on the degree of support and the relative importance of each child. The most important advantage of this system is that not only is the evidence combined but that the relative importance of the different sources is also considered.

To use the fuzzy integral as an information fusion technique in a system, it is necessary to understand the behavior of the fuzzy integral when the g_λ-fuzzy measure changes. In this paper, theoretical and experimental results have been derived that allows the prediction of the effects of changes in importance of nodes to the overall evaluation. The results obtained are intuitively reasonable for information fusion purposes.

The fuzzy integral algorithm was applied to image data in an automatic target recognition situation. The algorithms performed well at both the feature level and source level integration. The fuzzy integral provides a natural coupling of objective evidence and expectation.

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Comments on "An Optimal Multiple Threshold Scheme for Image Segmentation"

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***Abstract*—The paper by Reddi *et al.* presented a fast search scheme for finding single and multiple thresholds as a speed improvement to Otsu's scheme. The correspondence points out explicitly that their fast search procedure doesn't converge to the optimal threshold if the interclass variance has not the unique maximum. The limitations of Otsu's technique are also discussed briefly.**

Otsu [1] proposed a threshold selection method from histograms based on maximizing the separability of the resultant classes in gray levels. For a bimodal distribution case, the interclass variance $\sigma^2(k)$ is defined as

$$\begin{aligned}\sigma^2(k) &= p_D(m_D - m_0)^2 + p_B(m_B - m_0)^2 \\ &= p_D m_D^2 + p_B m_B^2 - m_0^2 \end{aligned} \tag{1}$$

where p_D is the probability of dark pixel whose gray level is less than k, p_B is the probability of bright pixel whose gray level is greater than k, m_D is the mean of dark pixels, m_B is the mean of bright pixels, and m_0 is the total mean. We can determine the optimal threshold k^* that maximizes $\sigma^2(k)$. It is also straightforwardly extended to multiple thresholding problems. However, the maximization procedure becomes more and more complicated computationally and even impossible to implement practically as the number of thresholds to be selected increases.

Thus, Reddi *et al.* [4] proposed a fast search scheme to overcome this limitation of Otsu's method with an assumption that $\sigma^2(k)$ has the unique maximum. They attempted to reduce the computation time equivalently by finding k such that $m_B + m_D = 2k$. However, the rigorous proof of the unimodality has not yet been obtained [1]. Also Kittler and Illingworth demonstrate that this conjecture of unimodality does not hold in general [2]. Kittler and Illingworth show that the criterion function may not only be multimodal, but more importantly, if it is multimodal, its global maximum is not guaranteed to give correct segmentation results. The purpose of this correspondence is to point out explicitly with experimental evidence that the speed improvement by Reddi *et al.* fails to converge to the global maximum, since Otsu's method and the method by Reddi *et al.* are not equivalent mathematically, especially for cases with $\sigma^2(k)$ nonunimodal.

According to several experimental results, the proposed method by Reddi *et al.* is often influenced by the local maxima or minima of the interclass variance. In that case, the selected threshold k is varied by changing the initial starting point. It is, therefore, obvious that the fast search procedure by Reddi *et al.* doesn't converge to the same optimal threshold k^* which maximizes $\sigma^2(k)$. Mathematically, the optimal threshold k^* for which $\sigma^2(k)$ is maximum is not equal to the gray level k such that $\delta\sigma^2(k)/\delta k = 0$ which gives local maximum, local minimum, or inflection points also. Also this is true for the multiple thresholds.

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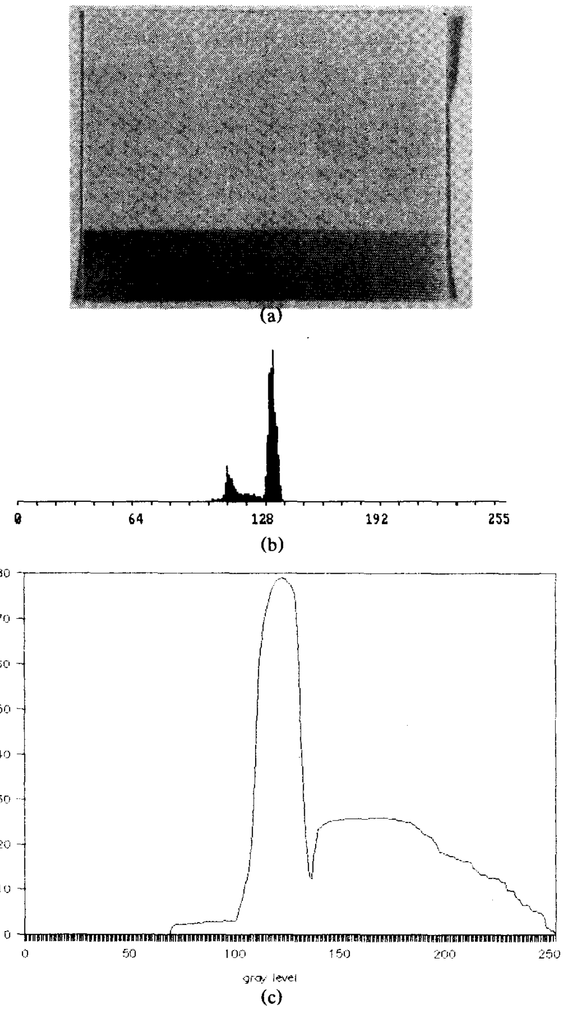

Fig. 1. Experimental results. (a) Test image. (b) Histogram. (c) Interclass variance $\sigma^2(k)$.

Experimental results are shown in Fig. 1. Fig. 1(a) is a 256×256 test image uniformly quantized to 8 bits, and its histogram and interclass variance $\sigma^2(k)$ are shown in Fig. 1(b) and (c), respectively. Simulation results show that $\sigma^2(k)$ has not unique maximum. The fast search procedure by Reddi *et al.* converges to several gray levels (ex., $123, 127, 137, 166, \ldots$) depending on the initial starting points, whereas the optimal threshold by Otsu is 123.

Furthermore, to see the performance of Otsu's method on various test images with different histogram shape, we vary the object size, the mean difference, and the noise power added to the original image. Here we define the object size to be the ratio of the object area to the entire image area and the mean difference to be the difference of the average intensities of the object and the background. Otsu's scheme exhibits the relatively good performance if the histogram can be assumed to have bimodal distribution and assumed to possess deep a and sharp valley between two peaks. But if the object area is small compared with the background area, the histogram no longer exhibits bimodality [2]. And if the variances of the object and the background intensities are large compared to the mean difference, or the image is severely corrupted by additive noise, the sharp valley of the gray level histogram is degraded. Then the possibly incorrect threshold determined by Otsu's scheme results in the segmentation error. From the experimental results, the performance of global thresholding techniques including Otsu's scheme is shown to be limited by the small object size, the small mean difference, the large variances of the object and the background intensities, the large amount of noise added, and so on [3].

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