Pumping lemma for context free languages :-

Pumping Lemma is used to prove a language as not context free.

'L' be context free language. 'n' be a integer constant.
Select a string 'z' from 'L' such that $|z| \geq n$.
Divide the string z into 5 parts uvwxy such that

$|vwx| \leq n$

$|vx| \geq 1$

for $i \geq 0$, uv^iwx^iy is in L.

Q. such that $L = \{a^n b^n c^n / n \geq 1\}$ is not a C.F Language

sol: let 'L' is a CFL

let $n = 3$

$L = \{abc, aabbcc, aaabbb, \dots\}$

$z = aaabbbccc$

$|z| \geq n$

$9 \geq 3$ ✓

$\underset{u}{aa}\,\underset{v}{a}\,\underset{w}{b}\,\underset{x}{b}\,\underset{y}{bccc}$

$u = aa$
$v = a$
$w = b$
$x = b$
$y = bccc$

① $|vwx| \leq n \Rightarrow |a.b.b| \leq 3$

$3 \leq 3$ ✓

② $|vx| \geq 1$

$|ab| \geq 1$

$2 \geq 1$ ✓

③ $i \geq 0$, $uv^iwx^iy \in L$

$i = 0 \Rightarrow aaa^0bb^0bccc$

$\Rightarrow aabbccc \notin L$

Here we got a string which is not present in the language
Hence proved.

The language is not a context free language.

Q. Prove that $L = \{a^p \mid p$ is a prime number$\}$ is not context free language.

Sol:- Let 'L' be a CFL

$L = \{aa, aaa, aaaaa, aaaaaaa, \ldots\}$

let $n=5$, $z = aaaaa$

$|z| \geq n$, $|aaaaa| \geq 5$

$5 \geq 5$ ✓

$\frac{a}{u}\ \frac{a}{v}\ \frac{a}{w}\ \frac{a}{x}\ \frac{a}{y}$

① $|vwx| \leq n$

$|aaa| \leq n$ $\quad 3 \leq 5$ ✓

② $|vx| \geq 1$

$|a \cdot a| \geq 1$

$2 \geq 1$ ✓

③ $i=0 \Rightarrow uv^iwx^iy$

$a \cdot a^0 \cdot a \cdot a^0 \cdot a \Rightarrow aaa \in L$

we need to repeat the process until we get a string which is not present in the language.

Let $i=1 \Rightarrow aa^1 \cdot a \cdot a^1a$

$aaaaa \in L$

$i=2 \Rightarrow aa^2aa^2a$

$aaaaaaa \in L$

$i=3 \Rightarrow aa^3aa^3a$

$aaaaaaaaa \notin L$

The string is not present in the language. so the language is not CFL.

Q. Prove that $L = \{a^{n^2} / n \geq 1\}$ is not a CFL.

Sol: Let 'L' be a CFL

$L = \{a, aaaa, aaaaaaaaa, \ldots\}$

Let $n = 2$, $z = aaaa$ $\quad |z| \geq n$

$|aaaa| \geq 2$

$4 \geq 2$ ✓

$\underset{u}{a}\ \underset{v}{a}\ \underset{w}{\epsilon}\ \underset{x}{a}\ \underset{y}{a}$

① $|vwx| \leq n$

$|a \cdot \epsilon\, a| \leq n$

$|aa| \leq 2$

$2 \leq 2$

② $|vx| \geq 1$

$|a \cdot a| \geq 1$

$2 \geq 1$

③ $i = 0 \Rightarrow a \cdot a^0 \cdot \epsilon \cdot a^0 \cdot a$

$a \cdot 1 \cdot \epsilon \cdot 1 \cdot a = aa \notin L$

This is not a context free language.

Q. Show that $L = \{a^i b^j \mid j = i^2\}$ is not a CFL.

Sol:- Let 'L' be a CFL

$L = \{ab, aabbbb, aaabbbbbbbbb, \ldots\}$

Set 'z' from the L such that $|z| \geq n$

$n = 4$, $z = aabbbb$

$|z| \geq n \Rightarrow |aabbbb| \geq 4$

$6 \geq 4$ ✓

$\underset{u}{a}\ \underset{v}{a}\ \underset{w}{bb}\ \underset{x}{b}\ \underset{y}{b}$

① $|vwx| \leq n$

$|a \cdot bb \cdot b| \leq 4$

$4 \leq 4$ ✓

② $|vx| \geq 1$

$|a \cdot b| \geq 1$

$2 \geq 1$

for $i \geq 0$, $uv^i wx^i y$ is in L.

$i=0 \Rightarrow a \cdot a^0 \cdot bb \cdot b^0 \cdot b$

$a \cdot bb \cdot b \notin L$

The language is not a CFL.

Differences between Regular lang. & Context free lang :-

Regular language	Context free language
1. one language which is generated by regular grammar.	1. One language is generated by context free grammar.
2. Regular language is accepted by Pushdown Automata such as DFA & NFA	2. Context free language is accepted by push down Automata (PDA).
3. Regular languages are closed under union, concatenation, kleen closure, intersection, complementation	3. CFL's are closed under union, concatenation, kleen closure
4. Regular languages are used in sequential circuits & text editors	4. C.F L's are used in compilers.

Closure properties of Context free languages :-

→ Context free languages are closed under

(a) union (b) concatenation (c) kleen closure

Not closed under

(a) intersection (b) complementation.

union :- Let L_1 & L_2 be CF's

$L_1 \cup L_2$ is also a CFL

$L_1 = \{a^n \mid n \geq 0\}$ $L_2 = \{b^n \mid n \geq 0\}$

$L_1 = \{\varepsilon, a, aa, \ldots\}$ $L_2 = \{\varepsilon, b, bb, \ldots\}$

$S_1 \to aS_1 \mid \varepsilon$

$S_2 \to bS_2 \mid \varepsilon$

$L_3 = L_1 \cup L_2$

$L_3 = \{\varepsilon, a, aa, b, bb, \ldots\}$

$S \to S_1 \mid S_2$

concatenation :- Let L_1 & L_2 be CF's

$L_1 \cdot L_2$ is also a CFL

$L_1 = \{a^n \mid n \geq 0\}$ $L_2 = \{b^n \mid n \geq 0\}$

$L_1 = \{\varepsilon, a, aa, \ldots\}$ $L_2 = \{\varepsilon, b, bb, \ldots\}$

$S_1 \to aS_1 \mid \varepsilon$

$S_2 \to bS_2 \mid \varepsilon$

$L_3 = L_1 \cdot L_2$

$L_3 = \{\varepsilon, a, aa, b, bb, \ldots\}$

$S \to S_1 \cdot S_2$

kleen closure :-

Let L is a CFL then

L^* is also a CFL

$L = \{a\}$

$L^* = \{\varepsilon, a, aa, \ldots\}$