

Image Thresholding by Maximizing the Index of Nonfuzziness of the 2-D Grayscale Histogram

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Image segmentation plays an important role in various image processing applications including robot vision and document image analysis and understanding. In contrast to classical set theory, fuzzy set theory, which takes into account the uncertainty intrinsic to various images, has found great success in the area of image thresholding. In this paper, an image thresholding approach based on the index of nonfuzziness maximization of the 2-D grayscale histogram is proposed. The threshold vector (T, S) , where T is a threshold for pixel intensity and S is another threshold for the local average of pixels, is obtained by an exhaustive searching algorithm. In this approach, the difference between these two components (T and S) is guaranteed to be within a relatively small range, which leads to reasonable results from the viewpoint of human vision perception. This cannot be achieved in certain entropy-based methods. Experimental results have shown that our proposed approach not only performs well and effectively but also is more robust when applied to noisy images. © 2002 Elsevier Science (USA)

1. INTRODUCTION

Over the past several decades, a number of approaches for automatic threshold selection have been proposed for image segmentation [1–6]. Because of the complexity of image segmentation, efforts to apply new ideas and concepts to image thresholding continued during the last decade [6–10]. The objective of most of the existing methods is to find the globally optimum threshold which depends on the first-order gray-level histogram of an image (1-D grayscale histogram). However, more information contained in the image can be utilized to obtain a better segmentation.

In recent years, fuzzy set theory has been applied to pattern classification and object recognition, and especially image thresholding [5–9, 11–14]. One-dimensional entropic thresholding was first introduced by Pun [15]. His contribution is that the optimal threshold will separate all pixels of a grayscale image into foreground and background classes by maximizing a posteriori entropy, which is defined as the sum of the entropies of the two classes. To further Pun's work, Kapur *et al.* refined Pun's method by deriving the two entropies from the original grayscale distribution of an image [16]. Huang and Wang applied a fuzzy entropy measure to image segmentation based on the 1-D grayscale histogram [17]. They defined the image pixel membership functions as being dependent on a threshold value. The membership functions were used to reflect the distributions of the pixels in the background and object classes. As a result, classification errors are reduced to a certain degree. The image thresholding methods mentioned above are solely based on the 1-D grayscale histogram. One drawback of these methods is that only the distribution of the pixel grayscales of an image is considered, whereas the spatial information—the correlation between different gray levels—is ignored.

More recently, two-dimensional entropic techniques using local neighborhood as well as pixel information have been proposed to optimize the global threshold. The concept of a 2-D grayscale histogram originated from the research work of Kapur *et al.* [16] and that of Kirby and Rosenfeld [18]. Abutaleb proposed the optimal selection by maximizing the total entropy defined on the 2-D (grayscale and local average grayscale) histogram [19]. It was reported that the separation between the background and the object classes could be achieved by maximizing a 2-D entropy criterion. In the work that followed, Brink improved Abutaleb's method by maximizing the class entropies, which was achieved by using a single threshold vector to maximize the entropy derived from both the background and the object classes [20]. To speed up the process, Chen *et al.* proposed a fast two-phase algorithm to reduce computing time from $O(L^4)$ to $O(L^{4/3})$ [21]. In the first phase, a set of quantized threshold vectors was obtained by Brink's method. In the second phase of the search process, the search space was greatly reduced while the quality of the thresholding for image segmentation was maintained. Subsequently, Gong *et al.* proposed a fast recursive algorithm to decrease the computational complexity from $O(L^4)$ to $O(L^2)$ to [22] based on the fast algorithm of Otsu method [23, 24].

The above-mentioned approaches based on the 2-D grayscale histogram employ an entropy measure. In the case of noisy images, we found that the fuzzy entropy-based method did not produce good results. In this paper, in order to adequately utilize the intrinsic information of an image, we employ the concept of the 2-D grayscale histogram and propose an index of nonfuzziness to optimize the threshold vector.

Our paper is organized as follows. Section 2 reviews the concept of a 1-D grayscale histogram and the thresholding methods based on the 1-D histogram. In this section, the

concept of the 2-D grayscale histogram is compared to the first-order gray-level histogram. Section 3 defines the membership functions for the 2-D histogram set. Section 4 proposes the index of nonfuzziness of the 2-D histogram and an optimization procedure of the index of the 2-D grayscale histogram after reviewing several existing measures. Experimental results of a great number of images are reported and compared with several typical fuzzy entropy-based approaches in Section 5. We also discuss in this section the rationale of using the index of nonfuzziness as a measure function and analyze the results on Gaussian noise corrupted images. Finally, concluding remarks are provided in Section 6.

2. 2-D GRAYSCALE HISTOGRAM

The conventional 1-D thresholding methods focus on the selection of the peaks or valleys by analyzing the gray-level histogram. There are two main approaches to locating the bottom of the valley: parametric and nonparametric. The former assumes that the probability density function (PDF) of the gray-level distribution of each class is known and a valley is located by Bayesian estimation and numerical analysis. The latter is interested in a kind of optimization based on some criteria and manages to manipulate the within-class variance, between-class variance, total variance, or entropy. Essentially, these methods seek to determine the best threshold T from the gray-level histogram. Assume that one grayscale image is $f(x, y)$, which contains $M \times N$ pixels with a gray level ranging from 0 to $L - 1$. The gray level 0 represents the darkest pixel and $L - 1$ the brightest one. A gray-level histogram is a function that shows, for each gray level, the number of pixels in the image that have that gray level. The abscissa is a gray level and the ordinate is the frequency of occurrence (number of pixels), as shown in Fig. 1c. As a result, the 1-D thresholding function $f_T(x, y)$ is defined as

$$f_T(x, y) = \begin{cases} b_0, & \text{if } f(x, y) \leq T \\ b_1, & \text{if } f(x, y) > T, \end{cases} \quad (1)$$

where T is a threshold that separates two classes. Gray values b_0 and b_1 are the predetermined values for the background and object classes, respectively. In general, $b_0 = 0$ and $b_1 = L - 1$.

From the viewpoint of information processing, the thresholding techniques that are based on the 1-D gray-level histogram alone, however, do not make full use of all the information available in the image. The drawback for not fully utilizing the information in an image becomes apparent as the signal-to-noise ratio (SNR) decreases. Thus, one could expect an improvement in image thresholding if the spatial relationships among pixels are exploited. Ahuja and Rosenfeld [3] proposed a prototype of the 2-D histogram—the *M matrix*—which is used to represent the probability of the co-occurrence of two gray-level values when the corresponding pixels are a specific distance apart. Consequently, we can say that a 2-D thresholding method utilizes not only the grayscale of each pixel, but also its neighborhood average grayscale.

DEFINITION. The 2-D gray-level histogram is defined as the matrix

$$H_{2d} = \{r_{ij} \mid r_{ij} = \text{number of bin } (i, j), 0 \leq r_{ij} \leq MN, i, j = 0, 1, \dots, L - 1\}, \quad (2)$$

where element r_{ij} stands for the number of grayscale bins with (gray level, local average) = (i, j) .

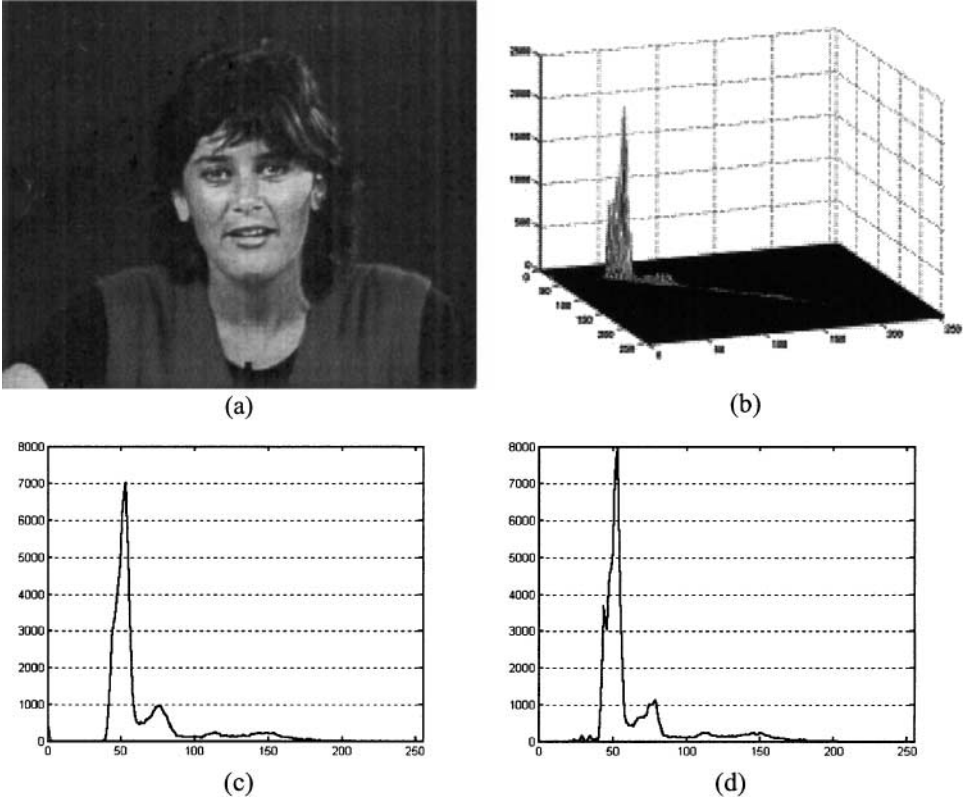


FIG. 1. Illustration of the 2-D grayscale histogram. (a) Original grayscale image (American Miss); (b) 2-D grayscale histogram; (c) 1-D grayscale histogram; (d) 1-D histogram of the local average grayscale.

The local average values of an image represent the spatial information except the precise point information reflected by each pixel. The local average in a small square window centered at the pixel (x, y) is defined as

$$g(x, y) = \frac{1}{(2w+1)^2} \sum_{k=-w}^w \sum_{l=-w}^w f(x+k, y+l), \quad x = 1, 2, \dots, M; y = 1, 2, \dots, N. \quad (3)$$

Hereby, we can define a 2-D thresholding function $f_{(T,S)}(x, y)$ as

$$f_{(T,S)}(x, y) = \begin{cases} b_0, & \text{if } f(x, y) \leq T \vee g(x, y) \leq S \\ b_1, & \text{if } f(x, y) > T \wedge g(x, y) > S, \end{cases} \quad (4)$$

where $0 \leq b_0, T, b_1 \leq L - 1$, and (T, S) is a 2-D threshold vector.

Figure 1b shows the 2-D grayscale histogram of an image with height M and width N , in which the t -axis represents the gray level and s -axis denotes the locally neighborhood average grayscale. Since the grayscale image $f(x, y)$ contains L levels (from 0 to $L - 1$), there are L^2 elements in the 2-D histogram. In the same figure, we draw the 1-D grayscale histogram and the 1-D histogram of the locally neighborhood average grayscale (Figs. 1c and 1d) to show the relationship between the 1-D and the 2-D histograms. Similar to the threshold in the 1-D histogram, an optimum threshold vector (T, S) should be determined to separate the two groups within the planar function. Consequently, the 2-D histogram is

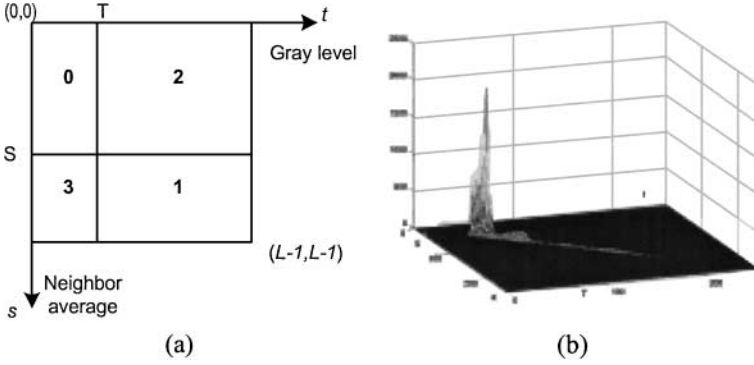


FIG. 2. The 2-D histogram partitioned by a threshold vector (T, S) . (a) The planner and (b) spatial representation of the division of a 2-D histogram by a threshold vector.

partitioned into four regions by the threshold vector (T, S) , as shown in Fig 2a. On the one hand, since the pixels belonging to the object or background class make more contributions to the diagonal quadrants, region 0 (upper left) and 1 (lower right) are mainly used to represent the distribution of the object and background classes. On the other hand, off-diagonal quadrants, regions 2 and 3, mainly reflect the distribution of the edge pixels and noise in an image.

3. MEMBERSHIP FUNCTION

It is important to define a proper membership function for a fuzzy set. The definition of membership functions is usually problem dependent and is often done heuristically and subjectively. In fuzzy sets, the membership functions can be sketched by four commonly used functions, which are triangular, trapezoidal, bell-shaped, and S -shape functions [7–9]. In this paper, we define a membership function similar to that applied in the 1-D histogram entropy-based thresholding method proposed by Huang and Wang [17]. The membership function was defined as

$$\mu_A[f(x, y); t] = \begin{cases} \frac{1}{1 + |f(x, y) - m_1(t)|/C}, & f(x, y) \leq t \\ \frac{1}{1 + |f(x, y) - m_2(t)|/C}, & f(x, y) > t, \end{cases} \quad (5)$$

where $m_1(t)$ and $m_2(t)$ are the average values of $f(x, y)$ (regarded as a fuzzy set) in two classes, respectively. C is a constant chosen in such a way that $0.5 \leq \mu_A[f(x, y); t] \leq 1$.

It is obvious that the pair (grayscale i , local average j) should be considered when we define the membership function $\mu(r_{ij}; t, s)$ for element r_{ij} in the 2-D grayscale histogram. Hereby, we give the definition of the membership function used in our approach and a detailed procedure for its computation.

3.1. Definition of Membership Function

The membership function for the 2-D gray-level histogram H_{2d} is defined as

$$\mu(r_{ij}; s, t) \hat{=} \min\{\mu_H(i; t), \mu_N(j; s)\}, \quad (6)$$

where $\mu_H(i; t)$ and $\mu_N(j; s)$ are derived from the histogram according to the criterion

proposed in [17], which will be discussed in detail in Section 3.2. Actually, the item $\mu(r_{ij}; t, s)$ denotes the probability that an arbitrary pixel, whose grayscale is i and whose neighborhood average level is j , belongs to the background or object class.

3.2. Computation of Membership

Step 1. Derive 2-D histogram H_{2d} from image $f(x, y)$. Note that the t -axis and s -axis represent the gray level and local average gray level, respectively.

Step 2. Project 2-D grayscale histogram H_{2d} onto the t -axis to obtain 1-D histogram $H(i)$, which denotes the number of occurrences at gray level i .

$$H(i) = \sum_{j=0}^{L-1} r_{ij}, \quad i = 0, 1, \dots, L-1 \quad (7)$$

Step 3. Given an arbitrary threshold t , to compute the membership function $\mu_H(i; t)$ for gray value i in the 1-D grayscale histogram $H(i)$,

$$\mu_H(i; t) = \begin{cases} \frac{1}{1 + |i - m_1^H(t)|/C_H}, & i \leq t \\ \frac{1}{1 + |i - m_2^H(t)|/C_H}, & i > t, \end{cases} \quad (8)$$

where C_H is a constant to ensure that $\mu_H(i; t)$ is within $[0.5, 1]$. $m_1^H(t)$ is the average value of the gray level of the background class and $m_2^H(t)$ is that of the object class, given a threshold t , respectively, obtained by

$$m_1^H(t) = \sum_{k=0}^t kH(k) / \sum_{k=0}^t H(k) \quad (9)$$

and

$$m_2^H(t) = \sum_{k=t+1}^{L-1} kH(k) / \sum_{k=t+1}^{L-1} H(k). \quad (10)$$

Step 4. As in Step 2, project 2-D histogram H_{2d} onto the s -axis in order to get the 1-D histogram $N(j)$ of the local average grayscale,

$$N(j) = \sum_{i=0}^{L-1} r_{ij}, \quad j = 0, 1, \dots, L-1. \quad (11)$$

Step 5. Given an arbitrary threshold s , to compute the membership function $\mu_N(j; s)$ of the average gray level j in the 1-D local average grayscale histogram $N(j)$,

$$\mu_N(j; s) = \begin{cases} \frac{1}{1 + |j - m_1^N(s)|C_N}, & j \leq s \\ \frac{1}{1 + |j - m_2^N(s)|C_N}, & j > s, \end{cases} \quad (12)$$

where C_N is also a constant to be chosen such that $0.5 \leq \mu_N(j; s) \leq 1$. $m_1^N(s)$ is the average value of the local average gray levels of the background class and $m_2^N(s)$ is that of the object

class, given a threshold s . They are defined as

$$m_1^N(s) = \sum_{k=0}^s kN(k) \bigg/ \sum_{k=0}^s N(k) \quad (13)$$

and

$$m_2^N(s) = \sum_{k=s+1}^{L-1} kN(k) \bigg/ \sum_{k=s+1}^{L-1} N(k). \quad (14)$$

Step 6. After computing the membership functions $\mu_H(i; t)$, $0 \leq t \leq L - 1$, and $\mu_N(j; s)$, $0 \leq s \leq L - 1$, in their 1-D histograms, the membership function $\mu(r_{ij}; t, s)$ of 2-D grayscale histogram H_{2d} is computed as the minimum of $\mu_H(i; t)$ and $\mu_N(j; s)$, as defined in Eq. (6).

Remark 1. The main reason we define the membership function as Eq. (6) is to consider the correlation between one pixel's grayscale and its locally average grayscale in the neighborhood. It means that we also consider the spatial relationship of the pixels in a local region besides their original grayscales. If the gray level of an arbitrary pixel is close to the mean of the grayscales of an arbitrary object class, and the local average grayscale also approximates the mean of the local average levels of this object class, the membership value at this pixel is large. Otherwise, the membership value is comparatively small.

Another reason for selecting the minimum of $\mu_H(i; t)$ and $\mu_N(j; s)$ as a membership score is that we should ensure that the value of the membership is not less than 0.5 so that it satisfies the requirement of the definition of the index of nonfuzziness. Since the values of $\mu_H(i; t)$ and $\mu_N(j; s)$ are not less than 0.5 as in our definition, the value of the membership function, $\mu(r_{ij}; t, s) = \min\{\mu_H(i; t), \mu_N(j; s)\}$, is definitely equal to or greater than 0.5. It is easy to see that the definition of $\mu(r_{ij}; t, s) = \mu_H(i; t) \times \mu_N(j; s)$ does not satisfy the above-mentioned constraint. According to Eq. (6), a pixel (with grayscale i and local average grayscale j) belongs to the background class if $0 \leq i \leq t$ and $0 \leq j \leq s$ whereas a pixel belongs to the object class if $t < i \leq L - 1$ and $s < j \leq L - 1$.

4. THE INDEX OF NONFUZZINESS AND OPTIMUM SOLUTION

Unlike the conventional thresholding methods, which can utilize any kind of fuzzy measure, our approach adopts the index of nonfuzziness of the 2-D grayscale histogram as a measure. We propose the index of nonfuzziness of the 2-D histogram in a way that is different from typical fuzzy measures such as the fuzzy entropy measure proposed in [13, 14, 17, 25], the index of fuzziness [7, 9, 11], the index of nonfuzziness [27], the Yager measure [26], fuzzy compactness, and the linear index of fuzziness [14]. From the viewpoint of the fuzzy set, $X = \{\mu_X(x_{mn}) = \mu_{mn}/x_{mn}, m = 1, \dots, M; n = 1, \dots, N\}$ is the fuzzy set representation of the pattern corresponding to an image, where $\mu_X(x_{mn})$ or μ_{mn}/x_{mn} ($0 \leq \mu_{mn} \leq 1$) denotes the grade of the (m, n) th pixel with an intensity of x_{mn} which possesses some property μ_{mn} . The index of nonfuzziness defined in [27] is

$$\eta(X) = \frac{1}{MN} \sum_m \sum_n |\mu_X(x_{mn}) - \mu_{\bar{X}}(x_{mn})|, \quad (15)$$

where (\bar{X}) is the complement of X .

4.1. Our Proposed Measure

As defined in Section 3.1, the membership function of the 2-D grayscale histogram H_{2d} , is $\mu(r_{ij}; t, s) = \min\{\mu_H(i; t), \mu_N(j; s)\}$. We define the index of nonfuzziness of the 2-D gray-level histogram as

$$\begin{aligned} \eta(t, s) = & \frac{1}{MN} \sum_{i=0}^t \sum_{j=0}^s [2 * \mu(r_{ij}; t, s) - 1] \times r_{ij} \\ & + \frac{1}{MN} \sum_{i=t+1}^{L-1} \sum_{j=s+1}^{L-1} [2 * \mu(r_{ij}; t, s) - 1] \times r_{ij}. \end{aligned} \quad (16)$$

Since the background and object classes mainly concentrate on the diagonal regions, the measure function $\eta(t, s)$ is composed of these two parts as well. Considering that $0.5 \leq \mu(r_{ij}; t, s) \leq 1$ and $0 \leq r_{ij} \leq MN$, the index of nonfuzziness $\eta(t, s)$ definitely lies in $[0, 1]$.



(a)

FIG. 3. Experimental results of benchmark images using our proposed method and those proposed by Abutaleb and Brink. Upper left, the original image; upper right, the bi-level image using Abutaleb's method; lower left, the bi-level image using Brink's method; lower right, the bi-level image using our approach. (a) Lena image; (b) Boat image; (c) Peppers image.



(b)

FIG. 3—Continued

The following is the computational procedure to maximize the index of nonfuzziness $\eta(t, s)$ to obtain optimum threshold vector (T, S) .

4.2. Computation of $\eta(t, s)$ and Optimum Solution of (T, S)

Step 1 [Initialization]. Compute membership function $\mu(r_{ij}; t, s)$ by Eq. (6) and accumulate the proposed measure $\eta(t, s)$ of the 2-D histogram of the input image in diagonal quadrants 0 and 1 (see Fig. 2), according to Eq. (16).

Step 2 [Iteration]. Increase t from f_{\min} to f_{\max} and s from g_{\min} to g_{\max} step by step in order to seek the optimal threshold vector (T, S) to maximize $\eta(t, s)$; that is,

$$\eta(T, S) = \max_{f_{\min} \leq t \leq f_{\max} \quad g_{\min} \leq s \leq g_{\max}} \{\eta(t, s)\}, \quad (17)$$

where

$$f_{\min} = \min_{all(x,y)} \{f(x, y)\}, \quad f_{\max} = \max_{all(x,y)} \{f(x, y)\} \quad (18)$$

and

$$g_{\min} = \min_{all(x,y)} \{g(x, y)\}, \quad g_{\max} = \max_{all(x,y)} \{g(x, y)\}. \quad (19)$$

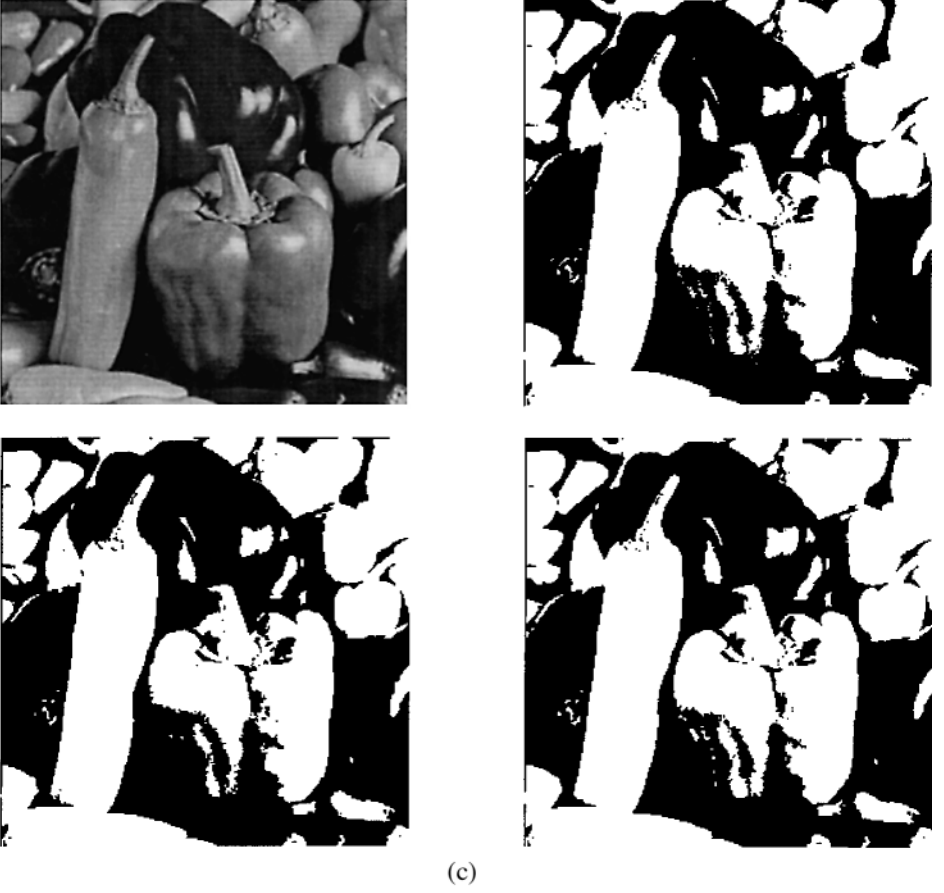


FIG. 3—Continued

5. EXPERIMENTAL RESULTS AND DISCUSSION

In our experiments, the local average grayscale $g(x, y)$ at pixel (x, y) in a 3×3 window is

$$g(x, y) = \frac{1}{3 \times 3} \sum_{k=-1}^1 \sum_{l=-1}^1 f(x+k, y+l), \quad x = 1, 2, \dots, M; y = 1, 2, \dots, N. \quad (20)$$

Additionally, two constants C_H, C_N in Eqs. (8) and (12) are initialized as $C_H = f_{\max} - f_{\min}$ and $C_N = g_{\max} - g_{\min}$, respectively, where f_{\min}, f_{\max} are computed from Eq. (18) and g_{\min}, g_{\max} are obtained by Eq. (19).

5.1. Experiment 1: Thresholding Results of Different Methods on Benchmark Images

Our proposed index of nonfuzziness of the 2-D histogram-based approach was compared to the approaches proposed by Abutaleb and Brink. Abutaleb took the sum of two entropies derived from the background and foreground classes as an objective function and searched

globally for the optimal solution by maximizing the 2-D entropy summation. Brink refined the objective function to be the minimum of the entropy of the background class and that of the object class. His algorithm maximizes the minimum of two entropies, which means the maximization of class entropies. In our experiments, we mainly compared the thresholding performance of our proposed index of nonfuzziness with that of the fuzzy entropy measure.

The thresholding results on benchmark images using our approach and the approaches proposed by Abutaleb and Brink are shown in Fig. 3. All these images have a size of 256×256 with 256 gray levels. In each group of four images, the one on the upper left is the original image, the one on the lower right is the thresholding image obtained by using our approach. The upper right and lower left images were obtained by using Abutaleb’s method and Brink’s method, respectively. In order to compare each method’s ability to deal with the ambiguous pixels, most of which are distributed near the off-diagonal quadrants, we reported the number of unprocessed pixels and the threshold vector obtained by each method in Table 1. The table shows that the results of our proposed method are very encouraging.

5.2. Experiment 2: Comparison of Different Methods on Noisy Images

In these experiments, the proposed thresholding approach based 2-D grayscale histogram and the index of nonfuzziness were compared to three existing methods when they were applied to several noisy images of different SNRs. The referenced methods include the 1-D fuzzy thresholding method proposed by Huang and Wang, the 2-D entropy-based thresholding method proposed by Abutaleb, and its improved version by Brink. Gaussian noise with various variances was added to these. Note that the SNR is defined as 10 times the logarithm of the ratio of the noise-free image power to the noisy image power.

The thresholding results of the noise-free American Miss image by our approach and other existing methods are shown in Fig. 4. The 2-D grayscale histogram and the 1-D grayscale and local average grayscale histograms can be found in Fig. 1. In terms of human perception, there is little difference among the thresholding results of a noise-free image produced by

TABLE 1

The Threshold Vectors and the Number of Unprocessed Pixels by Different Methods

Benchmark images	Abutaleb’s method		Brink’s method		Our method	
	Threshold vector	# pixels	Threshold vector	# pixels	Threshold vector	# pixels
Lena	(98, 91)	4388	(107, 94)	5941	(102, 101)	2109
Goldhill	(125, 117)	5327	(109, 106)	4396	(118, 116)	3892
Baboon	(111, 120)	9077	(119, 110)	9380	(117, 113)	7719
CameraMan	(145, 143)	2922	(123, 123)	4572	(72, 77)	1123
Boat	(126, 126)	2347	(107, 110)	2186	(93, 96)	2008
Barbara	(120, 117)	3273	(126, 126)	2981	(123, 122)	2969
Peppers	(105, 104)	1329	(109, 110)	1672	(104, 103)	1393
Zelda	(83, 84)	1784	(82, 83)	2004	(90, 90)	1565

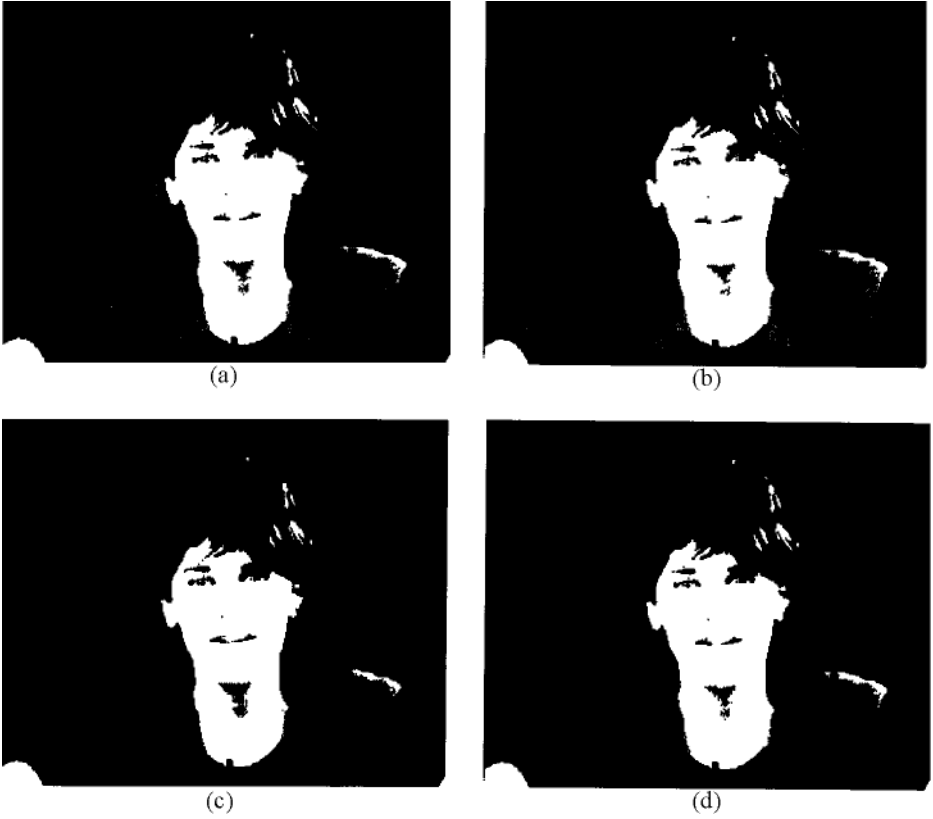


FIG. 4. Thresholding results of the American Miss image by different methods. (a) The bi-level image by Huang and Wang's algorithm with $T=87$; (b) the result of Abutaleb's method with $(T, S)=(89, 88)$; (c) the result of Brink's method with $(T, S)=(90, 88)$; (d) the bi-level image of our approach with $(T, S)=(87, 88)$.

the 1-D fuzzy measure and those produced by the three methods based on the 2-D grayscale histogram. However, quite different results were obtained when these thresholding methods were applied to noisy images. Figures 5, 6, and 7 show the thresholding results of Gaussian noise degraded images with an SNR of 30, 20, and 10 DB, respectively. It can be observed that the 1-D fuzzy entropy-based method is much less robust when dealing with noisy images. Abutaleb's method and Brink's method have a certain degree of robustness to low-level noise. However, when the level of Gaussian noise increases, the thresholding results are relatively poor. The experimental results show that our approach is rather robust to Gaussian noise.

5.3. Remaining Problems and Future Work

The exhaustive search algorithm is time-consuming in our approach. The computation complexity is $O(L^4)$ where L is the number of gray levels that an image has. Brink's method has the same problem of high computational complexity. In our future research, a fast algorithm will be developed. We have tried to use a wavelet analysis based technique to reduce the computational complexity, the results of which will be reported in the future.

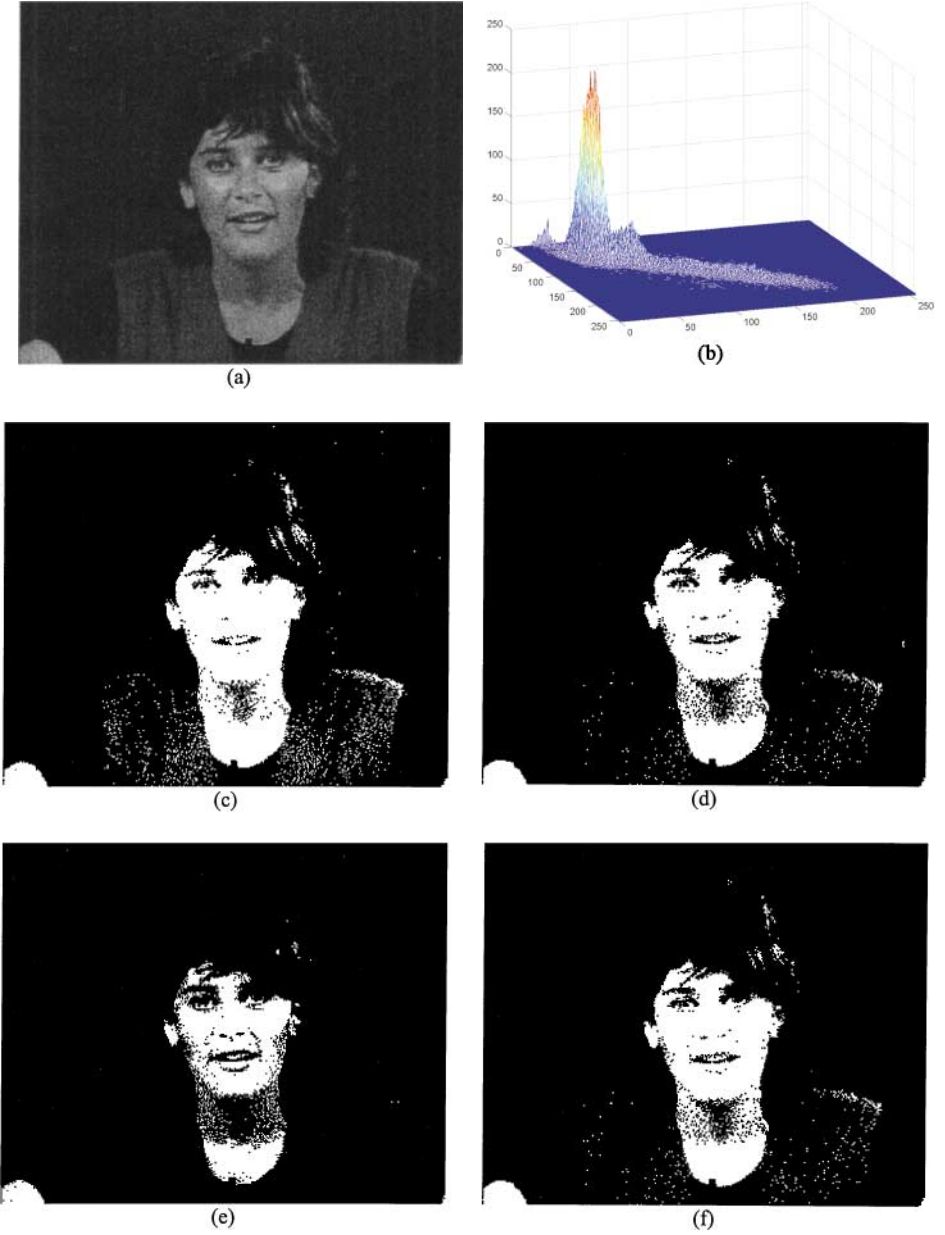
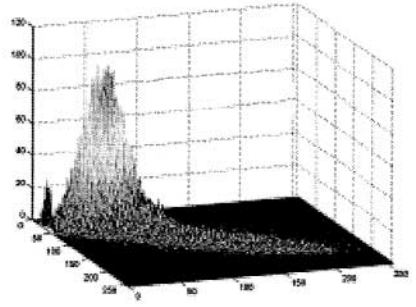


FIG. 5. Thresholding results on a noisy image with an SNR of 30 dB. (a) Noisy image; (b) the 2-D grayscale histogram; (c) the bi-level image obtained by Huang and Wang's algorithm with $T = 90$; (d) the result of Abutaleb's method with $(T, S) = (99, 98)$; (e) the result of Brink's method with $(T, S) = (120, 118)$; (f) the bi-level image of our approach with $(T, S) = (99, 98)$.

Our proposed method can be easily extended to multiple-class segmentation problems. For multiclass image segmentation, we should either know the number of objects in advance or estimate the number of objects based on an iterative algorithm. One solution is that we can partition quadrants 0 and 1 into smaller quadrants, such as 00, 01, 02, 03 (for quadrant 0) and 10, 11, 12, 13 (for quadrant 1), respectively. In each smaller quadrant, the same index of nonfuzziness and search procedure will be used to obtain the threshold vectors. Thus,



(a)



(b)



(c)



(d)



(e)



(f)

FIG. 6. Thresholding results on a noisy image with an SNR of 20 dB. (a) Noisy image; (b) the 2-D grayscale histogram; (c) the bi-level image obtained by Huang and Wang's algorithm with $T = 73$; (d) the result of Abutaleb's method with $(T, S) = (107, 101)$; (e) the result of Brink's method with $(T, S) = (108, 101)$; (f) the bi-level image of our approach with $(T, S) = (111, 110)$.

the whole image can be separated into four classes, including background and three object classes. This process can be repeated to obtain more classes if necessary. Another solution is to seek to the top $(K - 1)$ threshold vectors from the 2-D histogram matrix directly so that the K classes of objects (including the background) are distinguished by these vectors. Regardless of which multiclass thresholding criteria to be employed, the pixels that fall into the off-diagonal quadrants should be classified into correct classes. A misclassification will cause edge ambiguity and noise infection.

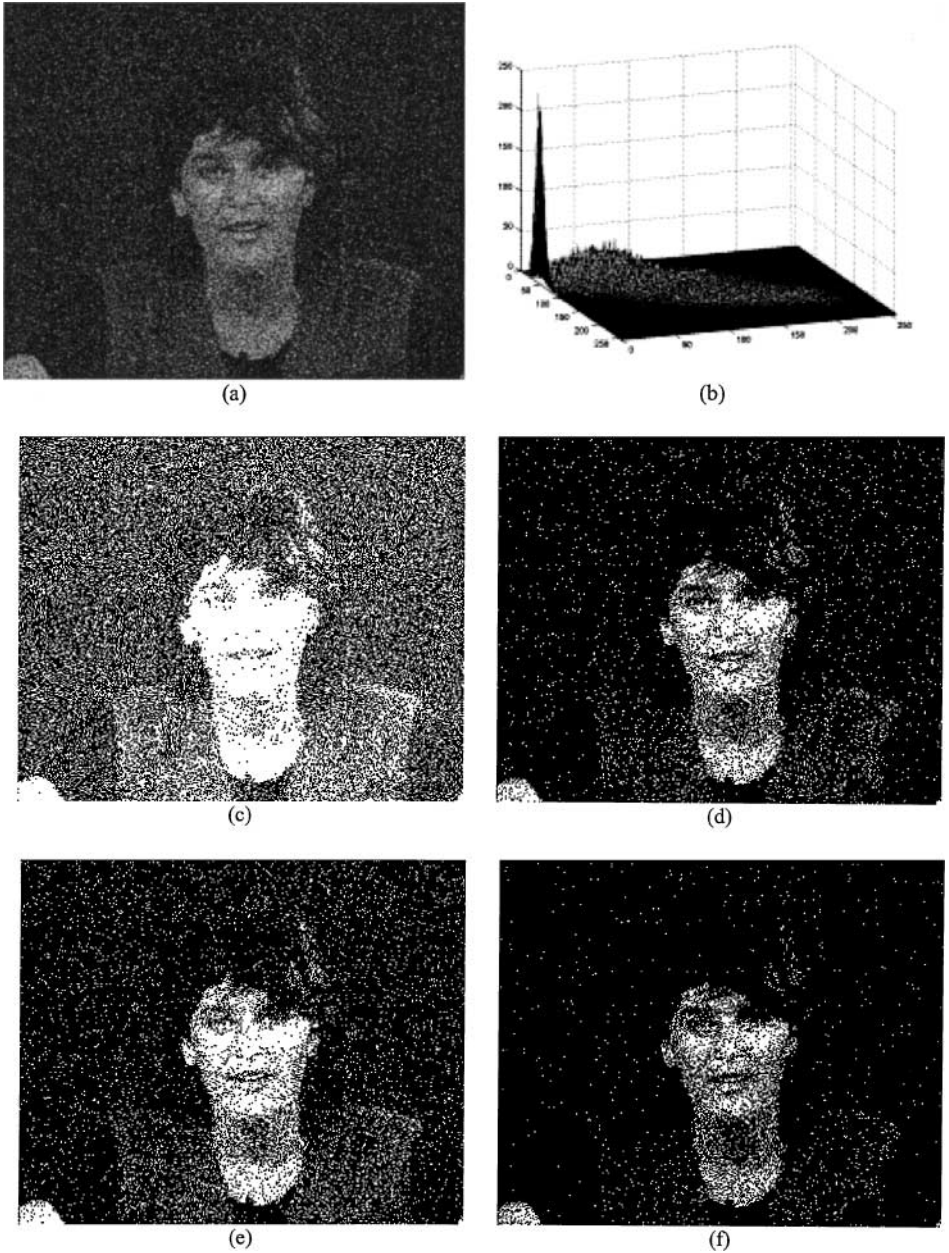


FIG. 7. Thresholding results on a noisy image with an SNR of 10 dB. (a) Noisy image; (b) the 2-D grayscale histogram; (c) the bi-level image obtained by Huang and Wang's algorithm with $T = 64$; (d) the result of Abutaleb's method with $(T, S) = (121, 107)$; (e) the result of Brink's method with $(T, S) = (107, 92)$; (f) the bi-level image of our approach with $(T, S) = (141, 138)$.

6. CONCLUSIONS

A new thresholding method is proposed based on an index of nonfuzziness on the 2-D gray-level histogram. In our approach, the threshold vector (T, S) is determined by an exhaustive searching algorithm within the quadrants. Experimental results have shown

that the difference between T and S is sometimes relatively large when the entropies and fuzziness index are used as a measure. This large difference between T and S will result in fewer pixels belonging to the quadrant 0 or 1 in the 2-D histogram. Based on the index of nonfuzziness, it is guaranteed that T and S can be within a relatively small range. As a result, better thresholding outcomes can be achieved by using our approach. Experimental results on noise-free gray-level images and Gaussian noise degraded images have shown the effectiveness of our approach for image thresholding. Our method has a better robustness when dealing with noisy images and can also be applied to the segmentation of multiclass grayscale images.

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