债券

债券收益率

- 名义收益率（票面收益率、息票率、债息、coupon rate）
  - 指债券票面上的固定利率
  - $CR = C_i/F$, F为债券面额（面值、par value、face value）

- 现期收益率（本期收益率、current yield）
  - 根据债券利息和债券市场价格计算出来的收益率
  - $CY = C_i/P$, P为债券市场价格

债券收益率

- 到期收益率（最终收益率、Yield to maturity、bond yield）
  - 投资购买债券的内部收益率（IRR）
  - 指投资者按照市场价格购买且一直持有到满期时可以获得的年平均收益率
  - 根据 $P = \sum_{i=1}^{n} \frac{C_i}{(1+y)^i} + \frac{F}{(1+y)^n}$ 计算 y
  - Excel function: IRR 或 YIELD

Yield to maturity calculation

- 债券价格：107.02
- 票面利息：8元
- 每年付息一次
- 债券面值：100元
- 期限为10年

	A	B
1	-107.2	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	108	
12	6.98%	-->IRR(A1:A11)

Yield to maturity calculation

- 债券价格：107.02
- 票面利息：8元
- 每年付息一次
- 债券面值：100元
- 期限为10年

Day count convention

Treasury Bonds: Actual/Actual (in period)

Corporate Bonds: 30/360

Money Market Instruments: Actual/360

Basis	Day count basis
0 or omitted	US (NASD) 30/360
1	Actual/actual
2	Actual/360
3	Actual/365
4	European 30/360

Yield to maturity assumption

- 到期收益率假设再投资收益率等于该到期收益率
- 如果**再投资收益率不等于到期收益率?**

	A	B
1	-107.2	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	108	
12	6.98%	-->MIRR(A1:A11,0, 0.0698)
13	5.99%	-->MIRR(A1:A11,0, 0.03)

债券收益率

- 赎回收益率（yield to call）
  - 当发行公司决定提前赎回，赎回收益率衡量投资者在提早赎回日的到期收益率
  - $P = \sum_{i=1}^{nc} \frac{C_i}{(1+y)^i} + \frac{P_c}{(1+y)^n}$，$P_c$为赎回价格。

- 已实现收益率（realized yield）
  - 当投资人决定把债券卖出，已实现收益率衡量投资者在卖出时的到期收益率
  - $P = \sum_{i=1}^{np} \frac{C_i}{(1+y)^i} + \frac{P_f}{(1+y)^n}$，$P_f$为卖出价格。

债券的特征

债券与股票的不同之处：

- **债券契约（debt covenant)**
- **债券评级（credit rating)**

Debt covenants

- 否定性条款(negative covenant)
  - 不允许或限制股东做某些事情的规定
- 肯定性条款(positive covenant)
  - 应该履行某些责任的规定，如要求营运资金、权益资本达到一定水平以上

肯定性条款（Positive Covenants）

- **List of the top 10 most common metrics lenders use as debt covenants for borrowers:**
  - Debt / EBITDA
  - Debt / (EBITDA – Capital Expenditures)
  - Interest Coverage (EBITDA or EBIT / Interest)
  - Fixed Charge Coverage (EBITDA / (Total Debt Service + Capital Expenditures + Taxes)
  - Debt / Equity
  - Debt / Assets
  - Total Assets
  - Tangible Net Worth

否定性条款（Negative Covenants）

- **典型的限制性条款包括**
  - 追加债务的限制
  - 不允许分红超过特定比例
  - Mergers and Acquisitions的限制

Cross default

- 有些债券还包括所谓“交叉违约”（Cross Default）条款，该条款规定，对于有多笔债务的公司，**只要对其中一笔违约，则认为公司对全部债务违约**。

Violation of Debt Covenants

- When a debt covenant is violated, depending on the severity, the lender can do several things:
  - Demand penalty payment
  - Increase the predetermined interest rate
  - Increase the amount of collateral
  - Demand full immediate repayment of the loan
  - Terminate the debt agreement

债券评级

- AAA (Aaa): “extremely strong capacity to meet financial commitments”.
- AA+, AA, AA- (Aa1, Aa2, Aa3): “very strong capacity to meet its financial commitments.” The differences from AAA are very small.
- From 1981 through 2010, only 1.3 percent of global corporate bonds originally rated AA eventually went into default.

债券评级

- **A+, A, A- (A1, A2, A3): “Strong capacity to meet financial commitments, but somewhat susceptible to adverse economic conditions and changes in circumstances.”**

债券评级

- BBB+, BBB, BBB- (Baa1, Baa2, Baa3): "adequate capacity to meet financial commitments, but are more subject to adverse economic conditions or changing circumstances."
- BBB- is the last tier at which a bond is still considered "investment grade."
- Bonds rated below this level are considered "below investment grade" or, more commonly, "high yield," a more risky segment of the market.

债券评级

- B+, B, B- (B1, B2, B3): “can meet their current financial commitments, but their **future outlook** is more vulnerable to adverse developments”.
- CCC+, CCC, CCC- (Caa1, Caa2, Caa3): “dependent on favourable business, financial and economic conditions to meet financial commitments."
  - Fitch uses a single CCC rating, without breaking it out into the plus and minus distinctions as S&P does.
- CC (Ca): Like bonds rated CCC, bonds in this tier are also vulnerable right now but face an even higher level of uncertainty.
- C: **most vulnerable to default**. Often, this category is reserved for bonds in special situations, such as **those in which the issuer is in bankruptcy but payments are continuing at present.**
- D (C): The worst rating, **assigned to bonds that are already in default.**

债券评级

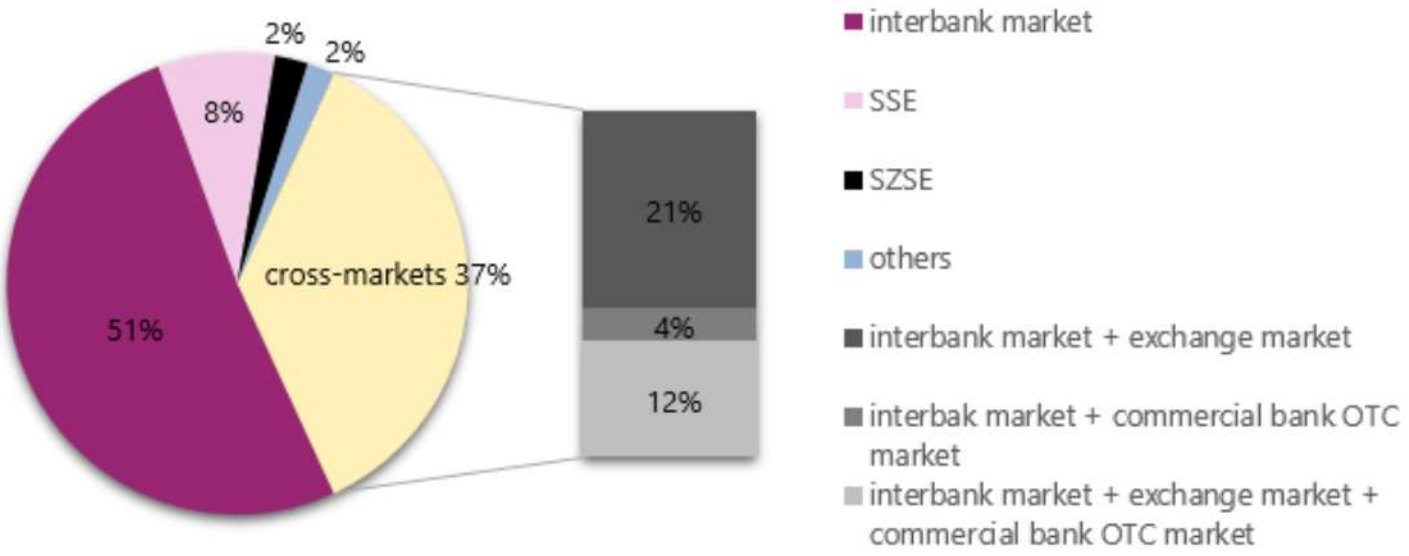
- 1992年，98家美国公司持有AAA信用评级
- 2016年，仅剩2家持有AAA信用评级：Microsoft 与 Johnson & Johnson.
- 2011年8月，美国被S&P调降至AA+等级。

债券评级

- **信用评级约高不代表债券收益率越低，微软的债券收益率还是会比美国国债收益率高。**
- **一般长期债券受到利率风险影响程度远大于信用风险。**

中国债券交易概况

Figure 4. Proportion of outstanding bonds of different sub-markets as of December 31, 2018

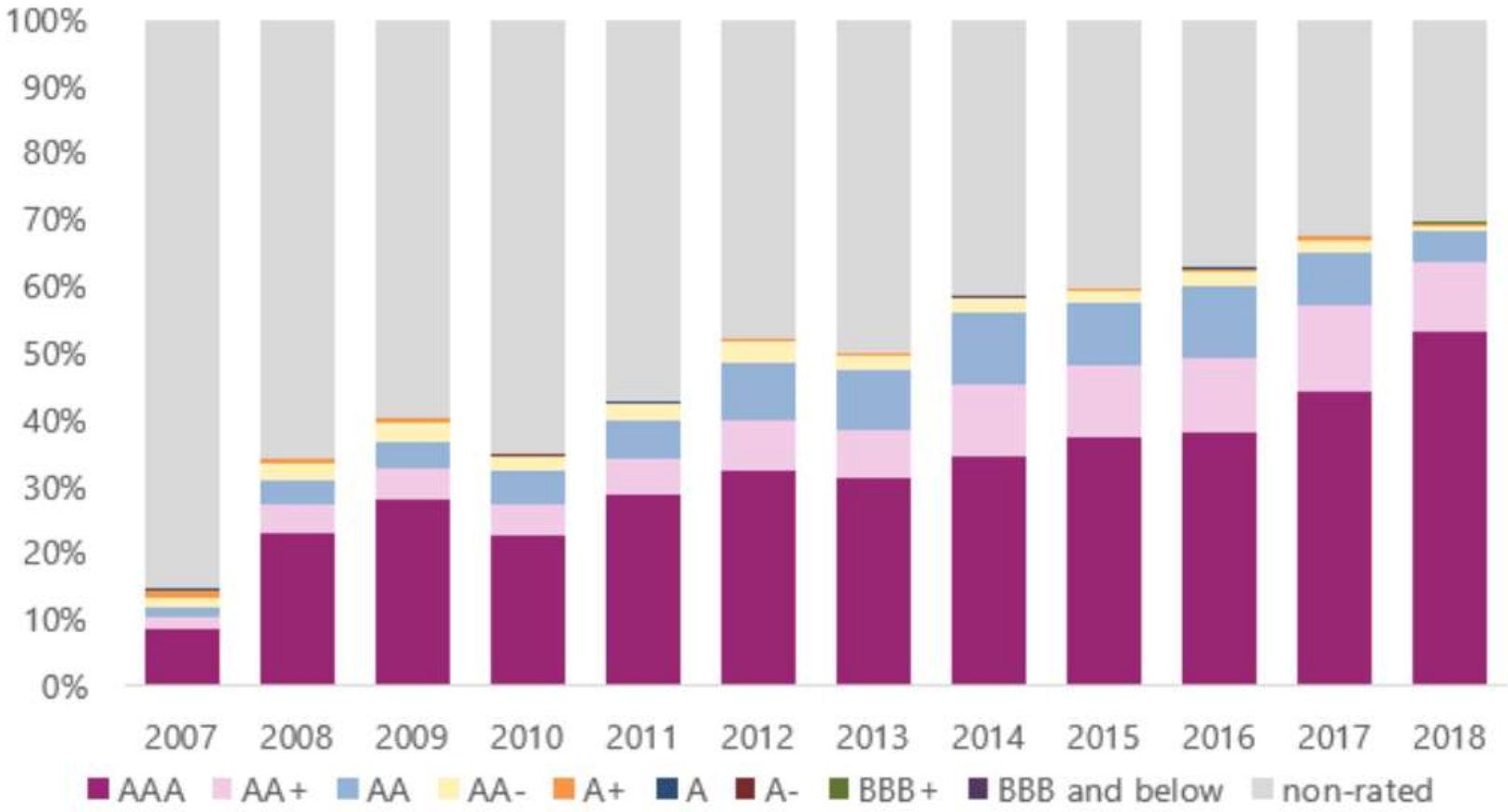


Source: WIND, Golden Credit

截至2018年底，政府债券占40%

Bond Varieties		Shares
Government bonds	Treasury bonds	**17.4%**
	Local government bonds	**21.1%**
	Government-backed agency bonds	1.9%
Financial bonds	Policy bank bonds	16.8%
	Financial bonds excluding policy bank bonds	6.9%
Corporate credit bonds	Enterprise bonds	3.0%
	Corporate bonds	6.8%
	Short-term commercial paper	2.3%
	Medium-term notes	6.6%
	Private placements	2.3%

国内债券信用的评级分布



Source: WIND, Golden Credit

Domestic bond vs Foreign bond

- Foreign bond: bond issued in a domestic market by a foreign entity in the domestic market's currency as a means of raising capital.
- 一般外国债券都有特别的名字:
  - Panda bond（熊猫债）：外国公司在中国发行的以人民币计价公司债。
  - Yankee bond：外国公司在美国发行的以美元计价公司债。
  - Arirang bond, Samurai bond, Kangaroo bond

2017年俄罗斯铝业联合公司发行熊猫债

Offering Overview	
Issuer	United Company RUSAL Plc
Bond Type	Corporate bond (private placement to eligible investors only)
Issuer/Bond Rating	AA+/AAA
Guarantee	Fully secured by an unconditional and irrevocable joint and several liability guarantees provided by the China United SME Guarantee Corporation
Offering Size	RMB 1.0 bln
Maturity	3 years, with a put option at the end of the 2nd year
Coupon rate	5.50%
Use of proceeds	After the deduction of issuance expenses, the funds raised were planned for total off-shoring through a special account expected to have up to 100% of raised funds used to replenish working capital (including, but not limited to, companies within the corporation for the procurement from Chinese suppliers), with the remainder to be used for repaying interest bearing debts.
Key Dates	Book building and price inquiry: March 16, 2017 First day of issuance: March 17, 2017 Value day: March 20, 2017
Market	Shanghai Stock Exchange

Source: CICC

中国公司于美国发行的扬基债

- **截至2019年第三季，中国公司在美国总共发行了72亿美元的债券（大部分为可转换债）。**
  - **该金额超越了2018年的$56亿。**
- **发行公司包括LexinFintech(乐信)、NIO(蔚来汽车)、爱奇艺、Bilibili。**

债券报价

Price quotation of T-Bills

$$P = \left(100 - Y\frac{360}{n}\right)$$

P is cash price per \$100 （实际支付的价格）

Y is quoted yield （报价）

n is the remaining life of the T-bill in **actual calendar** days

Price quotation of T-Bills

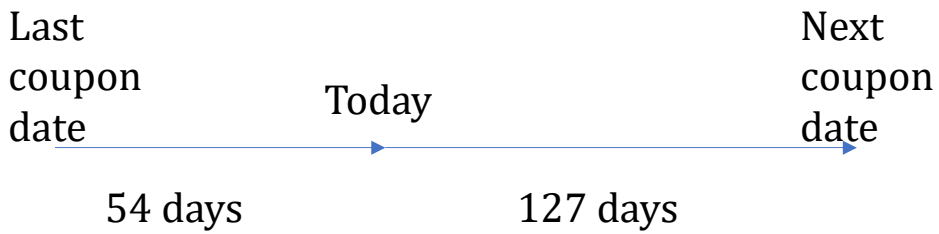
- Price of a 91-day T-bill is quoted as 8.
- The interest is $2.0222 (8*91/360).
- Cash price is $97.9778($100 - $2.0222).
- The actual rate of return for the 91-day period T-Bill: 2.064%

Calculating Cash Price (dirty price) of T-Bond

Cash price = Quoted price (报价或净价) +

Accrued Interest since last coupon date (应计利息)

- Quoted price: 95-16 or $95.50
- Coupon rate: 11%



- Cash price = $95.50 + 54/181×$5.50 = $97.14

债券定价

- 假设支付固定利息，每年支付两次（semi-annual）

$$B = \sum_{i=1}^{2T} \frac{F \times c/2}{(1 + y/2)^i} + \frac{F}{(1 + y/2)^{2T}}$$

$$= F\frac{c}{y}\left(1 - \frac{1}{(1 + y/2)^{2T}}\right) + F\left(\frac{1}{(1 + y/2)^{2T}}\right)$$

Impact of Compounding (複利)

When we compound m times per year at rate R an amount A grows to $A(1+R/m)^m$ in one year

Compounding frequency	Value of $100 in one year at 10%
Annual (m=1)	**110.00**
Semiannual (m=2)	**110.25**
Quarterly (m=4)	**110.38**
Monthly (m=12)	**110.47**
Weekly (m=52)	**110.51**
Daily (m=365)	**110.52**

Continuous Compounding 连续复利

- In the limit as we compound more and more frequently we obtain continuously compounded interest rates
- **\$100 grows to \$100e^{RT}** when invested at a continuously compounded rate R for time T
- **\$100 received at time T discounts to \$100e^{-RT} at time zero** when the continuously compounded discount rate is R

Conversion Formulas

Define

R_c : continuously compounded rate

R_m: same rate with compounding *m* times per year

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c / m} - 1\right)$$

债券定价

- 如果假设连续复利

$$B = \sum_{i=1}^{n} cf_i e^{-yt_i}$$

- cf_i是t_i期的现金流量

久期（Duration）

- Duration of a bond that provides cash flow cf_i at time t_i is

$$D = \sum_{i=1}^{n} t_i \left[\frac{cf_i e^{-yt_i}}{P} \right]$$

where P is its market price and y is its yield to maturity.

久期（Duration）

- 久期的第一个意涵：每个时期的加权平均，也可被解读为“**需要多久时间拿回本金**”

$$D = \sum_{i=1}^{n} t_i \left[\frac{cf_i e^{-yt_i}}{P} \right]$$

久期（Duration）

- 久期的第二个意涵：债券价值对利率的敏感度。

$$\frac{\Delta B}{B} = -D\Delta y$$

- 如果假设每年m次复利

$$\frac{\Delta B}{B} = -D' \frac{\Delta y}{1 + y/m}$$

- $D' \frac{1}{1+y/m}$ 也被称为modified duration (修正久期)

Duration的特征

- 相同到期日之下，固定利率债券 < 零息债券
- 相同到期日之下，5%固定利率债券 < 10%固定利率债券
- 相同利率，5年期债券 < 10年期债券
- 浮动利率债券的久期趋近于零
  - 假设每半年支付一次利息，当支付完最近一次利息时，久期等于0.5
- 债券组合的久期 = 每个债券的加权平均

反向浮动利率债券 （inverse floater）

- **假设一个五年期反向浮动利率债券，每一期支付的利率是（8%-LIBOR), 本金是 $100**
- **Inverse floater 每一期的现金流量：$4 - $50*LIBOR**
- **Inverse floater‘s duration = fixed rate bond’s duration – floating rate bond duration > 0**

债券定价原理

1. Bond prices move inversely to market interest changes.

2. **The variability in bond prices and term to maturity are positively related**. For a given change in the level of market interest rates, changes in the bond prices are greater for long-term maturities.

3. The **sensitivity to changes in market interest rates increases** at a diminishing rate **as the time remaining until the bond's maturity increases**.

4. **Absolute increases in market interest rates and subsequent bond price changes are not symmetrical**. For a given maturity, a decrease in the market interest rate causes a price rise that is larger than the price decline resulting from an equal increase in market interest rate.

5. Bond price volatility is related to its coupon rate. The **percentage change in a bond's price due to a change in the market interest rate will be smaller if its coupon rate is higher**.

Convexity（凸性）

- Duration：债券价格对债券收益率一次微分
- Convexity：债券价格对债券收益率二次微分

$$C = \frac{1}{P}\frac{d^2P}{dy^2}$$

$$\frac{\Delta P}{P} \approx -D\Delta y + \frac{1}{2}C\Delta y^2$$

Convexity（凸性）

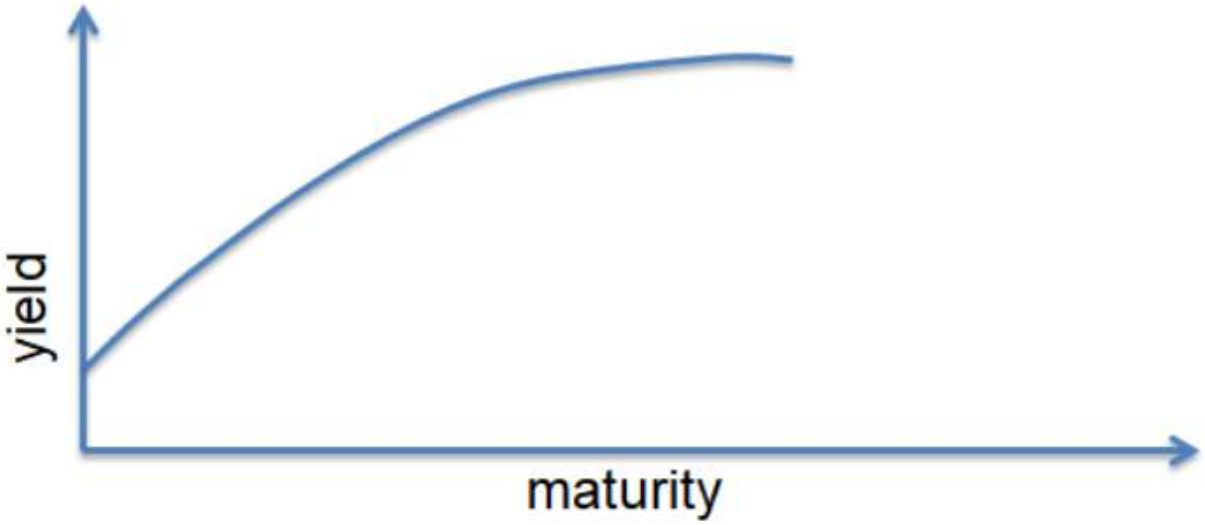
- Duration：债券价格对债券收益率一次微分
- Convexity：债券价格对债券收益率二次微分，也是衡量久期对利率变动的敏感度

$$C = \frac{1}{P}\frac{d^2P}{dy^2}$$

$$\frac{\Delta P}{P} \approx -D\Delta y + \frac{1}{2}C\Delta y^2$$

收益率曲线（yield curve）

- 收益率曲线代表每一期货币的时间价值
- 也称为zero curve



利率曲线的构建

Bond Principal	Time to Maturity (yrs)	Coupon per year ($)*	Bond price ($)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

$$100 = 97.5e^{R\times 0.25}$$

$$100 = 94.9e^{R\times 0.5}$$

$$100 = 90e^{R\times 1.0}$$

利率曲线的构建

Maturity (years)	*Zero rate (%) (cont. comp.)*
0.25	10.127
0.50	10.469
1.00	10.536

$$100 = 97.5e^{R\times 0.25}$$

$$100 = 94.9e^{R\times 0.5}$$

$$100 = 90e^{R\times 1.0}$$

利率曲线的构建

- 计算1.5 year 的利率

$$4e^{-0.10469\times 0.5}+4e^{-0.10536\times 1.0}+104e^{-R\times 1.5}=96$$

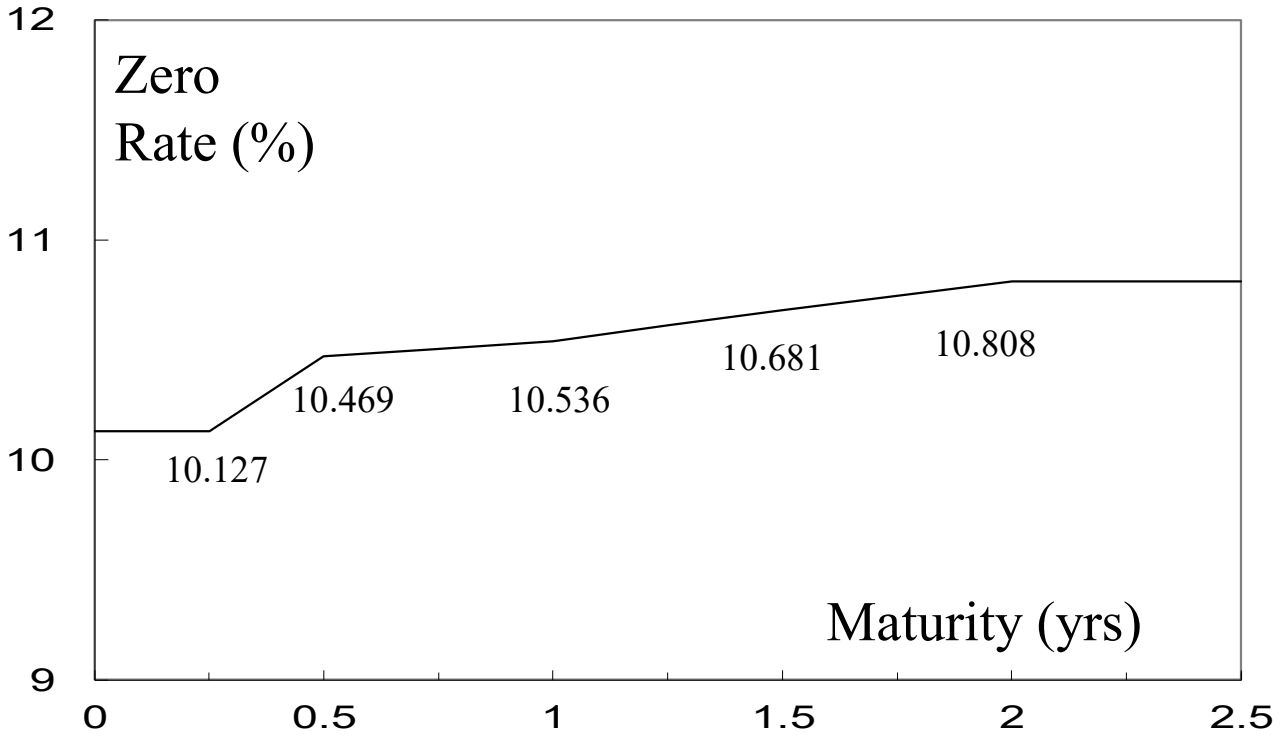
R = 0.10681 or 10.681%

- Similarly the two-year rate is 10.808%

利率曲线的构建

Maturity (*years*)	*Zero rate* (%) (*cont. comp.*)
0.25	10.127
0.50	10.469
1.00	10.536
1.50	10.681
2.00	10.808

利率曲线的构建


12
11
10
9
Zero
Rate (%)
10.127
10.469
10.536
10.681
10.808
Maturity (yrs)
0
0.5
1
1.5
2
2.5

Forward Rates（远期利率）

- Suppose that the zero rates for time periods T_1 and T_2 are R_1 and R_2 with both rates continuously compounded.
- The forward rate for the period between times T_1 and T_2 is $\dfrac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$
- or $R_F = R_2 + (R_2 - R_1)\dfrac{T_1}{T_2 - T_1}$
- **This formula is only approximately true when rates are not expressed with continuous compounding**

Forward Rates

Year (n)	Zero rate for n-year investment (% per annum)	Forward rate for n-th year (% per annum)
1	3.0	
2	4.0	5.0
3	4.6	5.8
4	5.0	6.2
5	5.5	6.5

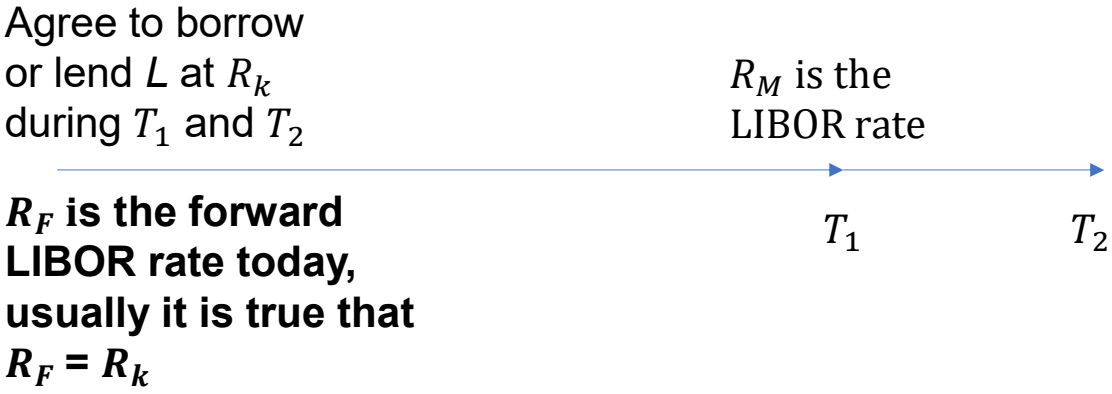
投资人认为n年后一年期的利率

远期利率

- **Borrow at “short-term” rate and lend at “long-term” rate if you think the implied forward rate is too high.**
- **一年期利率5%**
- **两年期利率7%**
- **远期利率≈9%，若认为一年后的利率低于9%：**
  - **借$100一年, 投资两年期零息债**
  - **一年后，再借$105,利率 R_M**
  - **两年后，投资收益率是 (114 – 105(1+ R_M))**
  - **假设$R_M = 8\%$，投资收益率 $= \$0.6$**

Forward Rate Agreement 远期利率协议

- A forward rate agreement (FRA) is an OTC agreement that a certain rate will apply to a certain principal during a certain future time period.



FRA

- Company X agrees to enter into FRA as lender. The extra interests that it will receive/pay at time T_2

$$L(R_K - R_M)(T_2 - T_1)$$

- At time T_1, the value of this payoff is

$$\frac{L(R_K - R_M)(T_2 - T_1)}{1 + R_M(T_2 - T_1)}$$

Theories of the Term Structure

- Market Segmentation Theory:
  - Short, medium and long rates are determined independently of each other
  - Short-term rate is determined by supply and demand of the short-term bond market, e.g. inter-bank liquidity trading
  - Long-term rate is determined by supply and demand of the long-term bond market, e.g. pension fund

Theories of the Term Structure

- Unbiased Expectations Theory:
  - Forward rates equal expected future zero rates
  - During expansion, investors expect interest rates to rise in the future, which accounts for the usual upward slope.

Liquidity Preference Theory

- Suppose that the outlook for rates is flat and you have been offered the following choices

Maturity	Deposit rate	Mortgage rate
1 year	**3%**	6%
5 year	3%	**6%**

- Which one would you choose as a depositor? Which one for your mortgage?

Theories of the Term Structure

- Liquidity Preference Theory:
  - Investors prefer liquidity, unwilling to tie up their fund for longer period.
  - Borrowers are risk averse, more willing to avoid refinancing risk.

Liquidity Preference Theory

- To match the maturities of borrowers and lenders, a bank has to increase long rates above expected future short rates
- In our example the bank might offer

Maturity	Deposit rate	Mortgage rate
1 year	3%	6%
5 year	4%	7%