_{1-\alpha} = -t_\alpha$.

Problem: Find $t_{\alpha,\nu}$ when (i) $\alpha = 0.025$, $\nu = 7$ (ii) $\alpha = 0.005$, $n = 11$ (iii) $\alpha = 0.05$, $\nu = 15$

(iv) $\alpha = 0.95$, $\nu = 15$

Solution:

(i) From t-table $t_{0.025,\,7} = 2.365$

(ii) When $n = 11$, $\nu = n-1 = 11-1 = 10$.

$$t_{\alpha,\nu} = t_{0.005,\,10} = 3.169$$

(iii)

(iv) For $\alpha = 0.95$ we do not have $t_{\alpha,\nu}$ value in the t- table. But using the relation $t_{1-\alpha} = -t_{\alpha}$,

we can write $t_{0.95} = -t_{1-0.95} = -t_{0.05}$

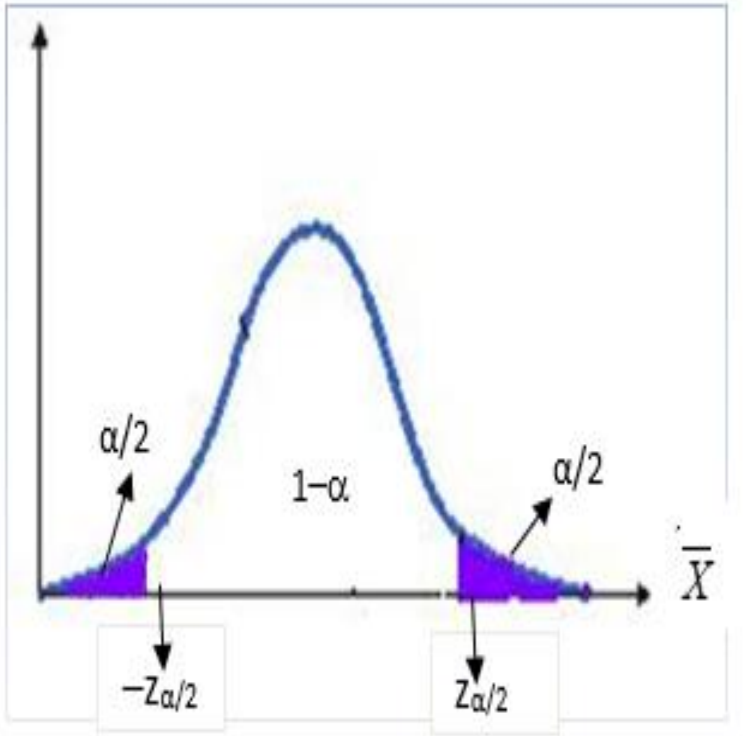
Therefore, $t_{0.95,\,15} = -t_{0.05,\,15} = -1.753$

Now, Z_α is the value of 'Z' such that the area under the standard normal curve to this value of 'Z' is 'α'.

Similarly, $Z_{\alpha/2}$ is the value of 'Z' such that the area under the standard normal curve to this value of 'Z' is 'α/2'.

By, symmetry of Z-curve the area left side to $-Z_{\alpha/2}$ is also α/2.

That is the area under the Z-curve between $-Z_{\alpha/2}$ and $Z_{\alpha/2}$ is $1-(\alpha/2+\alpha/2)=1-\alpha$.



It can be written as $-Z_{\alpha/2} \leq Z \leq Z_{\alpha/2}$ with probability (1–α).

Find the values of

$(i)\, z_{0.05}$ $\quad\quad\quad\quad$ $(ii)\, z_{0.025}$

$$(i)\ F(z) = 1 - \alpha$$

$$F(z) = 1 - 0.05$$

$$F(z) = 0.95 = 1.64$$

$$\therefore z_{0.05} = 1.64$$

$$(ii)\ F(z) = 1 - \alpha$$

$$F(z) = 1 - 0.025$$

$$F(z) = 0.975 = 1.96$$

$$\therefore z_{0.025} = 1.96$$