(a) BFS Affected Subgraph Expansion

A_0 (Initial) $d = 1$ $d = 2$ $d = 3$

$|A_0| = 47$

Algorithm 3
$A \leftarrow A_0$
for $d = 1..D$:
$\quad A \leftarrow A \cup \mathcal{N}(A)$
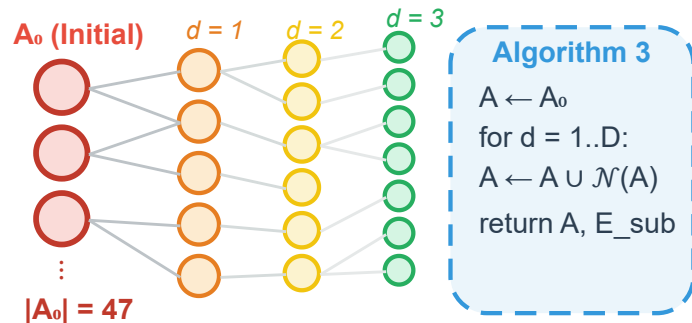return A, E_sub

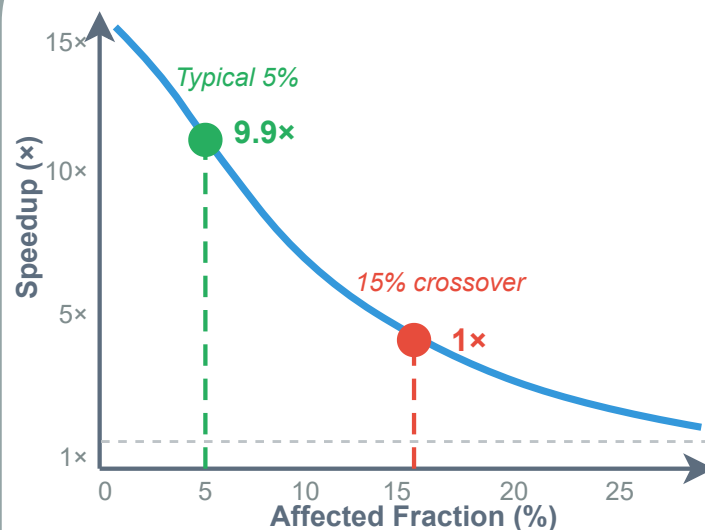**Expansion Results (Ethereum-S)**
$|A_0| = 47 \rightarrow |A| = 471$ (D = 3 hops)
Branching factor: $b = |A|^{1/D} / |A_0|^{1/D} = 2.34$

Definition 3: Affected subgraph $A = \{v \mid dist(v, A_0) \leq D\}$

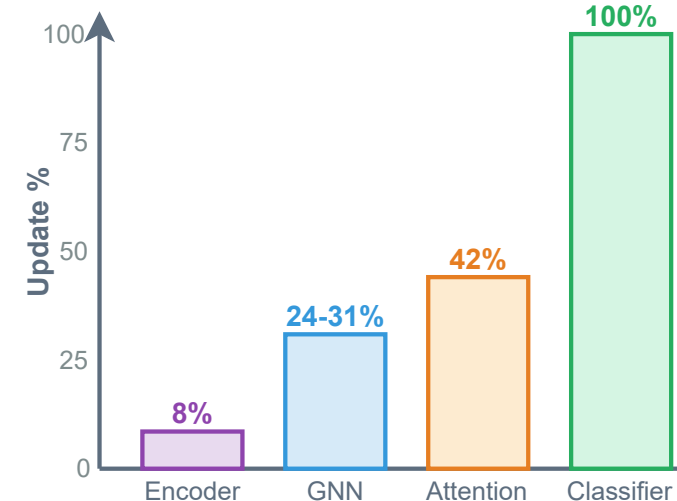*Typical: 5% graph affected per update cycle*

(b) Speedup vs. Affected Fraction

Typical 5%
9.9×

15% crossover
1×

Beyond 15%: Full retraining preferred

(c) Layer-wise Parameter Updates

8% — Encoder
24-31% — GNN
42% — Attention
100% — Classifier

Average: 4.04% of total parameters updated

(d) Accuracy Preservation Over Time

Δ = 0.2%

— Weekly retrain
-- TAIL incremental

F1 Score / Weeks

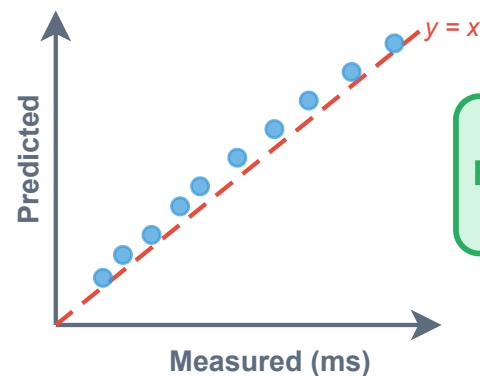**Accuracy within ±0.2% of weekly retraining**

(e) Theorem 3 Complexity Validation

Theorem 3 (TAIL Complexity)
$$T\_inc \in \mathcal{O}(|A_0| \cdot b^D + |A| \cdot p + |E\_sub| \cdot h)$$

$y = x$

Predicted / Measured (ms)

Validation
$R^2 = 0.9423$
$p < 10^{-8}$

Strong fit validates theoretical complexity bounds

(f) Comparison: TAIL vs. Baselines

SIEGE: 3.2×, 0.8%
SIT: 5.1×, -0.4%
TAIL: 9.9×, -0.17%

Speedup (×) / ΔF1 (%)

0% / -1%

■ Speedup
■ ΔF1

TAIL: Best speedup
with minimal degradation